# Supplementary information

## Supplementary results

### Velocity distributions

It is important to investigate the velocity distributions in the scroll chamber, bell mouth, and drop shaft, given that many previous studies had endeavoured to link geometrical design with velocity distributions by developing analytical models to facilitate the design of scroll drop shafts. This section provides insight into the velocity distributions in comparison with the experimental results from Guo (2012).

It is more convenient to set up a cylindrical coordinate system along the centreline of the drop shaft for vortex flows. In this study, the origin of the cylindrical coordinate system is located at the same point as the Cartesian coordinate system shown in Figure 1. The axial direction coincides with the vertical direction, and the angular (tangential) direction is counterclockwise when viewed from the positive vertical direction. The angular coordinate is set to zero on the positive *y*-axis and 180 degrees on the negative *y*-axis. Supplementary Figure 1 shows the contours of both mean tangential $V\_{t}$ and axial (vertical) $V\_{z}$velocity at various cross-sections. Note that the negative velocity represents the clockwise direction for tangential velocity and downward for vertical velocity. It is observed that the water flow shows higher clockwise tangential velocity away from the scroll chamber or drop shaft wall, except for the near centre area where the air core exists.

In the scroll chamber, the radial distributions of the magnitude of mean tangential velocity at different vertical locations and the same angular location are plotted in Supplementary Figure 2(a), where the LES results are close to the experimental distributions. These profiles agree very well with each other except at location *z* = 1.5 *mm*, which is very close to the chamber bottom wall and significantly affected by the presence of the wall. The similarity in the distribution of tangential velocity between different vertical locations reveals the potential of free vortex flow in the scroll chamber, which, as an assumption, has been extensively adopted by analytical models in previous studies (Mulligan et al., 2019). On the other hand, this may not be valid when we look at the distributions at the same height but different angular locations, as shown in Supplementary Figure 2(c). The velocity profiles at different angular locations do not coincide, and the profiles increase from $θ=180$ (at the entrance of the scroll chamber) to $θ=-90$, attributed to the decreasing radius from $θ=180$ to $θ=-90$ resulting in increased velocities. The velocity profiles at $θ=180$ present a plunging drop at a radial location of around 0.15, where the chamber tongue wall is present. This can also explain the interruption at a radial location of around 0.12 in Supplementary Figure 2(a), where the wake region caused by the wall still plays a role.

The tangential velocity distributions are translated into the flow circulation ($Γ=V\_{t}∙r$) along the radial direction, as shown in Supplementary Figures 2(b) and (d). It can be observed that the angular momentum (flow circulation) remains constant along the vertical direction at the same angular location, except for the region very close to the bottom wall. On the other hand, the assumption of free vortex flow, i.e., a constant circulation, does not apply between different angular locations. In other words, the angular momentum increases downstream (clockwise) until $θ=0$, and then decreases downstream.

In the bell mouth and drop shaft, a forced vortex flow is typically assumed in previous analytical models. Supplementary Figure 3 shows the tangential velocity distributions in the bell mouth and drop shaft. Similar to the distributions in the scroll chamber, the tangential velocity in the bell mouth and drop shaft also presents self-similarity between different vertical locations but non-self-similarity between different angular locations. The vertical velocity distributions are also presented in Supplementary Figure 4. It can be observed that the vertical velocity becomes significant in the bell mouth and drop shaft. All the results are in good agreement with the experiment, as shown in Supplementary Figures 3 and 4.

## Supplementary tables

Supplementary Table 1. Dimensions of the scroll vortex drop shaft.

|  |  |
| --- | --- |
| Parameter | Value (*m*) |
| Vortex chamber first arc radius *R*1 | 0.306 |
| Vortex chamber second arc radius *R*2 | 0.251 |
| Vortex chamber third arc radius *R*3 | 0.195 |
| Vortex chamber fourth arc radius *R*4 | 0.167 |
| Vortex chamber tongue length *s* | 0.027 |
| Width of inflow channel *B* | 0.167 |
| Length of inflow channel *L* | 1.130 |
| Height of inflow channel *H* | 0.200/0.300 |
| Shaft diameter *D* | 0.120 |
| Shaft eccentricity *e* | 0.028 |
| Bell mouth diameter *δ* | 0.052 |

Supplementary Table 2. Numerical runs.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Runs | *Q (L/s)* | $${Q}/{\sqrt{gD^{5}}}$$ | *ha/D* | *λm* |
| 1 | 10.0 | 0.640 | 1.58 | 0.32 |
| 2 | 8.0 | 0.513 | 1.27 | 0.43 |
| 3 | 6.0 | 0.384 | 0.94 | 0.51 |
| 4 | 4.0 | 0.256 | 0.64 | 0.65 |
| 5 | 15.0 | 0.960 | 2.35 | 0.20 |

## Supplementary figures

 

 (a) (b) (c)



 (d) (e) (f)

m/s

Supplementary Figure 1. Contours of tangential and vertical velocity in (a)-(d) scroll chamber (*z* = 30 *mm*), (b)-(e) bell mouth (*z* = -30 *mm*), and (c)-(f) drop shaft (*z* = -60 *mm*).



 (a) (b)



 (c) (d)

Supplementary Figure 2. Radial distribution in scroll chamber: magnitude of mean tangential velocity $V\_{t}$ and circulation $Γ=V\_{t}∙r$ (a)-(b) at different vertical locations (*θ* = 105°) and (c)-(d) different angular coordinates (*z* = 30 mm).



 (a) (b)



 (c) (d)

Supplementary Figure 3. Radial distribution of tangential mean velocity at different vertical locations (*θ* = 90°) in (a) bell mouth and (b) drop shaft, and at different angular coordinates in (c) bell mouth (*z* = -40 *mm*) and (d) drop shaft (*z* = -65 *mm*).



 (a) (b)



 (c) (d)

Supplementary Figure 4. Radial distribution of vertical mean velocity at different vertical locations (*θ* = 90°) in (a) bell mouth and (b) drop shaft, and at different angular coordinates in (c) bell mouth (*z* = -40 *mm*) and (d) drop shaft (*z* = -65 *mm*).

## Supplementary notation

*Aac* = the air core cross-sectional area,

*Amac* = the minimum air core cross-sectional area,

*B* = the width of the inflow channel,

$C\_{w}$ = the WALE constant,

*D* = the shaft diameter,

*e* = the shaft eccentricity,

*E* = the total energy head,

*E(κ)* = the energy spectrum,

*E0* = the initial energy head at Section 1-1,

*EMI* = the mass integral of energy head over cross-section,

$F\_{st,i}$ = the surface tension force,

***g*** = the gravitational acceleration,

*ha* = the water level height of approach flow at Section 1-1,

*H* = the height of approach channel,

*L* = the length of approach channel,

*L*v = the vortex length scale,

*p* = the pressure,

*Q* = the flow rate discharged from the inlet,

*r* = the radial coordinate,

*R*1 = the vortex chamber first arc radius,

*R*2 = the vortex chamber second arc radius,

*R*3 = the vortex chamber third arc radius,

*R*4 = the vortex chamber fourth arc radius,

*s* = the vortex chamber tongue length,

$\tilde{S}\_{ij}$ = the rate of strain tensor,

$S\_{ij}^{d}$ = traceless symmetric part of the square of the velocity gradient tensor,

*t* = the time,

$\tilde{u}$ = the velocity vector,

*Vr* = the radial velocity,

*Vt* = the tangential velocity,

*Vz* = the axial (vertical) velocity,

*x, y, z* = Cartesian coordinate,

*y*+ = the non-dimensional distance to a wall,

*α* = the volume fraction of the water phase

*δ* = the shaft transition diameter,

*λ* = the air core area percentage,

*λm* = the minimum air core percentage,

*μ* = the dynamic viscosity of fluid,

$μ\_{SGS}$ = the SGS turbulent viscosity,

*ρ* = the density of fluid,

*σ* = the surface tension coefficient,

$σ\_{ij}$ = the viscous stress tensor,

$τ\_{ij}$ = the sub-grid stress tensor,

*κ* = the wavenumber,

*κc* = the local curvature in VOF,

*Г* = the flow circulation,

$∆$ = the characteristic sub-grid length scale.