Likelihood Neglect Bias and the Mental Simulations Approach: An Illustration using the Old and New Monty Hall Problems

Supplementary Materials

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Appendix A: Training Materials

The following page features experiment 1's reading materials for training participants in the mental simulations approach.

Participant Materials

KEEP THIS TAB OPEN THROUGHOUT THE STUDY

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Background to the Study

We all make judgments about probabilities. You might choose one job rather than another because of your judgment that you will probably be happier in that job. Or you might take some medication because of your judgment that it is probably safe.

This study is about probabilities.

However, humans are susceptible to various *cognitive biases*—that is, errors in their judgment. This study aims to teach you a method to help you overcome a particular cognitive bias that you might fall prey to when reasoning about probabilities.

To teach you the method, we want to walk you through a hypothetical scenario. Note that you will be asked questions about this and another problem later, and your ability to give correct answers in this study will determine whether you receive the bonus payment (if you are completing this study for payment).

Also, this material asks you to do various tasks, such as drawing circles. We encourage you to do this with some paper and a pen or pencil, if you have these items. Otherwise, if you do not have these items, just follow these materials to the best of your ability.

Let us now consider the hypothetical scenario.

The Story of the Prisoners

Imagine that you and three other people—Alison, Billy and Carly—are in prison. Three of you will be imprisoned for life, and one of you will be set free. A lottery was used to randomly determine who will be set free. So each of you have an equal chance of being set free at the beginning of this story.

The prison warden knows who will be set free, and you ask him if he can tell you who it is. He says that he can tell you the names of ***<u>two</u>*** prisoners who will ***<u>not</u>*** be set free, but he cannot tell you whether you will be set free or not. We will also suppose he cannot lie about who will be set free.

He then tells you that Billy and Carly will ***<u>not</u>*** be set free. Consequently, either you or Alison will be set free.

Now, once the warden has given you this testimony—that is, his statement about who will ***<u>not</u>*** be set free—which of the following is true: you are more likely to be set free, Alison is more likely to be set free, or both of you are equally likely to be set free?

At this point, an intuitive answer is that you and Alison are equally likely to be set free. After all, only two options remain, and you both started off with an equal probability of being set free. This answer, however, is incorrect. It results from a cognitive bias—an error in human judgment. We want to teach you an approach to correct this bias.

Surprisingly, the correct answer is that **Alison is more likely to be set free**.

To see how this is so, we will use the *mental simulations* approach to probabilistic reasoning.

The Mental Simulations Approach

The core idea behind this approach is that we will run so-called *mental simulations* of the scenario in our mind—that is, we will imagine that the scenario with the prisoners happened a number of times. We will then ask ourselves the question: who is more likely to be set free? To correctly calculate the relevant probabilities with these simulations, we need to think about two kinds of probabilities: prior probabilities and the probability of the evidence. Let us consider these in more detail.

Prior Probabilities

We need to first consider the *prior probabilities* of who will be set free—that is, the probability of being set free *prior* to receiving some evidence. In this case, the evidence is the warden's testimony that Bill and Carly will ***not*** be set free.

We will then consider the probability of getting this testimony given the various possible outcomes for who will be set free. But for now, we are just considering the prior probabilities of the outcomes.

At the beginning of our story, then, there are four outcomes:

- Outcome 1 = You will be set free
- Outcome 2 = Alison will be set free
- Outcome 3 = Billy will be set free
- Outcome 4 = Carly will be set free

Remember that which prisoner will be set free is determined by a random lottery, so each person initially has an equal prior probability of being set free. For example, the prior probability that you will be set free is 1/4 or 25%, and it is the same with the other outcomes.

Let us then imagine a number of simulations of this situation, say, 12 simulations (we will explain exactly why the number 1**8. 2 was chosen later on). We can depict these simulations in different ways.

One way to depict them is with circles, supposing that each circle represents a time that the scenario happens. You can see this here:

12 ‘mental’ simulations of the Prisoners Story

The first step of the mental simulations approach is then to image some simulations.

We will now imagine that in some of these simulations, you will be set free, while in the other simulations, the others will be set free. The second step is then to **proportion the number of simulations** where a given outcome is true by the **prior probability of that outcome**.

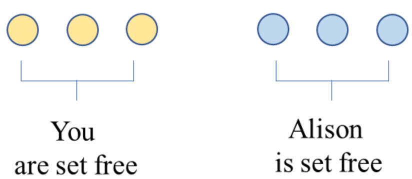
So, for instance, since you have a 25% prior probability of being set free, we will make it so that you are set free in 25% of the simulations—that is, in 3 of the 12 simulations.

To do this, if you have writing materials, go ahead and depict the 12 simulations using circles like how it was done above. Below, this proportioning has already been done for the outcomes where you or Alison are set free, but you need to do it for the other two outcomes: the outcome where Billy will be set free and the outcome where Carly will be set free.

So go ahead and proportion the 6 simulations for the remaining outcomes based on the prior probability of those outcomes. This could be done by making it so that for some circles, an outcome is true, as you can see below:

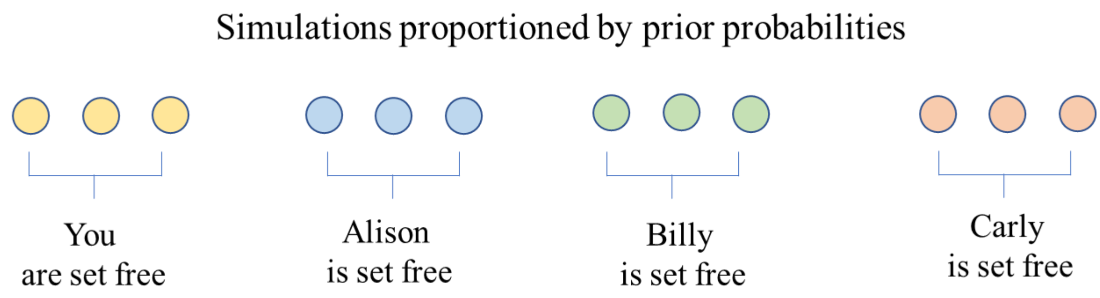
Simulations proportioned by prior probabilities

[Proportion the remaining simulations here]



Once you are done, answer the question on the webpage and continue to the next section.

If you did that exercise correctly, then a given prisoner will be set free in 25% of the simulations because they each have a 25% prior probability of being set free. What you should then have would look something like this:



So the first step of the mental simulations approach is to imagine some simulations, and the second step is to proportion those simulations by the prior probabilities. The third step is to then further proportion these simulations by the second kind of probability.

The Probabilities of the Testimony given the Outcomes

The second kind of probability that we need to consider is the probability of the testimony given the various outcomes. Recall that the testimony was this:

> Warden's testimony = the warden's statement that Billy and Carly will ***not*** be set free

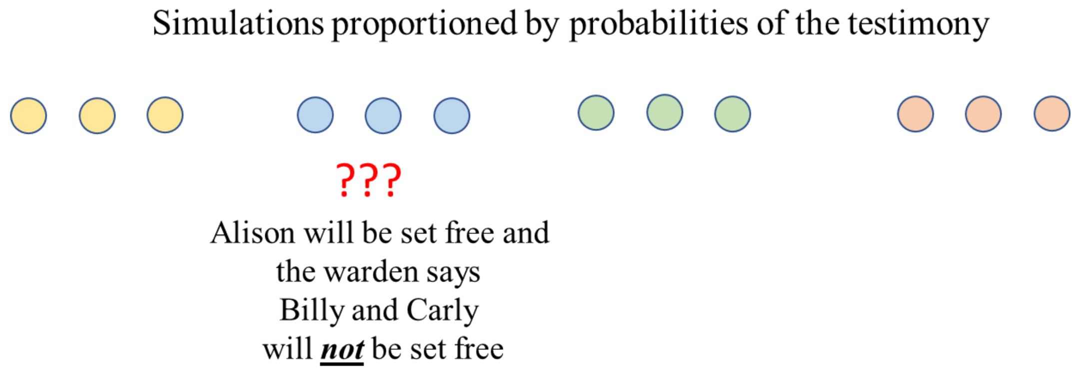
Also recall that the warden said he cannot tell you whether you will be set free, and he can tell you the names of *only* two people who would ***not*** be set free. He then gave you his truthful testimony that Billy and Carly will ***not*** be set free.

We now need to consider how probable this testimony would be given the various outcomes. Once we know how probable the evidence is for a given outcome, we then need to proportion the simulations by that probability.

For example, consider the outcome where Billy will be set free. If Billy was to be set free, then the warden would not have truthfully told you that Billy and Carly would not be set free. This is because we have supposed that the warden cannot lie. So there is a 0% probability that the warden would give you his testimony if Billy was to be set free. For that reason, we then proportion the simulations so that the warden gives you his testimony in 0% of the simulations where Billy will be set free.

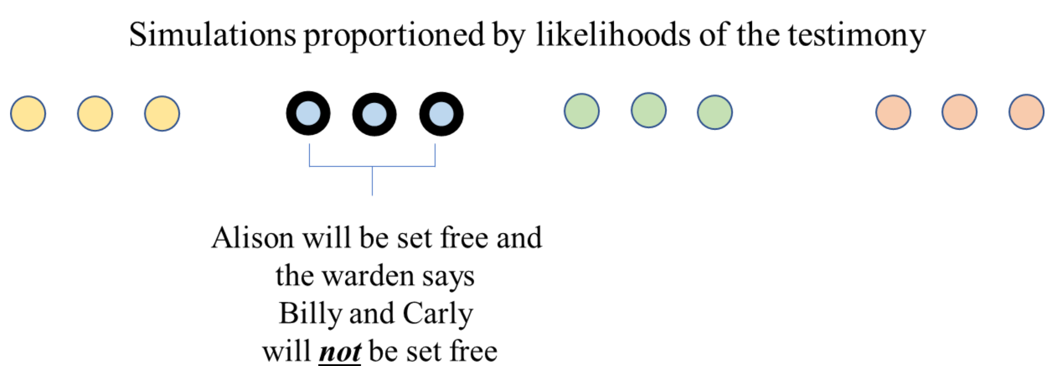
Similarly, we will make it so that the warden gives you his testimony in 0% of the simulations where Carly will be set free. Again, this is because the warden cannot lie and there is a 0% probability that the warden would give you his testimony if Carly was to be set free.

Now consider the simulations where Alison will be set free. In how many of these would the warden give you his testimony?

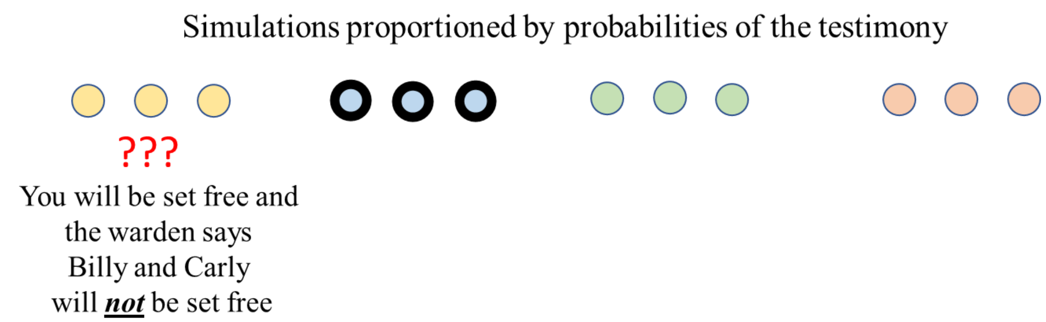


The correct answer is that the warden would give you his testimony in 100% of the simulations where Alison will be set free. This is because if Alison was to be set free, then there is a 100% probability that he would tell you that Billy and Carly will ***not*** be set free. And the reason for this is that he would **have** to tell you that Billy and Carly will ***not*** be set free: because he cannot lie, he would not say that Alison would ***not*** be set free if she was to actually be set free, and because he cannot tell you your fate, he cannot tell you that you will ***not*** be set free.

So if you have writing materials, go ahead and make it so that the warden gives you this testimony in 100% of the simulations where Alison will be set free. You can do this by circling the simulations as you can see here:



Now consider the simulations where you will be set free. In how many of these would the warden give you his testimony?



Enter your answer on the webpage and then proceed to the next section.

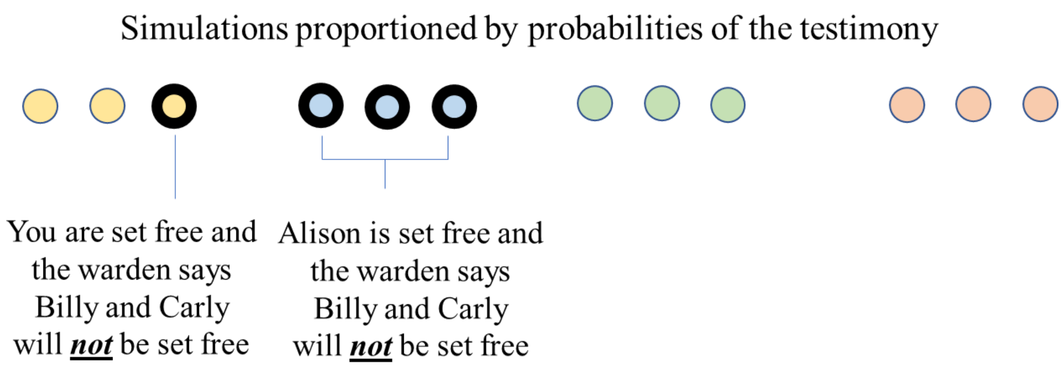
The correct answer is that the warden would give you his testimony in 1/3 or approximately 33% of the simulations where you will be set free.

To see how this is so, let us consider the probability that he would give you the same testimony if ***you*** were to be set free. So now imagine that you will be set free. Then, the warden could have given you one of any three combinations of names about who will ***not*** be set free. He could have said:

1) that Alison and Billy will ***not*** be set free
2) that Alison and Carly will ***not*** be set free, or
3) that Billy and Carly will ***not*** be set free

Since there are three combinations which the warden could have said, the probability that the warden would tell you that Billy and Carly will ***not*** be set free is 1/3 if you were to be set free. For this reason, we will make it so that the warden tells you that Billy and Carly will ***not*** be set free in 1 out of 3 of the simulations where you will be set free.

This is again depicted here:



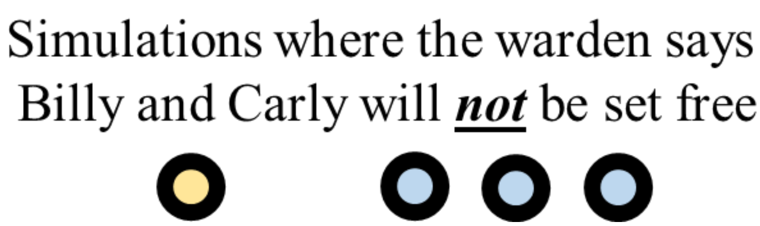
So we have seen the first three steps of the mental simulations approach: first, imagine some simulations; second, proportion the simulations by the prior probabilities; and third, then proportion the simulations by the probability of the evidence.

The fourth step is then to get rid of the simulations without the evidence. Let us explore this in more detail.

Eliminate Irrelevant Simulations and Calculate the Probabilities

Now, we can calculate the probability that you or Alison will be set free given the warden's testimony. To do that, we just consider only the simulations where the warden gave you his testimony. The rationale for this is intuitive: since, in the story, you are in a situation where you have been given this testimony, it makes sense to calculate probabilities only with reference to the simulations where the warden has given you this testimony.

We can depict the remaining simulations by crossing out or removing the circles where the evidence does not obtain, as you can see here:



Once we have eliminated the simulations without the evidence, we can carry out the fifth and final step: we can calculate the probabilities of the outcomes given the evidence by counting the remaining outcomes. Here, we can see that there are only 4 simulations where the warden gave you this testimony, and in 3 of those, Alison will be set free. For that reason, the probability that Alison will be set free is 3/4 or 75% and not 1/2 or 50%, as we might have initially thought. We can now see why it is more probable that Alison will be set free: in this case, the evidence is more probable given that outcome. In other words, the warden is more likely to give you the testimony that he did if Alison was to be set free, and this is why there are more simulations where Alison will be set free after we have eliminated the simulations that do not have the evidence.

FAQs about the Approach

We will now answer some frequently asked questions about the approach.

Why is the answer that this approach gives the correct one?

This approach not only aims to help us calculate the probabilities about who will be set free, but it also helps us to understand *why those probabilities are the correct ones.* This is because the mental simulations approach provides a snapshot of what would happen *if the scenario was to happen a large number of times*. More specifically, if the story of the prisoners was to happen, say, 100,000 times, then we can see that about 75% of the time, Alison would be released when the warden has given you that information. And we can see why this is the case. Those scenarios are first proportioned by the prior probabilities of the outcomes, and then by the probabilities of the testimony given those outcomes. This enables us to see the frequency with which Alison will be set free among the times when we have been given that information.

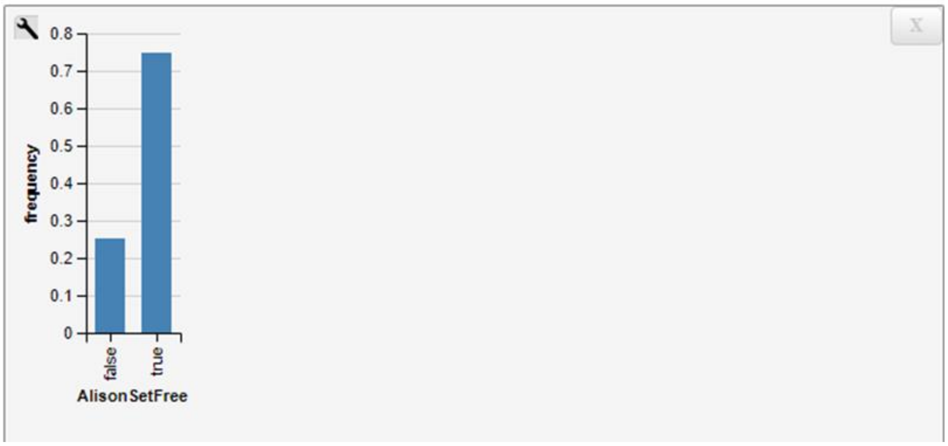
Of course, in real life, the frequency with which something happens does not always match the probability of that thing: a coin may have a 50% probability of landing heads, but if you toss it 10 times, it might land heads on 30% of those times. Nevertheless, we can correctly calculate probabilities if we suppose that the probability of something matches the frequency with which that thing happens in these simulations. For example, if an outcome has a prior probability of 25%, then we imagine that it will happen in 25% of the simulations.

We can also show that that the mental simulations approach delivers the right answers if we run computer simulations.

Here, for instance, we provide an example of a program in a programming language called *WebPPL.* We ran the program many times to generate 100,000 computer simulations where the warden gives you his testimony, and in approximately 75% of them, Alison was set free instead of you:

```
//Four prisoner story simulator

var Simulations = Infer({method: 'rejection', samples: 100000}, function (){

//Assigns prior probabilities to the possible outcomes
  var YouSetFree = flip(1/4)
  var AlisonSetFree = flip(1/4)
  var BillySetFree = flip(1/4)
  var CarlySetFree = flip(1/4)

//This is a constraint to make it so that only one prisoner will be set free
  condition(YouSetFree + AlisonSetFree + BillySetFree + CarlySetFree == 1)

//This assigns probabilities to the warden's testimony given the outcomes
  var WardensTestimony = (YouSetFree && flip(1/3))||
                         (AlisonSetFree && flip(3/3))||
                         (BillySetFree && flip(0/3))||
                         (CarlySetFree && flip(0/3))

  condition(WardensTestimony)

  return{AlisonSetFree: AlisonSetFree}
})

Simulations
```

run



Why does intuition give the wrong answer?

We can also see, from this approach, why the intuitive answer—that you and Alison are equally likely to be free—is the incorrect answer. The reason is that it fails to incorporate the probability of the evidence given the various possible outcomes: *it is more probable that the warden would give you his testimony if Alison was to be set free than if you were to be set free*. In this sense, most people do not properly account for the fact that the evidence is more probable given one possible outcome rather than another. We then display a *cognitive bias* when we fail to correctly consider these probabilities and their implications for the probability of the outcomes. The mental simulations approach helps us to avoid that cognitive bias and to see how the probability of the evidence affects the probability of the outcomes given that evidence.

So even though the intuitive answer is wrong, we can nevertheless replace the incorrect intuition with better intuitions. To do so, consider other cases where it is more obvious that some evidence favors one outcome over another if the evidence is more probable given that outcome than given the other.

Let us consider an analogy. Suppose you test positive for a disease. It is more likely that you would test positive if you had the disease than if you did not. So, intuitively, the positive test raises the probability that you have the disease!

We can now apply that same intuition to the story of the prisoners. Suppose the warden tells you that Billy and Carly will not be set free. As mentioned previously, it is more likely that he would give you that testimony if Alison was to be set free than if you were to be set free. So, intuitively, the warden's testimony should raise the probability that Alison will be set free.

How many simulations should we use with approach?

How many simulations do we need to imagine with the approach? The answer is this: whatever number lets you do the proportioning! In particular, there are two things to proportion. The first are the prior probabilities; in our story, these are each 1/4 or 25%. The second things to proportion are probabilities of the evidence given the outcomes; in our story, these varied from outcome to outcome. In our story, 12 simulations work.

But note that we did not need to run these mental simulations with *exactly* 12 simulations. For example, 36 simulations also work. We could have made each outcome true in 9 simulations before proportioning the simulations by the probabilities of the testimony so that Alison is free in 9 simulations where the warden gives you the testimony and you are set free in 3 of the simulations where the warden gives you the testimony. In this case, the probability that Alison will be set free is still $9/12 = 3/4 = 75\%$.

The only thing that matters is that the number of simulations—whatever it is—can be proportioned first by the prior probabilities and then by the probabilities of the testimony given the various outcomes.

Summary of the Approach

Here, then, is a summary of steps in the mental simulations approach:

1. Imagine some simulations:
    - Imagine *n* number of simulations (where *n* is any number that can be proportioned by the prior probabilities and then by probability of the evidence)
2. Proportion according to prior probabilities of the outcome:
    - For each possible outcome, make the proportion of simulations where that outcome is true correspond to the prior probability of that outcome
3. Proportion according to probabilities of the evidence:

- For each set of simulations for a given outcome, make the proportion of simulations where the evidence obtains correspond to the probability of that evidence given that outcome

4. Eliminate irrelevant simulations:
    - Remove the simulations where the evidence does not obtain
5. Calculate probabilities:
    - Determine the proportion of the remaining simulations where a particular outcome is true; this is the probability of that outcome given the evidence

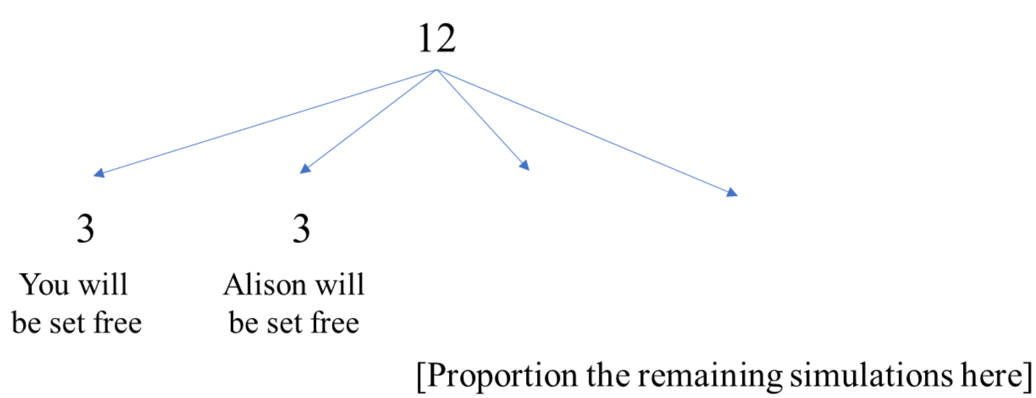
So that is the mental simulations approach to probabilistic reasoning.

Mental Simulations Using Numbers

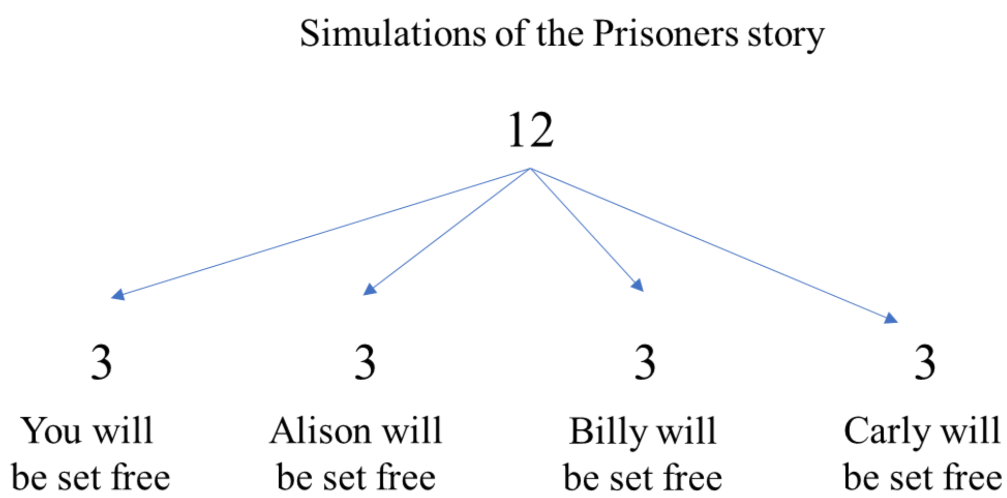
To help you further internalize the mental simulations approach, the below exercise asks you to repeat the above procedure, but by using an approach where the simulations are represented with numbers instead of circles.

First, proportion the remaining number of simulations where an outcome is true by the prior probability of that outcome. This has already been done for two outcomes, but not for the others.

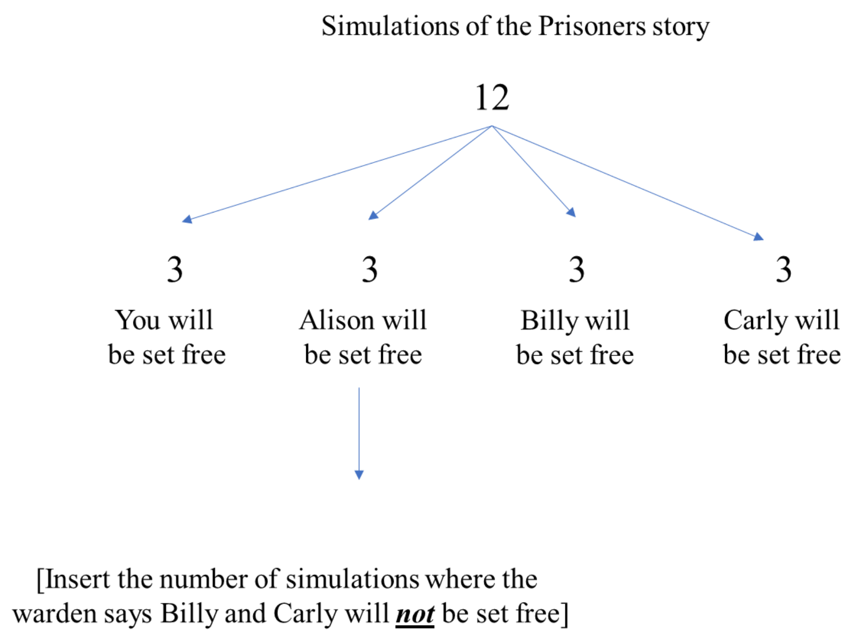
Simulations of the Prisoners story



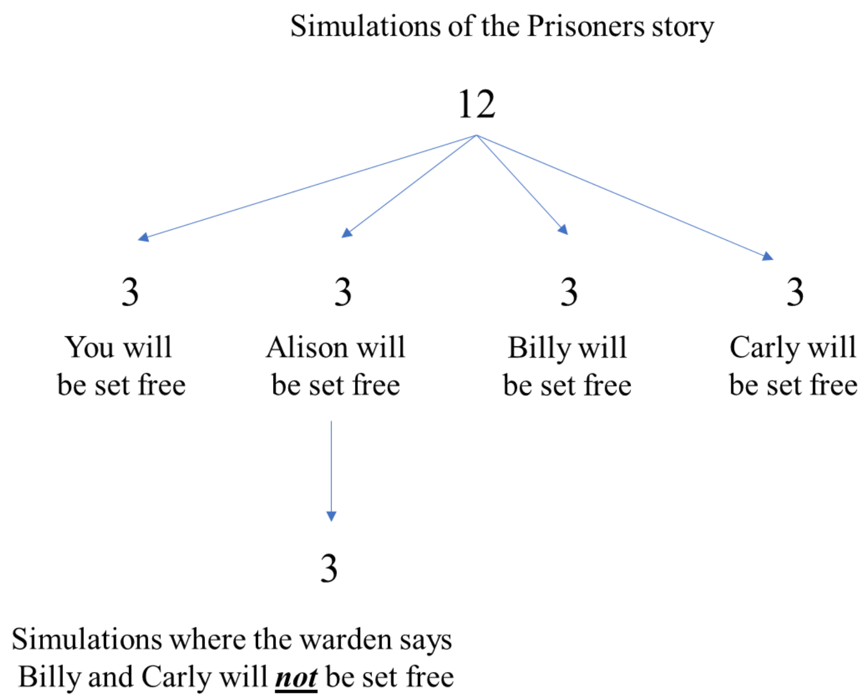
If you did that right, you should have something like what is on the following page.



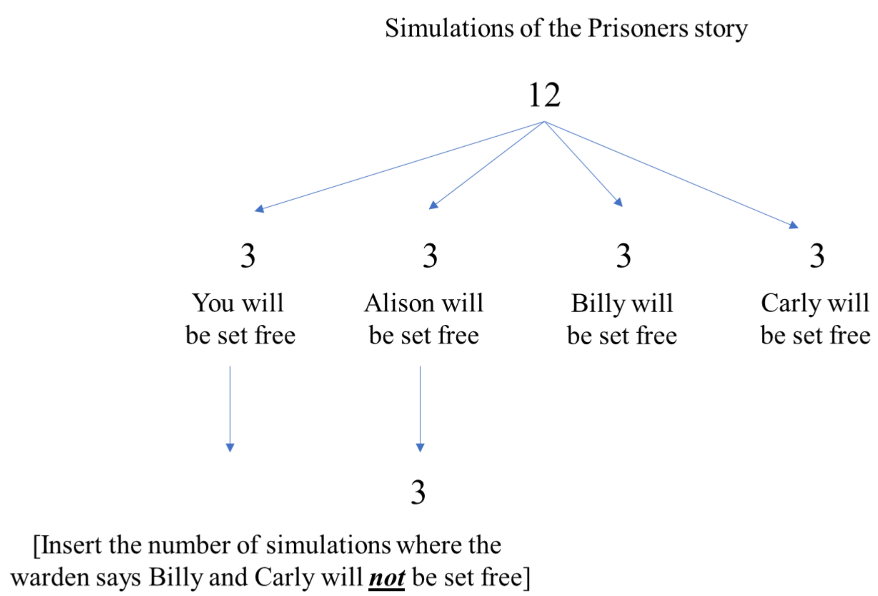
Now proportion the number of simulations where Alison will be set free by the probability of the warden's testimony that Billy and Carly will ***not*** be free if Alison was to be set free. (Remember, the probability is 100%.)



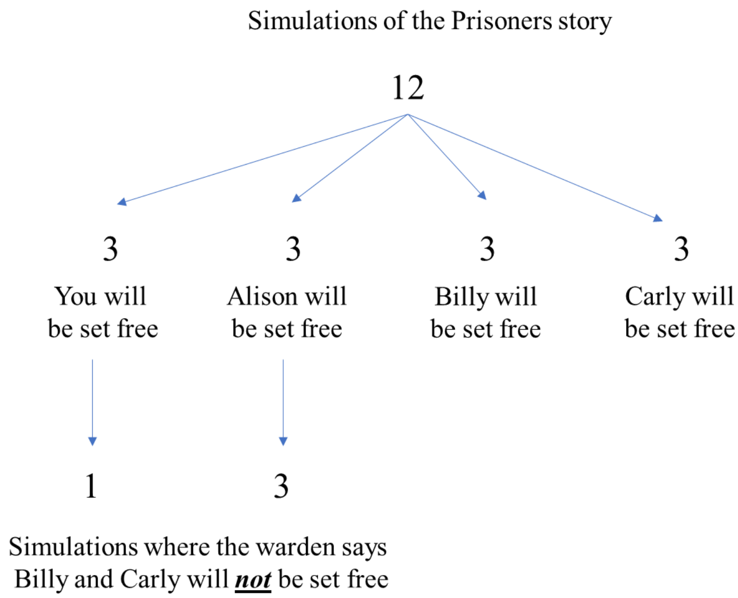
If you did that correctly, you should have something like what follows:



Now proportion the number of simulations where you are set free by the probability of the warden's testimony that Billy and Carly will ***not*** be free if you were to be set free. (Remember, the probability is 1/3.)



If you did that correctly, you should have something like this.



Now eliminate the irrelevant simulations, considering ***only*** the simulations where the wardens says Billy and Carly will ***not*** be set free:

If you did that correctly, you would have something like this:

You will be set free	Alison will be set free
1	3

Simulations where the warden says
Billy and Carly will ***<u>not</u>*** be set free

Now count the number of these simulations where Alison will be set free over the total number of remaining simulations where the warden says Billy and Carly will ***not*** be set free. This will give you the probability that Alison will be set free given the warden's testimony—a probability of 3/4.

So that is one way to use the mental simulations approach—with numerical representations.

We will now present you with a final problem involving probabilities. Please complete the problem by using the survey in the link. You are free to solve the problem in whatever way you think is fitting.

Appendix B: Proofs of Main Results

This appendix proves two results:

1. The law of likelihood
2. The equivalence of the results delivered by Bayes' theorem and the mental simulations approach

1. The Law of Likelihood

Our version of the law of likelihood states the following:

If $P(e|h_1) > P(e|h_2)$, then $\frac{P(h_1|e)}{P(h_2|e)} > \frac{P(h_1)}{P(h_2)}$

To prove this theorem of the probability calculus, we can prove the stronger theorem that:

$\frac{P(h_1|e)}{P(h_2|e)} > \frac{P(h_1)}{P(h_2)}$ if and only if $P(e|h_1) > P(e|h_2)$

To do this, first suppose the right-most condition holds—that is:

$$\frac{P(h_1|e)}{P(h_2|e)} > \frac{P(h_1)}{P(h_2)}$$

Then, by Bayes's theorem, this condition holds if and only if the following holds:

$$\frac{\frac{P(e|h_1)P(h_1)}{P(e)}}{\frac{P(e|h_2)P(h_2)}{P(e)}} > \frac{P(h_1)}{P(h_2)}$$

Next, we multiply both sides by $\frac{P(e)}{P(e)}$:

$$\frac{\frac{P(e|h_1)P(h_1)}{P(e)}}{\frac{P(e|h_2)P(h_2)}{P(e)}} \cdot \frac{P(e)}{P(e)} > \frac{P(h_1)}{P(h_2)} \cdot \frac{P(e)}{P(e)}$$

Which is the same as:

$$\frac{\frac{P(e|h_1)P(h_1)}{P(e)} \cdot P(e)}{\frac{P(e|h_2)P(h_2)}{P(e)} \cdot P(e)} > \frac{P(h_1)}{P(h_2)} \cdot 1$$

Which is equivalent to:

$$\frac{P(e|h_1)P(h_1)}{P(e|h_2)P(h_2)} > \frac{P(h_1)}{P(h_2)}$$

And this is equivalent to:

$$\frac{P(e|h_1)}{P(e|h_2)} \cdot \frac{P(h_1)}{P(h_2)} > \frac{P(h_1)}{P(h_2)}$$

And when both sides of the inequality are divided by $\frac{P(h_1)}{P(h_2)}$, we have the following:

$$\frac{P(e|h_1)}{P(e|h_2)} > 1$$

Then we can multiply both sides of the inequality by $P(e|h_2)$:

$$P(e|h_1) > P(e|h_2)$$

We have then proved that:

$\frac{P(h_1|e)}{P(h_2|e)} > \frac{P(h_1)}{P(h_2)}$ if and only if $P(e|h_1) > P(e|h_2)$

Our version of the law of likelihood then follows:

If $P(e|h_1) > P(e|h_2)$, then $\frac{P(h_1|e)}{P(h_2|e)} > \frac{P(h_1)}{P(h_2)}$

2. The Bayes' theorem/mental simulations equivalence

We can now turn to prove that the mental simulations approach delivers answers that always accord with Bayes' theorem.

To do this, we can first formally characterize the answer that is delivered by the mental simulations approach, and we show that this aligns exactly with the answer delivered by Bayes' theorem.

Formally Characterizing the Mental Simulations Approach:

First, mental simulations approach asks one to imagine N simulations. Let us denote the total number of simulations with N_T.

Further, it asks us to proportion the simulations where a given outcome is true by the prior probability of that outcome. Formally, let $\{h_1, \ldots, h_k\}$ be the set of *k* mutually exclusive outcomes, exactly one of which is true. Let N_{h_j} be the number of simulations where h_j is true for any h_j in $\{h_1, \ldots, h_k\}$. Then, by stipulation, the proportion of total simulations where a given outcome h_j is true is equal the prior probability of that outcome. So:

1) $\frac{N_{h_j}}{N_T} = P(h_j)$

The next step in the mental simulations approach is then to take all the simulations where a given outcome h_j is true, and to then proportion those N_{h_j} simulations by the probability of the evidence. Formally, let $N_{h_j \& e}$ be all the simulations for a given outcome h_j where *e* is true. Then, this step in the approach is to make the proportion of the simulations where h_j and *e* is true equal to the probability of the evidence given that outcome. So:

2) $\frac{N_{h_j \& e}}{N_{h_j}} = P(e|h_j)$

Then, we have to eliminate the simulations where the evidence is not true, considering only the total number of where the evidence is true. This number is given by the following equation:

3) $N_e = N_{h_1 \& e} + \cdots + N_{h_k \& e}$ where N_e is the total number of simulations where the evidence *e* is true

What this says is that the total number of simulations where the evidence is true is equal to the sum of all the simulations where the evidence is true across the simulations for all the exhaustive and mutually exclusive outcomes $\{h_1, \ldots, h_k\}$.

The mental simulations approach then tells us that we can calculate the probability of a given hypothesis h_j by calculating the proportion of simulations where h_j is true among all the simulations where *e* is true. Put formally, the mental simulations approach claims that:

4) $P(h_j|e) = \frac{N_{h_j \& e}}{N_e}$

So we have now characterized the steps in the mental simulations approach. Let us now aim to prove that, given the stipulations 1) and 2) in the earlier steps of the mental simulations approach, claim 4) is indeed true—that is to say, that the mental simulations approach agrees with Bayes' theorem:

$$P(h_j|e) = \frac{N_{h_j\&e}}{N_e} = \frac{P(e|h_j)P(h_j)}{P(e)}$$

Proof of the Bayes/Mental Simulations Agreement:

To prove this, let us start with Bayes' theorem:

$$P(h_j|e) = \frac{P(e|h_j)P(h_j)}{P(e)}$$

We can first prove that $P(e|h_j)P(h_j) = \frac{N_{h_j\&e}}{N_T}$ and then that $P(e) = \frac{N_e}{N_T}$. Since, by algebra, $\frac{\frac{N_{h_j\&e}}{N_T}}{\frac{N_e}{N_T}}$ is equivalent to $\frac{N_{h_j\&e}}{N_e}$, it will follow that $P(h_j|e) = \frac{P(e|h_j)P(h_j)}{P(e)} = \frac{\frac{N_{h_j\&e}}{N_T}}{\frac{N_e}{N_T}} = \frac{N_{h_j\&e}}{N_e}$.

First, to prove $P(e|h_j)P(h_j) = \frac{N_{h_j\&e}}{N_T}$. Recall stipulations 1) and 2) above:

1) $\frac{N_{h_j}}{N_T} = P(h_j)$

2) $\frac{N_{h_j\&e}}{N_{h_j}} = P(e|h_j)$

Given these stipulations, it follows that:

$$P(e|h_j)P(h_j) = \frac{N_{h_j\&e}}{N_{h_j}} \cdot \frac{N_{h_j}}{N_T}$$

By algebra then:

$$P(e|h_j)P(h_j) = \frac{N_{h_j\&e}}{N_{h_j}} \cdot \frac{N_{h_j}}{N_T} = \frac{N_{h_j\&e} \cdot N_{h_j}}{N_{h_j} \cdot N_T} = \frac{N_{h_j\&e}}{N_T}$$

We have then proved the first step—that $P(e|h_j)P(h_j) = \frac{N_{h_j\&e}}{N_T}$. Now to prove that $P(e) = \frac{N_e}{N_T}$.

Recall 3) above:

3) $N_e = N_{h_1\&e} + \cdots + N_{h_k\&e}$

Then, dividing both sides by N_T, we have the following:

$$\frac{N_e}{N_T} = \frac{N_{h_1\&e} + \cdots + N_{h_k\&e}}{N_T} = \frac{N_{h_1\&e}}{N_T} + \ldots + \frac{N_{h_k\&e}}{N_T}$$

Then recall the earlier theorem we proved:

$$P(e|h_j)P(h_j) = \frac{N_{h_j\&e}}{N_T}$$

It then follows that:

$$\frac{N_e}{N_T} = \frac{N_{h_1\&e}}{N_T} + \ldots + \frac{N_{h_k\&e}}{N_T} = P(e|h_1)P(h_1) + \cdots + P(e|h_k)P(h_k)$$

Yet we also know by the theorem of total probability that if $\{h_1, \ldots, h_k\}$ are exhaustive and mutually exclusive outcomes, then:

$$P(e) = P(e|h_1)P(h_1) + \cdots + P(e|h_k)P(h_k)$$

But since we have shown that:

$$\frac{N_e}{N_T} = P(e|h_1)P(h_1) + \cdots + P(e|h_k)P(h_k)$$

As desired, it follows that:

$$P(e) = \frac{N_e}{N_T}$$

We have now proved that $P(e|h_j)P(h_j) = \frac{N_{h_j\&e}}{N_T}$ and that $P(e) = \frac{N_e}{N_T}$. It then follows that:

$$P(h_j|e) = \frac{P(e|h_j)P(h_j)}{P(e)} = \frac{\frac{N_{h_j\&e}}{N_T}}{\frac{N_e}{N_T}} = \frac{N_{h_j\&e}}{N_e}$$

So we have proven that the mental simulations approach always agrees with Bayes' theorem—that is:

$$P(h_j|e) = \frac{P(e|h_j)P(h_j)}{P(e)} = \frac{N_{h_j\&e}}{N_e}$$

Appendix C: Computer Simulations of the New Monty Hall Problem

The article argues that there is a $\frac{10}{11}$ or 91% chance that door B conceals the prize in the new Monty Hall problem. Recall that the new Monty Hall problem is the same as the original Monty Hall problem, except that Monty Hall would have a 10% chance of opening door C when door A is selected and door A also conceals the prize.

This claim should be uncontroversial since it is derived directly from Bayes' theorem—the exact same mathematical machinery that tells us to switch doors in the original Monty Hall problem.

However, to further support this claim, the below program ran 100,000 computer simulations of the new Monty Hall problem. The results showed that door B concealed the prize approximately 91% of the time.

The code is included below so that the reader may replicate and verify this result for themselves.

The program runs on a probabilistic programming language called *WebPPL*—freely available at http://webppl.org/.

On it, the program implemented a method of computer simulation known as *rejection sampling*. This runs numerous simulations of the probabilistic setup, and it then rejects any simulations where the specified conditions are not met. In our case, the specified conditions were that exactly one of the three doors conceals the prize, that door A is selected, and that door C is then opened. In these conditions, door B concealed the prize 91% of the time.

Here is the code which the reader can implement and modify for their purposes. Note in particular the variable *L*—where *L* strands for 'the likelihood of opening the right-most door'. Adjusting this variable changes likelihood that Monty Hall would open the right-most door that is unselected and does not conceal the prize. If it is set to .5, then the setup is equivalent to the original Monty Hall problem, and the probability that door B conceals the prize is $\frac{2}{3}$. If it is set to .1, then the setup is equivalent to the New Monty Hall problem, and the probability that door B conceals the prize is $\frac{10}{11}$ or 91%.

Figure 4

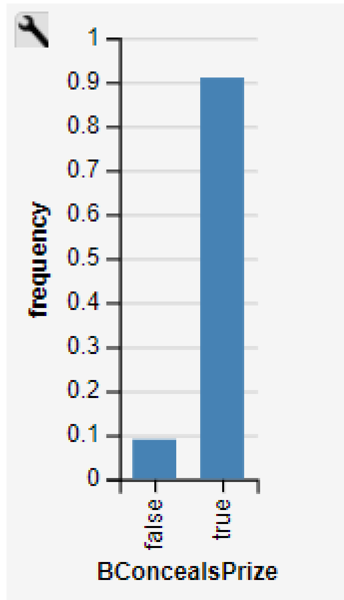
WebPPL Code for the New Monty Hall Problem Simulator

Not secure | webppl.org

```
//New Monty Hall Problem simulator

var SimulatorOutput = Infer(
//Enter "{method:'enumerate'}," as an argument to calculate exact probabilities,
//or use "{method:'rejection', 100000}" to run 100,000 simulations with rejection sampling.
  {method: 'rejection', samples:100000},
  function(){

//Prior probabilities for the prizes location
  var AConcealsPrize =  flip(1/3)
  var BConcealsPrize =  flip(1/3)
  var CConcealsPrize =  flip(1/3)

//This is a constraint to make A, B and C exhaustive and mutually exclusive hypotheses
//i.e. to make it so only one door conceals the prize
  condition(AConcealsPrize + BConcealsPrize + CConcealsPrize == 1)

//This causes the participant to randomly choose a door with a probability of one third

  var SelectedDoor = (flip(1/3) ? 'A':
                      flip(1/2) ? 'B':
                      'C')

//This variable specifies the likelihood that Monty Hall would open the rightmost door
//When set to .5, the simulator runs the original Monty Hall problem
//When set to .1, the simulator runs the new Monty Hall problem
  var L = .1

// Likelihoods for phase 2
  var DoorIsOpened = (SelectedDoor == 'A' && AConcealsPrize)? (flip(L)? 'C': 'B'):
                     (SelectedDoor == 'A' && BConcealsPrize)? 'C':
                     (SelectedDoor == 'A' && CConcealsPrize)? 'B':
                     (SelectedDoor == 'B' && AConcealsPrize)? 'C':
                     (SelectedDoor == 'B' && BConcealsPrize)? (flip(L)? 'C': 'A'):
                     (SelectedDoor == 'B' && CConcealsPrize)? 'A':
                     (SelectedDoor == 'C' && AConcealsPrize)? 'B':
                     (SelectedDoor == 'C' && BConcealsPrize)? 'A':
                     (flip(L)? 'B':'A')

//This makes the simulator consider only cases where door A is selected and
//door C is opened
  condition(SelectedDoor == 'A' && DoorIsOpened == 'C')

//This tells the simulator to return the frequency or probability with which door B conceals
//the prize according to the above conditions
  return{BConcealsPrize: BConcealsPrize}
 })

SimulatorOutput
```

Figure 5

WebPPL Code for the New Monty Hall Problem Simulator



Appendix D: Critical evaluation of explanations of experimental differences

The experimental results indicate that the participants in both experimental groups are more likely than participants in the control groups to give correct posteriors. But what are the various explanations for what it is about the training interventions that help them do better, and what is the evidence bearing on these explanations? This section reviews possible explanations in much greater detail.

Greater time and attention

One explanation is that participants in the experimental condition considered the details of the Monty Hall problem more slowly and carefully. Yet it is doubtful that slowness and attentiveness explain the responses for several reasons. First, as is visible in Appendix E, numerous participants rationales clearly describe the process of the mental simulations approach, thus suggesting they actually used the approach. Second, it is doubtful that participants can deduce the correct answers merely with more time and attention. After all, even bright minds like mathematician Paul Erdõs and many academics famously gave the wrong answers to the Monty Hall problem, and this is presumably despite them giving plenty of time and attention to the topic (Piattelli-Palmarini, 1994; Schechter, 1998; vos Savant, 1997). It is then improbable that participants in the experiments would, unlike these bright minds, get the right answer merely with more time and attention.

Internet searches

Another explanation is that the training material prompted participants to do internet searches which revealed the correct probabilities. This explanation appears to be plausible for at least three participants in Experiment 1, since they specified the correct posteriors but reported not using the mental simulations approach (their rationales also indicate no use of the mental simulations approach).

Yet this explanation cannot explain all participants' correct responses in the experimental conditions. This is for two reasons. First, in both experiments, the rationales of some participants clearly show detailed use of the mental simulations approach. Despite this, no online material provides instruction in how to use the approach, simply because the approach is an innovation of these experiments. Second, in the second experiment, no online material provides answers to the new Monty Hall problem, simply because, again, the new variation is an innovation of that experiment. So online searches do not explain the use of the mental simulations approach in some rationales nor correct answers to the new Monty Hall problem. Therefore, something else must explain why particular participants got the correct answers.

In any case, future experiments could better control for this variable by (1) restricting participants internet access, perhaps through in-person participation, or by (2) including other experimental conditions featuring nonsensical training and explanations of the problem of the prisoners, thereby seeing whether any kind of training can produce correct posteriors (perhaps via internet searches) or whether the mental simulations training is unique.

Mindless modification of training content, including analogical transfer

Another explanation is that participants got the correct answers by modifying and applying content from the training materials, but without any real understanding of the mental simulations approach. There are at least two mechanisms for how this could occur.

The first is that, in experiment 1, participants could somehow produce answers by analogizing from the problem of prisoners without understanding the mental simulations approach. To be sure, the problem of the prisoners could be an important part of the mechanism by which the mental simulations approach could have an effect even in the best-case scenario. After all, it is the very scenario by which the schema of the mental simulations approach is illustrated in experiment 1 to thereby impart an understanding of the approach. Consequently, it is unsurprising if it is part of the mechanism. This mechanism may receive some support from the rationales of some particular participants. For example, participant 442319305 wrote “However, I thought back to what I learned with the Prisoners Story. I simply replaced the prison guard with Monty in this situation.” Similar support may also come from participants 868847741 and 449787739, since they both explicitly mention the problem of the prisoners too.

However, the main question is whether the problem of the prisoners suffices to help participants answer the Monty Hall problem *without understanding the mental simulations approach,* and it seems that that is where the support for such an explanation ends. Instead, there are two challenges for this explanation. First, there is no other indication that this explanation is true, indications we might have expected. For example, nothing in the rest of participant 442319305’s comment suggests they duplicated the structure of the prisoners’ problem without understanding the mental simulations approach. In providing their answer, they said “Here is where I had to assume Monty knew which door he was going to open, which left the probability that the door I had chosen had the other goat (since I'm guessing he didn't pick 50/50). This means that the odds are actually in favor of the prize being behind door B, so I decided I should probably switch. I at that point thought I either had a 1/3 chance of being correct in choosing door A, or a 2/3 change [sic] of being correct if I switched and chose door B.” Now, it is not clear that this rationale makes sense or reflects an understanding of the mental simulations approach, but what is clear is that it does not look like this person straightforwardly tailored the structure of the problem of the prisoners to get a 2/3 probability that Monty Hall would switch. A similar point applies for participants 868847741 and 449787739 as well.

Second, it is not clear that participants would know how to modify the structure of the problem of the prisoners to get an adequate answer in the Monty Hall problem. As was mentioned in the methods section, “participants could not solve the Monty Hall problem merely by mindlessly repeating answers to the story of the prisoners” and some “additional understanding is needed”. This is because the Monty Hall problem and the problem of the prisoners are importantly different.

They have a different number of possible outcomes (3 vs. 4), different prior probabilities (1/3 vs. 1/4), different likelihoods ({1, 1/2} vs. {1, 1/3}) and consequently different posterior probabilities ({1/3, 2/3} vs {1/4, 3/4}). If some participants were neither smart nor motivated enough to understand the mental simulations approach, it seems improbable that they were smart and motivated enough to create a correct solution to the Monty Hall approach despite these significant differences—and this in the space of a short experimental task. The rationales for the participants also do not suggest any obvious way that they could have got the right answers via a simple modification of the problem of the prisoners either. At best, this might happen in a minority of cases, but it is still very unlikely.

Instead, it is more probable that participants either did internet searches as per the previous explanation (and perhaps created a fictional rationalization for their answers in terms of the problem of the prisoners) or instead have some understanding of the mental simulations approach. Furthermore, as we shall consider in more detail shortly, the analogizing explanation is further challenged by the fact that the multiple rationales depict use of the mental simulations procedures, something that would be both surprising and redundant if they merely analogized the problems without understanding the mental simulations approach. But regardless, just because they have some understanding of the approach, it does not entail that they have a full or adequate understanding.

That is why participants could have applied content from the training materials via a second and more likely mechanism: they could have understood the mental simulations approach well enough to apply it, but not well enough to understand why the answers it generates are the right ones. This mechanism competes with the third explanation we will consider: that some participants used the mental simulations, and actually understood it. We will consider evidence bearing on their understanding in the next section.

Understanding of the mental simulations approach

There is strong evidence that at least some participants used the mental simulations approach. This can be seen from the highlighted rationales in the next appendix, since all display some evidence of using the approach, albeit some rationales (e.g. participant 866122681) show clearer evidence than others (e.g. participant 749649506). Other participants may also understand the approach, even if their rationales give no clear evidence of using the mental simulations approach (e.g. participants 646622749 and 326363669).

The question is, then, did these participants really understand the mental simulations approach? To answer this question, it is useful to ask what it takes to *really* understand the mental simulations approach. We could consider there to be three levels of understanding to the mental simulations approach.

One level is the algorithmic level: participants can follow the algorithm to generate correct answers using the mental simulations approach. At this level, a participant can apply the approach but does not necessarily understand why it delivers the correct answers that it does. Participant

400307542 may have understood the approach merely at this level, since their self-reported "thought process involved trying to utilize the mental simulation method and not quite grasping it in this given scenario in all honesty".

However, a second level is the likelihood level: they can follow both the algorithm, and they realize that the algorithm delivers correct answers because it incapsulates the fact that one hypothesis makes the evidence more likely than the other. Some participants seem to show this kind of understanding. For example, participant 646622749 reported understanding "Moderately well" why their answer was correct: "If A had the door [sic], B and C are equally likely to be opened. However, if B has the door [sic] then C has a 100 percent chance of being opened. This evidence makes it statistically more likely for B to have the prize behind it." Similarly, participant 326363669 reported understanding "Moderately well" why their answer was correct: "When Monty Hall eliminates a door, the door I did not pick now has a 2/3 chance of the prize being behind it, because there is a 100% chance that he would have elimated [sic] the door he did if the prize was behind it, while there is only a 50% chance he would have eliminated the door he did if the prize was behind the door I chose".

The third level is the hypothetical frequency level: they can follow the algorithm, understand that the likelihoods make one hypothesis more probable than the other, and they also understand that the likelihoods do this in virtue of making (along with the priors) the relevant hypothesis more frequently true if the probabilistic situation were to happen a number of times. Some rationales appear to be consistent with the possibility. For example, user 19922931 reported understanding "Very well" why their answer was correct: "I imagined six simulations, two for each door. Then I figured out how many of those two would likely result in the evidence (that C was not the prize)." This is consistent with using the simulations exercise to determine the hypothetical frequency with which the evidence involves a given outcome.

The rationales would seem to suggest that at least some participants had a deeper understanding. That said, it is difficult to test human understanding, as anyone familiar with Chinese room thought experiments or the Turning test can testify. Perhaps future experiments could better probe participants' understanding by utilizing interview methods or multiple problems.

Causal Bayesianism

Following Krynski and Tenenbaum (2007), another explanation of the results is that humans are conforming to a causal Bayesian framework of reasoning. Krynski and Tenenbaum (2007) theorize that humans are better able to address particular reasoning tasks when those tasks are phrased in ways that encourage them to perform causal Bayesian inference—that is, in ways that encourage them to generate a correct causal model of the scenario, to assign values to the parameters of that model and to perform Bayesian inference over that model to obtain the posteriors.

Some of their experimental manipulations are ones where, in their words, the existence of an alternative causal "mechanism seems plausible" (Krynski and Tenenbaum, 2007, p. 439). For example, in experiments 1 and 2, they find participants give better answers when they consider the likelihood of a false positive test given that one has a *benign cyst* rather than given that one simply does not have cancer. In experiment 3, they find participants give better answers when they consider the likelihood of a false identification of a taxi's color given that it has *faded paint* rather than given that there was some unspecified perceptual error. The idea is that a causal mechanism seems plausible, or at least more plausible, in the former cases where a causal mechanism is specified (e.g. benign cysts or faded paint) compared to the latter cases where no causal mechanism is specified (e.g. no cancer or an unspecified perceptual error). (Experiment 4 also involves variations in the specification of causal mechanisms.) They claim that participants perform better under these circumstances where causal mechanisms (and hence a causal model) are clearly specified.

Yet the results presented in this paper's experiments are not like these manipulations. The experimental and control conditions do not differ at all in terms of the specification of causal mechanisms or models: for example, both participants are merely told the likelihoods of Monty Hall opening a given door without *any differences in mechanisms*. Additionally, most participants in both conditions are aware of the likelihoods (with participants in the Experiment 1's control group being even more so!); consequently, differing awareness of the likelihoods does not explain differences in correct posteriors. Instead, what differs between the participants is solely their training in the mental simulations approach.

Of course, one might think Krynski and Tenenbaum's (2007) framework is somewhat similar to the mental simulations approach. Arguably, the mental simulations approach involves generating a causal model that can produce particular kinds of evidence given various causes (such as door C being opened given either door A or door B concealing the prize), assigning values to the parameters of that model (such as prior probabilities and likelihoods), and performing Bayesian-like inferences over that model. To that extent, one might think it is consistent with a causal Bayesian framework.

However, the mental simulations approach is significantly different to their experimental manipulations for various reasons. First, it furthermore involves calculating the posteriors by simulating hypothetical frequencies—especially the frequency with which a given outcome is true among the simulations where the evidence has been generated. Second, the approach explicitly highlights the importance of likelihoods, not just base rates. Third, as mentioned, unlike Krynski and Tenenbaum's (2007) experimental manipulations, improvements in reasoning do not result from the introduction of specified causal mechanisms; instead, improvements result just from the simulation procedure.

Appendix E: Rationales for those with correct posteriors

This appendix presents the unedited rationales from participants who specified the correct posteriors in both experiments. This helps estimate participants' use of the mental simulations approach, their understanding and the reasons for their responses. Rationales have been highlighted that appear to display the mental simulations approach. Note, however, that participants may also have used the mental simulations approach, even if their rationales do not clearly display this.

Table 8

Self-reported rationales in Experiment 1

Participant ID	Reported Using Mental Simulations Approach	Rationale (unedited)
537835730	Yes	I had a bit of trouble wrapping my mind around the process, but I gave it my best shot. I first drew out the 3 doors, initially giving each a 33% chance. Upon being told that door C did not contain the prize, I eliminated the option and left doors A and B each with a 50% probability. However, we were told that the door which did not conceal the prize would open. This made me think that door B was less likely to contain the prize, so out of the trials, door A was most likely to be correct.
442319305	Yes	So originally I thought the answer must be 50/50 in regards to which of the remaining doors has the prize, since Monty revealed one of the doors which had a goat, going from 1/3 probability to 1/2 probability. However, I thought back to what I learned with the Prisoners Story. I simply replaced the prison guard with Monty in this situation. Since I had chosen a door already, it meant Monty had no choice but to open a door with the goat- he was not going to reveal the door with the prize even though he knew which one it was, just like the prison guard knew who was going to be set free, but could not outright reveal who. In this case, since I had chosen door A, Monty either knew exactly which of the remaining doors he was going to open (B or C) or head had to simply choose 50/50 between them- if I chose the door with the prize (A). Here is where I had to assume Monty knew which door he was

		going to open, which left the probability that the door I had chosen had the other goat (since I'm guessing he didn't pick 50/50). This means that the odds are actually in favor of the prize being behind door B, so I decided I should probably switch. I at that point thought I either had a 1/3 chance of being correct in choosing door A, or a 2/3 change of being correct if I switched and chose door B.
744418722	Yes	If I did the lesson correct; Given the scenario of me picking A and the host opening C. There was only one way it worked that A was the correct answer. Because the other option was the host opening B (which didn't happen). But with B being the correct answer and me picking A then the host has to open C both times (if we simulate it twice to account for the two options if it is A). The result is 1 vs 2. Or 1/3 vs 2/3. Attempting to follow the lesson; I broke it into the three prior possibilities of A, B, or C. Then with the information of the scenario I figured there were only two outcomes. For A, only one worked. And for B both worked because it would have to always be the same. At least that's what I'm getting out of it anyway.
199229318	Yes	I imagined six simulations, two for each door. Then I figured out how many of those two would likely result in the evidence (that C was not the prize). For A, this was 1/2. For B it was 2/2 (because A could not be revealed), and for C it was 0/2. So eliminating the simulations that did not support the evidence, we get 1 case of A being the prize and 2 cases of B being the prize. So it looks like B is more likely, with a 2/3 probability.
646622749	Yes	I was thinking that initially it is equally likely for any of the doors to have the prize. Once I pick a door (A), nothing has changed beside the fact that Monty cannot open A. Of course, A could always have the prize in it regardless, but Monty's next move can affect probability based on the evidence received. If A had the door, B and C are equally likely to be opened. However, if B has the door then C has a 100 percent chance of being opened. This evidence makes it statistically more likely for B to have the prize behind it.
749649506	Yes	First, I struggled to grasp what steps I'd need to take and how the proportioning would work in this scenario. I considered the possible outcomes for the first scenario (A) and determined the likelihood of Door C being opened, then did the same for Door B, with the added information that he wouldn't open the door you chose no matter what.
866122681	Yes	First, I se tup 6 simulations, then I made each door equally possible to have the prize so I placed two circles under each door simulating 1/3 possibility of having the prize. I then crossed out one circle under door A because that was chosen, next, I crossed

		out two circles under door C as Monty revealed that to be a goat. That left 1/3 possibility of door A having the prize and 2/3 possibility of door B having the prize.
568797575	Yes	This is horrible. You should have hired someone to explain things more concretely without reference to terms like outcome. anyway, if you know C is eliminated it yields possibilities of a and b. Now i llost it. there should have been more examples. I believe the simulation is 2/3 for B and 1/3 for a after eliminating C but cannot be sure. I just don't understand thsi after tolerating those lenghty videos.
868847741	Yes	So there were 3 doors initially and whatever door I choose first, the host will choose a door without a prize to show me a goat. That was the prior 50% thinking. Now that 1 door has been eliminated out of the probability, I have two options like the prisoner situation. In that, there was a group that was low probability (me being free at 25%) and one that was high (Allison being free at 75%). If I combine the doors into 2 groups, the low probability group has got to be 1/3 (1 door choice) while the other one was 2/3 (2 door choices including the goat door sort of). I think I'm doing it wrong, though because it still makes no sense to me.
449787739	Yes	YIKES. OKAY I BASED IT ON THE PRISONERS PROBLEM. ME BEING FREED EQUALS DOOR A. DOOR B EQUALS ALLISON, DOOR C EQUALS CARLY AND BILLY. SINCE I ALREADY KNEW THE OUTCOME OF THE PRISONER PROBLEM, I JUST ASSUMED THE OUTCOME WITH MONTY TO BE THE SAME BASED ON A REDUCED NUMBER OF SIMULATIONS... 9 VS 12 OR 2/3 VS 3/4
326363669	Yes	The prior probability of the prize being behind each door is 1/3 for each door. When Monty Hall eliminates a door, the door I did not pick now has a 2/3 chance of the prize being behind it, because there is a 100% chance that he would have elimated the door he did if the prize was behind it, while there is only a 50% chance he would have eliminated the door he did if the prize was behind the door I chose.
400307542	Yes	The thought process involved trying to utilize the mental simulation method and not quite grasping it in this given scenario in all honesty.
185541643	Yes	I tried to think through the simulation scenarios according to the materials provided. I eliminated door C and then thought through the scenarios of how the doors selection is done by the host depending on where the prize was located.

Table 9

Self-reported rationales in Experiment 2

Participant ID	Experimental Condition	Rationale (unedited)
338237458	Mental simulations	Just did the math like you said before: one out of ten chances so 1 for A ten out of ten chances so 10 for B 1 + 10 = 11 then it's just 1/11 for A and 10/11 for B
598284154	Mental simulations	Out of 30 runs with A picked, A will have the prize 10 times. Of those 10 times, C will be opened once. B will also have the prize 10 times, and of those 10 times C will be opened all the times. C cannot be opened if C has the prize, so an open door C must reflect either A or B. It will reflect a B prize 10 times and an A prize 1 time.
451237683	Mental simulations	I visualized the plot of thirty simulations. I noted that there was a 10% chance that Monty would open door C if the prize was behind door A. This is one of thirty dots. If the prize is behind door B there is a 100% chance that Monty will open door C. This is ten of thirty dots. Monty revealed a goat was behind door C. I can eliminate those ten dots. Of the remaining twenty dots, Monty would have opened door C in 11 situations. In the other 9 he would have opened door B and revealed nothing. Of the eleven situations in which door C is opened 10 are the the prize is concealed behind door B.
578638111	Mental simulations	If there is a ten percent chance, that means that one out of ten times door c will be opened if door a is chosen and has the prize. However door c will be opened ten out of ten times if door a is chosen but door b holds the prize. this is a total of eleven, with 1/11 chance of the prize being a and 10/11 chance of it being in b.
110491721	Mental simulations	If I selected door A and the prize is hidden behind door A, there was a 10% probability that he would have opened the rightmost door, C. If I selected door A and the prize was hidden behind door B, then he would have had to open door C 100% of the time. Therefore, if door C is revealed, then I know that he would have only opened door C 10% of the time if the prize is behind door A and he would have opened it 100% of the time if the prize is behind B. So I know there is a much higher chance (10/11X) of the prize being behind B when he reveals C.
291572899	Mental simulations	I broke it down into 10 trials for each A, B, and C like shown in the examples. All 10 of will choose C if behind Door B, versus only 1/10 will choose C if behind Door A. So 10/11 if behind B, and 1/11 if behind A.

480108439	Mental simulations	C will be opened 1 in 10 of the times A is correct. C will be opened 10 in 10 times if B is correct. So, of all of the instances where C is opened (1 + 10) only 1 will occur if A is correct. Hence, the probability A is correct is 1/11. The probability of B being correct would be the compliment of the probability A is correct, or 10/11.
662965787	Mental simulations	Considering the reduced probability of Monty opening the furthest right door after door A is selected(30% to 10%), in combination with the certainty that Monty opens door C to avoid revealing that door B actually contains the prize, leads me to believe that door C containing a goat increases the probability that I was initially correct in selecting door A.
992298791	Mental simulations	I used the same approach. If the prize is in A, and we chose A, Monty most likely would have opened B, not C (10% chance to open C). But if it's behind B, and we chose A, he has to open C (100%). So doing the same approach with 30 simulations, eliminating 10 where the prize is behind C, then eliminating the ones where the prize is behind A and B is opened takes away 9, so there are total 11. So 1/11 and 10/11 respectively.
818754973	Probability accrual	At first, I thought it was easy and chose what I remembered from the last game which was 1/3 for door A and 2/3 for door B. But then I started thinking about the new information, which would mean that it is only around a 10% chance that the prize is behind door A, and 90% chance that it is behind door B. I thought about this because in the new instructions it says that if you choose door A and door A conceals the prize, Monty will still open door C with a 10% probability. If you select door A and door B conceals the prize, Monty must open door C. So that would mean that since he opened door C, there is a 10% chance that door A has the prize leaving a 90% chance that door B has the prize. So I figured that the 2/3 and 1/3 option couldn't be right given these new instructions. 1 of 11 is around 9-10%, and 10/11 is around 90-91%. I thought those were the closest numbers to exactly 10 and 90%, so I chose them. This was really confusing, though. This really threw me for a loop.
352429990	Possible models	Using this scenario, I have chosen A and a goat is revealed in C. If I am correct, there is a 10% chance that C will be revealed. If I am wrong, then C is the only door that Monty could have opened as it is the only one not selected/with a prize. As C being chosen is a low percentage (10) in the chance of being right, there is a greater probability that I am wrong and should switch my choice to door B. Considering the fact that regardless of revealing, I have a greater probability to win if I switch, the probability of it being A is relatively low.
464562185	Control	I didnt quite know how to calculate the odds, i just figured it is a lot more likely that the price will be behind door B, since Monty

has to select the non winning door if i picked the door that does not have the prize behind it

References

Bar-Hillel, M. (1980). The base-rate fallacy in probability judgments. *Acta Psychologica*, *44*(3), 211–233. https://doi.org/10.1016/0001-6918(80)90046-3

Bramwell, R., West, H., & Salmon, P. (2006). Health professionals' and service users' interpretation of screening test results: Experimental study. *BMJ*, *333*(7562), 284.

Burns, B. D., & Wieth, M. (2004). The Collider Principle in Causal Reasoning: Why the Monty Hall Dilemma Is So Hard. *Journal of Experimental Psychology: General*, *133*(3), 434–449. https://doi.org/10.1037/0096-3445.133.3.434

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). L. Erlbaum Associates.

Cohen, J. (1994). *The Earth Is Round (p < .05)*.

De Neys, W., & Verschueren, N. (2006). Working Memory Capacity and a Notorious Brain Teaser. *Experimental Psychology*, *53*(2), 123–131. https://doi.org/10.1027/1618-3169.53.1.123

Eddy, D. M. (1982). *Probabilistic reasoning in clinical medicine: Problems and opportunities* (pp. 249–267).

Falk, R., & Greenbaum, C. W. (1995). Significance Tests Die Hard: The Amazing Persistence of a Probabilistic Misconception. *Theory & Psychology*, *5*(1), 75–98. https://doi.org/10.1177/0959354395051004

Gardner, M. (1959). Mathematical Games: Problems involving questions of probability and ambiguity. *Scientific American*, 174–182. https://doi.org/10.1038/scientificamerican1059-174

Gigerenzer, G., Gaissmaier, W., Kurz-Milcke, E., Schwartz, L. M., & Woloshin, S. (2007). Helping doctors and patients make sense of health statistics. *Psychological Science in the Public Interest*, *8*(2), 53–96.

Gigerenzer, G., & Hoffrage, U. (1995). How to improve Bayesian reasoning without instruction: Frequency formats. *Psychological Review*, *102*(4), 684.

Gigerenzer, G., & Hoffrage, U. (1999). *Overcoming difficulties in Bayesian reasoning: A reply to Lewis and Keren (1999) and Mellers and McGraw (1999)*.

Gilovich, Thomas., Griffin, D. W., & Kahneman, D. (2002). *Heuristics and biases: The psychology of intuitive judgment*. Cambridge University Press.

Granberg, D., & Dorr, N. (1998). Further Exploration of Two-Stage Decision Making in the Monty Hall Dilemma. *The American Journal of Psychology*, *111*(4), 561. https://doi.org/10.2307/1423551

Hacking, I. (2016). *Logic of statistical inference*. Cambridge University Press.

Hammerton, M. (1973). A case of radical probability estimation. *Journal of Experimental Psychology*, *101*(2), 252–254. https://doi.org/10.1037/h0035224

Hawthorne, J. (2018). *Supplement to Inductive Logic: Likelihood Ratios, Likelihoodism, and the Law of Likelihood*. The Stanford Encyclopedia of Philosophy (Spring 2018 Edition). https://plato.stanford.edu/entries/logic-inductive/sup-likelihood.html

Hoffrage, U., Gigerenzer, G., Krauss, S., & Martignon, L. (2002). Representation facilitates reasoning: What natural frequencies are and what they are not. *Cognition*, *84*(3), 343–352.

Hoffrage, U., Hafenbrädl, S., & Bouquet, C. (2015). Natural frequencies facilitate diagnostic inferences of managers. *Frontiers in Psychology*, *6*, 642.

Hoffrage, U., Krauss, S., Martignon, L., & Gigerenzer, G. (2015). Natural frequencies improve Bayesian reasoning in simple and complex inference tasks. *Frontiers in Psychology*, *6*, 1–14. https://doi.org/10.3389/fpsyg.2015.01473

Hoffrage, U., Lindsey, S., Hertwig, R., & Gigerenzer, G. (2000). Communicating statistical information. In *Science* (Vol. 290, Issue 5500, pp. 2261–2262). American Association for the Advancement of Science.

Joyce, J. M. (2010). A defense of imprecise credences in inference and decision making. *Philosophical Perspectives*, *24*(1), 281–323. https://doi.org/10.1111/j.1520-8583.2010.00194.x

Kalinowski, P., Fidler, F., & Cumming, G. (2008). Overcoming the inverse probability fallacy: A comparison of two teaching interventions. *Methodology*, *4*(4), 152–158. https://doi.org/10.1027/1614-2241.4.4.152

Kaplan, M. (1996). *Decision theory as philosophy*. Cambridge University Press.

Krauss, S., & Wang, X. T. (2003). The psychology of the Monty Hall problem: Discovering psychological mechanisms for solving a tenacious brain teaser. *Journal of Experimental Psychology: General*, *132*(1), 3–22. https://doi.org/10.1037/0096-3445.132.1.3

Krynski, T. R., & Tenenbaum, J. B. (2007). The Role of Causality in Judgment Under Uncertainty. *Journal of Experimental Psychology: General*, *136*(3), 430–450. https://doi.org/10.1037/0096-3445.136.3.430

Kurzenhäuser, S., & Hoffrage, U. (2002). Teaching Bayesian reasoning: An evaluation of a classroom tutorial for medical students. *Medical Teacher*, *24*(5), 516–521.

Liu, A.-Y. (1975). Specific information effect in probability estimation. *Q Perceptual and Motor Skills*, *4*(1), 475–478.

Meacham, C. J. G. (2014). Impermissive Bayesianism. *Erkenntnis*, *79*(6), 1185–1217. https://doi.org/10.1007/s10670-013-9547-z

Phillips, L. D., & Edwards, W. (1966). Conservatism in a simple probability inference task. *Journal of Experimental Psychology*, *72*(3), 346.

Piattelli-Palmarini, Massimo. (1994). *Inevitable illusions: How mistakes of reason rule our minds*. Wiley.

Prasanta S. Bandyopadhyay. (2011). *Philosophy of Statistics.* North Holland.

Rescorla, M. (2020). A Realist Perspective on Bayesian Cognitive Science. In A. Nes & T. Chan (Eds.), *Inference and Consciousness* (pp. 40–73).

Saenen, L., Heyvaert, M., Van Dooren, W., Schaeken, W., & Onghena, P. (2018). Why Humans Fail in Solving the Monty Hall Dilemma: A Systematic Review. *Psychologica Belgica*, *58*(1), 128–158. https://doi.org/10.5334/pb.274

Saenen, L., Van Dooren, W., & Onghena, P. (2015). A randomised Monty Hall experiment: The positive effect of conditional frequency feedback. *Thinking & Reasoning*, *21*(2), 176–192. https://doi.org/10.1080/13546783.2014.918562

Schechter, B. (1998). *My brain is open: The mathematical journeys of Paul Erdõs*. Simon and Schuster.

Tubau, E., Aguilar-Lleyda, D., & Johnson, E. D. (2015a). Reasoning and choice in the Monty Hall Dilemma (MHD): Implications for improving Bayesian reasoning. *Frontiers in Psychology*, *6*, 353. https://doi.org/10.3389/fpsyg.2015.00353

Tubau, E., Aguilar-Lleyda, D., & Johnson, E. D. (2015b). Reasoning and choice in the Monty Hall Dilemma (MHD): Implications for improving Bayesian reasoning. *Frontiers in Psychology*, *6*, 353. https://doi.org/10.3389/fpsyg.2015.00353

Tubau, E., Alonso, D., & Alonso, D. (2003). Overcoming illusory inferences in a probabilistic counterintuitive problem: Th role of explicit representations. *Memory & Cognition*, *31*(4), 596–607. https://doi.org/10.3758/BF03196100

Tversky, A., & Kahneman, D. (1973). Availability: A Heuristic for Judging Frequency and Probability122. In *COGNlTIVE PSYCHOLOGY* (Vol. 5).

Villejoubert, G., & Mandel, D. R. (2002). The inverse fallacy: An account of deviations from Bayes's theorem and the additivity principle. *Memory and Cognition*, *30*(2), 171–178. https://doi.org/10.3758/BF03195278

vos Savant, M. (1997). *The power of logical thinking*. St. Martin's Press.

White, R. (2010). Evidential Symmetry and Mushy Credence. In *Oxford Studies in Epistemology* (pp. 161–186). Oxford University Press.

Wilcox, J. (2019). An Argument for the Principle of Indifference and Against the Wide Interval View. *The Journal for General Philosophy of Science*, *50*(4).