Appendices

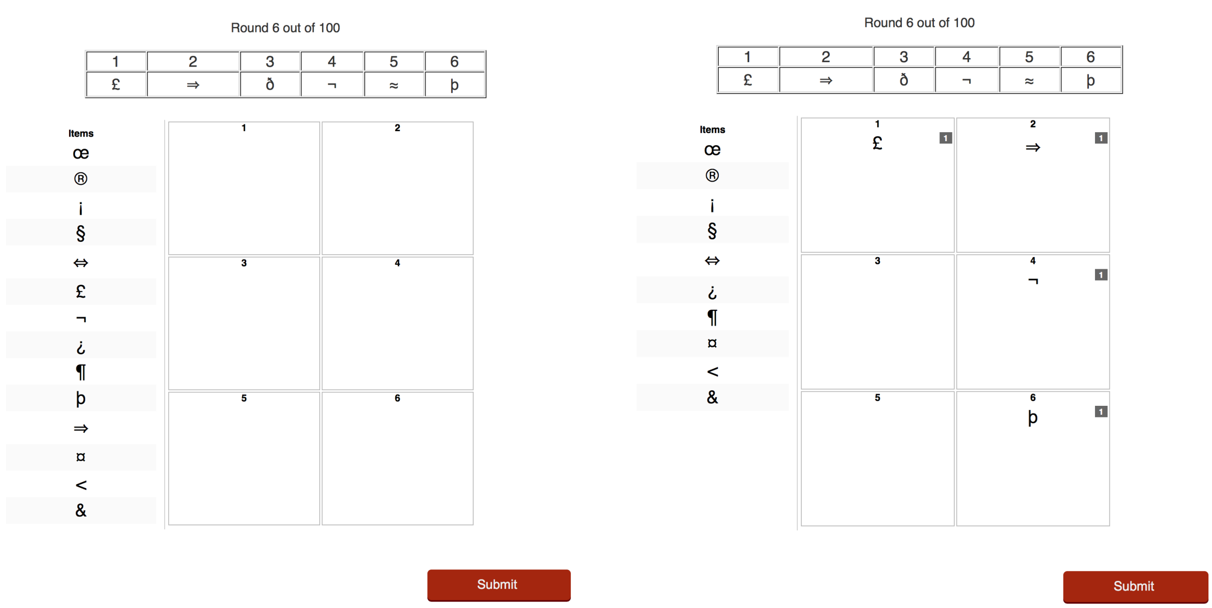
A Experimental Instructions (Online)

This task is based on finding correct correspondences between numbers and symbols. In each round, you will see 6 pairs of number–symbol combinations (the key) arranged in a table at the upper part of the screen, see Figure A.1a for an example. Below the key, there will be 6 empty numbered boxes.

You will use the key to fill in the boxes with the symbols located in a column to the left of the boxes. You will do this by dragging the symbols into the boxes. If a symbol from the column is in the key, drag it to the corresponding numbered box. Some of the symbols will not be listed in the key. In this case, you should not use them in any of the boxes. Some of the numbers will not have corresponding symbols. In this case, you should leave those boxes empty. Each box, therefore, can contain either one or no symbols. Figure A.1b shows an example of a correctly solved round.

After filling all the boxes as you see fit, click “Submit”, and you will proceed to the next round. You can proceed with each round at your own pace, there is no time limit. We ask that you complete all 100 rounds of the task. We will show you your score at the end of the task. You will receive $20 for completing this task.

You will have 3 practice rounds before the actual task begins. This will give you a chance to familiarize yourself with the interface. During the practice, you will receive feedback if you make a mistake.



(a) Decision screen (b) Correct solution

Figure A.1: Digit-Symbol Task

B Math Appendix (Online)

The discounted value function $h(E)$ of the problem (2) must satisfy the following Hamiltonian-Jacobi-Bellman (HJB) equation:

$$0 = -\rho h - k + \alpha h' + \frac{\sigma^2}{2} h'', \tag{B.1}$$

where k is the cost of a unit of effort, assumed to be equal to 1. The general solution to the HJB equation (B.1) is

$$h(E) = Ae^{\beta_1 E} + Be^{\beta_2 E} - \frac{k}{\rho}, \tag{B.2}$$

where $\beta_{1,2}$ are the roots of the characteristic equation

$$\frac{\sigma^2}{2}\beta^2 + \alpha\beta - \rho = 0. \tag{B.3}$$

The two roots are

$$\beta_{1,2} = \frac{-\alpha \pm \sqrt{\alpha^2 + 2\rho\sigma^2}}{\sigma^2}. \tag{B.4}$$

It is worth noting that the term with the negative root β_1 is explosive and thus needs to be eliminated, hence $A = 0$. The HJB equation then becomes

$$h(E) = Be^{\beta_2 E} - \frac{k}{\rho}. \tag{B.5}$$

To determine the optimal threshold E^*, two conditions are used: the value-matching condition $h(E^*) = \mu p(E^*)$ and the smooth-pasting condition $h'(E^*) = \mu p'(E^*)$. From the smooth-pasting condition it follows that

$$Be^{\beta E^*} = \frac{\mu p'(E^*)}{\beta}, \tag{B.6}$$

where $\beta \equiv \beta_2$. Plugging it into the value-matching condition yields

$$\frac{\mu p'(E^*)}{\beta} - \frac{k}{\rho} = \mu p(E^*), \tag{B.7}$$

which after re-arranging the terms becomes

$$\frac{\rho}{\beta}p'(E^*) - \rho p(E^*) = \frac{k}{\mu}. \tag{B.8}$$

The limiting result in the case of $\rho \to 0$ follows from (B.8) after noting that

$$\begin{aligned}
\lim_{\rho\to 0} \frac{\rho}{\beta} &= \lim_{\rho\to 0} \frac{\rho\sigma^2}{-\alpha + \sqrt{\alpha^2 + 2\rho\sigma^2}} \\
&= \lim_{\rho\to 0} \frac{\sigma^2}{\frac{2\sigma^2}{2\sqrt{\alpha^2+2\rho\sigma^2}}} \text{ (l'Hopital's rule)} \\
&= \lim_{\rho\to 0} \sqrt{\alpha^2 + 2\rho\sigma^2} \\
&= \alpha.
\end{aligned}$$

Equation (B.8) then becomes

$$p'(E^*) = \frac{k}{\alpha\mu}. \tag{B.9}$$

Assuming that $p(E) = 1 - e^{-E}$ and noting that $p'(E) = e^{-E}$, one obtains from (B.9)

$$e^{-E^*} = \frac{k}{\alpha\mu} \tag{B.10}$$

or

$$E^* = \ln\alpha + \ln\frac{\mu}{k}. \tag{B.11}$$

Consider the average response time $\bar{\tau}^* = E^*/\alpha = \ln(\alpha\mu)/\alpha$. The marginal effect of ability on $\bar{\tau}^*$ is given by

$$\begin{aligned}
\tau^*_\alpha &= \frac{\alpha\frac{1}{\alpha\mu}\mu - \ln\alpha\mu}{\alpha^2} \\
&= \frac{1 - \ln(\alpha\mu)}{\alpha^2}.
\end{aligned}$$

This expression is positive if the optimal threshold is sufficiently low or when $\alpha\mu < e$. If the optimal threshold is high, or $\alpha\mu \geqslant e$, the effect of ability is negative.

C Additional Analysis (Online)

C.1 Monte Carlo Simulations

First, I consider the validity of the estimation procedure using a simulation exercise. Simulations are conducted for the five different parameter vectors listed in Table C.1. Parameter values are drawn from a uniform distribution on $[1, 10]$. For each parameter vector, the data (100 observations) on outcomes and response times is simulated using the theoretical model 1000 times. The resulting distributions of the parameter estimates are presented on Figure C.1. The vertical lines indicate the true values of the parameters. As expected, the distributions of parameter estimates are well-centered around the true values.

Table C.1: True Parameter Values and Mean Estimates

	α	μ	σ
Panel A. *True Values*			
1	3.39	4.35	6.16
2	2.66	7.32	6.16
3	2.51	8.27	4.46
4	6.27	1.08	3.64
5	2.80	7.17	9.25
Panel B. *Mean Estimates*			
1	3.65	4.87	6.31
2	2.88	8.53	6.34
3	2.67	9.68	4.60
4	6.49	1.12	3.69
5	3.20	8.40	9.52

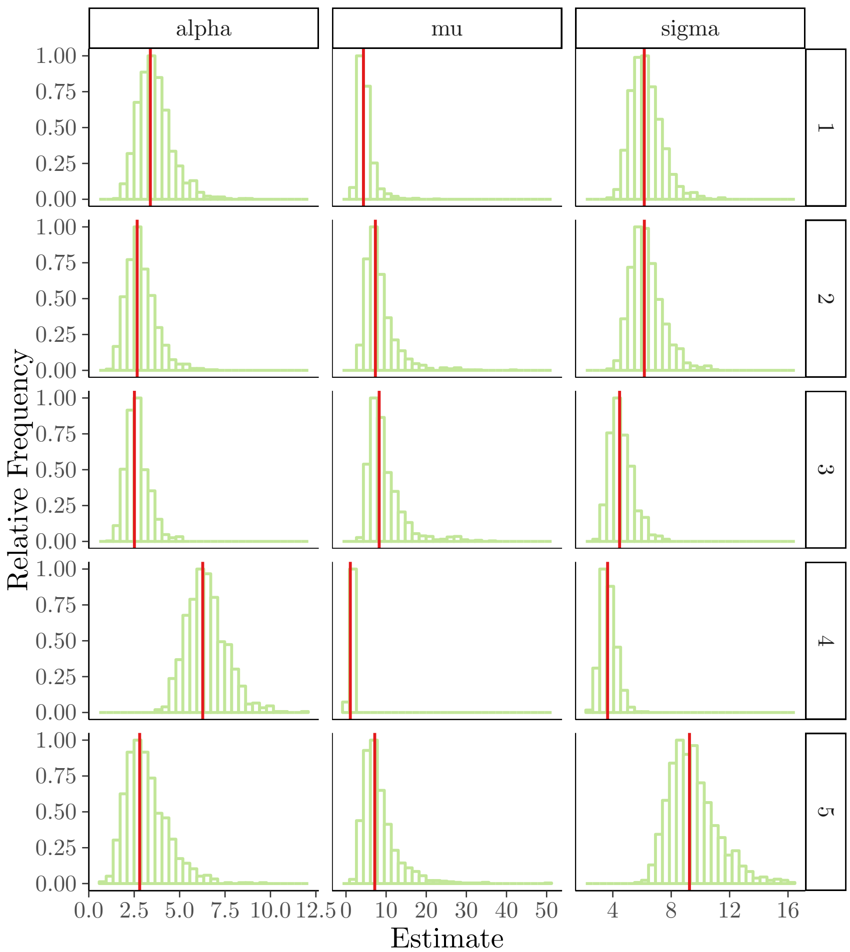
Notes: Panel A shows the five different true parameter vectors drawn from a uniform distribution. Panel B shows the corresponding mean estimates of parameters from the simulated data.

Second, I consider the consequences of using performance as a proxy for ability. I assume that the true data generating process for some outcome of interest y is

$$y_i = \beta_0 + \beta_1 \alpha_i + \beta_2 \mu_i + \epsilon_i, \tag{C.12}$$

where i indexes a subject, α_i is ability of a subject i, μ_i is motivation of a subject i, and ϵ_i is an error term. I further assume that a researcher estimates a model in which only performance p_i is

Figure C.1: Monte Carlo Simulations



Note: The figure shows the histograms of the distributions of parameter estimates $(\hat{\alpha}, \hat{\mu}, \hat{\sigma})$ for five different values of the true vector of parameters. The vertical lines correspond to the true value of parameters.

observed, but not ability and motivation:

$$y_i = \gamma_0 + \gamma_1 p_i + \eta_i. \tag{C.13}$$

I then study how the true values of β_1 and β_2 affect the estimates of γ_1. In the simulation, the values of α_i are drawn from a truncated normal distribution with the mean 7 and the standard deviation 1 and the lower bound of 1. The values of the logarithm of μ_i are drawn from a truncated normal distribution with the mean 1, the standard deviation 1, bounded between 0 and 3. These distributional assumption are made to roughly match the observed distributions of ability and motivation in the experiment. Performance as a function of ability and motivation is then computed using the model as $p_i = 1-(\alpha_i\mu_i)^{-1}$. The noise term ϵ_i is drawn from a normal distribution with the mean 0 and the standard deviation 3. The generated data consists of 1000 observations. Table C.2 shows the true values of β_1 and β_2 and the corresponding estimates of γ and its standard error. The table makes it clear that issues arise whenever $\text{sgn}(\beta_1\beta_2) \neq 1$: the sign of the estimated coefficient on performance does not coincide with the sign of the ability coefficient in the true model, which would lead to wrong conclusions about the effect of ability on the outcome.

Table C.2: True Parameter Values and Estimates

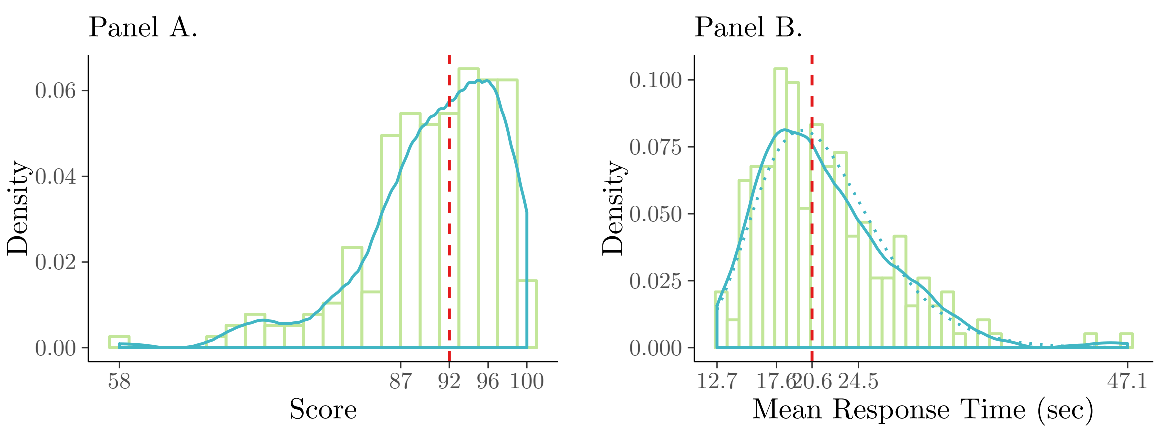
β_1	β_2	$\hat{\gamma}_1$	$\hat{\gamma}_{1se}$
0	1	72.52	3.30
0	−1	−71.72	3.43
1	1	78.81	3.29
1	−1	−65.43	3.69
1	0	6.69	2.79
−1	1	66.23	3.55
−1	−1	−78.01	3.39
−1	0	−5.89	2.76

Notes: The table reports the coefficients on ability (β_1) and motivation (β_2) in the true model, and the corresponding estimates of the coefficient on performance ($\hat{\gamma}_1$) and its standard error from the estimated model.

C.2 Summary Statistics of the DST

Figure C.2 (Panel A) shows the distribution of the raw scores from the DST. The subjects perform very well on the DST with 75% of the subjects scoring 87 and above. Figure C.2 (Panel B) shows the distribution of the mean response times, averaged across all rounds for each subject. The distribution is tightly concentrated around the median of 20.6 seconds and has a relatively fat right tail. The actual distribution (solid line) matches closely the reference inverse Gaussian distribution (dotted line) with the parameters matching the sample moments. In fact, one cannot reject the null hypothesis that the sample of mean response times comes from the inverse Gaussian distribution (Kolmogorov-Smirnov test $p-$value $= 0.393$).

Figure C.2: Distributions of Scores and Mean Response Times on DST

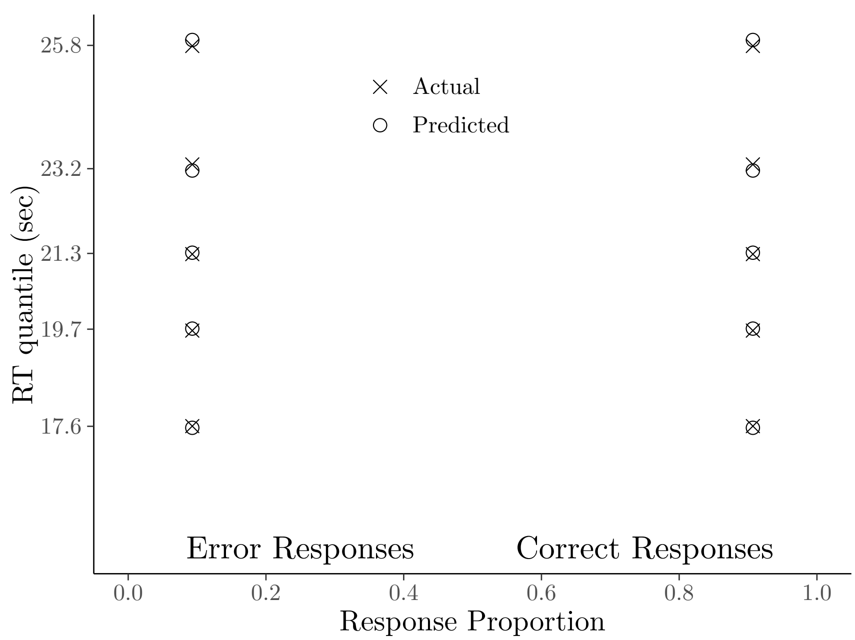


Note: Panel A shows the distribution of the scores on the DST. Panel B shows the distribution of the mean response times on the DST. The smooth solid line is the kernel density estimate, the vertical bars are the histogram, and the vertical dashed line is the sample median. The breaks on the horizontal axis correspond to the quintiles of the distribution. On Panel B, the dotted line is the reference density of an inverse Gaussian distribution with the parameters matching the sample moments.

C.3 Additional Estimates

Figure C.3 shows the quantile probability plot adopted from Ratcliff and McKoon (2007).[23] The data are pooled across all subjects. The graph shows the proportion of correct and incorrect response (horizontal axis) against the quantiles of the distribution of response times (vertical axis). The quantiles of response times are $0.1, 0.3, 0.5, 0.7$, and 0.9. The circles represent the predicted values from the estimated model, and the crosses represent the actual values. As is clear from the picture, the model does a good job at jointly predicting outcomes and response times in the pooled data.

Figure C.3: Quantile Probability Graph for Pooled Data

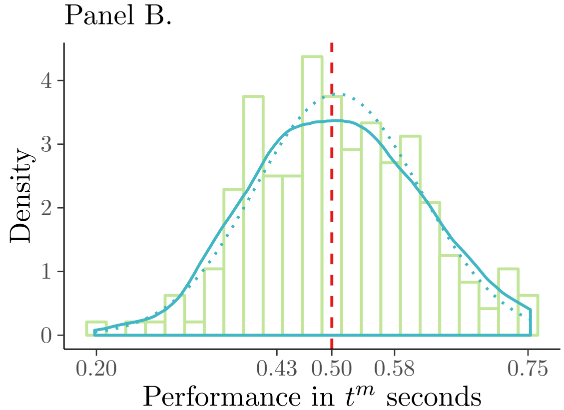


Note: The figure shows the quantile probability plot from the pooled data (averaged across all subjects). Points on the right (left) correspond to success (failure) rates. Circles represent the predicted values from the estimated model, crosses represent the observed data.

A useful alternative way of looking at the ability differences across subjects is to convert the ability estimates into performance. To translate the estimates of ability into performance, I compute the probability of success at the average accumulated effective effort in t^m seconds, where t^m ($\approx$ 5.71 seconds) is calibrated such that it is the time for a person with median ability to reach a 0.5

[23] Since only one treatment was used and there was no variation in difficulty, it is not possible to draw the complete lines as in Ratcliff and McKoon (2007).

Figure C.4: Distributions of Raw and Transformed Ability



Note: The figure shows the distribution of the probability of success in t^m seconds. The smooth solid line is the kernel density estimate, the vertical bars are the histogram, the dotted line is the reference density of a normal distribution with the parameters matching the sample moments, and the vertical dashed line is the sample median. The breaks on the horizontal axis correspond to the quintiles of the distribution.

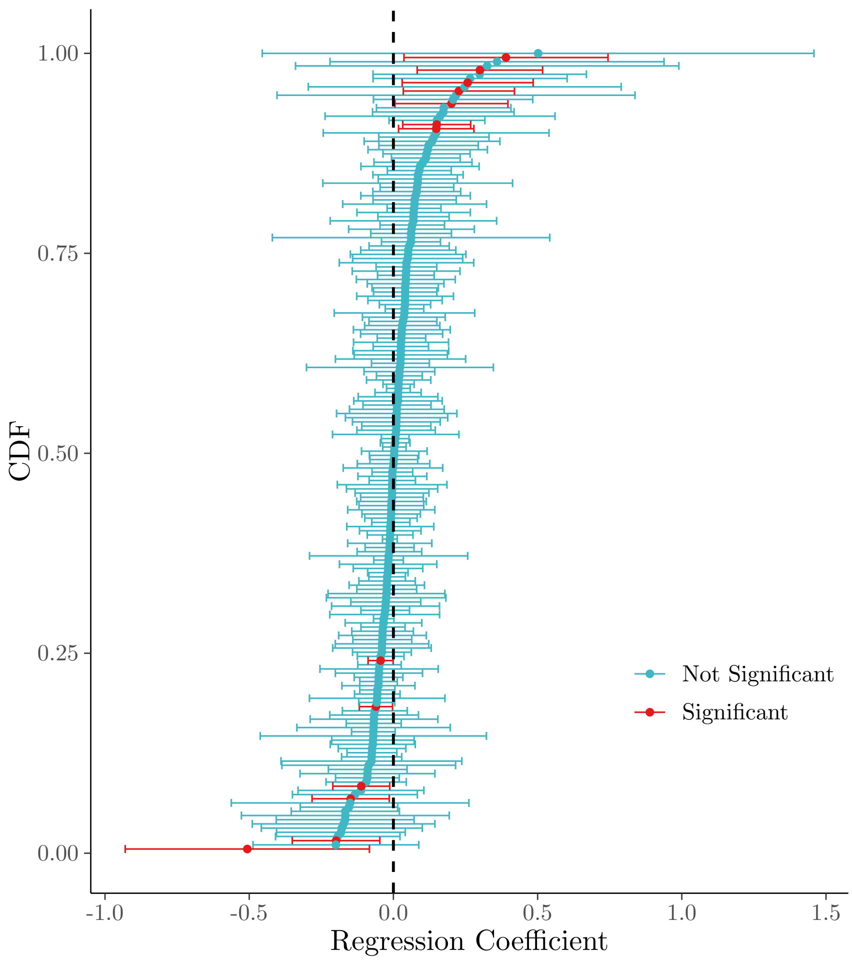
probability of success.[24] Due to variation in ability, subjects will have different levels of accumulated effective effort in t^m seconds, which will then translate into different probabilities of success. Figure C.4 shows the distribution of the resulting performance. This distribution is more symmetric than the distribution of raw ability estimates. In fact, one cannot reject the null hypothesis of the distribution of performance in t^m seconds being normal (Shapiro-Wilk test p-value $= 0.574$). A subject at the 75th percentile would have a 1.4 times higher performance in t^m seconds than a subject at the 25th percentile. A subject with the highest ability would have a 1.5 times higher performance than a median subject and a 3.8 times higher performance than a subject with the lowest ability.

On Figure C.5, I address the point about whether a success on a trial of the DST is significantly correlated with the response time in that trial. I estimate a logistic regression of the outcome of a trial on the response time, for each subject individually. I then present the estimated regression coefficients on the response time graphically by ordering them from lowest to highest. The graph shows the point estimates and the 95% confidence intervals around them. As can be seen from the graph, in the overwhelming majority of the cases, the null hypothesis about no significant

[24] The resulting performance is counterfactual in the sense that it is generated using the model and ability estimates from a hypothetical scenario. This counterfactual performance is of course different from the observed performance on the test, which is a conventional measure of ability. The benefit of this transformation is that it converts ability into familiar performance terms. Both the counterfactual performance and raw ability estimates can be viewed as the same quantity (ability) expressed in different units.

effect of the response time on the success cannot be rejected. The effect is significant only for 13 subject, which represents 7% of the sample, and even among these subjects, there is no systematic relationship between response times and outcomes.

Figure C.5: Response Times and Success



Note: The graph shows the regression coefficients from individual-level logistic regressions of outcomes on response times. Each point on the graph represents an individual-level estimate, and the points are ordered from lowest to highest. The error bars show 95% confidence intervals. Significance is determined based on a 0.05 cutoff for the p-value.