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SUPPLEMENTARY MATERIAL

Supplementary material for "Downscaling land surface temperature data using edge detection and block-diagonal Gaussian process regression"

A. Estimating the ECOSTRESS variance parameter

In this section, we justify (2.1) using elementary Fourier analysis (e.g., Stoica and Moses 2005). Assuming that there is no blurring at the edges, we can compute the power spectral density (PSD) of $f \coloneqq f_{R_r}^{\mathrm{ES}}$ as

$$S_f(\omega) = A \exp(-4\pi^2 (\ell_r^{\mathrm{ES}})^2 \, \|\omega\|^2)$$

for some constant $A > 0$. Similarly, we can compute the energy spectral density of the blur kernel $\kappa \coloneqq \kappa_{\mathrm{blur}}$ as

$$|\hat{\kappa}(\omega)|^2 = \exp(-4\pi^2 \sigma_{\mathrm{blur}}^2 \|\omega\|^2),$$

where $\hat{\kappa}$ denotes the Fourier transform of κ. The PSD of the convolution factorizes as a product, so we obtain

$$S_{f*\kappa}(\omega) = S_f(\omega) \, |\hat{\kappa}(\omega)|^2 = A \exp\left(-4\pi^2 \left((\ell_r^{\mathrm{ES}})^2 + \sigma_{\mathrm{blur}}^2\right) \|\omega\|^2\right).$$

In particular, integrating the PSD over all frequencies yields the variance:

$$\begin{aligned}
\mathrm{Var}(f * \kappa) &= \iint S_{f*\kappa}(\omega) \, d\omega \\
&= \iint A \exp\left(-4\pi^2 \left((\ell_r^{\mathrm{ES}})^2 + \sigma_{\mathrm{blur}}^2\right) \|\omega\|^2\right) d\omega \\
&= \frac{A}{4\pi \left((\ell_r^{\mathrm{ES}})^2 + \sigma_{\mathrm{blur}}^2\right)}.
\end{aligned}$$

Similarly, we have

$$(\sigma_r^{\mathrm{ES}})^2 \coloneqq \mathrm{Var}(f) = \iint S_f(\omega) \, d\omega = \iint A \exp(-4\pi^2 (\ell_r^{\mathrm{ES}})^2 \, \|\omega\|^2) \, d\omega = \frac{A}{4\pi (\ell_r^{\mathrm{ES}})^2}.$$

Note that $\mathrm{Var}(y_{R_r}^{\mathrm{ES}}) = \mathrm{Var}(f * \kappa) + \sigma_{\mathrm{sensor}}^2$ due to independence between the (blurry) signal and the sensor noise. Putting the pieces together, we obtain

$$(\sigma_r^{\mathrm{ES}})^2 = \mathrm{Var}(f * \kappa) \left(\frac{(\ell_r^{\mathrm{ES}})^2 + \sigma_{\mathrm{blur}}^2}{(\ell_r^{\mathrm{ES}})^2}\right) = (\mathrm{Var}(y_{R_r}^{\mathrm{ES}}) - \sigma_{\mathrm{sensor}}^2) \left(\frac{(\ell_r^{\mathrm{ES}})^2 + \sigma_{\mathrm{blur}}^2}{(\ell_r^{\mathrm{ES}})^2}\right).$$

Finally, the empirical variance $\widehat{\mathrm{Var}}(y_{R_r}^{\mathrm{ES}})$ is a consistent estimator of $\mathrm{Var}(y_{R_r}^{\mathrm{ES}})$ because the kernel is squared exponential (and hence stationary and ergodic). Thus, it follows that the proposed estimator (2.1) for σ_r^{ES} is consistent.

B. Converting full-width at half-maximum to standard deviation

In this section, we justify the conversion from full-width at half-maximum (FWHM) to standard deviation for a Gaussian kernel. Recall that the FWHM is defined as the width of the kernel at half of its maximum value. For a Gaussian kernel with standard

deviation σ, the maximum value occurs at the center of the kernel, and is equal to $1/(\sigma\sqrt{2\pi})$. To find the FWHM, we need to solve for the points where the kernel value is equal to half of its maximum value:

$$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right) = \frac{1}{2}\,\frac{1}{\sigma\sqrt{2\pi}}.$$

Simplifying this equation, we obtain

$$\exp\left(-\frac{x^2}{2\sigma^2}\right) = \frac{1}{2} \implies x = \pm\sigma\sqrt{2\ln(2)}.$$

Since the FWHM is defined as the distance between the two points where the kernel value is half of its maximum value, we have

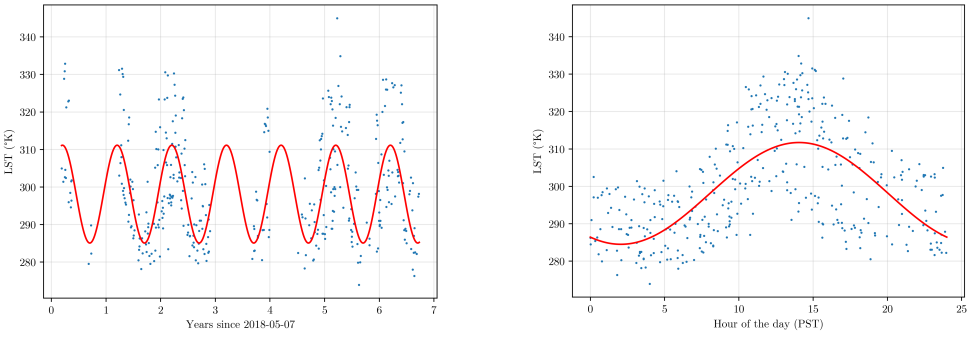
$$\text{FWHM} = 2\sigma\sqrt{2\ln(2)}.$$

Rearranging this equation gives us the desired conversion:

$$\sigma = \frac{\text{FWHM}}{2\sqrt{2\ln(2)}}.$$

C. Additional figures

In this appendix, we collect several additional figures (Figure 1 and Figure 2).



(a) Annual cycle

(b) Diurnal cycle

Figure 1: Plots of the fitted temperature cycle at a randomly selected pixel for (a) the annual cycle from ECOSTRESS data and (b) the diurnal cycle from ECOSTRESS data.

D. Verification of methods

To verify that our method works, we manually blur a high-resolution Landsat image (Figure 3). We set σ_{blur} to 0.97 ECOSTRESS pixels (70 m), matching the true blur in the data. Then, we set $\sigma_{\text{sensor}} = 10^{-4}$, which is reasonable because we expect much smaller sensor noise in Landsat compared to ECOSTRESS. We use the same estimated parameters from the real data in this toy example. Using these parameters, we reconstruct the image using a simplified version of our pipeline (Figure 4). Note that the kriging mean does de-blur the image, recovering some of the features present in the original image. Our choice of a squared exponential kernel results in a relatively smooth reconstruction; one could obtain a rougher reconstruction using a different kernel, but we leave this for future work.

Lastly, in Figure 5, we compare our method against a baseline of using standard Gaussian process regression with a sliding window of 40 pixels by 40 pixels, with a 2 pixel overlap between windows (e.g., Gregory et al 2024). Where the windows overlap, we average the predictions. As expected, this naive approach leads to an oversmoothed prediction. We note in Figure 6 that our method leads to much more evenly distributed errors than the blurred image or the baseline method, compared to the original image. We quantify the performance of our method using the mean squared error of the blurred image, BDGP reconstruction, and the naive approach in Table 1.

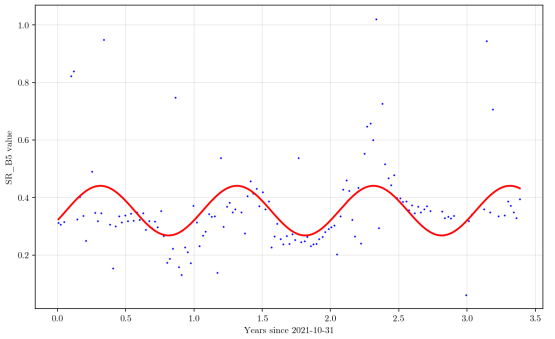
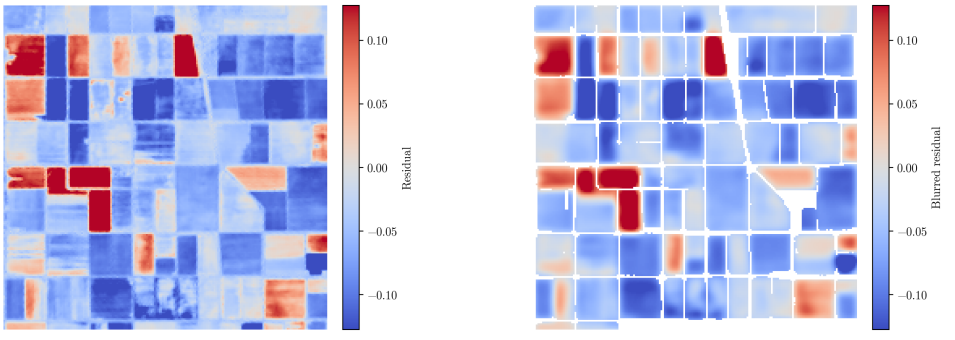
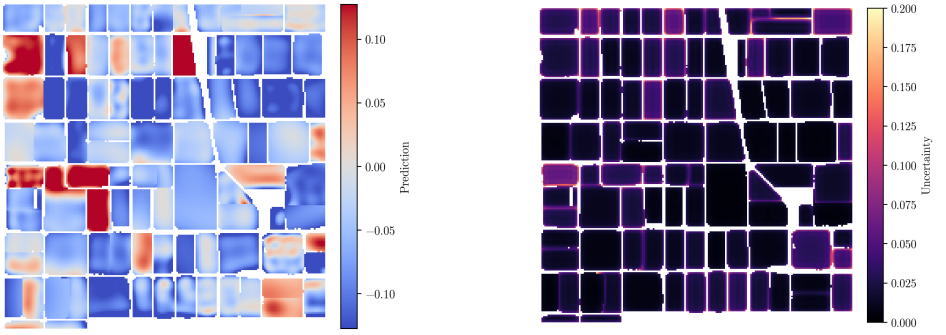


Figure 2: Plot of the annual cycle of the SR_B5 channel of the Landsat data at a randomly selected pixel. This channel measures the surface reflectance in the near-infrared spectrum of light.



(a) Original image (b) Blurred image

Figure 3: (a) the original high resolution image and (b) the manually blurred image used as input to the toy example, with the learned region mask applied.



(a) Kriging mean (b) Kriging per-pixel 2σ

Figure 4: Verification of the proposed method on the toy example of Figure 3: (a) the kriging mean prediction and (b) two per-pixel standard deviations for the kriging predictions.

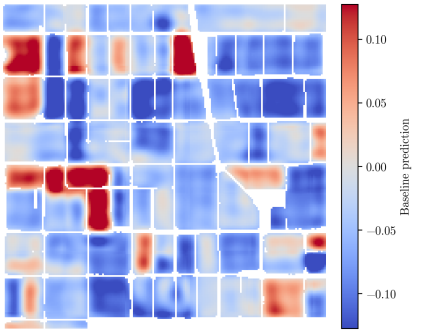
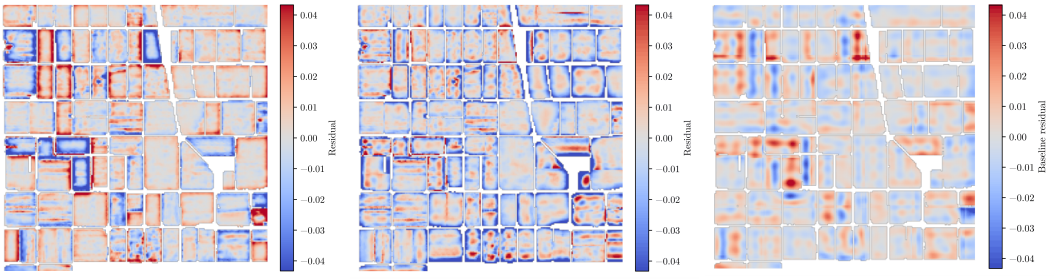


Figure 5: Baseline method (kriging mean) for the toy example of Figure 3 using a squared exponential kernel and a sliding window. After obtaining the predictions, we apply our learned region mask.



(a) Blurred residuals (b) BDGP residuals (c) Baseline residuals

Figure 6: Comparison of residuals to the original image of (a) the blurred image, (b) the BDGP kriging predictions, and (c) the baseline kriging predictions.

Table 1: Mean squared error of the blurred image, BDGP reconstruction, and the naive approach relative to the original image in the toy example of Figure 3.

Method	**Mean squared error (MSE)**
Blurred image (base)	4.898×10^{-3}
Block diagonal GP regression (ours)	4.971×10^{-4}
Stationary GP regression w/ sliding window (baseline)	7.305×10^{-4}

E. Refining the oversegmented image

In this appendix, we provide our algorithm to refine the oversegmented image provided by the Segment Anything Model (SAM).

- Discard all segments with fewer than 100 pixels.
- Sort the remaining segments in increasing order of area.
- Iterate through the segments s from largest to smallest:
  - Subtract s from all previously kept segments wherever they overlap.
  - Add the current segment to the list of kept segments.

References

Gregory W, MacEachern R, Takao S, Lawrence IR, Nab C, Deisenroth MP and Tsamados M (2024) Scalable interpolation of satellite altimetry data with probabilistic machine learning. *Nature Communications* **15**(1), 7453.

Stoica P and Moses RL (2005) *Spectral analysis of signals*. vol 452. Prentice Hall.