The Accountability of Politicians in International Crises and the Nature of Audience Cost: **Supplementary Materials**

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Contents

1 Proposition 4: Maximally Extractive Retention Strategies

Proof of Proposition 4. The Home citizen's payoff is bounded above by a function that has two terms.The first is the payoff from types who keep the status quo: $\underline{p}y$. The second comes from types who initiate a crisis and (for the upper bound) fully extract: $(1-\underline{p})\min\left\langle 1, \frac{1+\underline{p}}{2}+c_F\right\rangle$. All together, the payoff at $\underline{p}$ is

$$\mathcal{F}(\underline{p}) = \underline{p}y + (1-\underline{p})\min\left\langle 1, \frac{1+\underline{p}}{2}+c_F\right\rangle.$$

We will proceed in two steps. First, we will find the values of $\underline{p}$ that maximize $\mathcal{F}$. Second, we will extend the construction from the proof of Proposition 2 to a complete retention strategy.

Step 1: Unpacking the minimization and simplifying gives

$$\mathcal{F}(\underline{p}) = \begin{cases} \underline{p}y + (1-\underline{p}) & \text{if } \underline{p} \geq 1-2c_F \\ \frac{1}{2}+c_F+(y-c_F)\underline{p}-\frac{1}{2}\underline{p}^2 & \text{if } \underline{p} < 1-2c_F \end{cases}.$$

The function $\mathcal{F}$ is continuous since the two defining expressions are equal when $\underline{p} = 1-2c_F$. It is decreasing on $(1-2c_F, 1]$ since $\mathcal{F}'(\underline{p}) = y-1 < 0$ on that interval.

The function $p\frac{1}{2}+c_F+(y-c_F)\underline{p}-\frac{1}{2}\underline{p}^2$ is strictly concave, and is maximized at $p = y-c_F$. This gives three cases for $\mathcal{F}$:

(i) $y \leq c_F$. In this case, $\mathcal{F}$ is decreasing on its entire domain, and is maximized at $\underline{p} = 0$.

(ii) $c_F < y \leq 1-2c_F$. In this case, $\mathcal{F}$ is maximized at $\underline{p} = y-c_F$.

(iii) $1 - 2c_F < y$. In this case, $\mathcal{F}$ is increasing on the interval $[0, 1 - 2c_F)$. Thus the function is maximized at $\underline{p} = 1 - 2c_F$.

Step 2: From the previous step, we have to consider three cases: (i) $\underline{p} = 0$, (ii) $\underline{p} = 1 - 2c_F$, and (iii) $\underline{p} = y - c_F$ with $c_F < y < 1 - c_F$. In each, we must extend the pair (r_S, r_W) to a complete retention strategy in such a way that the appropriate $\underline{p}$ is part of the equilibrium.

The first case, $\underline{p} = 0$, is easy—set $r_Q = 0$.

For the second and third cases, we will use:

Lemma 9. Suppose the citizen uses a fully extractive cutoff reward scheme (r_S, r_W) with cutoff given by $x^\dagger$. This strategy can be extended to a complete retention strategy (r_Q, r_S, r_W) that induces an interior entry threshold $\underline{p}$ if and only if $x^\dagger - y + \overline{r} \leq 1$.

Proof. Notice that type $\underline{p}$ is indifferent between the status quo and initiation followed by acceptance if and only if $y + r_Q = x^\dagger + \overline{r}$, which can be rearranged to give $r_Q = x^\dagger - y + \overline{r}$. This is a feasible retention probability only if $x^\dagger - y + \overline{r} \leq 1$. □

Now we consider the two remaining cases in turn.

(ii) $\underline{p} = 1 - 2c_F$. From the proof of Proposition **??**, $\overline{r} = c_F$. Thus the critical condition from Lemma **??** is $1 - y + c_F \leq 1$, or $y > c_F$.

(iii) $\underline{p} = y - c_F$ with $c_F < y < 1 - c_F$. From the proof of Proposition **??**, $\overline{r} = \frac{1-\underline{p}}{2}$. Thus the critical condition from Lemma **??** is $\frac{1+\underline{p}}{2} + c_F - y + \frac{1-\underline{p}}{2} \leq 1$, or $y > c_F$.

So in each case, Proposition 2 and Lemma 9 imply that there is a retention strategy that is maximally extractive with the indicated properties. □

2 Distinguishing Between Victory and Defeat

In this appendix, we sketch the argument that our analysis in the main text also cover the case in which the Voter can condition retention on the outcome of a war. We do so for the case of secret settlements.

A retention strategy is now a 4-tuple (r_Q, r_S, r_V, r_D), where r_V is the probability of retention in the event that Home wins a war and r_D is the probability of retention in the event that Home loses a war. Otherwise, the model is as in Section 1.

For a fixed retention strategy, the analysis of equilibrium is similar to that laid out in Section 2. The key difference is in the Home Leader's acceptance decision. She accepts offer x if and only if her type p satisfies

$$\begin{aligned} p \leq \overline{p}^*(x) &= \frac{1}{1 + r_V - r_D} \left(x + r_S - r_D + \gamma c_H\right) \\ &= \alpha x + \Delta, \end{aligned}$$

where $\alpha = \frac{1}{1+r_V-r_D}$ and $\Delta = \frac{r_S-r_D+\gamma c_H}{1+r_V-r_D}$.

An argument like that leading to Lemma 2 shows that Foreign's optimal offer is

$$x^* = \min \left\langle \frac{\underline{p} + \alpha c_F + \Delta(\alpha - 1)}{2\alpha - \alpha^2}, \frac{1 - \Delta}{\alpha} \right\rangle.$$

Now we can give an example to show that rewarding losers—$r_D > r_V$—can increase the Home citizen's payoff, relative to treating the war outcomes symmetrically—$r_D = r_V$.

2.1 Example 1

Example 1. Let $c_H = \frac{1}{2}$, $c_F = \frac{1}{10}$, and let $y = \frac{1}{10}$. Then Lemma 6 says that, when r_V is constrained to equal r_D, the optimal retention strategy induces settlement with probability 1, conditional on initiation. From the proof of Lemma 7, we can calculate the Home citizen's payoff in this case as

$$\begin{aligned} V &= \underline{p}y + (1-\underline{p})x^* \\ &= (y+c_H)y + (1-(y+c_H)y)\frac{1}{2}(1+y+c_F) \\ &= \frac{78}{125} \\ &< \frac{2}{3}. \end{aligned}$$

To show that this is no longer optimal, we do not have to solve the complete optimization problem; it suffices to display a sample strategy that's better. Since the optimal strategy with unrestricted $\mathbf{r}$ must give payoffs at least this great, this shows that allowing leaders to be rewarded for losing can make the citizen better off.

Here's the strategy. Set $\Delta = 0$ and have everyone enter, i.e. set $\underline{p} = 0$. Then the equilibrium offer from Foreign is

$$x^* = \min\langle \frac{c_F}{2-\alpha}, \frac{1}{\alpha} \rangle.$$

The first argument of min is strictly increasing in α while the second argument is decreasing in α, so x^* is maximized where they are equal:

$$\frac{1/3}{2-\alpha} = \frac{1}{\alpha}.$$

Solving this equation gives $\alpha = 3/2$. Since every type of Home enters and accepts the offer, Home's payoff is $x^* = 2/3 > V$. As $\alpha > 1$, this strategy rewards losers.

The idea of rewarding losers seems very strange, but there is actually a compelling intuition for the idea. The leader's payoff to war as a function of his signal is

$$p(1+r_V)+(1-p)r_D-\gamma c_H=(1+r_V-r_D)p+r_D-\gamma c_H.$$

This is, of course, increasing in strength measured by p. The rate of this increase with p is governed by the difference $r_V - r_D$: the smaller is this difference, the less does the war payoff increase with p. This implies the fraction of types bought out of war when Foreign increases the offer by a fixed increment is greater the lower is the difference $r_V - r_D$. Consequently, increasing the reward to losing (relative to the reward to winning) increases Foreign's marginal incentive to make larger offers.

So, within the current model, rewarding losers makes a lot of intuitive sense. But doing so may be a bad idea for reasons neglected by the model. For example, rewarding losers of wars might create perverse incentives for leaders to mismanage conflicts.[1] Thus we turn to the question of what is the optimal strategy subject the constraint that $r_V \geq r_D$. We show that the answer to that question is identical to the answer we derived before when the citizen could not make rewards contingent on the war's outcome.

2.2 Proposition 1: Re-selection with war outcome contingent schemes

Proposition 1. *Assume the citizen can distinguish victory from defeat, but is constrained to retain victors at least as often as losers ($r_V \geq r_D$). The optimal incentive strategy is that described in Appendix B.*

Sketch of the Proof. In this new setup, the intermediate program from the Proof

[1] While it is not obvious that the leader has bad incentives when losers are rewarded–he prefers to win as long as $1+r_V > r_D$, which is consistent with $r_V < r_D$– no one has worked out the incentives to manage a war and, therefore, we cannot be sure of the consequences.

of Lemma 5 becomes:

$$\max_{\underline{p},\hat{p},x,\Delta,\alpha} \underline{p}y + (\hat{p} - \underline{p})x + \int_{\hat{p}}^{1} (t - c_H)\, dt$$

$$\text{st } \hat{p} = \alpha x + \Delta$$

$$x \leq \frac{\underline{p} + \alpha c_F + \Delta(\alpha - 1)}{2\alpha - \alpha^2}$$

$$x \leq \frac{1 - \Delta}{\alpha}$$

Lemma 1. *At a solution,*

$$x = \frac{\underline{p} + \alpha c_F + \Delta(\alpha - 1)}{2\alpha - \alpha^2}.$$

Proof. If neither inequality constraint binds, the solution includes $\underline{p} = 0$, $x = 1$, $\alpha = 1$, and $\Delta = 0$. But then we have

$$x = 1 > c_F = \frac{\underline{p} + \alpha c_F + \Delta(\alpha - 1)}{2\alpha - \alpha^2},$$

and the first inequality constraint is violated.

A similar argument works if the second inequality constraint binds but the first does not. □

Thus the relaxed program becomes:

$$\max_{x,\Delta,\alpha} \left((2\alpha - \alpha^2)x - \alpha c_F - \Delta(\alpha - 1)\right) y + (\alpha(\alpha x + \Delta) - \alpha x + \alpha c_F)\, x + \int_{\alpha x + \Delta}^{1} (t - c_H)\, dt$$

$$\text{st } x \leq \frac{1 - \Delta}{\alpha}$$

$$\frac{1}{2} \leq \alpha \leq 1$$

If neither constraint binds, the first-order conditions imply that $\alpha = 0$. And if only the first constraint binds, the first-order conditions imply that $\alpha = 0$. In neither case is the second constraint satisfied.[2]

Suppose neither constraint binds. Then the first-order conditions are:

$$\frac{\partial \tilde{U}}{\partial \alpha}(\alpha, \Delta, x) = 0$$
$$\frac{\partial \tilde{U}}{\partial \Delta}(\alpha, \Delta, x) = 0$$
$$\frac{\partial \tilde{U}}{\partial x}(\alpha, \Delta, x) = 0$$

Substitute for the derivatives to get:

$$(2 - 2\alpha)xy + (x - y)(c_F + \Delta) + (2\alpha - 1)x^2 - (\alpha x + \Delta - c_H)x = 0$$
$$c_H + y - \alpha y - \Delta = 0$$
$$\alpha^2 x - \alpha^2 y + \alpha c_F + \alpha c_H - 2\alpha x + 2\alpha y = 0$$

This system has two solutions:

$$(\alpha, \Delta, x) = (0, c_H + y, y + c_F + c_H) \quad \text{and} \quad (\alpha, \Delta, x) = (0, c_H + y, y).$$

Neither satisfies the second constraint.

[2] See Sage code at Sage calculations.

Suppose only the first constraint binds. Then the Kuhn-Tucker conditions are:

$$\frac{\partial \tilde{U}}{\partial \alpha}(\alpha, \Delta, x) - \lambda_1 x = 0$$
$$\frac{\partial \tilde{U}}{\partial \Delta}(\alpha, \Delta, x) - \lambda_1 = 0$$
$$\frac{\partial \tilde{U}}{\partial x}(\alpha, \Delta, x) - \lambda_1 \alpha = 0,$$

where λ_1 is the multiplier on the the constraint $1 - \Delta - \alpha x \geq 0$.

Again, there are two solutions:

$$(\alpha, \Delta, x, \lambda_1) = (0, 1, y, c_H + y - 1) \quad \text{and} \quad (\alpha, \Delta, x, \lambda_1) = (0, 1, c_F + 1, c_H + y - 1).$$

And again, neither satisfies the second constraint.

Thus one of the inequalities in the second constraint must be an equality.If $\alpha = \frac{1}{2}$, then the Kuhn-Tucker conditions are:

$$\frac{\partial \tilde{U}}{\partial \alpha}(\alpha, \Delta, x) - \lambda_1 x + \lambda_2 = 0$$
$$\frac{\partial \tilde{U}}{\partial \Delta}(\alpha, \Delta, x) - \lambda_1 = 0$$
$$\frac{\partial \tilde{U}}{\partial x}(\alpha, \Delta, x) - \lambda_1 \alpha = 0,$$

with $\lambda_1(1 - \Delta - \alpha x) = 0$ and $\lambda_2 > 0$.

If the first constraint is slack, then there is no solution with λ_2 nonnegative. If the first constraint binds, then

$$\lambda_2 = -\frac{1}{4}c_F^2 + \frac{1}{2}(c_F + 1)y - \frac{1}{4}y^2 - \frac{1}{2}c_F - \frac{1}{4}$$

The right-hand side is strictly decreasing in c_F and strictly increasing in y. Thus λ_2 is bounded above by 0.

So the only candidates for a solution have $\alpha = 1$. But this is the same as $r_V = r_D$. □