Online Appendix I to "Economic Evaluation under Ambiguity and Structural Uncertainties" [Supplemental Information]

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A. Omitted Proofs

Proof of Theorem 1: Suppose that the alternative is the dominant strategy under CBA in all possible states of the world. The result is straightforward to show for the alternative: $g(\Delta Q) - \Delta C > 0 \;\forall (g, \Delta Q, \Delta C) \in (\mathbb{G}, \mathbb{Q}, \mathbb{C}) \Leftrightarrow g > \frac{\Delta C}{\Delta Q} \;\forall (g, \Delta Q, \Delta C) \in (\mathbb{G}, \mathbb{Q}, \mathbb{C})$ because $\underline{Q} > 0$. The proof for the status quo follows by reversing the inequality. ■

Proof of Theorem 2: The welfare under the status quo is $W_s = 0$ for CBA and $V_s = 0$ for CEA. The welfare under the alternative is $W_a = g(\Delta Q) - \Delta C$ under CBA and $V_a = g - \frac{\Delta C}{\Delta Q}$ under CEA.

i. Under CBA, a Bayesian planner puts a prior π on g and selects the alternative if and only if $E_\pi[W_a] > E_\pi[W_s]$. Consequently, the Bayesian selects the alternative if and only if $E_\pi[g] > \frac{\Delta C}{\Delta Q}$. Under CEA, the Bayesian planner uses the same prior π on g and selects the alternative if and only if $E_\pi[V_a] > E_\pi[V_s]$. Therefore, the Bayesian selects the alternative if and only if $E_\pi[g] - \frac{\Delta C}{\Delta Q} > 0$. These rules are exactly the same.

ii. Under CBA, a MM planner selects the alternative if and only if $\min_{g \in \mathbb{G}} W_a > \min_{g \in \mathbb{G}} W_s$. Consequently, the MM planner selects the alternative if and only if $g_L \Delta Q - \Delta C > 0$ or equivalently $g_L > \frac{\Delta C}{\Delta Q}$. Under CEA, the MM planner selects the alternative if and only if $\min_{g \in \mathbb{G}} V_a > \min_{g \in \mathbb{G}} V_s$. Therefore, the MM planner selects the alternative if and only if $g_L - \frac{\Delta C}{\Delta Q} > 0$. These rules are exactly the same.

iii. Under CBA, a MMR planner selects the alternative if and only if it has lower maximum regret than the status quo. The regret of the status quo is $R_s = \max\{W_a, W_s\} - W_s$ and the regret of the alternative is $R_a = \max\{W_a, W_s\} - W_a$. It follows that the maximum regret of the status quo is $g_H(\Delta Q) - \Delta C$ and that the maximum regret of the alternative is $-(g_L(\Delta Q) - \Delta C)$. Consequently, the planner selects the alternative if and only if $-(g_L(\Delta Q) - \Delta C) < g_H(\Delta Q) - \Delta C$ or $\frac{\Delta C}{\Delta Q} < \frac{g_L + g_H}{2}$. Under CEA, the regret of the status quo is $R_s = \max\{V_a, V_s\} - V_s$ and the regret of the alternative is $R_a = \max\{V_a, V_s\} - V_a$. It follows that the maximum regret of the status quo is $g_H - \frac{\Delta C}{\Delta Q}$ and that the maximum regret of the alternative is $-\left(g_L - \frac{\Delta C}{\Delta Q}\right)$. Consequently, the planner selects the alternative if and only if $-\left(g_L - \frac{\Delta C}{\Delta Q}\right) < g_H - \frac{\Delta C}{\Delta Q}$ or $\frac{\Delta C}{\Delta Q} < \frac{g_L + g_H}{2}$. These rules are exactly the same. ■

Proof of Theorem 3: The welfare under the status quo is $W_s = 0$ for CBA and $V_s = 0$ for CEA. The welfare under the alternative is $W_a = g(\Delta Q) - \Delta C$ under CBA and $V_a = g - \frac{\Delta C}{\Delta Q}$ under CEA. Under CBA, a Bayesian planner will place a distribution π on the state space $(\mathbb{G}, \mathbb{Q}, \mathbb{C})$ and select the alternative if and only if $E_\pi[W_a] > E_\pi[W_s]$. Consequently, the planner will select the alternative if and only if $E_\pi[g(\Delta Q) - \Delta C] = E_\pi[g\Delta Q] > E_\pi[\Delta C]$. Under CEA, a Bayesian planner selects the alternative if and only if $E_\pi[V_a] > E_\pi[V_s]$. Consequently, the planner will select the alternative if and only if $E_\pi[g] > E_\pi\left[\frac{\Delta C}{\Delta Q}\right]$. These are not generally equivalent rules.

For (a), if the marginal distributions on $\mathbb{G}$ and $\mathbb{Q}$ are independent under π, then $E_\pi[g(\Delta Q) - \Delta C] = E_\pi[g]E_\pi[\Delta Q] - E_\pi[\Delta C]$ and so the planner using CBA selects the alternative if and only if $E_\pi[g] > E_\pi\left[\frac{\Delta C}{\Delta Q}\right]$. For (b), if the marginal distributions on $\mathbb{Q}$ and $\mathbb{C}$ are independent under π, then $E_\pi\left[\frac{\Delta C}{\Delta Q}\right] = E_\pi[\Delta C]E_\pi\left[\frac{1}{\Delta Q}\right]$ and so the planner using CEA selects the alternative if and only if $E_\pi[g] > E_\pi[\Delta C]E_\pi\left[\frac{1}{\Delta Q}\right]$. For (c), if the Bayesian planner selects the alternative under CEA, $E_\pi[g] > E_\pi[\Delta C]E_\pi\left[\frac{1}{\Delta Q}\right] > \frac{E_\pi[\Delta C]}{E_\pi[\Delta Q]}$ by Jensen's inequality. Consequently, the Bayesian planner also selects the alternative under CBA. ■

Proof of Theorem 4: The welfare under the status quo is $W_s = 0$ for CBA and $V_s = 0$ for CEA. The welfare under the alternative is $W_a = g(\Delta Q) - \Delta C$ under CBA and $V_a = g - \frac{\Delta C}{\Delta Q}$ under CEA. Under CBA, a MM planner selects the alternative if and only if we have $\min_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})} W_a > \min_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})} W_s$. Thus, the planner selects the alternative if $\min_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})}[g(\Delta Q) - \Delta C] > 0$. When $(g, \underline{Q}, \overline{C}) \in (\mathbb{G}, \mathbb{Q}, \mathbb{C})$, this reduces to $g_L > \frac{\overline{C}}{\underline{Q}}$.

Under CEA, a MM planner selects the alternative if and only if $\min_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})} V_a > \min_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})} V_s$. Thus, the planner selects the alternative if $\min_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})}[g - \frac{\Delta C}{\Delta Q}] > 0$. If the expression in either (i) or (ii) holds then the alternative is a dominant strategy. Consequently, if there is no dominant strategy, a MM planner selects the status quo and a MM planner selects the alternative if and only if it is a dominant strategy. ■

Proof of Theorem 5: Under CBA, the maximum regret from selecting the status quo is

$$\max_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})} \{g\Delta Q - \Delta C\}$$

The maximum regret from selecting the alternative is

$$-\min_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})} \{g\Delta Q - \Delta C\}$$

The planner therefore chooses the alternative if and only if $-\min_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})}\{g\Delta Q - \Delta C\} < \max_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})}\{g\Delta Q - \Delta C\}$ or equivalently

$$\max_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})} \{g\Delta Q - \Delta C\} + \min_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})} \{g\Delta Q - \Delta C\} > 0$$

If $\{(g_H, \overline{Q}, \underline{C}), (g_L, \underline{Q}, \overline{C})\} \in (\mathbb{G},\mathbb{Q},\mathbb{C})$ then the maximum regret from selecting the status quo is $g_H\overline{Q} - \underline{C}$ and the maximum regret from choosing the alternative is $-[g_L\underline{Q} - \overline{C}]$. Therefore, the planner chooses the alternative if and only if $-g_L\underline{Q} + \overline{C} < g_H\overline{Q} - \underline{C}$ or equivalently $(\overline{C} + \underline{C}) < (g_H\overline{Q} + g_L\underline{Q})$.

Under CEA, $\max_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})}\left[g - \frac{\Delta C}{\Delta Q}\right]$ is the maximum regret from selecting the status quo. The maximum regret from choosing the alternative is $\max_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})}\left[\frac{\Delta C}{\Delta Q} - g\right]$. The planner therefore selects the alternative if and only if

$$\max_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})}\left[\frac{\Delta C}{\Delta Q} - g\right] < \max_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})}\left[g - \frac{\Delta C}{\Delta Q}\right]$$

or equivalently $\max_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})}\left[g-\frac{\Delta C}{\Delta Q}\right]+\min_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})}\left[g-\frac{\Delta C}{\Delta Q}\right]>0$.

If $\{(g_H,\overline{Q},\underline{C}),(g_L,\underline{Q},\overline{C})\}\in(\mathbb{G},\mathbb{Q},\mathbb{C})$ then the maximum regret from selecting the status quo is $g_H-\frac{\underline{C}}{\overline{Q}}$ and the maximum regret from the alternative is $\frac{\overline{C}}{\underline{Q}}-g_L$. The planner therefore chooses the alternative if and only if $\frac{\overline{C}}{\underline{Q}}-g_L<g_H-\frac{\underline{C}}{\overline{Q}}$ or equivalently $\frac{\overline{C}\overline{Q}+\underline{C}\underline{Q}}{2\underline{Q}\overline{Q}}<\frac{g_L+g_H}{2}$. ∎

Proof of Theorem 6: *Statement (i).* By Theorem 5, a MMR planner using CEA selects the alternative if and only if $\max_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})}\left[g-\frac{\Delta C}{\Delta Q}\right]+\min_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})}\left[g-\frac{\Delta C}{\Delta Q}\right]>0$. Then because $(\mathbb{G},\mathbb{Q},\mathbb{C})\equiv\mathbb{G}\times(\mathbb{Q},\mathbb{C})$ this expression becomes $\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[g_H-\frac{\Delta C}{\Delta Q}\right]+\min_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[g_L-\frac{\Delta C}{\Delta Q}\right]>0$ and we can rewrite the statement $\exists(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C}):\Delta Q=\overline{Q}\ \wedge\ (g,\Delta Q,\Delta C)\in\operatorname{argmax}_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})}\left[g-\frac{\Delta C}{\Delta Q}\right]$ as:

$$\exists(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C}):\Delta Q=\overline{Q}\ \wedge\ (\Delta Q,\Delta C)\in\operatorname*{argmax}_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[g_H-\frac{\Delta C}{\Delta Q}\right]$$

Then using this fact, $\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[g_H-\frac{\Delta C}{\Delta Q}\right]+\min_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[g_L-\frac{\Delta C}{\Delta Q}\right]>0$ can be rewritten: $\max_{\Delta C\in\mathbb{C}:(\overline{Q},\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[g_H-\frac{\Delta C}{\overline{Q}}\right]+\min_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[g_L-\frac{\Delta C}{\Delta Q}\right]>0$. Multiplying all terms by $\overline{Q}>0$ then yields the following condition which must hold:

$$\max_{\Delta C\in\mathbb{C}:(\overline{Q},\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[g_H\overline{Q}-\Delta C\right]+\overline{Q}\left(\min_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[g_L-\frac{\Delta C}{\Delta Q}\right]\right)>0$$

We wish to show that this implies an MMR planner selects the alternative under CBA, which occurs if and only if $\max_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})}\{g\Delta Q-\Delta C\}+\min_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})}\{g\Delta Q-\Delta C\}>0$ by Theorem 5. Then using the facts that $(\mathbb{G},\mathbb{Q},\mathbb{C})\equiv\mathbb{G}\times(\mathbb{Q},\mathbb{C})$ and $\Delta Q>0\ \forall\Delta Q\in\mathbb{Q}$ we can rewrite the left-hand side of this condition as $\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\{g_H\Delta Q-\Delta C\}+\min_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\{g_L\Delta Q-\Delta C\}$. It remains to show that:

$$\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\{g_H\Delta Q-\Delta C\}+\min_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\{g_L\Delta Q-\Delta C\}$$
$$\geq\max_{\Delta C\in\mathbb{C}:(\overline{Q},\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[g_H\overline{Q}-\Delta C\right]+\overline{Q}\left(\min_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[g_L-\frac{\Delta C}{\Delta Q}\right]\right)$$

to establish that $\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\{g_H\Delta Q-\Delta C\}+\min_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\{g_L\Delta Q-\Delta C\}>0$. This holds if both: (a) $\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\{g_H\Delta Q-\Delta C\}\geq\max_{\Delta C\in\mathbb{C}:(\overline{Q},\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[g_H\overline{Q}-\Delta C\right]$ and (b) $\min_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\{g_L\Delta Q-\Delta C\}\geq\overline{Q}\left(\min_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[g_L-\frac{\Delta C}{\Delta Q}\right]\right)$.

Using the fact that $\Delta Q>0\ \forall\Delta Q\in\mathbb{Q}$, we can rewrite (b) as:

$$-\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\{\Delta C-g_L\Delta Q\}\geq\overline{Q}\left(-\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[\frac{\Delta C}{\Delta Q}-g_L\right]\right)$$
$$\overline{Q}\left(\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[\frac{\Delta C}{\Delta Q}-g_L\right]\right)\geq\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\{\Delta C-g_L\Delta Q\}$$
$$\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[\frac{\Delta C}{\Delta Q}-g_L\right]\geq\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[\frac{\Delta Q}{\overline{Q}}\left\{\frac{\Delta C}{\Delta Q}-g_L\right\}\right]$$

Next, note that $\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[\frac{\Delta C}{\Delta Q}-g_L\right]\geq 0$. Suppose not. Then:

$$0<\min_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[g_L-\frac{\Delta C}{\Delta Q}\right]=\min_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q},\mathbb{C})}\left[g-\frac{\Delta C}{\Delta Q}\right]$$

because $(\mathbb{G},\mathbb{Q},\mathbb{C})\equiv\mathbb{G}\times(\mathbb{Q},\mathbb{C})$. Then approving the alternative is a dominant strategy, and we have a contradiction. Consider now that $0<(\Delta Q/\overline{Q})\leq 1\ \forall\Delta Q\in\mathbb{Q}$ by construction and $\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[\frac{\Delta C}{\Delta Q}-g_L\right]\geq 0$. Consequently: $\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[\frac{\Delta C}{\Delta Q}-g_L\right]=\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[(1)\left\{\frac{\Delta C}{\Delta Q}-g_L\right\}\right]\geq\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[\frac{\Delta Q}{\overline{Q}}\left\{\frac{\Delta C}{\Delta Q}-g_L\right\}\right]$ and condition (b) holds.

For condition (a), the result is almost immediate: $\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\{g_H\Delta Q-\Delta C\}=\max\left(\max_{\Delta C\in\mathbb{C}:(\overline{Q},\Delta C)\in(\mathbb{Q},\mathbb{C})}\{g_H\overline{Q}-\Delta C\},\ \max_{(\Delta Q,\Delta C):\{\Delta Q\neq\overline{Q}\ \wedge\ (\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})\}}\{g_H\Delta Q-\Delta C\}\right)\geq\max_{\Delta C\in\mathbb{C}:(\overline{Q},\Delta C)\in(\mathbb{Q},\mathbb{C})}\left[g_H\overline{Q}-\Delta C\right]$.

Statement (ii). By Corollary B2, if an MMR planner under CEA will select the alternative, they would have selected in the context of Theorem B2 where $g=(g_L+g_H)/2$ under CEA. Then by Corollary O.B1 in the Online Appendix, the MMR planner using CBA in the context of Theorem B2 where $g=(g_L+g_H)/2$ would also have selected the alternative. The result follows by applying Corollary B3. ■

Proof of Theorem 7: For (a), consider the statement of Theorem 3(i) and note that

$E_\pi[g\Delta Q] > E_\pi[\Delta C]$ becomes $E_\pi[g] > \frac{E_\pi[\Delta C]}{\Delta Q}$. Further, the expression in 3(ii) reduces to this same inequality. Consequently, the Bayesian planner selects the alternative if and only if $E_\pi[g] > \frac{E_\pi[\Delta C]}{\Delta Q}$ under both CBA and CEA. For (b), CBA and CEA solutions are generally equivalent by Theorem 4. For (c), consider the statement of Theorem 5(ii) and note that $\max_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q}=\{\Delta Q\},\mathbb{C})}\left[g - \frac{\Delta C}{\Delta Q}\right] + \min_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q}=\{\Delta Q\},\mathbb{C})}\left[g - \frac{\Delta C}{\Delta Q}\right] > 0$ is satisfied if and only if $\max_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q}=\{\Delta Q\},\mathbb{C})}\{g\Delta Q - \Delta C\} + \min_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q}=\{\Delta Q\},\mathbb{C})}\{g\Delta Q - \Delta C\} > 0$ is satisfied (obtained through multiplication of both sides of the inequality by $\Delta Q > 0$). Thus, the MMR planner selects the alternative if and only if $\max_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q}=\{\Delta Q\},\mathbb{C})}\{g\Delta Q-\Delta C\}+\min_{(g,\Delta Q,\Delta C)\in(\mathbb{G},\mathbb{Q}=\{\Delta Q\},\mathbb{C})}\{g\Delta Q - \Delta C\} > 0$ under both CBA and CEA. ∎

B. Additional Results

Theorem B1 *If there is no dominant strategy, $\mathbb{G} = \{g\}$, $\Delta Q \in \mathbb{Q}$, and $\Delta C \in \mathbb{C}$, then the solutions used by a Bayesian with distribution π on $(\mathbb{Q}, \mathbb{C})$ planner are not generally equivalent under CBA and CEA:*

i. Under CBA, a Bayesian planner selects the alternative if and only if $g > \frac{E_\pi[C]}{E_\pi[Q]}$.

ii. Under CEA, a Bayesian planner selects the alternative if and only if $g > E_\pi\left[\frac{\Delta C}{\Delta Q}\right]$.

If the marginal distributions on $\mathbb{Q}$ and $\mathbb{C}$ are independent under π, then under CEA a Bayesian planner selects the alternative if and only if $g > E_\pi[\Delta C]E_\pi\left[\frac{1}{\Delta Q}\right]$. Using CBA, the solution still takes the form in (i).

Proof: The welfare under the status quo is $W_s = 0$ for CBA and $V_s = 0$ for CEA. The welfare under the alternative is $W_a = g(\Delta Q) - \Delta C$ under CBA and $V_a = g - \frac{\Delta C}{\Delta Q}$ under CEA. Under CBA, a Bayesian planner will place a distribution π on the state space $(\mathbb{Q}, \mathbb{C})$ and select the alternative if and only if $E_\pi[W_a] > E_\pi[W_s]$. Consequently, the planner will select the alternative if and only if $E_\pi[g(\Delta Q) - \Delta C] = gE_\pi[Q] - E_\pi[C] > 0$ or equivalently $g > \frac{E_\pi[C]}{E_\pi[Q]}$. Under CEA, a Bayesian planner selects the alternative if and only if $E_\pi[V_a] > E_\pi[V_s]$.

Consequently, the planner will select the alternative if and only if $g > E_\pi\left[\frac{\Delta C}{\Delta Q}\right]$. These are not generally equivalent rules. If the marginal distributions on $\mathbb{Q}$ and $\mathbb{C}$ are independent under π, then $E_\pi\left[\frac{\Delta C}{\Delta Q}\right] = E_\pi[\Delta C]E_\pi\left[\frac{1}{\Delta Q}\right] > \frac{E_\pi[C]}{E_\pi[Q]}$ by Jensen's inequality. ■

Corollary B1 *If there is no dominant strategy,* $\mathbb{G} = \{g\}$, $\Delta Q \in \mathbb{Q}$, *and* $\Delta C \in \mathbb{C}$, *and the marginal distributions on* $\mathbb{Q}$ *and* $\mathbb{C}$ *are independent under* π, *then a Bayesian planner with a distribution* π *on* $(\mathbb{Q}, \mathbb{C})$ *selects the alternative under CBA if they select the alternative under CEA.*

Proof: If the Bayesian DM selects the alternative under CEA, $g > E_\pi[\Delta C]E_\pi\left[\frac{1}{\Delta Q}\right] > \frac{E_\pi[C]}{E_\pi[Q]}$ by Jensen's inequality. The Bayesian DM thus also selects the alternative under CBA. ■

Theorem B2 *If there is no dominant strategy,* $\mathbb{G} = \{g\}$, $\Delta Q \in \mathbb{Q}$, *and* $\Delta C \in \mathbb{C}$, *then the solutions used by a MMR planner are not generally equivalent under CBA and CEA:*

i. Under CBA, a MMR planner selects the alternative if and only if

$$\max_{(\Delta Q, \Delta C) \in (\mathbb{Q}, \mathbb{C})} \{g\Delta Q - \Delta C\} + \min_{(\Delta Q, \Delta C) \in (\mathbb{Q}, \mathbb{C})} \{g\Delta Q - \Delta C\} > 0$$

ii. Under CEA, a MMR planner selects the alternative if and only if

$$\frac{\min_{(\Delta Q, \Delta C) \in (\mathbb{Q}, \mathbb{C})} \frac{\Delta C}{\Delta Q} + \max_{(\Delta Q, \Delta C) \in (\mathbb{Q}, \mathbb{C})} \frac{\Delta C}{\Delta Q}}{2} < g$$

If the state space is "rectangular" such that $\{(\overline{Q}, \underline{C}), (\underline{Q}, \overline{C})\} \in (\mathbb{Q}, \mathbb{C})$ *then:*

a. Under CBA, a MMR planner selects the alternative if and only if $\frac{\underline{C} + \overline{C}}{\overline{Q} + \underline{Q}} < g$

b. Under CEA, a MMR planner selects the alternative if and only if $\frac{\overline{C}\,\overline{Q} + \underline{C}\,\underline{Q}}{2\underline{Q}\,\overline{Q}} < g$

Proof: Under CBA, $\max_{(\Delta Q, \Delta C) \in (\mathbb{Q}, \mathbb{C})}\{g\Delta Q - \Delta C\}$ is the maximum regret from selecting the status quo. The maximum regret from selecting the alternative is $-\min_{(\Delta Q, \Delta C) \in (\mathbb{Q}, \mathbb{C})}\{g\Delta Q -$

$\Delta C\}$. The planner therefore chooses the alternative if and only if $-\min_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\{g\Delta Q - \Delta C\} < \max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\{g\Delta Q - \Delta C\}$ or equivalently:

$$\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\{g\Delta Q - \Delta C\} + \min_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\{g\Delta Q - \Delta C\} > 0$$

If $\{(\overline{Q},\underline{C}),(\underline{Q},\overline{C})\} \in (\mathbb{Q},\mathbb{C})$ then the maximum regret from selecting the status quo is $g\overline{Q}-\underline{C}$ and the maximum regret from choosing the alternative is $-[g\underline{Q}-\overline{C}]$. Therefore, the planner chooses the alternative if and only if $-g\underline{Q}+\overline{C} < g\overline{Q}-\underline{C}$ or $\frac{\underline{C}+\overline{C}}{\overline{Q}+\underline{Q}} < g$. Under CEA, the maximum regret from selecting the status quo is $g - \min_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\frac{\Delta C}{\Delta Q}$. The maximum regret from choosing the alternative is $\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\frac{\Delta C}{\Delta Q} - g$. The planner therefore selects the alternative if and only if $\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\frac{\Delta C}{\Delta Q} - g < g - \min_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\frac{\Delta C}{\Delta Q}$ or equivalently $\frac{\min_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\frac{\Delta C}{\Delta Q}+\max_{(\Delta Q,\Delta C)\in(\mathbb{Q},\mathbb{C})}\frac{\Delta C}{\Delta Q}}{2} < g$. If $\{(\overline{Q},\underline{C}),(\underline{Q},\overline{C})\} \in (\mathbb{Q},\mathbb{C})$ then the maximum regret from selecting the status quo is $g - \frac{\underline{C}}{\overline{Q}}$ and the maximum regret from the alternative is $\frac{\overline{C}}{\underline{Q}} - g$. The DM therefore chooses the alternative if and only if $\frac{\overline{C}}{\underline{Q}} - g < g - \frac{\underline{C}}{\overline{Q}}$ or $\frac{\overline{C}\overline{Q}+\underline{C}\underline{Q}}{2\underline{Q}\overline{Q}} < g$. ■

Corollary B2 *If there is no dominant strategy,* $g \in \mathbb{G}$, $\Delta Q \in \mathbb{Q}$, *and* $\Delta C \in \mathbb{C}$ *and the state space is "rectangular" as defined in the statement of Theorem 5, then the solution used by a MMR planner under CEA is equivalent to the solution used by an MMR planner under CEA when there is no dominant strategy,* $\mathbb{G} = \left\{\frac{g_L+g_H}{2}\right\}$, $\Delta Q \in \mathbb{Q}$, $\Delta C \in \mathbb{C}$, *and the state space is "rectangular" as defined in the statement of Theorem B2.*

Proof: Follows from Theorems B2(b) and 5(b). ■

Lemma B1 *Part (a) of Theorem 5 can be rewritten as follows: Under CBA, a MMR planner selects the alternative if and only if* $\frac{\overline{C}+\underline{C}}{\overline{Q}+\underline{Q}} < \frac{g_L+g_H}{2} + \phi$, *where* $\phi \equiv \frac{g_H-g_L}{2}\left(\frac{\overline{Q}-\underline{Q}}{\overline{Q}+\underline{Q}}\right) \geq 0$

Proof of Lemma B1: Expand and then simplify the RHS of the inequality:

$$\begin{aligned} g_H\overline{Q} + g_L\underline{Q} &= \frac{g_H}{2}\overline{Q} + \frac{g_L}{2}\underline{Q} + \frac{g_H}{2}\overline{Q} + \frac{g_L}{2}\underline{Q} + \frac{g_L}{2}\overline{Q} + \frac{g_H}{2}\underline{Q} - \frac{g_L}{2}\overline{Q} - \frac{g_H}{2}\underline{Q} \\ &= \frac{g_H+g_L}{2}(\overline{Q}+\underline{Q}) + \frac{g_H}{2}\overline{Q} + \frac{g_L}{2}\underline{Q} - \frac{g_L}{2}\overline{Q} - \frac{g_H}{2}\underline{Q} \end{aligned}$$

$$= \frac{g_H + g_L}{2}(\overline{Q} + \underline{Q}) + \frac{g_H - g_L}{2}\overline{Q} + \frac{g_L - g_H}{2}\underline{Q}$$

$$= \frac{g_H + g_L}{2}(\overline{Q} + \underline{Q}) + \frac{g_H - g_L}{2}(\overline{Q} - \underline{Q})$$

Dividing both sides by $(\overline{Q} + \underline{Q})$, the inequality can be rewritten as in the statement of the Lemma. The fact that $\phi \geq 0$ follows from $g_H \geq g_L$, $\overline{Q} \geq \underline{Q}$, and $\underline{Q} > 0$. ■

Corollary B3 *If there is no dominant strategy, $g \in \mathbb{G}$, $\Delta Q \in \mathbb{Q}$, and $\Delta C \in \mathbb{C}$, then an MMR planner using CBA will select the alternative if an MMR planner under CBA selects the alternative when there is no dominant strategy, $\mathbb{G} = \left\{\frac{g_L + g_H}{2}\right\}$, $\Delta Q \in \mathbb{Q}$, and $\Delta C \in \mathbb{C}$.*

Proof of Corollary B3: Follows from Lemma B1 and Theorems 5(a) and B2(a). ■