## Supplemental Text

### Detecting Outliers in Kiva Size Distributions

The power-law analysis package works best on large datasets reporting populations rather than samples. Unavoidably, our data represent a convenience sample. This leads to some strange distributions, as is evident in the All-PIII kiva data, as well as the NSJ PII and PIII kiva data. Clauset (personal communication 6/10/2016) suggests that these results (in which the “compare distribution” and “test statistic” indicate power-law distributions despite power-law probabilities of 0) likely reflect some of the oddities of our sample, which probably includes over-sampling of larger kivas from the population.

To remedy this we attempted to identify outliers, here defined as those kivas that were more than 1.5 midspreads in size from the upper bound on a boxplot (Drennan 2009:39). This markedly improves the fit of the NSJ and All-PIII kiva-size distributions to a power law distribution (Supplemental Table 3). This same technique does not need to be applied to our other datasets, which are either populations (as in the case of the simulation territory sizes) or, in the case of our site sizes, are in most cases derived from block or transect surveys in which all recognized sites were recorded.

Taking this additional analysis into account does not affect our agreement with Clauset’s assessment (personal communication 6/10/2016) that the NSJ PII, NSJ PIII and All-PIII datasets correspond weakly to power-law distributions.

### Model: The Public Goods Game

To the household/ecosystem dynamics represented in the base version of “Village” we added (in Kohler et al. 2012, and retained here) the concept of social groups defined as sets of households playing an annually repeated public goods game among themselves. A set of households that internally plays an annual public goods game is called a simple group. The exact nature of the public good in question is left unspecified in the model; the costs and benefits in the game are denominated solely in maize. In the Pueblo world, plausible examples of public goods include defense (Rusch 2014) or construction of public resources such as reservoirs (Wilshusen et al. 1997) and great kivas, and perhaps great houses themselves.

Households within a simple group may choose to work under a leader who can greatly reduce the likelihood of failures in cooperation due to free-riding or lack of coordination among households in the group. As in any public goods game, if every household contributes to the public good, the return to each is higher than its contribution. Yet there is a temptation to defect, since the reduction in returns to defecting households is generally less than the costs of contributing to the public good. If some agents choose to defect (not contribute), the returns to all households decrease. In the Kohler et al. (2012) model, leaders ensure that defection is punished, and when the cost of supporting a leader is less than the losses incurred by defection, groups who choose to support a leader prosper. The model is evolutionary, so that strategies yielding higher returns slowly replace less-rewarding strategies. Whereas simple groups may be either hierarchical, or non-hierarchical, a robust result of the model is that as group size grows to the point where “mutual monitoring” for defectors becomes too costly, households in groups submit to the greater costs, but even greater benefits, of supporting a leader, thus becoming hierarchical.

### *Model: Parameter Selection, Group Formation and Territoriality*

In a further extension of this model we made these social groups territorial and allow them to subsume other groups by warfare, creating hierarchies of groups that we called “complex groups” (Kohler et al. in review). The model presented below differs from that in two main ways, both related to fission: first, once groups are subsumed by dominant groups, they may attempt to remove themselves from the hierarchy (revolt); second, when group population reaches its maximal group size (parameterized in Table 3) the group divides in two. While many parameters could be varied, in the model presented here we were concerned specifically with how maximum basal group size, fatalities from warfare and parameters surrounding tribute affect the growth of complex groups. Consequently the parameters we varied reflected these goals (Table 3). *S* indexes the proportion of fatalities among all its combatants the smaller group is willing to suffer before conceding defeat; β is the proportion of a subordinate group’s net benefit from the public goods game paid to its direct dominant as tribute; and μ is the proportion of that tribute passed through an intermediate group to a dominant group (1-­μ therefore being the tax kept on that pass­‐through). We explored all combinations of unique values for these four parameters resulting in 2 x 2 x 3 x 3 = 36 combinations, each of which was run 15 times for a total of 540 runs, allowing us to recognize variability within and between parameter combinations.

In the model discussed here, groups maintain territories that grow as group membership grows and new members colonize new farmland. Group territories are calculated by drawing the smallest possible polygon around all agents within each group. If group territories come in contact hindering further expansion, groups may choose to engage in violent conflict. The concepts of groups, public goods games, territoriality, merging, warfare, and tribute are central to this model. We describe them below but for more detail please see Kohler et al. (in review).

All agents are able to act in ways that are appropriate to either a hierarchical or a non-hierarchical setting, as described in Kohler et al. (in review:Table 1). Agents track their ancestry to one of 200 foundational households and express hierarchical or non-hierarchical behaviors based on majority preference within each simple group. While in Kohler et al. (2012) groups were formed by agents who wanted to live in groups that were similar with respect to their preference for living in a hierarchical vs. non-hierarchical group, in the model reported here (and in Kohler et al. [in review]) groups are composed of kin who may have similar preferences relative to hierarchy due to inheritance. Hierarchical preferences may still change through time given a slow evolutionary dynamic in which agents in groups making the higher-return choice out-reproduce those in groups making the lower-return choice, and a faster social learning dynamic in which agents emulate the hierarchical preference of the household with the most storage in its neighborhood (which may include some households in other groups).

### *Model: Conflict, Merging, and Tribute*

Agents require arable land to produce maize to provide their basic caloric intake. New households (formed by marriage) may not be able to locate productive fields within their group’s territory, or established households may need to add plots if production is declining or the kids keep arriving. New households must seek new locations, and expanding households may as well if adding plots locally is not possible. In seeking new locations agents keep track of “frustrations” which occur any time they cannot move to a cell because that cell is already fully occupied, in another group’s territory, or because such a move would cause the territories of their group and another group to overlap. In low-population settings (typically within the first 300 years of beginning the simulation) an appropriate unoccupied cell, not in another group’s territory, can often be found within the allowed search radius.

If however an agent needs to move and all the more productive cells are in another group’s territory or claiming the cell would cause the two group territories to overlap, that is termed a “frustration that hurts.” A group with a household needing to expand into another group’s territory (because no other options are available) will choose to confront one of the groups on its periphery that is causing a “frustration that hurts.” Groups maintain a list of the groups with which each of its households has frustrations. Each year the group sorts this list according to both the distance between the two groups and the number of frustrations it has catalogued. The focal group iterates through its frustrations list, calculating its likelihood of winning a battle against each by comparing the attrition thresholds (*s*)of each group (the probability that the smaller group will reach its attrition threshold sooner than the larger group) which depends (with some stochasticity) on the relative number of fighters in each group. Generally larger groups are willing to confront smaller groups.

When this happens, the focal group first tenders an offer of merger, which, if accepted, will subsume the frustrating group as a subordinate, creating a “complex group.” (We discuss the implications of merging below.) The group offered a merger likewise calculates its likelihood of winning in a fight against this opponent; with some stochasticity, if it is smaller, it will likely not win, and therefore accepts the merger. If the frustrating group declines the offer to merge (because its probability of winning is favorable), the groups may fight. Assuming that fighting does not result in a stand-off, the winning group subsumes the loser. Our use of stochastic Lanchester’s Laws in making these decisions is explained in algorithmic detail in Kohler et al. (in review).

If conflict occurs, groups calculate the number of casualties they would expect in a conflict. Ancient warfare was generally less lethal than modern warfare, with fighting ending once a group suffered a relatively small quantity of casualties, often around two percent of its fighters (Keeley 1996:91). In the simulation here we set a parameter of attrition, which we call *s* (Table 3), at values of 0.02 and 0.05 of the smaller number of fighters as the acceptable quantity of casualties in a battle.

Ability to levy tribute is a defining characteristic of power in complex societies (Steponaitis 1981). Once groups are in a hierarchical relationship subordinate groups pay tribute to their dominant group, which here is calculated as a tax on the net benefit of each subordinate’s public goods game. The proportion of that benefit rendered as tribute is the 𝛽 parameter in Table 3. If there are more than two groups in a chain then each subservient group pays a tribute in this proportion to its directly dominant group. Groups with a dominant that receive these flows from a subordinate pass along this tribute, retaining (1 - μ) of it for the favor. Since β and μ index proportions of the net benefit to the public goods game, a dominant group might receive no tribute, especially if a subordinate group is non-hierarchical. Even in that case, however, another benefit of hierarchy is defense. When a group is attacked, or attacks another group, that group calls on its direct dominant and all subordinates for assistance. Dominant groups receive the benefit of tribute from their subordinates (if the public goods game is successful) in addition to the benefit of defense. The model we present here makes two main changes to that described by Kohler et al. (in review), one giving simple groups the possibility of revolting from polities, and the second allowing simple groups that grow too large to split in two (fission).

### *Model: Revolt in Complex Groups*

In Kohler et al. (in review) we did not allow for simple groups to leave a complex group, despite abundant ethnographic and archaeological evidence that doing so is common (e.g., Flannery and Marcus 2012:170; Turchin and Gavrilets 2009). In the previous model, if the group on top of the complex hierarchy became small in population it would still receive tribute from its subordinate groups who had no mechanism to protest. In the model reported here, simple groups may decide whether to attempt to secede (revolt) from the complex group.

Every year after they play the public goods game, subservient groups assess whether they should try to revolt, using the same algorithms as for merging/fighting, considering each group’s number of fighters (males from 15 to 50 years old). Revolts are undertaken if the odds of success (in terms of the number of fighters on each side) appear favorable. The group considering whether to revolt considers both its own fighters and those in each of its subordinate groups if any, since it will take those subordinates along if its revolt is successful.

Some of the dynamics in complex group formation are illustrated in Supplemental Figure 8. Even with the change in fissioning noted above, it can still happen that smaller groups dominate larger. In the case where a dominant group has more than one chain of subordinates, those from one chain will be enlisted to help suppress a revolt from members of the other, so that polities with chains branching just below the top are most invulnerable to revolt (Supplemental Figure 8, panel 6).

### *Model: Fission of Simple Groups*

In our earlier work on this model, when a simple group reached its maximum group size, individual households would bud-off to form their own groups. This process is meant to reflect the fact that simple groups, even those with leaders able to induce cooperation in a public goods game, eventually reach size thresholds such as “Dunbar’s number” (2008) beyond which growth is difficult. The previous process however created many groups of size 1 that would often become immediately subordinated to the nearest larger group.

In the model version reported here, when groups reach 50 or 100 agents (Table 2), they will fission into two groups of approximately equal size, with new groups dictated by propinquity. We employ a k-means clustering algorithm to divide the group into 2 polygons. This algorithm creates 2 candidate centroids at random and associates each household in the group with the nearest centroid. New centroids are then calculated for each group and new assignments are made, and so forth in an iterative process terminating when group assignments no longer change.

On fission the smaller group loses its subordinate groups, but keeps its dominant group. The larger of the two groups keeps its subordinate groups as well as its dominant group. Fission is therefore costly to the group that leaves.