

Appendix: The rough derivation of Eq. (4, 5).

Bases on the Eqs. (21, 28) in [27], we can derive the amplitude modulation ratio of the output fluences at spatial frequency $k_{\perp max}$ to the one at ideal case. The ratio is:

$$A_t(\tau) = \frac{\int U(t)U^*(t - \tau)dt}{\int |U(t)|^2 dt}$$

where τ is defined by (21) in [27]. This ratio $A_t(\tau)$ can also be considered as $\frac{PTA_{out} - PTA_{ideal}}{PTA_{in} - PTA_{ideal}}$.

From $A_t(\tau) = \frac{PTA_{out} - PTA_{ideal}}{PTA_{in} - PTA_{ideal}}$ and Eq. (32) in [27], we can find that

$$\frac{PTA_{out} - PTA_{ideal}}{PTA_{in} - PTA_{ideal}} = T = \exp\left(-\left(\frac{L_2 - L_1}{l}\right)^2\right)$$

Or

$$\begin{aligned} PTA_{out} &= e^{-\left(\frac{\lambda \Delta L}{\lambda c \tau p}\right)^2} (PTA_{in} - PTA_{ideal}) + PTA_{ideal} \\ &= e^{-\left(\frac{L_2 - L_1}{l}\right)^2} (PTA_{in} - PTA_{ideal}) + PTA_{ideal} \end{aligned}$$

Based on Eqs. (8, 21) in [27], we can achieve that: $\frac{\omega_0}{A} = \frac{2\pi c}{\lambda_0}$, $\frac{1}{A} = \frac{\cos^3 \beta}{\cos \alpha (\sin \alpha + |\sin \beta|)}$.

Based on Eq. (32), we can deduce that: $l = \frac{c \tau_{FWHM} \Lambda}{\sqrt{\ln 2} \lambda}$

Hence

$$l = \frac{c \tau_{FWHM} \Lambda}{\sqrt{\ln 2} \lambda} = \frac{\tau_{FWHM} c}{\sqrt{\ln 2} \lambda k_{\perp max} \lambda_0} \frac{2\pi \lambda}{\lambda_0}$$

$$l = \frac{\tau_{FWHM}}{\sqrt{\ln 2}} \frac{1}{k_{\perp max}} \frac{2\pi c}{\lambda_0}$$

$$l = \frac{\tau_{FWHM}}{\sqrt{\ln 2}} \frac{1}{k_{\perp max}} \frac{\omega_0}{A}$$

As a result: $l = \frac{c \tau_{FWHM} \Lambda}{\sqrt{\ln 2} \lambda} = \frac{\omega_0 \tau_{FWHM}}{\sqrt{\ln 2} \cdot k_{\perp max} \cdot A} = \frac{\omega_0 \tau_{FWHM}}{\sqrt{\ln 2} \cdot k_{\perp max}} \frac{\cos^3 \beta}{\cos \alpha (\sin \alpha + |\sin \beta|)}$.