Supplemental Material (not copyedited or formatted) for: Caitlin Ainsley, Clifford Carrubba, Georg Vanberg. 2021.
"The future matters: Judicial preferences and decision-making on collegial courts."
Journal of Law and Courts 9(1). DOI: https://doi.org/10.1086/712652.

Supplementary Online Appendix

"The future matters: Judicial preferences over legal rules and decision-making on collegial courts"

Result 1a. If potential cases are distributed uniformly and judges experience constant losses for misclassified cases, judicial preferences over legal rules are described by symmetric linear loss functions.

Proof. Given a judge with ideal rule x_i who receives a utility of $\ell(c) = -1$ if a case receives a different disposition under rule r than under x_i, we can write the judge's expected utility $\forall r > x_i$ as

$$EU_i(r) = \int_r^{x_i} -(1) \cdot \tfrac{1}{b-a} dc = -\tfrac{x_i - r}{b-a}.$$

Similarly, $\forall r < x_i$, the expected utility is given by,

$$EU_i(r) = \int_{x_i}^{r} -(1) \cdot \tfrac{1}{b-a} dc = -\tfrac{r - x_i}{b-a}.$$

Therefore, taken together the expected utility over the entire case space can be written as:

$$EU_i(r) = -\frac{|x_i - r|}{b-a}, \tag{1}$$

This is a symmetric linear loss function defined over the support of a uniform distribution $f(c) = \frac{1}{b-a}$. ■

Result 1b. If potential cases are distributed uniformly and judges experience linear losses for misclassified cases, judicial preferences over legal rules are described by symmetric quadratic loss functions.

Proof. Given a judge with ideal rule x_i who receives a utility of $\ell(c) = -|x_i - c|$ if a case receives a different disposition under rule r than under x_i, we can write the judge's expected utility $\forall r < x_i$ as

$$EU_i(r) = \int_r^{x_i} -|x_i - c| \cdot \tfrac{1}{b-a} dc = -\tfrac{(x_i - r)^2}{b-a},$$

and $\forall r > x_i$ as

$$EU_i(r) = \int_{x_i}^{r} -|x_i - c| \cdot \tfrac{1}{b-a} dc = -\tfrac{(x_i - r)^2}{b-a}.$$

Taken together the expected utility over the entire case space can be written as

$$EU_i(r) = -\frac{(x_i - r)^2}{b-a}, \tag{2}$$

This is a symmetric quadratic loss function defined over the support of a uniform distribution $f(c) = \frac{1}{b-a}$. ■

Result 1c. If judicial preferences over legal rules are derived from judicial aversion to the potential that future cases are decided incorrectly from the judge's point of view, then any case distribution other than a uniform distribution will result in single-peaked, asymmetric rule preferences, with greater utility loss in high density regions of the case space.

Proof. This proof proceeds in three parts. (1) We show preferences given by the expected utility calculation defined in Equation 1 are single-peaked. (2) Next, derived preferences are only symmetric about a given ideal rule x_i when we assume a uniform distribution. (3) Finally, it is shown utility losses are greater in high density regions of the case space.

(1) Single-Peakedness. Let the preferences of a judge with ideal rule x_i be defined by the expected utility function given in Equation 1, where $EU_i(x_i) = 0$. For this function to be single-peaked, it must be true that $\forall r < x_i$, $\frac{\partial EU_i(r)}{\partial r} > 0$ and $\forall r > x_i$, $\frac{\partial EU_i(r)}{\partial r} < 0$. Consider $r \geq x_i$ and any loss function for wrongly decided cases $\ell(\cdot)$ that imposes a constant or increasing loss as $|c - x_i|$ increases. By definition of the cumulative distribution function, $\forall \varepsilon > 0$, $F(r + \varepsilon) - F(x_i) > F(r) - F(x_i)$. Taken together, this implies that it must be true that $\frac{\partial EU_i(r)}{\partial r} < 0$. A similar argument applies for $r \leq x_i$.

Supplemental Material (not copyedited or formatted) for: Caitlin Ainsley, Clifford Carrubba, Georg Vanberg. 2021.
"The future matters: Judicial preferences and decision-making on collegial courts."
Journal of Law and Courts 9(1). DOI: https://doi.org/10.1086/712652.

(2) Asymmetric Preferences. For a judge with ideal rule x_i, we can say that preferences are symmetric iff $\forall\delta > 0$, (a) $EU_i(x_i - \delta) = EU_i(x_i + \delta)$ and (b) $\ell(c = x_i - \delta) = \ell(c = x_i + \delta)$. We can expand condition (a) with the definition of derived preferences:

$$\int_{x_i-\delta}^{x_i} \ell(c)f(c)dc = \int_{x_i}^{x_i+\delta} \ell(c)f(c)dc.$$

Given that $\ell(c = x_i - \delta) = \ell(c = x_i + \delta)$, preferences are therefore only symmetric if:

$$\int_{x_i-\delta}^{x_i} f(c)dc = \int_{x_i}^{x_i+\delta} f(c)dc.$$

Thus, any case distribution $f(c)$ which does not have equal density on every set of arbitrary intervals $\{[x_i - \delta], [x_i + \delta]\}$ above and below the ideal rule (x_i) will result in asymmetric preferences over legal rules.

(3) Losses and Case Density. By definition of expected utility over r, it is straightforward to see that because $f(c)$ is strictly positive and $\ell(c) < 0$ if a case receives a different disposition under rule r than under x_i, increased density between x_i and r results in greater utility losses.

■

Lemma 1. For a single-peaked distribution of future cases, the width of the join region $[\underline{x}_i, \overline{x}_i]$ for a judge with ideal rule x_i is decreasing as x_i approaches the mode of the underlying distribution. Further, the join region is shorter in the direction towards the mode of the case distribution than in the direction away from the mode.

Proof. We focus here on the case in which a judge's vote is pivotal to the existence of a majority opinion. The case in which judges are not pivotal is analogous. Define judges' preferences over legal rules by their ideal rule x_i. Let $\kappa > 0$ denote the cost of concurrence. A judge's join region — denoted $[\underline{x}_i, \overline{x}_i]$ — is defined by the fact that

$$U(\underline{x}_i) + \alpha U(\underline{x}_i) = U(\overline{x}_i) + \alpha U(\overline{x}_i) = -\kappa \tag{3}$$

Re-writing these expressions by making the expected utilities explicit yields:

$$(1+\alpha)\int_{x_i}^{\overline{x}_i} \ell(c)f(c)dc = -\kappa \quad \text{and} \quad (1+\alpha)\int_{\underline{x}_i}^{x_i} \ell(c)f(c)dc = -\kappa.$$

For $\ell(c) = -1$, and using the cumulative case distribution $(F(c))$, we can rewrite these as the following:

$$F(\overline{x}_i) - F(x_i) = \frac{\kappa}{1+\alpha} \tag{4}$$

$$F(x_i) - F(\underline{x}_i) = \frac{\kappa}{1+\alpha} \tag{5}$$

Let $\overline{x}_i^*$ and $\underline{x}_i^*$ define the upper and lower bounds, respectively, for a given $F_1(c)$, x_i and κ. Holding x_i and κ constant, consider the effect of a change in the case distribution (from $F_1(c)$ to $F_2(c)$) such that $F_2(x_i) - F_2(\underline{x}_i^*) > F_1(x_i) - F_1(\underline{x}_i^*)$. It follows that for a given κ and x_i, the $\underline{x}_i^{**}$ that solves Equation 5 under case distribution $F_2(c)$ must be greater than $\underline{x}_i^*$: $||x_i - \underline{x}_i^{**}|| < ||x_i - \underline{x}_i^*||$. In words, the width of the join region shrinks.

When the ideal rule of a judge (x_i) moves towards the mode of the underlying distribution (from either direction), the fact that the case distribution is single-peaked implies that case density is increasing around their ideal rule. Therefore, for a constant κ, the width of the join region must contract. Further, because the greatest density around the ideal rule is contained in the direction of the mode, the distance between the ideal rule and upper boundary will be shortest towards the mode. ■

Lemma 2. For two judges with ideal rules $x_i < x_{i+1}$, if there exists overlap in their join regions, an increase in the density of the case distribution between their ideal rules decreases the size of the overlap region. For sufficiently large increases in the case density, the overlap disappears.

Proof. We again focus on the case in which a judge's vote is pivotal to the existence of a majority opinion. The case in which judges are not pivotal is analogous. Define judges preferences over legal rules by their ideal rule x_i. Let $\kappa > 0$ denote the cost of writing separately. A judge's join region — denoted $[\underline{x}_i, \overline{x}_i]$ — is defined by the fact that:

$$U(\underline{x}_i) + \alpha U(\underline{x}_i) = U(\overline{x}_i) + \alpha U(\overline{x}_i) = -\kappa.$$

Re-writing these expressions by making the expected utilities explicit yields:

$$(1+\alpha)\int_{x_i}^{\overline{x}_i} \ell(c) f(c) dc = -\kappa \quad \text{and} \quad (1+\alpha)\int_{\underline{x}_i}^{x_i} \ell(c) f(c) dc = -\kappa.$$

For $\ell(c) = -1$, and using the cumulative case distribution ($F(c)$), we can rewrite these as the following:

$$F(\overline{x}_i) - F(x_i) = \tfrac{\kappa}{1+\alpha}$$
$$F(x_i) - F(\underline{x}_i) = \tfrac{\kappa}{1+\alpha}$$

We can solve for the boundaries of the join region by working with the cumulative case distribution ($F(c)$) and the inverse of the cumulative case distribution ($F^{-1}(c)$):

$$\overline{x}_i = F^{-1}\left(\frac{\kappa}{1+\alpha} + F(x_i)\right) \tag{6}$$

$$\underline{x}_i = F^{-1}\left(F(x_i) - \frac{\kappa}{1+\alpha}\right). \tag{7}$$

Consider two arbitrary judges with ideal rules such that $x_i < x_{i+1}$. By definition, judges x_i and x_{i+1} will only form a coalition when $\overline{x}_i \geq \underline{x}_{i+1}$. Using our definitions of the boundaries of the join region given in Equations 6 and 7 above, we can rewrite this condition as:

$$F^{-1}\left(\frac{\kappa}{1+\alpha} + F(x_i)\right) \geq F^{-1}\left(F(x_{i+1}) - \frac{\kappa}{1+\alpha}\right)$$
$$F\left(F^{-1}\left(\frac{\kappa}{1+\alpha} + F(x_i)\right)\right) \geq F\left(F^{-1}\left(F(x_{i+1}) - \frac{\kappa}{1+\alpha}\right)\right)$$
$$\frac{\kappa}{1+\alpha} + F(x_i) \geq F(x_{i+1}) - \frac{\kappa}{1+\alpha}$$
$$\frac{2\kappa}{1+\alpha} \geq F(x_{i+1}) - F(x_i) \tag{8}$$

Therefore, as the density between the ideal rules ($F(x_{i+1}) - F(x_i)$) increases, the overlap in the two judges join regions decreases for any given κ and α, and the overlap disappears completely once the condition in (8) is no longer met. ■

Result 2. An increase in the density of cases within the Pareto set of the judges on a collegial court weakly decreases the size of majority coalitions, provided that the cost of writing separately is sufficiently low.

Proof. For an N-member court $\{x_1, ..., x_N\}$ and case distribution $f(c)$, assume there exists a majority coalition of judges $\{x_i, ..., x_{i+n}\}$, where $1 \leq i < N$ and $\frac{N+1}{2} \leq n \leq N - i$. For this coalition to exist, it must be true that $\overline{x}_i > \underline{x}_{i+n}$, which by Lemma 2 (Equation 8) implies the following condition must hold:

$$\frac{2\kappa}{1+\alpha} \geq F(x_{i+n}) - F(x_i) \tag{9}$$

Let $g(c)$ denote a distribution of cases with a cumulative distribution $G(c)$ such that,

$$\int_{x_i}^{x_{i+n}} g(c)dc > \int_{x_i}^{x_{i+n}} f(c)dc, \tag{10}$$

meaning for case distribution $g(c)$ there exists a greater density of cases on the interval $[x_i, x_{i+n}]$ than there does for the case distribution $f(c)$. For this same coalition $\{x_i, ..., x_{i+n}\}$ to exist under case distribution $g(c)$, Lemma 2 implies the following condition must hold:

$$\frac{2\kappa}{1+\alpha} \geq G(x_{i+n}) - G(x_i) \tag{11}$$

Rewriting Equation 10, the following inequality must be true by definition:

$$G(x_{i+n}) - G(x_i) > F(x_{i+n}) - F(x_i), \tag{12}$$

which implies that for sufficiently large increases in density within the range of ideal rules $[x_i, x_{i+n}]$, the overlap in join regions for $\{x_i, ..., x_{i+n}\}$ will narrow and eventually breakdown as the condition given in Equation 11 is no longer met.

The possible coalition under the new case distribution $g(c)$ must be weakly smaller than the coalition under $f(c)$ so long as $\overline{x}_{i+1} < \underline{x}_{i+n+1}$ and $\overline{x}_{i-1} < \underline{x}_i$. While by definition of our minimum winning coalition both of these conditions hold under the case distribution $f(c)$, they also hold under $g(c)$ when:

$$G^{-1}\left(\frac{\kappa}{1+\alpha} + G(x_{i+1})\right) < G^{-1}\left(G(x_{i+n+1}) - \frac{\kappa}{1+\alpha}\right)$$
$$\frac{2\kappa}{1+\alpha} < G(x_{i+n+1}) - G(x_{i+1}) \tag{13}$$

and

$$G^{-1}\left(\frac{\kappa}{1+\alpha} + G(x_{i-1})\right) < G^{-1}\left(G(x_i) - \frac{\kappa}{1+\alpha}\right)$$
$$\frac{2\kappa}{1+\alpha} < G(x_i) - G(x_{i-1}). \tag{14}$$

Therefore, increasing density within the range of ideal rules contained in the minimum winning coalition $[x_i, ..., x_{i+n}]$ results in smaller coalitions so long as κ is sufficiently small.

Finally, it is necessary to show that increased density outside of the majority coalition but within the pareto set of N judges results in weakly smaller coalitions. Here, the increased density has no effect on the likelihood of any member of $\{x_i, ..., x_{i+n}\}$ joining that coalition. Further, given that by definition $\overline{x}_{i-1} < \underline{x}_i$ under $f(c)$, increased density to the left of the coalition such that

$$\int_{i-1}^{i} g(c)dc > \int_{i-1}^{i} f(c)dc$$

only further increases the distance between $\overline{x}_{i-1}$ and $\underline{x}_i$. Therefore, this cannot increase the size of the coalition by adding judges to the left. By the same logic, increased density to the right of the coalition such that

$$\int_{i+n}^{i+n+1} g(c)dc > \int_{i+n}^{i+n+1} f(c)dc$$

does not affect the overlap in the join regions of judges $\{x_i, ..., x_{i+n}\}$ and further increases the distance between $\overline{x}_{i+n}$ and $\underline{x}_{i+n+1}$. Therefore, this cannot increase the size of the coalition by adding judges to the right. ■

Result 3. For a sufficiently large increase of the density of the case distribution between the ideal rule of the median judge and the judges adjacent to the median, existing majority coalitions will break down, and no majority opinion can emerge, provided that the cost of writing separately is sufficiently low.

Supplemental Material (not copyedited or formatted) for: Caitlin Ainsley, Clifford Carrubba, Georg Vanberg. 2021.
"The future matters: Judicial preferences and decision-making on collegial courts."
Journal of Law and Courts 9(1). DOI: https://doi.org/10.1086/712652.

Proof. For an (odd) N-member court, any majority coalition must include the median justice, whose ideal rule is given by x_m where $m = \frac{N+1}{2}$. For a distribution of cases $f(c)$, assume there exists a majority coalition. For a majority coalition to exist, there must exist overlap in the join regions of the median judge and one or both of his nearest neighbors (i.e., $\overline{x}_{m-1} > \underline{x}_m$ and/or $\overline{x}_m > \underline{x}_{m+1}$). Formally, this implies it is a necessary but not sufficient condition for at least one of the following two inequalities to hold:

$$F^{-1}\left(\frac{\kappa}{1+\alpha} + F(x_{m-1})\right) > F^{-1}\left(F(x_m - \frac{\kappa}{1+\alpha})\right)$$

$$\implies F(x_m) - F(x_{m-1}) < \frac{2\kappa}{1+\alpha} \tag{15}$$

$$F^{-1}\left(\frac{\kappa}{1+\alpha} + F(x_m)\right) > F^{-1}\left(F(x_{m+1} - \frac{\kappa}{1+\alpha})\right)$$

$$\implies F(x_{m+1}) - F(x_m) < \frac{2\kappa}{1+\alpha}. \tag{16}$$

For any distribution of cases $g(c)$ such that,

$$\int_{x_{m-1}}^{x_{m+1}} g(c)dc > \int_{x_{m-1}}^{x_{m+1}} f(c)dc,$$

$$\implies \int_{x_{m-1}}^{x_m} g(c)dc + \int_{x_m}^{x_{m+1}} g(c)dc > \int_{x_{m-1}}^{x_m} f(c)dc + \int_{x_m}^{x_{m+1}} f(c)dc \tag{17}$$

a majority coalition containing the median judge can exist if one or both of the following conditions hold:

$$G(x_m) - G(x_{m-1}) < \frac{2\kappa}{1+\alpha} \tag{18}$$

$$G(x_{m+1}) - G(x_m) < \frac{2\kappa}{1+\alpha}. \tag{19}$$

For sufficiently large increases in density around the ideal rule of the median judge, these conditions will not be satisfied. Therefore, increased density around the ideal rule of the median justice weakly increases the occurrence of plurality opinions as the join region of the median judge narrows. ∎