Supplementary Material

On the benefits of pension plan consolidation: Understanding the impact of full plan mergers

Abstract

This study investigates the benefits and drawbacks of pension plan consolidation by quantifying the impact of mergers of heterogeneous plans on different stakeholders in a unique Canadian implementation of defined benefit plans. Using a comprehensive framework that combines a realistic economic scenario generator, a stochastic mortality model that captures differences among subpopulations, a cost model with economies of scale, and a dynamic asset allocation methodology, we evaluate the combined effect of asset- and liability-side changes on three groups of measures: plan-related risk measures assessing profits from an economic capital perspective, consumption-based metrics to understand the impact on members, and contribution risk measures capturing the risk from the employer's viewpoint. We apply the framework to a hypothetical and empirically relevant merger and find that consolidation is favourable under most circumstances: the positive impacts of better diversification and economies of scale continue to outweigh the negative effects of heterogeneity even when the merging plans have different mortality expectations, different maturity levels, or modest differences in initial funded ratios.

Keywords: Retirement, Defined benefit pension schemes, Economies of scale, Simulation models, Economic capital

JEL Classification: J3, C63, C50

SM.A Additional details on the economic scenario generator

Bégin (2021) recently proposed a new economic scenario generator (ESG) for the actuarial profession based on three main principles: (1) a cascade structure, (2) observable regime-switching dynamics based on the monetary policy, and (3) observable dynamic variances given by nonlinear asymmetric GARCH dynamics in the spirit of Engle and Ng (1993).[1] The ESG used in this study is a modification of this model. Note that each time series is observed on a monthly basis (and months are denoted by t'). We append an asterisk to some processes introduced below to signal that these are monthly values, and they should not be confused with the annual values used in the article.

Unlike the (latent) Markov chains typically used in the literature, the chain used in this model is observed. Let $\{m^*_{t'}\}_{t'=0}^{T}$ be a discrete-time observable Markov chain with three states: tightening or *upward* (u), status quo (s), and accommodating or *downward* (d). The transition probabilities of this Markov chain are given as follows:

$$\mathbf{\Pi} = \begin{bmatrix} p_{\mathrm{uu}} & 1 - p_{\mathrm{uu}} & 0 \\ p_{\mathrm{su}} & 1 - p_{\mathrm{su}} - p_{\mathrm{sd}} & p_{\mathrm{sd}} \\ 0 & 1 - p_{\mathrm{dd}} & p_{\mathrm{dd}} \end{bmatrix}.$$

The states of this Markov chain are inferred from a reference rate $R^*_{t'}$—fixed by the central bank—following Engle et al. (2017).

The price inflation dynamics are similar to the autoregressive model of order one—or AR(1) for short—used in virtually all previous models proposed in the literature, with one main distinction: the long-run level of the price inflation is regime-dependent (i.e., $\mu_{q,\mathrm{u}}$ in the upward regime, $\mu_{q,\mathrm{s}}$ in the status quo regime, and $\mu_{q,\mathrm{d}}$ in the downward regime). A similar strategy is applied to the wage inflation and the dividend yield dynamics. The correlation between price and wage inflation innovations is assumed to be non-zero.

The short rate model relies on a piecewise transformation that deals differently with high and low rates: it uses a linear transform for higher rates and a logarithmic transform for lower rates.[2] Like the price inflation, wage inflation, and dividend yield models discussed above, the transformed short rate is modelled by an AR(1) model with a regime-dependent mean level. However, the homoskedastic variance is replaced by a GARCH dynamic variance. Non-nil correlations are also

[1] Monetary policy is known to impact multiple facets of the economy, such as the short rate (see, e.g., Engle et al., 2017) and stock returns (see, e.g., Ioannidis and Kontonikas, 2008).

[2] This approach is at the interface of the two most popular approaches for interest rate modelling: Wilkie (1986, 1995) uses a logarithm transformation to ensure that real interest rates are positive, whereas Ahlgrim et al. (2005) allow for negative rates.

Table SM.1: **Summary of the economic scenario generator.**

Monetary Policy

$$m^*_{t'} = \begin{cases} \text{u} & \text{if } \exists\, u \in [t'-3, t'] \text{ and } v \in [t', t'+3] \text{ such that } R^*_{t'} - R^*_u > 0 \text{ and } R^*_v - R^*_{t'} > 0 \\ \text{d} & \text{if } \exists\, u \in [t'-3, t'] \text{ and } v \in [t', t'+3] \text{ such that } R^*_{t'} - R^*_u < 0 \text{ and } R^*_v - R^*_{t'} < 0 \\ \text{s} & \text{otherwise} \end{cases}$$

where $R^*_{t'}$ is the month-t' policy rate

Data: Bank of Canada policy rate (Source: Bloomberg)

Price Inflation

$$q^*_{t'} = \mu_{q,m^*_{t'}} + a_q(q^*_{t'-1} - \mu_{q,m^*_{t'}}) + \sigma_q\, \varepsilon_{q,t'}, \text{ where } \varepsilon_{q,t'} \sim \mathcal{N}(0,1)$$

Data: Monthly consumer price index, not seasonally adjusted (Source: Statistics Canada)

Wage Inflation

$$w^*_{t'} = \mu_{w,m^*_{t'}} + a_w(w^*_{t'-1} - \mu_{w,m^*_{t'}}) + \sigma_w\, \varepsilon_{w,t'}, \text{ where } \varepsilon_{w,t'} \sim \mathcal{N}(0,1)$$

Data: Total hourly remuneration for Canada (Source: Bloomberg)

Short Rate

$$\tilde{r}^*_{0,t'} \equiv \mathrm{T}_r\,(r^*_{0,t'}) = \begin{cases} r^*_{0,t'} & \text{if } r^*_{0,t'} > 0.005 \\ c_{r,0} + c_{r,1}\log\left(r^*_{0,t'}\right) & \text{if } r^*_{0,t'} \leq 0.005 \end{cases}, \text{ where } r^*_{0,t'} \text{ is month-}t' \text{ short rate}$$

$$\tilde{r}^*_{0,t'} = \mu_{r,m^*_{t'}} + a_r\left(\tilde{r}^*_{0,t'-1} - \mu_{r,m^*_{t'}}\right) + \sigma_{r,t'}\,\varepsilon_{r,t'}, \text{ where } \varepsilon_{r,t'} \sim \mathcal{N}(0,1)$$

$$\sigma^2_{r,t'+1} = \sigma^2_r + \alpha_r\left(\left(\sigma_{r,t'}\,\varepsilon_{r,t'} - \sigma_{r,t'}\gamma_r\right)^2 - \sigma^2_r\left(1+\gamma^2_r\right)\right) + \beta_r\left(\sigma^2_{r,t'} - \sigma^2_r\right)$$

Data: 3-month zero-coupon bond risk-free yields (Source: Bank of Canada)

Forward Rates

$$f^*_{i,t'} = \frac{1}{\tau_i - \tau_{i-1}}\left(\tau_i\, r^*_{\tau_i,t'} - \tau_{i-1}\, r^*_{\tau_{i-1},t'}\right), \text{ where } r^*_{\tau_i,t'} \text{ is the } i^{\text{th}} \text{ yield with maturity } \tau_i \text{ in month } t'$$

$$\tilde{f}^*_{i,t'} \equiv \mathrm{T}_f\,(f^*_{i,t'}) = \begin{cases} f^*_{i,t'} & \text{if } f^*_{i,t'} > 0.005 \\ c_{f,0} + c_{f,1}\log\left(f^*_{i,t'}\right) & \text{if } f^*_{i,t'} \leq 0.005 \end{cases}, \text{ where } f^*_{i,t'} \text{ is the } i^{\text{th}} \text{ forward rate}$$

$$\tilde{\boldsymbol{f}}^*_{t'} = \mathbf{1}_n\,\tilde{r}_{t'} + \boldsymbol{\mu}_f + \boldsymbol{A}_f\,\boldsymbol{F}^*_{t'} + \boldsymbol{\Sigma}_f\,\boldsymbol{\varepsilon}_{f,t'}, \text{ where } \boldsymbol{\varepsilon}_{f,t'} \sim \mathcal{N}_7(\mathbf{0}_7, \mathbf{I}_7)$$

$$\boldsymbol{F}^*_{t'} = \boldsymbol{\mu}_F + \boldsymbol{A}_F\left(\boldsymbol{F}^*_{t'-1} - \boldsymbol{\mu}_F\right) + \boldsymbol{\Sigma}_F\,\boldsymbol{\varepsilon}_{F,t'}, \text{ where } \boldsymbol{\varepsilon}_{F,t'} \sim \mathcal{N}_2(\mathbf{0}_2, \mathbf{I}_2)$$

Data: 1-, 2-, 3-, 5-, 7-, 10-, and 20-year zero-coupon risk-free yields (Source: Bank of Canada)

Dividend Yield

$$\log(d^*_{t'}) = \log(\mu_{d,m^*_{t'}}) + a_d\left(\log\left(d^*_{t'-1}\right) - \log(\mu_{d,m^*_{t'}})\right) + \sigma_d\,\varepsilon_{d,t'}, \text{ where } \varepsilon_{d,t'} \sim \mathcal{N}(0,1)$$

Data: Dividends paid out on the stocks and level of the S&P/TSX Composite index (Source: Compustat)

Stock Index Returns

$$y^*_{t'} = \frac{r^*_{0,t'}}{12} + \mu_{y,m^*_{t'}} + \sigma_{y,t'}\,\varepsilon_{y,t'}, \text{ where } \varepsilon_{y,t'} \sim \mathcal{N}(0,1)$$

$$\sigma^2_{y,t'+1} = \sigma^2_y + \alpha_y\left(\left(\sigma_{y,t'}\,\varepsilon_{y,t'} - \sigma_{y,t'}\gamma_y\right)^2 - \sigma^2_y\left(1+\gamma^2_y\right)\right) + \beta_y\left(\sigma^2_{y,t'} - \sigma^2_y\right)$$

Data: S&P/TSX Composite index (Source: Compustat)

assumed with price inflation, wage inflation, and the dividend yield.

We build a term structure of interest rates on top of the short rate model. We rely on (continuously compounded) forward rates constructed from yields to construct the term structure; we use the one-, two-, three-, five-, seven-, ten-, and 20-year rates.[3] To prevent the chance of any forward

[3]We interpolate between the simulated rates to obtain rates not readily available in our model. For instance, the eight-year rate is obtained by combining the seven- and ten-year rates.

Table SM.1: **Summary of the economic scenario generator, continued.**

Alternative Asset Returns (Proxied by Private Equity Returns)

$$p^*_{t'} = \frac{r^*_{0,t'}}{12} + \mu_{p,m^*_{t'}} + \sigma_{p,t'}\,\varepsilon_{p,t'}, \text{ where } \varepsilon_{p,t'} \sim \mathcal{N}(0,1)$$

$$\sigma^2_{p,t'+1} = \sigma^2_p + \alpha_p\left(\left(\sigma_{p,t'}\,\varepsilon_{p,t'} - \sigma_{p,t'}\gamma_p\right)^2 - \sigma^2_p\left(1+\gamma^2_p\right)\right) + \beta_p\left(\sigma^2_{p,t'} - \sigma^2_p\right)$$

Data: Thomson Reuters private equity buyout index in Canadian dollars (Source: Bloomberg)

Investment Grade Corporate Bond Yield

$$\tilde{i}^*_{t'} \equiv \mathrm{T}_i\,(i^*_{t'} - r^*_{7,t'}) = \begin{cases} i^*_{t'} - r^*_{7,t'} & \text{if } i^*_{t'} - r^*_{7,t'} > 0.005 \\ c_{i,0} + c_{i,1}\log\left(i^*_{t'} - r^*_{7,t'}\right) & \text{if } i^*_{t'} - r^*_{7,t'} \leq 0.005 \end{cases}, \text{ where } r^*_{7,t} \text{ is the 7-year yield}$$

$$\tilde{i}^*_{t'} = \mu_{i,m^*_{t'}} + a_i\left(i^*_{t'-1} - \mu_{i,m^*_{t'}}\right) + \sigma_{i,t'}\,\varepsilon_{i,t'}, \text{ where } \varepsilon_{i,t'} \sim \mathcal{N}(0,1)$$

$$\sigma^2_{i,t'+1} = \sigma^2_i + \alpha_i\left(\left(\sigma_{i,t'}\,\varepsilon_{i,t'} - \sigma_{i,t'}\gamma_i\right)^2 - \sigma^2_i\left(1+\gamma^2_i\right)\right) + \beta_i\left(\sigma^2_{i,t'} - \sigma^2_i\right)$$

Data: Yield on S&P Canada investment grade corporate bond index (Source: Refinitiv Eikon)

Note: This table summarizes the various models included in our economic scenario generator. We assume the following correction structure between the price inflation, wage inflation, short rate, and dividend yield innovations:

$$\mathrm{Corr}\left(\left[\varepsilon_{q,t'}, \varepsilon_{w,t'}, \varepsilon_{r,t'}, \varepsilon_{d,t'}\right]^\top\right) = \begin{bmatrix} 1 & \rho_{q,w} & \rho_{q,r} & \rho_{q,d} \\ \rho_{q,w} & 1 & \rho_{w,r} & \rho_{w,d} \\ \rho_{q,r} & \rho_{w,r} & 1 & \rho_{r,d} \\ \rho_{q,r} & \rho_{w,r} & \rho_{r,d} & 1 \end{bmatrix}.$$

The transformation parameters for the short rate, forward rates, investment grade, and high yield corporate bond yields are as follows: $c_{r,0} = c_{f,0} = c_{i,0} = 0.005 - 0.005\log(0.005)$ and $c_{r,1} = c_{f,1} = c_{i,1} = 0.005$. The dividend yield is obtained by taking the total dividends paid out over the last 12 months and dividing this amount by the value of the index at the end of the period of interest. We also assume the following correction structure between the contemporaneous innovations of the stock index return, the alternative asset return, and the investment grade corporate bond yield:

$$\mathrm{Corr}\left(\left[\varepsilon_{y,t'}, \varepsilon_{p,t'}, \varepsilon_{i,t'}\right]^\top\right) = \begin{bmatrix} 1 & \rho_{y,p} & \rho_{y,i} \\ \rho_{y,p} & 1 & \rho_{p,i} \\ \rho_{y,i} & \rho_{p,i} & 1 \end{bmatrix}.$$

rates becoming negative, we use a similar transformation to that proposed above. We model the term structure by assuming that the spreads—defined as the difference between the transformed forward rates and the transformed short rate—are generated by observable factors: the slope and the curvature, as commonly done in the interest rate literature. We further assume that the observable factors summarizing the spreads follow a two-dimensional autoregressive model.[4]

The logarithm of the dividend yield is modelled via the AR(1); again, the long-run level of the dividend is regime-dependent. The correlation between price inflation, wage inflation, and short rate innovations is assumed to be non-zero.

The stock index returns are given by a process reminiscent of that used by Hardy (2001).

[4]The factor model is reminiscent of the vector autoregressive (VAR) model commonly used in econometrics to model the interaction between economic variables. One of their main advantages is that they do not require a detailed theory to estimate the relationship between economic variables (Sims, 1980). They were used by Campbell and Viceira (2002) in the investment context and Chan (2002) in the ESG context.

Instead of considering latent regimes, however, this model uses observable monetary regimes, which capture the changing nature of the average return. In addition, we add a GARCH structure to capture the changing nature of volatility over time. The alternative asset dynamics are also defined similarly. We assume correlation between the two asset returns.

The difference between investment grade corporate bond yield and the seven-year government risk-free yield is transformed based on the piecewise transformation similar to that used for the short rate and the forward rates.[5] This transformed difference is then assumed to follow a modified AR(1) process with regime-dependent long-run levels and GARCH variance. Investment grade corporate bond yields are correlated with stock index and alternative asset returns.

Table SM.1 summarizes the various models and data used to fit the framework. Please refer to Bégin (2021) for additional information on the ESG. Figure SM.1 reports monthly funnels of doubt taken at the end of our sample for the price and wage inflation rates, the short rate, the seven- and 20-year interest rate yields, the dividend yield, the stock index returns, the alternative asset returns, and the investment grade corporate bond yields.

From the monthly data generated by the model, we can obtain annual quantities needed to simulate the pension plan operation over time. Assuming that T is the last month of the sample, one can compute year-t (continuously compounded) rates and returns for the various economic quantities and financial asset classes as follows:

- Price and wage inflation:
$$q_t = \sum_{t'=T+12(t-1)+1}^{T+12t} q^*_{t'} \quad \text{and} \quad w_t = \sum_{t'=T+12(t-1)+1}^{T+12t} w^*_{t'},$$
respectively.

- Short and long rates:
$$r_t = r^*_{0,T+12(t-1)} \quad \text{and} \quad l_t = r^*_{20,T+12(t-1)},$$
respectively, where the long rate corresponds to the longest rate available in the model (i.e., 20-year).

- Long-term government bond portfolio return:
$$R_t^{(\mathrm{RF})} = \log\left(\frac{\sum_{s=1}^{10} \exp\left(-r^*_{\tau-1,T+12t}\,(s-1)\right)}{\sum_{s=1}^{10} \exp\left(-r^*_{\tau,T+12(t-1)} s\right)}\right),$$
assuming a portfolio of ten government zero-coupon bonds for simplicity's sake.

[5] We use a seven-year government risk-free yield in the investment grade corporate bond yield model because the average duration of the bonds used to compute this yield is approximately seven years.

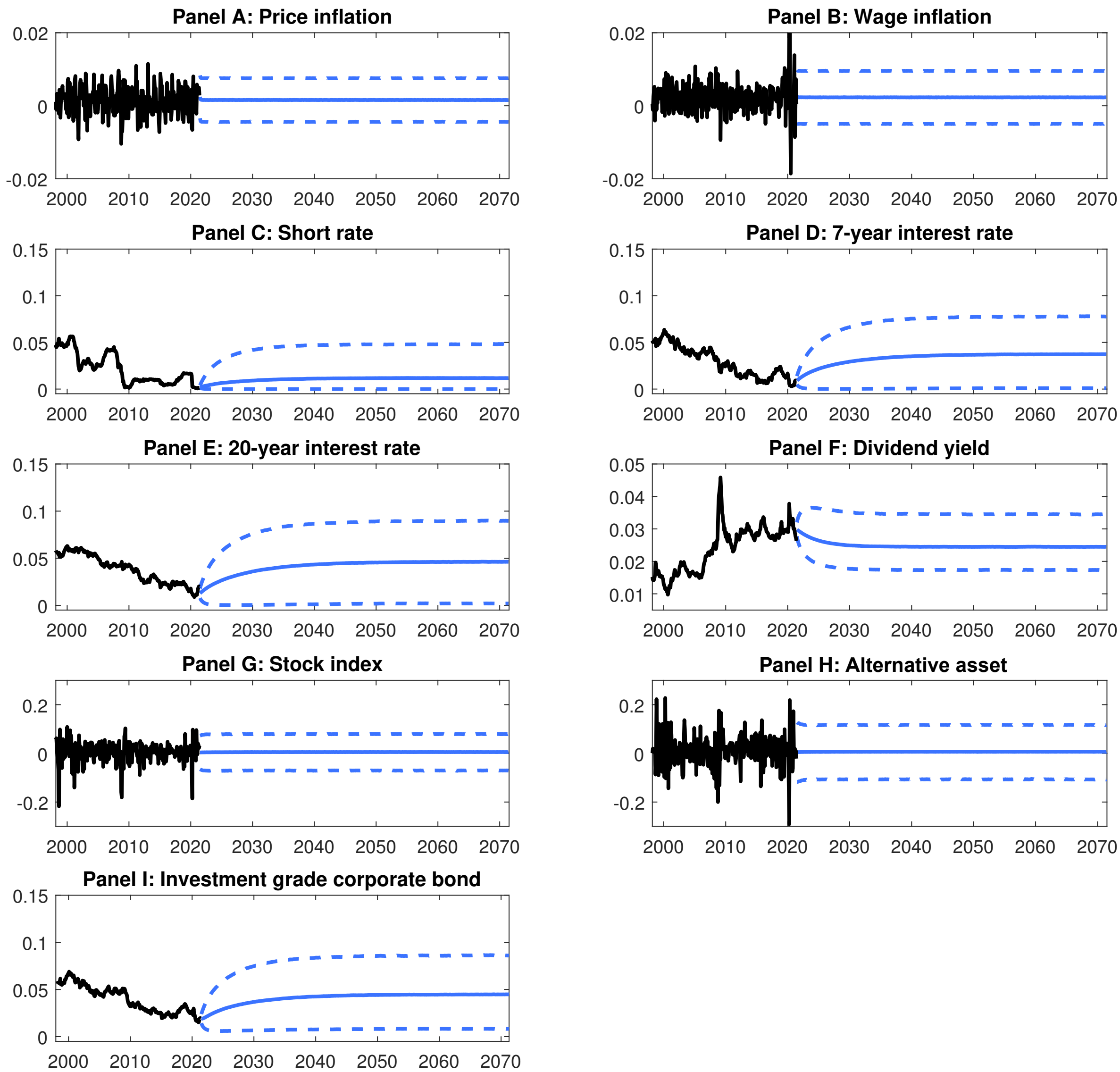

Figure SM.1: **Funnels of doubt of relevant economic and financial variables.**
Note: This figure shows the past series along with the funnels of doubt over the next 50 years. The figure reports monthly values for price inflation (Panel A), wage inflation (Panel B), the short rate (Panel C), the seven-year interest rate (Panel D), the 20-year interest rate (Panel E), the dividend yield on the S&P/TSX Composite index (Panel F), the S&P/TSX Composite index returns (Panel G), the alternative asset returns (Panel H), and the investment grade corporate bond yield (Panel I). Solid lines represent median and dashed lines the 90% confidence interval.

– Investment grade bond portfolio return:

$$R_t^{(\mathrm{IG})} = \log\left(\frac{\sum_{s=1}^{10} \exp\left(-i^*_{T+12t}\,(s-1)\right)}{\sum_{s=1}^{10} \exp\left(-i^*_{T+12(t-1)}\,s\right)}\right),$$

assuming a portfolio of ten investment grade zero-coupon bonds (and a flat investment grade bond yield term structure) for the sake of simplicity.

– (Total) stock index return:

$$R_t^{(S)} = \sum_{t'=T+12(t-1)+1}^{T+12t} \left(y_{t'}^* + \log\left(1 + \frac{d_{t'}^*}{12}\right)\right).$$

– Alternative asset return:

$$R_t^{(P)} = \sum_{t'=T+12(t-1)+1}^{T+12t} p_{t'}^*.$$

Table SM.2 reports some statistics of the annual economic and financial quantities mentioned above. These quantities and their distributions are aligned with past realizations.

Table SM.2: **Unconditional statistics of annual economic and financial variables.**

	Moments		**Percentiles**				
	Average	Std. dev.	10^{th}	25^{th}	50^{th}	75^{th}	90^{th}
Rates							
Price inflation	0.019	0.015	−0.001	0.009	0.019	0.029	0.038
Wage inflation	0.028	0.017	0.006	0.016	0.028	0.039	0.049
Short rate	0.016	0.017	0.000	0.001	0.012	0.027	0.040
Long rate	0.046	0.026	0.010	0.027	0.046	0.064	0.080
Returns							
Long-term government bond portfolio	0.043	0.054	−0.019	0.003	0.039	0.079	0.113
Investment grade bond portfolio	0.049	0.062	−0.027	0.005	0.044	0.089	0.129
Stock index	0.101	0.178	−0.110	0.003	0.107	0.205	0.302
Alternative asset	0.107	0.258	−0.210	−0.046	0.110	0.261	0.412

Note: This table reports the unconditional average and standard deviation (Std. dev. in the table) as well as the 10^{th}, 25^{th}, 50^{th}, 75^{th}, and 90^{th} percentiles of the annual distributions of various economic and financial variables. The 20-year government yield rate is used as a proxy for the long rate. The long-term government bond portfolio returns and the investment grade bond portfolio returns are obtained from a hypothetical portfolio of ten zero-coupon bonds with maturities one year to ten years.

SM.B More on the mortality and longevity modelling

The estimation of the model relies on three different datasets. First, we obtain sex-specific death counts and exposures for the total Canadian population and for each of the ten provinces from the Canadian Human Mortality Database, a satellite website of the well-known Human Mortality Database. We consider individuals aged 65 through 109 at the start of the year and years from 1980 to 2015; we denote the age and period sets by $\mathcal{X} \equiv \{65, 61, ..., 109\}$ and $\mathcal{T} \equiv \{1980, 1981, ..., 2015\}$, respectively.

Second, sex-specific death counts and exposures for 11 socioeconomic groups were provided by the Quebec Pension Plan (QPP). QPP covers individuals who live in the province of Quebec. We use these groups as proxies for socioeconomic status. Note that this dataset covers individuals who participated in the workforce. It means that, by construction, we have a 12^{th} group; that is, people who never participated in the workforce. We do not have explicit data for those, but they still are part of our modelling.

Third, the data obtained from the Canada Pension Plan (CPP) are structured in a similar fashion to that of QPP but for all Canadian residents except for Quebec: sex-specific death counts and exposures for the 11 socioeconomic groups defined above through the cohort maximum pension. Note that CPP data are aggregated across all provinces except Quebec, meaning that we cannot isolate the effect of the socioeconomic status for the four other regions from this dataset alone.

Following Dowd et al. (2011) and Cairns et al. (2019), we model the first stratum period effects via a common long-term trend for all sexes i_1 combined with a short-term gravity effect. Specifically, $\kappa_1^{(i_1)}(t)$ and $\kappa_2^{(i_1)}(t)$ are given by the following dynamics:

$$\kappa_1^{(i_1)}(t) = \mu_1 + \kappa_1^{(i_1)}(t-1) - \psi^{(\mathcal{I}_1)}\left(\kappa_1^{(i_1)}(t-1) - \overline{\kappa}_1^{(\mathcal{I}_1)}(t-1)\right) + \varepsilon_1^{(i_1)}(t), \tag{SM.1}$$

$$\kappa_2^{(i_1)}(t) = \mu_2 + \kappa_2^{(i_1)}(t-1) - \psi^{(\mathcal{I}_1)}\left(\kappa_2^{(i_1)}(t-1) - \overline{\kappa}_2^{(\mathcal{I}_1)}(t-1)\right) + \varepsilon_2^{(i_1)}(t), \tag{SM.2}$$

where

$$\overline{\kappa}_1^{(\mathcal{I}_1)}(t) = \sum_{i_1 \in \mathcal{I}_1} \omega_{i_1}\, \kappa_1^{(i_1)}(t), \quad \overline{\kappa}_2^{(\mathcal{I}_1)}(t) = \sum_{i_1 \in \mathcal{I}_1} \omega_{i_1}\, \kappa_2^{(i_1)}(t),$$

and ω_{i_1} are determined by sex-specific exposures and are approximated by $\frac{1}{2}$ for each sex in this study. Parameter $\psi^{(\mathcal{I}_1)}$ captures the speed at which male and female improvements go back to the national average. We assume that this parameter is between 0 and 1 to avoid any oscillatory behaviour. We also assume that it is the same for both sexes; indeed, preliminary tests show that sex-specific speed of improvement parameters are not statistically different from one another. Parameters μ_1 and μ_2 are the common drifts of the first and second period effect, respectively.

The random innovations in Equations (SM.1) and (SM.2) are assumed to be normal with mean zero and covariances

$$\mathrm{Cov}\left[\varepsilon_p^{(i_1)}(t), \varepsilon_{p'}^{(i_1')}(t)\right] = \begin{cases} \nu_p^{(\mathcal{I}_1)} & \text{if } i_1 = i_1' \text{ and } p = p' \\ \rho_{12}^{(\mathcal{I}_1)} \sqrt{\nu_p^{(\mathcal{I}_1)}\, \nu_{p'}^{(\mathcal{I}_1)}} & \text{if } i_1 = i_1' \text{ and } p \neq p' \\ \rho^{(i_1, i_1')}\, \nu_p^{(\mathcal{I}_1)} & \text{if } i_1 \neq i_1' \text{ and } p = p' \\ \rho^{(i_1, i_1')}\, \rho_{12}^{(\mathcal{I}_1)} \sqrt{\nu_p^{(\mathcal{I}_1)}\, \nu_{p'}^{(\mathcal{I}_1)}} & \text{if } i_1 \neq i_1' \text{ and } p \neq p' \end{cases}, \tag{SM.3}$$

where $p, p' \in \{1, 2\}$, $i_1, i_1' \in \mathcal{I}$, $-1 < \rho_{12}^{(\mathcal{I}_1)} < 1$, and $-1 < \rho^{(i_1, i_1')} < 1$.

For consistency's sake, we should be able to write the aggregated sex-specific death rates as follows:

$$m^{(i_1)}(x,t) = \frac{D^{(i_1)}(x,t)}{E^{(i_1)}(x,t)} = \exp\left(\beta_0^{(i_1)}(x) + \kappa_1^{(i_1)}(t) + \kappa_2^{(i_1)}(t)\,(x - \overline{x})\right), \tag{SM.4}$$

Interestingly, we can approximately obtain Equation (SM.4) if we sum the (weighted) death rates over every region and socioeconomic status if

$$\sum_{i_2 \in \mathcal{I}_2} \omega_{i_2} \kappa_1^{(i_2)}(t) = 0, \sum_{i_2 \in \mathcal{I}_2} \omega_{i_2} \kappa_2^{(i_2)}(t) = 0, \sum_{i_3 \in \mathcal{I}_3} \omega_{i_3} \kappa_1^{(i_3)}(t) = 0, \text{ and } \sum_{i_3 \in \mathcal{I}_3} \omega_{i_3} \kappa_2^{(i_3)}(t) = 0, \qquad \text{(SM.5)}$$

where weights ω_{i_2} are determined by the region-specific exposure of the total population and weights ω_{i_3} are determined by the socioeconomic group-specific exposure of the total population, on average. We use the average total exposure for each region as a mean to construct the ω_{i_2} weights in this study. For ω_{i_3}, we use the average total exposure for each group. The constraints above allow for consistency between subnational mortality and that of the whole Canadian population. As mentioned in Alexander et al. (2017), it is important that subnational mortality rates be consistent with the mortality rates observed at the national level when aggregated. It also allows for better identification of the parameters as we now create some dissociation between the sex-specific, region-specific, and socioeconomic-specific mortality improvements.

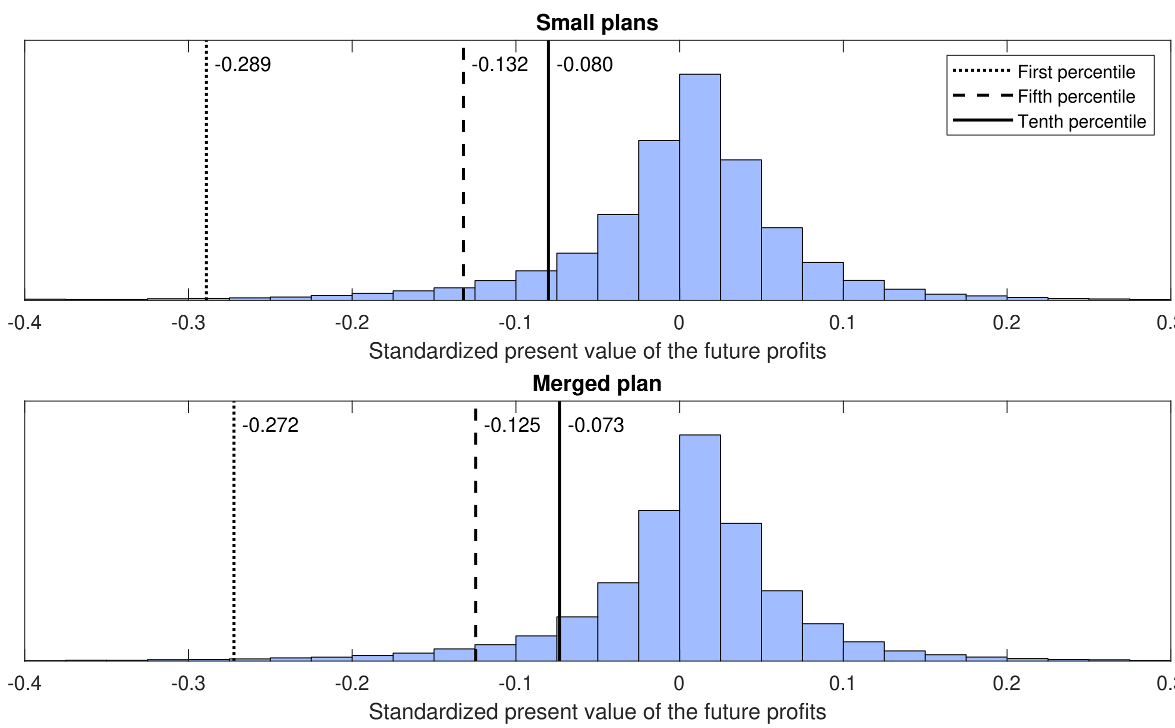

Figure SM.2: **Distribution of standardized present value of future profits for the merger of three small plans.**
Note: This figure reports the distribution of the standardized present value of future profits as given by Equation (13) for the small plan in the top panel and for the merged plan in the bottom panel. The first, fifth, and tenth quantiles of each distribution are reported with dotted, dashed, and solid lines, respectively.

Equation (SM.5) implies additional constraints on the second stratum period effect processes: we only need to model card($\mathcal{I}_2$) − 1 groups as we can get the last one by construction (i.e., we obtain the processes for British Columbia as a function of the other regional mortality improve-

Table SM.3: **Economic capital risk measures and improvements for the large plan.**

	$\mathbf{VaR}_p^{\mathbf{Plan}}$			$\mathbf{ES}_p^{\mathbf{Plan}}$		
	0.01	0.05	0.10	0.01	0.05	0.10
Base	−0.287 (0%)	−0.124 (0%)	−0.075 (0%)	−0.427 (0%)	−0.227 (0%)	−0.162 (0%)
Initial funded ratio at 0.8	−0.287 (0%)	−0.124 (–1%)	−0.075 (–1%)	−0.427 (0%)	−0.227 (0%)	−0.162 (0%)
Initial funded ratio at 1.2	−0.287 (0%)	−0.124 (0%)	−0.075 (0%)	−0.427 (1%)	−0.227 (0%)	−0.162 (0%)
Less mature plan	−0.287 (0%)	−0.124 (0%)	−0.075 (0%)	−0.427 (0%)	−0.227 (0%)	−0.162 (0%)
No retirees at inception	−0.287 (0%)	−0.124 (0%)	−0.075 (0%)	−0.427 (0%)	−0.227 (0%)	−0.162 (0%)
British Columbia	−0.287 (0%)	−0.124 (0%)	−0.075 (0%)	−0.427 (0%)	−0.227 (0%)	−0.162 (0%)
Nova Scotia	−0.287 (0%)	−0.124 (0%)	−0.075 (0%)	−0.427 (0%)	−0.227 (0%)	−0.162 (0%)
Higher starting salary	−0.287 (0%)	−0.124 (0%)	−0.075 (0%)	−0.427 (0%)	−0.227 (0%)	−0.162 (0%)
Lower starting salary	−0.287 (0%)	−0.124 (0%)	−0.075 (0%)	−0.427 (0%)	−0.227 (0%)	−0.162 (0%)
More female	−0.287 (0%)	−0.124 (0%)	−0.075 (0%)	−0.427 (0%)	−0.227 (0%)	−0.162 (0%)
More male	−0.287 (0%)	−0.124 (0%)	−0.075 (0%)	−0.427 (0%)	−0.227 (0%)	−0.162 (0%)
Retirement at 70 years old	−0.287 (0%)	−0.124 (0%)	−0.075 (0%)	−0.427 (0%)	−0.227 (0%)	−0.162 (0%)
Larger plan	−0.287 (0%)	−0.124 (0%)	−0.075 (0%)	−0.427 (0%)	−0.227 (0%)	−0.162 (0%)
Smaller plan	−0.287 (0%)	−0.124 (0%)	−0.075 (0%)	−0.427 (0%)	−0.227 (0%)	−0.162 (0%)
No rounding	−0.287 (0%)	−0.124 (0%)	−0.075 (0%)	−0.427 (0%)	−0.227 (0%)	−0.162 (0%)

Note: This table reports economic capital risk measures for the large plan and its post-merger improvements (in percentage) in parentheses. The value-at-risk and the expected shortfall are computed using Equations (14) and (15), respectively, for probability levels of 1%, 5%, and 10%. In addition to the base case, the economic capital risk measures are also computed for all the robustness cases mentioned in the caption of Table 4.

ments). The second dimension's period effects are modelled through dynamics similar to those of Equations (SM.1) and (SM.2), but without explicit gravity effects as $\overline{\kappa}_1^{(\mathcal{I}_2)}(t) = \sum_{i_2 \in \mathcal{I}_2} \omega_{i_2}\, \kappa_1^{(i_2)}(t) = 0$ and $\overline{\kappa}_2^{(\mathcal{I}_2)}(t) = \sum_{i_2 \in \mathcal{I}_2} \omega_{i_2}\, \kappa_2^{(i_2)}(t) = 0$ from Equation (SM.5). We assume that the first stratum captures the trends in the period effects and that they do not need to be included in the other strata. Implicitly, this means that all regions will converge to the national averages over the very long term, which turns out to be reasonable on biological grounds (this convergence might take several decades, if not centuries). All the parameters have a similar interpretation to that already explained for the first stratum. Moreover, the innovations are also normal, with zero mean and a similar correlation structure. Like $\psi^{(\mathcal{I}_1)}$, we assume that $\psi^{(\mathcal{I}_2)}$ is constant throughout the different regions. Letting this parameter be different for each region does not change the fit as most of the parameters are statistically similar.

Equation (SM.5) also adds extra constraints regarding the third stratum: we only need to model $\text{card}(\mathcal{I}_3) - 1$ groups as we can get the last one by construction. The period effects for the third stratum are modelled via dynamics similar to those of Equations (SM.1) and (SM.2). More details about the model and the datasets are available in Bégin et al. (2023).

SM.C Additional results on the merger of a small plan and a large plan

This section reports results that compare the large plan to the merged plan. Table SM.3 mirrors Table 4 but for the large plan instead of the small plan. Table SM.4, similar to Table 5, does the same for the CECs. Finally, Table SM.5 reports the employer contribution risk measures, similar to the results reported in Table 6 but for the large plan instead of the small.

As noted in the text, although a single merger between a large and small plan may bring negligible benefits to the large plan, multiple successive mergers may produce material benefits. This may justify some large plans' aggressive agenda for expansion through repeated mergers. To test this assumption, we consider a merger of a large plan of 25,000 members similar to that of Table 2 with 25 small plans of 1,000 members each. Table SM.6 reports the economic capital risk measures, the certainty equivalent consumptions, and the employer contribution risk measures (as well as their improvements) for this merger and both the small and large plans. Interestingly, these results support the hypothesis that multiple consecutive mergers benefit the large plan—and this from the perspective of the plan, the members, and the employer. The improvements are all strictly positive, implying a clear benefit even for large plans.

SM.D Merger of three small-sized plans

This section focuses on the second case study—a merger between three small-sized Ontario public sector employers. The features of this second case study are summarized in Table SM.7.

We assume that the pre-merger initial funded ratio is 1 for all plans and their service and complexity scores are 0. After the merger, the complexity score increases to 1. The investment universe for both the small and merged plans includes only long-term risk-free government bonds, investment grade corporate bonds, and the stock index. We also assume that the small plans round their asset allocation to the nearest 5%, similar to the assumption used in subsection 5.1, whereas the merged plan has no rounding. The plan membership is similar to that used in the article, except that the starting salary is \$80,000 at time 0.

Table SM.4: **Certainty equivalent consumption and improvements for the large plan.**

	Male				**Female**			
	25	35	45	55	25	35	45	55
Base	35,023	32,291	29,111	25,709	34,767	31,917	28,642	25,202
	(–3)	(–1)	(–1)	(0)	(–3)	(–1)	(–1)	(0)
Initial funded ratio at 0.8	35,023	32,291	29,111	25,709	34,767	31,917	28,642	25,202
	(–10)	(–5)	(–3)	(–1)	(–10)	(–5)	(–2)	(–1)
Initial funded ratio at 1.2	35,023	32,291	29,111	25,709	34,767	31,917	28,642	25,202
	(5)	(2)	(1)	(1)	(4)	(2)	(1)	(1)
Less mature plan	35,023	32,291	29,111	25,709	34,767	31,917	28,642	25,202
	(–4)	(–2)	(–1)	(0)	(–4)	(–2)	(–1)	(0)
No retirees at inception	35,023	32,291	29,111	25,709	34,767	31,917	28,642	25,202
	(2)	(1)	(1)	(1)	(2)	(1)	(1)	(0)
British Columbia	35,023	32,291	29,111	25,709	34,767	31,917	28,642	25,202
	(–3)	(–2)	(–1)	(0)	(–3)	(–2)	(–1)	(0)
Nova Scotia	35,023	32,291	29,111	25,709	34,767	31,917	28,642	25,202
	(–1)	(0)	(0)	(0)	(–1)	(0)	(0)	(0)
Higher starting salary	35,023	32,291	29,111	25,709	34,767	31,917	28,642	25,202
	(–6)	(–3)	(–1)	(–1)	(–6)	(–3)	(–1)	(0)
Lower starting salary	35,023	32,291	29,111	25,709	34,767	31,917	28,642	25,202
	(–2)	(–1)	(0)	(0)	(–2)	(–1)	(0)	(0)
More female	35,023	32,291	29,111	25,709	34,767	31,917	28,642	25,202
	(–3)	(–2)	(–1)	(0)	(–3)	(–2)	(–1)	(0)
More male	35,023	32,291	29,111	25,709	34,767	31,917	28,642	25,202
	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)
Retirement at 70 years old	35,023	32,291	29,111	25,709	34,767	31,917	28,642	25,202
	(4)	(2)	(1)	(0)	(4)	(2)	(1)	(0)
Larger plan	35,023	32,291	29,111	25,709	34,767	31,917	28,642	25,202
	(1)	(0)	(0)	(0)	(1)	(0)	(0)	(0)
Smaller plan	35,023	32,291	29,111	25,709	34,767	31,917	28,642	25,202
	(–2)	(–1)	(0)	(0)	(–2)	(–1)	(0)	(0)
No rounding	35,023	32,291	29,111	25,709	34,767	31,917	28,642	25,202
	(–3)	(–1)	(–1)	(0)	(–3)	(–1)	(–1)	(0)

Note: This table reports certainty equivalent consumptions (CECs) for the large plan and its post-merger improvements (in dollar) in parentheses. The CECs are computed using Equation (17) for males and females aged 25, 35, 45, and 55 years old. In addition to the base case, the CECs are also computed for all the robustness cases mentioned in the caption of Table 4.

From the assumptions mentioned above (denoted by ‘Base’), we generate 25,000 scenarios via the procedure explained in Section 3. Similar to our analysis of subsection 5.1, we compare the merged plan’s behaviour to that of the small plans. Again, when we refer to small plans, we consider hypothetical versions of these plans that have not merged.

Table SM.8 reports statistics about the long-run behaviour of the small plans (Panel A) and merged plan (Panel B); again, we report statistics on relevant quantities such as the distribution of the portfolio weights, the total asset return, the funded ratio, the valuation rate, and the contribution rate, all at the end of the horizon of $\tau = 50$ years.

Table SM.5: **Employer contribution risk measures and improvements for the large plan.**

	$\mathbf{VaR}_p^{\mathbf{Employer}}$			$\mathbf{ES}_p^{\mathbf{Employer}}$		
	0.05	0.10	0.50	0.05	0.10	0.50
Base	−2.777 (0%)	−2.262 (0%)	−1.000 (0%)	−3.492 (0%)	−2.991 (0%)	−1.789 (0%)
Initial funded ratio at 0.8	−2.777 (0%)	−2.262 (0%)	−1.000 (–1%)	−3.492 (0%)	−2.991 (0%)	−1.789 (0%)
Initial funded ratio at 1.2	−2.777 (0%)	−2.262 (0%)	−1.000 (0%)	−3.492 (0%)	−2.991 (0%)	−1.789 (0%)
Less mature plan	−2.777 (–1%)	−2.262 (–1%)	−1.000 (–1%)	−3.492 (–1%)	−2.991 (–1%)	−1.789 (–1%)
No retirees at inception	−2.777 (–1%)	−2.262 (–1%)	−1.000 (–1%)	−3.492 (–1%)	−2.991 (–1%)	−1.789 (–1%)
British Columbia	−2.777 (0%)	−2.262 (0%)	−1.000 (0%)	−3.492 (0%)	−2.991 (0%)	−1.789 (0%)
Nova Scotia	−2.777 (0%)	−2.262 (0%)	−1.000 (0%)	−3.492 (0%)	−2.991 (0%)	−1.789 (0%)
Higher starting salary	−2.777 (0%)	−2.262 (0%)	−1.000 (0%)	−3.492 (0%)	−2.991 (0%)	−1.789 (0%)
Lower starting salary	−2.777 (0%)	−2.262 (0%)	−1.000 (0%)	−3.492 (0%)	−2.991 (0%)	−1.789 (0%)
More female	−2.777 (0%)	−2.262 (0%)	−1.000 (0%)	−3.492 (0%)	−2.991 (0%)	−1.789 (0%)
More male	−2.777 (0%)	−2.262 (0%)	−1.000 (0%)	−3.492 (0%)	−2.991 (0%)	−1.789 (0%)
Retirement at 70 years old	−2.777 (1%)	−2.262 (1%)	−1.000 (1%)	−3.492 (0%)	−2.991 (0%)	−1.789 (1%)
Larger plan	−2.777 (0%)	−2.262 (0%)	−1.000 (0%)	−3.492 (0%)	−2.991 (0%)	−1.789 (0%)
Smaller plan	−2.777 (0%)	−2.262 (0%)	−1.000 (0%)	−3.492 (0%)	−2.991 (0%)	−1.789 (0%)
No rounding	−2.777 (0%)	−2.262 (0%)	−1.000 (0%)	−3.492 (0%)	−2.991 (0%)	−1.789 (0%)

Note: This table reports employer contribution risk measures for the large plan and their post-merger improvements in percentage (in parentheses). The value-at-risk and the expected shortfall are computed using Equations (18) and (19), respectively, for probability levels of 5%, 10%, and 50%. In addition to the base case, the employer contribution risk measures are also computed for all the robustness cases mentioned in the caption of Table 4.

The asset allocation for the small plans is similar to that of the merged plan, with the main difference being the suboptimal—rounded—allocation of the small plan. For all plans, about 43% of the assets are invested in the stock index, 40% in the investment grade bond portfolio, and 17% in the long-term government bond portfolio, on average. The allocation is also changing as a function of the economic conditions and the plans' funded status, implying wide ranges of possible allocations, as shown in Table SM.8.

The small plan's suboptimal asset allocation translates into total asset returns that are 0.1% higher, on average, for the merged plan. The valuation rate is also 0.1% higher for the merged

Table SM.6: **Economic capital risk measures and improvements, certainty equivalent consumptions and improvements, and employer contribution risk measures and improvements when considering a merger with 25 small plans.**

Panel A: Economic capital risk measures and improvements

	$\mathbf{VaR}_p^{\mathbf{Plan}}$			$\mathbf{ES}_p^{\mathbf{Plan}}$		
	0.01	0.05	0.10	0.01	0.05	0.10
Small plans	−0.294 (3%)	−0.140 (11%)	−0.085 (12%)	−0.449 (5%)	−0.244 (7%)	−0.177 (8%)
Large plan	−0.295 (2%)	−0.127 (2%)	−0.077 (2%)	−0.443 (4%)	−0.235 (3%)	−0.167 (3%)

Panel B: Certainty equivalent consumption and improvements

	Male				**Female**			
	25	35	45	55	25	35	45	55
Small plans	34,755 (265)	32,150 (139)	29,045 (65)	25,682 (27)	34,512 (252)	31,788 (129)	28,584 (58)	25,179 (23)
Large plan	34,939 (81)	32,247 (43)	29,089 (21)	25,700 (9)	34,687 (77)	31,877 (39)	28,623 (18)	25,195 (8)

Panel C: Employer contribution risk measures and improvements

	$\mathbf{VaR}_p^{\mathbf{Employer}}$			$\mathbf{ES}_p^{\mathbf{Employer}}$		
	0.05	0.10	0.50	0.05	0.10	0.50
Small plans	−3.113 (11%)	−2.596 (13%)	−1.316 (24%)	−3.832 (9%)	−3.329 (10%)	−2.115 (15%)
Large plan	−2.937 (5%)	−2.407 (6%)	−1.127 (11%)	−3.698 (6%)	−3.169 (6%)	−1.930 (7%)

Note: This table reports economic capital risk measures, consumption equivalent consumptions, and employer contribution risk measures for the large plan and its post-merger improvements in parentheses. The value-at-risk and the expected shortfall are computed using Equations (14) and (15), respectively, for probability levels of 1%, 5%, and 10%. The CECs are computed using Equation (17) for males and females aged 25, 35, 45, and 55 years old. The value-at-risk and the expected shortfall are computed using Equations (18) and (19), respectively, for probability levels of 5%, 10%, and 50%.

plan. This increase in the merged plan's average asset return makes the funded ratio higher (from 1.074 to 1.079, on average) and the contribution rate lower (from 11.3% to 10.6%, on average) when compared to the small plans. As for merger of a small and large plans, other factors, such as better mortality pooling, higher asset returns, and administrative and investment costs differences, also explain the higher funded ratio reported under the merged plan. For example, the costs per member are 2.7% higher for the small plans when compared to the merged plan.

Figure SM.2 shows histograms of the standardized present value of the future profits for the small plans (top panel) and the merged plan (bottom panel) used to assess economic capital. Similar to the merger of a small and large plans, the small plan's left tail is fatter; the tenth percentile decreases from −0.080 to −0.073 by merging—a 9% reduction. Similar improvements are reported for other probability levels and the ES statistics (see first rows denoted by 'Base' in Table SM.9).

Table SM.7: **Summary of pension plan features and assumptions for the merger of three small plans.**

	Small plans	Merged plan
Plan features		
Initial funded ratio	1	–
Service score	0	0
Complexity score	0	1
Other school employees	1	1
Corporate pension fund	0	0
All other dummies	0	0
Investment-related assumptions		
Investment universe	RF, IG, S	RF, IG, S
Rounding	Yes	No
Membership assumptions		
Starting age	25	–
Retirement age	65	–
Starting salary at time 0	\$80,000	–
Initial membership assumptions		
Numbers	2,000	–
Plan maturity	Stationarity	–
Proportion of females	50%	–
Province	Ontario	–

Note: This table summarizes the various plan features, investment-related, membership, and initial membership assumptions. This case corresponds to a merger involving three small-sized Ontario universities of about 2,000 members each.

Table SM.10 mirrors Table 5 and reports CEC measures for male and female members aged 25, 35, 45, and 55. The first rows of the table show that, in the base case, there are sizeable benefits for small plan members to merge. The CEC improvement is about \$200 per annum for members aged 25. The reductions in the level and uncertainty of the contributions mainly explain these improvements.

The reduction in both the level and the uncertainty of contributions is also reflected in the employer's contribution risk measures (see the first rows of Table SM.11). The median of the standardized present value of the employer contributions is 9% lower for the merged plan when compared to the small plans—moving from –1.185 to –1.084. We obtain similar decreases for other probability levels p and the employer's ES measures.

Our conclusions remain robust even when we change some of the underlying assumptions, including cases where the small plans are equally underfunded (with funded ratios of 0.8) or overfunded (with funded ratios of 1.2) before the merger, where the small plans are less mature. Allowing the merged plan to invest in alternative assets brings additional benefits.

Table SM.8: **Long-run statistics of asset allocation as well as asset- and liability-related quantities for the merger of three small plans.**

Panel A: Small plans.

	Moments		Percentiles				
	Average	Std. dev.	10^{th}	25^{th}	50^{th}	75^{th}	90^{th}
Asset allocation							
Long-term government bond portfolio	0.170	0.159	0.000	0.050	0.100	0.250	0.450
Investment grade bond portfolio	0.406	0.192	0.100	0.250	0.450	0.550	0.600
Stock index	0.424	0.125	0.300	0.350	0.400	0.500	0.600
Alternative asset	–	–	–	–	–	–	–
Asset-related quantity							
Total asset return	0.065	0.082	−0.032	0.019	0.068	0.115	0.161
Liability-related quantities							
Funded ratio	1.074	0.244	0.810	0.914	1.038	1.194	1.378
Valuation rate	0.065	0.006	0.057	0.061	0.065	0.068	0.072
Contribution rate	0.113	0.284	−0.246	−0.046	0.140	0.305	0.441

Panel B: Merged plan.

	Moments		Percentiles				
	Average	Std. dev.	10^{th}	25^{th}	50^{th}	75^{th}	90^{th}
Asset allocation							
Long-term government bond portfolio	0.171	0.155	0.012	0.058	0.120	0.242	0.433
Investment grade bond portfolio	0.398	0.189	0.116	0.261	0.437	0.552	0.604
Stock index	0.431	0.125	0.295	0.351	0.425	0.505	0.593
Alternative asset	–	–	–	–	–	–	–
Asset-related quantity							
Total asset return	0.066	0.083	−0.033	0.019	0.068	0.116	0.162
Liability-related quantities							
Funded ratio	1.079	0.247	0.812	0.914	1.042	1.200	1.389
Valuation rate	0.066	0.006	0.058	0.061	0.066	0.068	0.072
Contribution rate	0.106	0.283	−0.251	−0.053	0.134	0.297	0.432

Note: This table reports the long-run average and standard deviation (Std. dev. in the table) as well as the 10^{th}, 25^{th}, 50^{th}, 75^{th}, and 90^{th} percentiles of the weights invested in each asset class, the total asset return, the funded ratio, the valuation rate, and the contribution rate. Long-run quantities are taken at a horizon of 50 years, at which time the random quantities have converged to their long-run distributions. This is done for the small plans (Panel A) and the merged plan (Panel B). Note that the allocations sum to one in all scenarios.

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Table SM.9: **Economic capital risk measures and improvements for the merger of three small plans.**

	$\mathbf{VaR}_p^{\mathbf{Plan}}$			$\mathbf{ES}_p^{\mathbf{Plan}}$		
	0.01	0.05	0.10	0.01	0.05	0.10
Base	−0.289 (6%)	−0.132 (6%)	−0.080 (9%)	−0.439 (3%)	−0.235 (4%)	−0.168 (5%)
Initial funded ratio at 0.8	−0.375 (5%)	−0.171 (5%)	−0.105 (8%)	−0.563 (3%)	−0.304 (4%)	−0.219 (5%)
Initial funded ratio at 1.2	−0.230 (4%)	−0.107 (7%)	−0.063 (8%)	−0.363 (3%)	−0.191 (4%)	−0.136 (5%)
Less mature plan	−0.332 (4%)	−0.154 (5%)	−0.091 (6%)	−0.505 (3%)	−0.274 (4%)	−0.196 (5%)
No retirees at inception	−0.441 (1%)	−0.207 (2%)	−0.123 (4%)	−0.686 (1%)	−0.370 (2%)	−0.264 (2%)
Alternative asset in universe	−0.289 (4%)	−0.132 (9%)	−0.080 (10%)	−0.439 (6%)	−0.235 (7%)	−0.168 (7%)

Note: This table reports economic capital risk measures for the small plans and their post-merger improvements in percentage (in parentheses). The value-at-risk and the expected shortfall are computed using Equations (14) and (15), respectively, for probability levels of 1%, 5%, and 10%. In addition to the base case, the risk measures are also computed for all relevant robustness cases: an initial funded ratio of 0.8 and 1.2, less mature plans in which members and retirees are younger than 71 years old at time 0, plans with no retirees at time 0 (i.e., only active members), and a merged plan that can invest in an alternative asset.

Table SM.10: **Certainty equivalent consumptions and improvements for the merger of three small plans.**

	Male				**Female**			
	25	35	45	55	25	35	45	55
Base	69,465 (199)	64,132 (103)	57,821 (46)	51,068 (17)	69,047 (191)	63,497 (96)	57,018 (41)	50,196 (13)
Initial funded ratio at 0.8	67,925 (165)	63,293 (90)	57,413 (41)	50,883 (13)	67,567 (159)	62,714 (84)	56,650 (37)	50,034 (11)
Initial funded ratio at 1.2	70,158 (170)	64,494 (85)	57,993 (37)	51,145 (10)	69,710 (162)	63,835 (79)	57,173 (33)	50,263 (9)
Less mature plan	69,556 (67)	64,191 (36)	57,863 (15)	51,094 (5)	69,133 (64)	63,552 (33)	57,055 (13)	50,218 (4)
No retirees at inception	69,708 (53)	64,281 (26)	57,913 (9)	51,120 (2)	69,279 (51)	63,636 (24)	57,101 (8)	50,241 (2)
Alternative asset in universe	69,465 (403)	64,132 (210)	57,821 (96)	51,068 (34)	69,047 (386)	63,497 (195)	57,018 (86)	50,196 (30)

Note: This table reports certainty equivalent consumptions (CECs) for the small plans and their post-merger improvements (in dollar) in parentheses. The CECs are computed using Equation (17) for males and females aged 25, 35, 45, and 55 years old. In addition to the base case, the CECs are also computed for all the robustness cases mentioned in the caption of Table SM.9.

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Table SM.11: **Employer contribution risk measures and improvements for the merger of three small plans.**

	$\mathbf{VaR}_p^{\text{Employer}}$			$\mathbf{ES}_p^{\text{Employer}}$		
	0.05	0.10	0.50	0.05	0.10	0.50
Base	−2.943 (4%)	−2.429 (5%)	−1.185 (9%)	−3.662 (3%)	−3.160 (4%)	−1.965 (6%)
Initial funded ratio at 0.8	−3.191 (3%)	−2.708 (4%)	−1.493 (6%)	−3.891 (3%)	−3.408 (3%)	−2.253 (5%)
Initial funded ratio at 1.2	−2.777 (4%)	−2.218 (5%)	−0.878 (11%)	−3.601 (3%)	−3.034 (4%)	−1.723 (6%)
Less mature plan	−4.004 (2%)	−3.283 (2%)	−1.536 (4%)	−5.039 (2%)	−4.321 (2%)	−2.631 (3%)
No retirees at inception	−5.910 (2%)	−4.758 (2%)	−2.124 (2%)	−7.579 (2%)	−6.418 (2%)	−3.781 (2%)
Alternative asset in universe	−2.943 (10%)	−2.429 (11%)	−1.185 (20%)	−3.662 (9%)	−3.160 (10%)	−1.965 (13%)

Note: This table reports employer contribution risk measures for the small plans and their post-merger improvements in percentage (in parentheses). The value-at-risk and the expected shortfall are computed using Equations (18) and (19), respectively, for probability levels of 5%, 10%, and 50%. In addition to the base case, the employer contribution risk measures are also computed for all the robustness cases mentioned in the caption of Table SM.9.

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