Supplementary Material: Numerical study of sand particles transporting in the atmospheric Ekman boundary layer

Yixiang Wang,1 Cruz Y. Li,2\* Daniel Ziyue Peng,1 Tim K.T. Tse1

1 Department of Civil and Environmental Engineering, the Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, China

2 School of Civil Engineering, Chongqing University, No. 174 Shazheng Street, Shapingba District, Chongqing, 400044, China

\*Corresponding author: cruzli@cqu.edu.cn

**A Vertical profiles of sand particle velocity**

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| **Figure S1.** Profiles of averaged streamwise velocity of sand particles against time. (a) St=0.2; (b) St=3; (c) St=48. | |
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| ***Figure S2.*** *Profiles of particle vertical velocity. (a) gravity and (b) gravity .* | |

In Figure S2, it is apparent to see that particle vertical velocity for gravity deactivated cases is almost zero illustrating the statistically steady state has been reached and for gravity activated cases it is small enough when compared with two horizontal velocities in Figure 1 in the manuscript. Thus, we believe the time interval we chose is sufficient to attain reliable statistical results which are good enough for being used as a guideline.

**B Validation with previous studies**

We compare our DNS results with some previous studies in Figure S3. It is obvious that our DNS results are consistent with results from previous studies of Ekman layer, which indicates that the Coriolis is correctly introduced to the air phase. We also provide the vertical profile of in Figure S4. It is clear to find from these profiles that two transitions can be clearly observed at and , indicating the Ekman boundary layer being divided into three layers.

A snapshot of typical vortex structures of Ekman boundary layer is depicted in Figure S5, which is identified by *Q*-criterion and depicted with iso-surface of *Q* = 0.015. It is clear to observe the tangled vortex filaments appearing in some places near the boundary wall while other places barely could find vortex filaments indicating weak turbulence intensity locally. Another distinct feature is the vortex structures in Ekman boundary layer is oblique with respect to the streamwise direction *x*, which is generated by the Coriolis force.

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| ***Figure S3.*** *Comparison with previous DNS results.* | |
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| ***Figure S4.*** *Profile of mean horizontal velocities.* | |

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| **A close-up of a graph  Description automatically generated** |
| ***Figure S5.*** *Vortex structures of Ekman boundary layer with iso-surface of Q = 0.015.* |

**C Slip velocity between air and sand particles**

Firstly, we provide a qualitative analysis of the instantaneous velocities of the air and sand particle phases in Figure S6 and Figure S7. Here, we only present the horizontal and vertical results from Case in the absence of gravity since other cases could show very similar results. As evidenced in the vertical snapshots of Figure S6, we could see that within the turbulent Ekman layer , except in the near-wall regions, sand particles with high horizontal velocities tend to accumulate in regions with high-speed horizontal wind. Moreover, the vertical movement of sand particles is positively correlated with the zones featuring large vertical wind speed, showing that sand particles moving upward are in the regions having upward wind speed, and vice versa.

The results from the horizontal slices in Figure S7 reveal that sand particles form strip shapes near the ground wall at , and similar to the vertical snapshots, the large wind speed regions contain more sand particles. However, at the separation between the buffer layer and log-law region, where sand particles exhibit higher fluctuations (see Figure 2 and 3), the distinct difference between wind speed and sand speed is not immediately apparent. To obtain quantitative results, statistical calculations are necessary to analyze the velocity difference between air and sand particles.

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| ***Figure S6.****Vertical snapshots of fluid velocity contour and velocity vectors of sand particles at Case with gravity . Contours are (a) ; (b) ; and (c) , respectively.* |

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| ***Figure S7.*** *Horizontal snapshots of fluid velocity contour and velocity vectors of sand particles at Case with gravity . Contours are* *(a) at ; (b) at ; (c) at , (d) at ; (e) at ; and (f) at , respectively.* |

An additional investigation is carried out to explore the quantitative relationship between the velocity of sand particles and the wind velocity at the particle's location, as depicted in Figure S8. This examination utilizes solely the outcomes from Case for illustrative purposes. When the gravitational force is deactivated, it becomes evident that the majority of sand particles situated at exhibit their velocities are consistent with those of the surrounding air. However, the velocities of a minority of sand particles, whose velocities exceed , are higher than those of the adjacent air. As the sand particles move further away from the ground wall, the discrepancy between and gradually diminishes, showing dots gathering around the 45-degree matching line.

Upon activation of gravity, it is discernible that the discrepancy between and is greatest for sand particles positioned at , moderate at , and minimal at . This observation suggests that gravity promotes the development of particle velocity in the viscous sublayer and buffer layer but has minimal impact in the turbulent region. This is predominantly due to the vertical gravitational acceleration, which heightens the particle vertical velocities. Since the weak viscous force dominates in these two layers, gravity can exert a more substantial influence on the particle vertical kinetic energy. However, in the turbulent region, the inertial force is potent enough to counteract the effect of gravity.

Furthermore, it is also noted that the velocity distribution in the case with gravity is narrower in comparison to the case without gravity, as a greater number of sand particles settle on the ground wall under the force of gravity. Additionally, in the buffer layer and viscous sublayer, sand particles have a higher likelihood of exhibiting lower wind velocities in the presence of gravity, signifying that sand particles tend to congregate in regions with diminished wind velocity.

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| ***Figure S8.*** *Relationship between sand particle velocity and its corresponding wind velocity at the particle location. Only results from Case are presented: (a), (b) and (c) are sand particles with gravity at , respectively; (d), (e), and (f) are sand particles with gravity at , respectively. Dots are coloured by probability with (a) and (d), (b) and (e), (c) and (f) having the same colorbar interval, respectively.* | |



***Figure S9.*** *The hodograph of mean horizontal slip velocities normalized by particle diameters.*

**D Coriolis force included in the particle dynamic equation**

We note that the Coriolis force should be exerted on the sand particles. However, if we add this force in the particle dynamic equation (Eq. 5), it is very difficult to investigate the effect of gravity and fluid flow on the particles, as shown in Figure S10 and Figure S11, which are the results with Coriolis force included in Eq. 5 and without gravity.

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From these figures, it is evident to find that the spanwise velocity is influenced significantly by the Coriolis force especially when larger than 10 compared to Figure 1 and 4 in the manuscript. As the consequence, we cannot observe threes sublayers and the interaction between fluid flow and sand particles which is the main part we want to provide in this article. Therefore, in all the results of manuscript, we neglect this force in Eq. 5 for the sake of providing clear effect of gravity and turbulence.

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| ***Figure S10.*** *Profiles of particle horizontal velocity when Coriolis force is included in Eq. 5 and gravity is ignored.* | | |
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| ***Figure S11.*** *Profiles of mean slip velocities when Coriolis force is included in Eq. 5 and gravity is ignored.* | | |

**E Three-Dimensional Voronoï diagram analysis of sand particles**

Figure S12 depicts an example of 3D Voronoï volume of sand particles in the buffer layer () in the absence of gravity at . The Voronoï volume is color-coded based on the normalized Voronoï volume , with yellow indicating values of , green indicating values of , and cyan representing values between these two extremes. Notably, the accumulation of sand particles in streaks is observed near the ground wall, as evidenced by the cyan and yellow regions in the proximity of .

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| A diagram of a blue and green rectangular object  Description automatically generated |
| ***Figure S12.*** *3D view of Voronoï volume of sand particles with in the buffer layer* *() with gravity .* |

The impact of gravity on the clustering level of sand particles is depicted in Figure S13. Our findings indicate that for sand particles with *St* smaller than 3, by comparing Figure S13(a) with Figure 8(a), gravity does not significantly affect their clustering level. However, for sand particles with large *St*, their PDFs deviate more from the uniform distribution, which is indicative of a higher clustering level. Notably, this phenomenon is not clearly discernible in the results of sand particles within the slabs of the interfaces of the three subregions by comparing Figure S13(b), (c), and (d) with Figure 8(b), (c), and (d).

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| ***Figure S13.*** *PDF of Voronoï volume with gravity . Four subfigures represent PDF results of sand particles in (a) whole domain, (b) slab of , (c) slab of , and (d) slab of , respectively.* | |

Numerous previous studies (Wang & Maxey 1993) have demonstrated the tendency of heavy particles to accumulate in regions of low vorticity, a mechanism termed preferential concentration in the absence of gravity. In this study, we present the joint probability density function (PDF) of the magnitude of vorticity and Voronoï volume, depicted in Figure S14 for increasing values of the Stokes number () ranging from 0.2 to 48. Our results indicate that when is less than 3, the joint PDF exhibits an egg-shaped distribution with a maximum located at and . For joint PDF values greater than approximately 0.4, the Voronoï volume of sand particles typically ranges from 0.5 to 1.4, with corresponding vorticity magnitudes ranging from 0.08 to 0.5. This finding suggests that while most sand particles tend to locate in regions of relatively low vorticity, their occupied spatial volume is roughly equal to the overall mean value, indicating that sand particles with less than 3 are not aggregated into clusters despite their potential to accumulate in low vorticity regions.

With exceeding 6, the joint PDF exhibits an elongated egg-shaped contour with its maximum shifted from and to and . Moreover, the region where the joint PDF values exceed approximately 0.4 undergoes a change, manifesting as a shift to and . It is evident that the majority of sand particles continue to accumulate in regions of relatively low vorticity, meanwhile they aggregate into clusters that occupy spatial volumes smaller than the overall mean value. It is worth noting that the formation of these clusters with increasing should not be related to the turbophoresis mechanism, which posits that inertial particles tend to migrate toward near-wall regions of low turbulence intensity (Marchioli & Soldati 2002; Sardina et al. 2012). Because we use the normalized Voronoï volume to quantify the clustering level instead of the absolute particle numbers and from Figure 8 and Figure S13 we know that the PDF of Voronoï volume in slabs is similar to that in the entire domain, manifesting that turbophoresis mechanism is different from clustering level conceptually.

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| ***Figure S14.*** *Joint PDF of Vorono**ï volume and vorticity magnitude at sand particle locations with gravity . Stokes number for each case: (a) , (b) , (c) , (d) , (e) , (f) , (g) , and (h) , respectively.* | |

We use the case of as an example to illustrate the relationship between Voronoï volume and Voronoï surface area depicted in Figure S15. Our findings indicate that, irrespective of whether they are clusters or voids, small and large Voronoï polygons exhibit an exponential law between and  with a single fractal factor . However, our results differ from those of Monchaux et al. (2010), who showed that clusters with smaller Voronoï areas have a smaller fractal factor, while those with larger Voronoï areas exhibit other fractal behaviors. We attribute this discrepancy to the differences in the background turbulent flows. Specifically, the turbulence generated by grids installed in the wind tunnel in Monchaux et al. (2010) was homogeneous, whereas our Ekman boundary layer was affected by the ground wall and Coriolis force.

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| ***Figure S15.*** *Geometrical characteristics of Voronoï polygons for .* |

In this following part, we investigate the temporal characteristics of Voronoï volume. Figure S16(a) illustrates the time variation of Voronoï volume for selected sand particles over a single cycling time (). Our results indicate that the time variation of Voronoï volume is smooth for the sampling frequency employed. The transition of Voronoï volume from small to large values signifies the transformation of sand particles from clusters to voids, and vice versa. This transformation is not necessarily related to the process of moving away from or towards the ground wall. Additionally, we observe that some sand particles maintain their Voronoï volume unchanged during one cycling time, indicating that they remain at a similar clustering level within this period.

We are interested in quantifying the duration for which sand particles can maintain their clustering status. To this end, we calculate the time autocorrelation coefficient of Voronoï volume using all the recorded data within five cycling times, as shown in Figure S16(b). Our findings demonstrate that each sand particle has a unique process of evolution of clustering status owing to the turbulence generated by the ground wall. The autocorrelation coefficient curve for each particle is represented by a gray line in Figure S16(b). The ensemble average results of the time autocorrelation coefficient reveal that Voronoï volume becomes uncorrelated after , which is approximately 1.3 times one cycling time. This value is smaller than our sampling time of , indicating that our chosen sampling period is sufficient for obtaining reasonable statistical results. For simplicity, we present the results of case with only, as other cases yield similar outcomes.

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| ***Figure S16.*** *(a) Examples of time variation of* *Voronoï volume for Case with . Circle markers represent the maximum Voronoï volume is larger than and triangle markers represent the maximum Voronoï volume is smaller than . (b) Time autocorrelation coefficient of Voronoï volume for Case with .* *Gray lines are examples of 20 individual results and blue line is the ensemble average results with dark shading denoted as half of rms deviation and light shading* *denoted as one rms deviation, respectively.* |

Reference

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