Heterogeneous Trembles and Model Selection in the Strategy Frequency Estimation Method

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— Supplementary appendix —

Contents

A Likelihood function

The probability of observing individual i's data, y_i, conditional on tremble probability γ_i, strategy s_k, and model parameters θ is:

$$p(y_i \mid \gamma_i, s_k, \theta) = \gamma_i^{m_{i,k}}(1-\gamma_i)^{T_i - m_{i,k}} \tag{A.1}$$

Integrating out γ_i:

$$p(y_i \mid s_k, \theta) = \int_{-\infty}^{\infty} p(y_i \mid \gamma, s_k, \theta) p(\gamma \mid \theta) \mathrm{d}\gamma \tag{A.2}$$

(A.3)

where $p(\gamma \mid \theta)$ is the probability density function of the $N(\mu, \sigma^2)$ distribution. I compute this integral using Monte Carlo integration.

Marginalizing over the strategy frequencies yields the likelihood of observing participant i's data conditional on parameters θ:

$$p(y_i \mid \theta) = \sum_{k=1}^{S} p(y_i \mid s_k, \theta)\rho_{k,j(i)} \tag{A.4}$$

And so the overall log-likelihood of the model is:

$$\log p(y \mid \theta) = \sum_{i=1}^{N} \log\left(p(y_i \mid \theta)\right) \tag{A.5}$$

I solve this using Matlab's `fmincon` solver, with constraints ensuring mixing probabilities lie on the unit interval and sum to 1, and that $\sigma^2 \geq 0$.

B Heterogeneity bias

Consider a SFEM with two strategies. The overall log-likelihood function is:

$$\mathcal{L}(\rho, \gamma) = \sum_{i=1}^{N} \log\left(\rho\gamma_i^{m_{i,1}}(1-\gamma_i)^{T_i - m_{i,1}} + (1-\rho)\gamma_i^{m_{i,2}}(1-\gamma_i)^{T_i - m_{i,2}}\right) \quad \text{(B.1)}$$

The partial derivative of this with respect to the mixing probability ρ is:

$$\frac{\partial \mathcal{L}(\rho, \gamma)}{\partial \rho} = \sum_{i=1}^{N} \frac{\gamma_i^{m_{i,1}}(1-\gamma_i)^{T_i - m_{i,1}} - \gamma_i^{m_{i,2}}(1-\gamma_i)^{T_i - m_{i,2}}}{\gamma_i^{m_{i,1}}\rho(1-\gamma_i)^{T_i - m_{i,1}} - \gamma^{m_{i,2}}(1-\gamma_i)^{T_i - m_{i,2}}(\rho - 1)} \quad \text{(B.2)}$$

Taking the partial derivative of this with respect to γ_i yields:

$$\frac{\partial^2 \mathcal{L}(\rho, \gamma)}{\partial \rho \partial \gamma_i} = \frac{(m_{i,1} - m_{i,2})\,\gamma_i^{m_{i,1}+m_{i,2}}(1-\gamma_i)^{m_{i,1}+m_{i,2}}}{\gamma_i\,(1-\gamma_i)\,\left(\gamma_i^{m_{i,2}}(1-\gamma_i)^{m_{i,1}} + \gamma_i^{m_{i,1}}\rho(1-\gamma_i)^{m_{i,2}} - \gamma^{m_{i,2}}\rho(1-\gamma_i)^{m_{i,1}}\right)^2} \quad \text{(B.3)}$$

which has the same sign as $(m_{i,1} - m_{i,2})$, which is the difference between the number of trembles implied by each strategy. This expression is also non-zero unless $m_{i,1} = m_{i,2}$, which would only be true if both strategies implied exactly the same number of trembles.

Assume that a SFEM assuming homogeneous trembles (i.e. γ_i = a constant for all i) estimates mixing probability $\hat{\rho}'$ and tremble probability $\hat{\gamma}'$. Then, assuming interior solutions, substituting $\rho = \hat{\rho}'$ and $\gamma_i = \hat{\gamma}'$ for all i

into (B.2) must yield:

$$\left.\frac{\partial \mathcal{L}(\rho,\gamma)}{\partial \rho}\right|_{\rho=\hat{\rho}',\gamma=\hat{\gamma}'} = 0 \tag{B.4}$$

Now suppose that increasing γ_i for just one subject i increases the log-likelihood beyond that achieved in the homogeneous-trembles model. That is:

$$\left.\frac{\partial \mathcal{L}(\rho,\gamma)}{\partial \gamma_i}\right|_{\rho=\hat{\rho}',\gamma=\hat{\gamma}'} > 0 \tag{B.5}$$

By the cross partial derivative (B.3) it follows that the first-order condition no longer holds:

$$\text{sign}\left(\frac{\partial \mathcal{L}(\rho,\gamma)}{\partial \rho}\right)_{\rho=\hat{\rho}',\ \gamma_{k\neq i}=\hat{\gamma}',\ \gamma_i>\hat{\gamma}'} = \text{sign}\left(m_{i,1}-m_{i,2}\right) \tag{B.6}$$

and so the log-likelihood function can be further increased by adjusting ρ in the opposite direction of $\text{sign}\left(m_{i,1}-m_{i,2}\right)$. That is, given SFEM estimates $(\hat{\rho}',\hat{\gamma}')$, if the log-likelihood function can be increased by increasing subject i's tremble probability, then the log-likelihood function can be further increased by placing more weight on the strategy that implies fewer trembles by that subject.

See file `HetBiasWorking.m` in the data appendix for symbolic computation of these derivatives.

C Simulation

In order to assess the performance of the SFEM with and without heterogeneous trembles, I simulate the sampling distribution of the SFEM. I keep the true strategy frequencies fixed at:

$$(\rho_{\text{ALLC}}, \rho_{\text{ALLD}}, \rho_{\text{G}}, \rho_{\text{TFT}}, \rho_{\text{WSLS}}, \rho_{\text{T2}}) = (0, 0, 0.25, 0.25, 0.25, 0.25) \tag{C.1}$$

and simulate an experiment of $T = 10$ matches, with continuation probability $\delta = 0.75$ (that is, there is only one treatment).

I simulate the sampling distributions for four sample sizes (number of participants):

$$N \in \{40, 100, 200, 400\} \tag{C.2}$$

and four different distributions of trembles:

$$\mu = 0, \quad \sigma^2 \in \{0.001, 0.10, 0.100, 1.00\} \tag{C.3}$$

That is, $\mu = 0$ pins down the median tremble probability to $\gamma = 0.25$. Note that for $\sigma^2 = 1$, trembles are uniformly distributed on the interval $(0, 0.5)$.

The results of these simulations are shown in Tables S.1-S.4. Table S.1 shows the means of these distributions. Note that even for the smallest sample size and minimal heterogeneity in trembles, the heterogeneous trembles models are much more able to recognize that the frequencies of ALLC and ALLD are close to zero. Table S.2 shows the distributions' standard deviations. Here we see the bias-variance tradeoff between the constant and homogeneous trembles models: standard deviations are generally higher for the heterogeneous trembles model for strategies G, TFT, WSLS, and T2, even with substantial heterogeneity in trembles, or larger sample sizes. Taken together, the estimates from the heterogeneous trembles models are less biased for most data-generating processes under consideration, but have larger variances.

Table S.3 shows the root-mean squared errors (RMSE) for each estimator, and Table S.4 shows the total RMSE for each model's frequency estimators (i.e. summing each row in Table S.3). In general, for the heterogeneous model to outperform the constant trembles model by this metric, one needs at least about 100 subjects (fewer than in Dal Bó and Fréchette, 2011), and $\sigma^2 = 0.100$. For perspective, this corresponds to a standard deviation in tremble probability of about 0.06, which is below three of the six standard deviations estimated in Table 4.

N	σ^2	Assumption	ALLC	ALLD	G	TFT	WSLS	T2
TRUE VALUES			0.000	0.000	0.250	0.250	0.250	0.250
40	0.001	Constant	0.026	0.028	0.239	0.238	0.249	0.221
		Heterogenerous	0.002	0.002	0.255	0.249	0.249	0.242
	0.010	Constant	0.025	0.026	0.236	0.238	0.249	0.225
		Heterogenerous	0.001	0.001	0.250	0.249	0.249	0.251
	0.100	Constant	0.024	0.027	0.237	0.239	0.249	0.224
		Heterogenerous	0.001	0.000	0.251	0.249	0.249	0.250
	1.000	Constant	0.024	0.026	0.237	0.239	0.249	0.225
		Heterogenerous	0.000	0.000	0.250	0.250	0.249	0.250
100	0.001	Constant	0.026	0.027	0.240	0.238	0.249	0.220
		Heterogenerous	0.002	0.002	0.258	0.249	0.250	0.240
	0.010	Constant	0.025	0.026	0.237	0.238	0.249	0.225
		Heterogenerous	0.001	0.001	0.251	0.248	0.249	0.250
	0.100	Constant	0.025	0.027	0.236	0.239	0.249	0.225
		Heterogenerous	0.001	0.000	0.250	0.250	0.250	0.250
	1.000	Constant	0.024	0.027	0.236	0.239	0.249	0.225
		Heterogenerous	0.000	0.000	0.250	0.250	0.250	0.251
200	0.001	Constant	0.028	0.029	0.239	0.237	0.248	0.218
		Heterogenerous	0.002	0.002	0.258	0.249	0.248	0.240
	0.010	Constant	0.026	0.029	0.236	0.238	0.249	0.222
		Heterogenerous	0.001	0.001	0.251	0.249	0.249	0.249
	0.100	Constant	0.026	0.029	0.235	0.238	0.249	0.222
		Heterogenerous	0.001	0.000	0.250	0.250	0.249	0.250
	1.000	Constant	0.027	0.029	0.235	0.238	0.248	0.223
		Heterogenerous	0.000	0.000	0.250	0.250	0.249	0.251
400	0.001	Constant	0.047	0.054	0.227	0.226	0.243	0.204
		Heterogenerous	0.002	0.001	0.260	0.251	0.248	0.238
	0.010	Constant	0.046	0.053	0.223	0.227	0.244	0.208
		Heterogenerous	0.001	0.000	0.251	0.250	0.249	0.248
	0.100	Constant	0.046	0.053	0.222	0.226	0.243	0.209
		Heterogenerous	0.000	0.000	0.251	0.250	0.248	0.250
	1.000	Constant	0.046	0.053	0.222	0.226	0.243	0.209
		Heterogenerous	0.000	0.000	0.250	0.250	0.249	0.251

Table S.1: Means of simulated sampling distributions.

N	σ^2	Assumption	ALLC	ALLD	G	TFT	WSLS	T2
40	0.001	Constant	0.030	0.030	0.125	0.085	0.076	0.133
		Heterogeneous	0.009	0.007	0.142	0.095	0.085	0.156
	0.010	Constant	0.020	0.019	0.072	0.053	0.047	0.075
		Heterogeneous	0.004	0.003	0.077	0.059	0.052	0.080
	0.100	Constant	0.014	0.014	0.050	0.037	0.034	0.052
		Heterogeneous	0.002	0.002	0.054	0.042	0.037	0.056
	1.000	Constant	0.010	0.010	0.036	0.026	0.024	0.037
		Heterogeneous	0.001	0.001	0.038	0.030	0.026	0.039
100	0.001	Constant	0.030	0.029	0.123	0.086	0.075	0.130
		Heterogeneous	0.009	0.007	0.144	0.096	0.084	0.159
	0.010	Constant	0.020	0.020	0.073	0.053	0.048	0.075
		Heterogeneous	0.004	0.003	0.078	0.059	0.053	0.081
	0.100	Constant	0.014	0.014	0.050	0.038	0.034	0.052
		Heterogeneous	0.002	0.002	0.054	0.042	0.038	0.057
	1.000	Constant	0.010	0.010	0.036	0.027	0.024	0.036
		Heterogeneous	0.001	0.001	0.038	0.030	0.026	0.039
200	0.001	Constant	0.031	0.031	0.122	0.085	0.076	0.132
		Heterogeneous	0.009	0.007	0.141	0.097	0.085	0.158
	0.010	Constant	0.020	0.020	0.072	0.053	0.048	0.074
		Heterogeneous	0.004	0.003	0.078	0.060	0.054	0.081
	0.100	Constant	0.015	0.015	0.051	0.037	0.033	0.052
		Heterogeneous	0.002	0.002	0.054	0.042	0.037	0.055
	1.000	Constant	0.010	0.010	0.035	0.026	0.024	0.037
		Heterogeneous	0.001	0.001	0.038	0.029	0.027	0.040
400	0.001	Constant	0.041	0.041	0.119	0.083	0.076	0.127
		Heterogeneous	0.008	0.006	0.150	0.100	0.090	0.172
	0.010	Constant	0.026	0.026	0.069	0.051	0.047	0.069
		Heterogeneous	0.003	0.003	0.078	0.060	0.056	0.078
	0.100	Constant	0.018	0.018	0.048	0.036	0.034	0.048
		Heterogeneous	0.002	0.001	0.054	0.043	0.039	0.054
	1.000	Constant	0.013	0.013	0.033	0.026	0.024	0.034
		Heterogeneous	0.001	0.001	0.038	0.031	0.028	0.038

Table S.2: Standard deviations of simulated sampling distributions

N	σ^2	Assumption	ALLC	ALLD	G	TFT	WSLS	T2
40	0.001	Constant	0.040	0.041	0.125	0.086	0.076	0.136
		Heterogenerous	0.010	0.008	0.142	0.095	0.085	0.156
	0.010	Constant	0.032	0.033	0.073	0.054	0.047	0.079
		Heterogenerous	0.004	0.003	0.077	0.059	0.052	0.080
	0.100	Constant	0.028	0.030	0.051	0.039	0.034	0.058
		Heterogenerous	0.002	0.002	0.054	0.042	0.037	0.056
	1.000	Constant	0.026	0.028	0.038	0.029	0.024	0.045
		Heterogenerous	0.001	0.001	0.038	0.030	0.026	0.039
100	0.001	Constant	0.039	0.040	0.124	0.087	0.075	0.134
		Heterogenerous	0.009	0.007	0.144	0.096	0.084	0.160
	0.010	Constant	0.031	0.033	0.074	0.054	0.048	0.079
		Heterogenerous	0.004	0.003	0.078	0.059	0.053	0.081
	0.100	Constant	0.028	0.030	0.052	0.039	0.034	0.058
		Heterogenerous	0.002	0.002	0.054	0.042	0.038	0.057
	1.000	Constant	0.026	0.028	0.038	0.029	0.024	0.044
		Heterogenerous	0.001	0.001	0.038	0.030	0.026	0.039
200	0.001	Constant	0.042	0.043	0.123	0.086	0.076	0.136
		Heterogenerous	0.009	0.007	0.142	0.097	0.085	0.159
	0.010	Constant	0.033	0.035	0.074	0.055	0.048	0.080
		Heterogenerous	0.004	0.003	0.078	0.060	0.054	0.081
	0.100	Constant	0.030	0.032	0.053	0.039	0.033	0.058
		Heterogenerous	0.002	0.002	0.054	0.042	0.037	0.055
	1.000	Constant	0.028	0.031	0.038	0.029	0.024	0.046
		Heterogenerous	0.001	0.001	0.038	0.029	0.027	0.040
400	0.001	Constant	0.062	0.068	0.121	0.086	0.076	0.135
		Heterogenerous	0.008	0.007	0.151	0.100	0.090	0.173
	0.010	Constant	0.053	0.059	0.074	0.056	0.048	0.081
		Heterogenerous	0.003	0.003	0.078	0.060	0.056	0.078
	0.100	Constant	0.050	0.056	0.055	0.043	0.034	0.063
		Heterogenerous	0.002	0.001	0.054	0.043	0.039	0.054
	1.000	Constant	0.048	0.055	0.044	0.035	0.025	0.053
		Heterogenerous	0.001	0.001	0.038	0.031	0.028	0.038

Table S.3: Root mean squared errors of simulated sampling distributions

	Assuming $\sigma^2 = 0$				Assuming $\sigma^2 \geq 0$			
σ^2	$N = 40$	$N = 100$	$N = 200$	$N = 400$	$N = 40$	$N = 100$	$N = 200$	$N = 400$
0.001	0.0916	0.0909	0.0915	0.0953	0.1008	0.1024	0.1016	0.1086
0.010	0.0560	0.0563	0.0568	0.0629	0.0556	0.0563	0.0565	0.0561
0.100	0.0416	0.0419	0.0424	0.0512	0.0391	0.0393	0.0390	0.0392
1.000	0.0325	0.0325	0.0334	0.0446	0.0276	0.0276	0.0277	0.0278

Table S.4: Total root mean squared error of SFEM estimators.