A. Appendix. Dataset

A.1. Autoconversion Rates

We have adopted the following procedure to acquire the necessary information to compute the autoconversion rates:

1. Air density (ρ): We can compute the air density by adopting the procedure outline in Czernia and Szyk [2021], which involves utilising the temperature, pressure, and relative humidity (RH) of the air. The procedure includes various steps as follows:
    (a) The vapor pressure at temperature T can be derived as follows:

    $$p_v = \left(6.1078\ \text{hPa} \times 10^{\frac{7.5T}{T+237.3\ ^\circ\text{C}}}\right) \times \text{RH} \tag{A.1}$$

    where T represents the temperature in °C. The term in the bracket is the saturation vapor pressure computed as a function of the ambient temperature using an empirical integration of the Clausius-Clapeyron equation.

    (b) The pressure of dry air (p_d) can then be derived by computing the difference between the total air pressure (p) and vapor pressure (p_v) as follows:

    $$p_d = p - p_v, \tag{A.2}$$

    (c) Finally, the air density (ρ) can be calculated by using the following equation:

    $$\rho = \frac{p_d}{R_d \times T} + \frac{p_v}{R_v \times T}, \tag{A.3}$$

    where p_d represents the pressure of dry air. T is now given in K, R_d is the specific gas constant for dry air, with a value of 287.058 $\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$, and R_v is the specific gas constant for water vapor, with a value of 461.495 $\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$. These values are fundamental constants used to calculate the air density through thermodynamic principles.
2. Cloud water content (L_c): The value of L_c, representing the mass density of cloud droplets, can be calculated by utilising the mixing ratio of cloud liquid water (q_l) and air density (ρ). This calculation can be performed as follows:

    $$L_c = q_l \times \rho \tag{A.4}$$
3. Rain water content (L_r): The variable L_r, which represents the mass density of raindrops, is not directly available in our datasets, but it can be estimated as an (arbitrary) fraction of L_c. To estimate this variable, we make the assumption that L_r is equal to a fixed percentage of L_c, the mass density of cloud droplets. Specifically, we assume that L_r is equal to 10% of L_c. However, this assumption serves as a useful starting point for our analysis, and can be refined or adjusted.

B. Appendix. Additional Experiments

B.1. Different Loss Functions

To facilitate the comparison of different loss functions, we conducted additional experiments using Mean Absolute Error (MAE) and Quantile as the loss functions for a shallow neural network model tested on Cloud-top ICON-LEM Germany on 2 May 2013, at 1:20 pm, as depicted in Table 5. The results suggest that the various loss functions showed no significant differences, with MSE remaining the most effective among them, albeit with very slight differences.

Table 5. *Evaluation of the autoconversion rates prediction results on the ICON-LEM simulation model using various loss functions – Mean Absolute Error (MAE), Quantile, Mean Squared Error (MSE) – over Germany.*

Method	R^2	MAPE	RMSPE	SSIM	PSNR (dB)
MAE	89.38%	11.17%	14.47%	89.63%	25.68
Quantile	89.39%	11.16%	14.44%	89.64%	25.69
MSE (Main)	89.87%	10.88%	13.99%	89.97%	25.89