Online Appendix: Inefficient Unemployment and Bargaining Friction

C. Y. Kelvin Yuen
Hong Kong University of Science and Technology (Guangzhou);
Hong Kong University of Science and Technology

Online Appendix

A Derivations

A.1 Present-discounted value vs. flow value

The original Mortensen-Pissarides model uses the flow value of wages in defining the value of being employed, which in the model considered here is given by

$$E_t(\varepsilon_t) = w_t(\varepsilon_t) + \beta \mathbb{E}_t \left\{ \begin{array}{c} \left[s + (1-s) \left(\begin{array}{c} \lambda \chi_{t+1}(\varepsilon_{t+1}) \\ +(1-\lambda)\chi_{t+1}(\varepsilon_t) \end{array} \right) \right] U_{t+1} \\ +(1-s) \left[\begin{array}{c} \lambda(1-\chi_{t+1}(\varepsilon_{t+1}))E_{t+1}(\varepsilon_{t+1}) \\ +(1-\lambda)(1-\chi_{t+1}(\varepsilon_t))E_{t+1}(\varepsilon_t) \end{array} \right] \end{array} \right\}$$

If we also define the present value of wages as

$$W_t(\varepsilon_t) = w_t(\varepsilon_t) + \beta(1-s)\mathbb{E}_t \left[\begin{array}{c} \lambda(1-\chi_{t+1}(\varepsilon_{t+1}))W_{t+1}(\varepsilon_{t+1}) \\ +(1-\lambda)(1-\chi_{t+1}(\varepsilon_t))W_{t+1}(\varepsilon_t) \end{array} \right]$$

Then it is easy to see that

$$V_t(\varepsilon_t) = E_t(\varepsilon_t) - W_t(\varepsilon_t)$$

Moreover, the value of a matched firm is

$$\begin{aligned} F_t(\varepsilon_t) &= P_t(\varepsilon_t) - W_t(\varepsilon_t) \\ &= p_t(\varepsilon_t) - w_t(\varepsilon_t) \\ &\quad +\beta(1-s)\mathbb{E}_t\left[\lambda(1-\chi_{t+1}(\varepsilon_{t+1}))F_{t+1}(\varepsilon_{t+1}) + (1-\lambda)(1-\chi_{t+1}(\varepsilon_t))F_{t+1}(\varepsilon_t)\right] \end{aligned}$$

which is consistent with the standard MP model with flow wages. Hence, all the analytical results still hold if wages are expressed in terms of present discounted value $W_t(\varepsilon_t)$ instead of flow value $w_t(\varepsilon_t)$.

A.2 Steady-state flow wage and free-entry condition

This subsection derives the steady-state flow wage equation and the corresponding free-entry condition with bargaining wedge.

For a continuing match with $\varepsilon \geq \varepsilon^s$, the firm's surplus satisfies

$$F(\varepsilon) = p(\varepsilon) - w^{ss}(\varepsilon) + \beta(1-s)\left[(1-\lambda)F(\varepsilon) + \lambda \int_{\varepsilon^s}^{\varepsilon_{\max}} F(\varepsilon')\, dG(\varepsilon')\right] \tag{A.1}$$

where $w^{ss}(\varepsilon)$ is the flow wage and ε^s is the separation cutoff. Free entry condition of the firm implies

$$c = \beta q(\theta^{ss}) \int_{\varepsilon^s}^{\varepsilon_{\max}} F(\varepsilon)\, dG(\varepsilon) \tag{A.2}$$

so that the cost of vacancy equals expected profit.

On the worker side, the employment value is

$$E(\varepsilon) = w^{ss}(\varepsilon) + \beta\{[s + (1-s)\lambda G(\varepsilon^s)]U + (1-s)[(1-\lambda)E(\varepsilon) + \lambda \int_{\varepsilon^s}^{\varepsilon_{\max}} E(\varepsilon')\, dG(\varepsilon')]\} \tag{A.3}$$

and the unemployment value is given by

$$U = b + \beta\{\theta^{ss} q(\theta^{ss}) \int_{\varepsilon^s}^{\varepsilon_{\max}} E(\varepsilon')\, dG(\varepsilon') + [1 - \theta^{ss} q(\theta^{ss})(1 - G(\varepsilon^s))]U\} \tag{A.4}$$

Subtracting (A.4) from (A.3) yields

$$E(\varepsilon) - U = w^{ss}(\varepsilon) - b + \beta(1-s)(1-\lambda)\left[E(\varepsilon) - U\right] + \beta\left[\lambda(1-s) - \theta^{ss} q(\theta^{ss})\right] \int_{\varepsilon^s}^{\varepsilon_{\max}} \left(E(\varepsilon') - U\right)\, dG(\varepsilon') \tag{A.5}$$

The generalized Nash bargaining problem in steady state implies

$$(1-\eta)\left(E(\varepsilon) - U - \Omega^w\right) = \eta\left(F(\varepsilon) - \Omega^f\right) \tag{A.6}$$

or

$$(1-\eta)\left(E(\varepsilon) - U\right) = \eta F(\varepsilon) + \Omega^{net} \tag{A.7}$$

Integrating over the continuation region gives

$$(1-\eta) \int_{\varepsilon^s}^{\varepsilon_{\max}} \left(E(\varepsilon) - U\right) dG(\varepsilon) = \eta \int_{\varepsilon^s}^{\varepsilon_{\max}} F(\varepsilon) dG(\varepsilon) + \Omega^{net}(1 - G(\varepsilon^s)) \tag{A.8}$$

Combining (A.1), (A.2), (A.5), (A.7), (A.8), and after rearranging, we arrive at

$$w^{ss}(\varepsilon) = \eta\left(p(\varepsilon) + \theta^{ss} c\right) + (1-\eta)\, b + \left[1 - \beta(1-s)\left[1 - \lambda G(\varepsilon^s)\right] + \beta\theta^{ss} q(\theta^{ss})\left(1 - G(\varepsilon^s)\right)\right] \Omega^{net} \tag{A.9}$$

To derive the free-entry condition, integrating (A.1) yields

$$\int_{\varepsilon^s}^{\varepsilon_{\max}} F(\varepsilon)dG(\varepsilon) = \int_{\varepsilon^s}^{\varepsilon_{\max}} \left[p(\varepsilon) - w^{ss}(\varepsilon)\right] dG(\varepsilon) + \beta(1-s)\left[1-\lambda G(\varepsilon^s)\right] \int_{\varepsilon^s}^{\varepsilon_{\max}} F(\varepsilon)dG(\varepsilon) \tag{A.10}$$

Using (A.2), we have the free-entry condition for the flow model:

$$\frac{c}{\beta q\left(\theta^{ss}\right)} = \frac{\int_{\varepsilon^s}^{\varepsilon_{\max}} \left[p(\varepsilon) - w^{ss}(\varepsilon)\right] dG(\varepsilon)}{1-\beta(1-s)\left[1-\lambda G(\varepsilon^s)\right]} \tag{A.11}$$

A.3 Proof of Proposition 1

By putting $\varepsilon_t = \varepsilon_t^e$ in (13), we have

$$V_t\left(\varepsilon_t^e\right) + W_t\left(\varepsilon_t^e\right) - U_t = \left(1-\eta\right)\Omega_t^w - \eta\Omega_t^f \tag{A.12}$$

$$P_t\left(\varepsilon_t^e\right) - W_t\left(\varepsilon_t^e\right) = -\left[\left(1-\eta\right)\Omega_t^w - \eta\Omega_t^f\right] \tag{A.13}$$

Hence, if $\left(1-\eta\right)\Omega_t^w - \eta\Omega_t^f = 0$, then from (A.12) and (A.13), clearly $\varepsilon_t^w = \varepsilon_t^e = \varepsilon_t^f$. On the other hand, if $\left(1-\eta\right)\Omega_t^w - \eta\Omega_t^f > 0$, then from (A.12) and using the fact that $V_t\left(\varepsilon_t\right)$ and $W_t\left(\varepsilon_t\right)$ are strictly increasing, we have $\varepsilon_t^e > \varepsilon_t^w$. Also, from (A.13) and using the fact that $P_t\left(\varepsilon_t\right) - W_t\left(\varepsilon_t\right)$ is also strictly increasing in ε_t, we have $\varepsilon_t^f > \varepsilon_t^e$. Combining, we have $\varepsilon_t^w < \varepsilon_t^e < \varepsilon_t^f$. Similarly, if $\left(1-\eta\right)\Omega_t^w - \eta\Omega_t^f < 0$, then again from (A.12) and (A.13) respectively, we have $\varepsilon_t^e < \varepsilon_t^w$ and $\varepsilon_t^f < \varepsilon_t^e$.

A.4 Special case

Here I consider a special case where $\lambda = 1$ and $p_t\left(\varepsilon_t\right) = y_t\varepsilon_t$. Notice first that when $\lambda = 1$, the value of working V_t is independent of the current idiosyncratic shock. Putting $\varepsilon_t = \varepsilon_t^f$ into (2) and using the definition of ε_t^f, we have

$$W_t\left(\varepsilon_t^f\right) = P_t\left(\varepsilon_t^f\right) = p_t\left(\varepsilon_t^f\right) + \beta\left(1-s\right)\mathbb{E}_t\left(1-\chi_{t+1}\left(\varepsilon_{t+1}\right)\right)P_{t+1}\left(\varepsilon_{t+1}\right) \tag{A.14}$$

Also, putting $\varepsilon_t = \varepsilon_t^e$ into (2) and using the definition of ε_t^e, we have

$$U_t - V_t = P_t\left(\varepsilon_t^e\right) = p_t\left(\varepsilon_t^e\right) + \beta\left(1-s\right)\mathbb{E}_t\left(1-\chi_{t+1}\left(\varepsilon_{t+1}\right)\right)P_{t+1}\left(\varepsilon_{t+1}\right) \tag{A.15}$$

Combining (A.14) and (A.15) yields

$$V_t + W_t\left(\varepsilon_t^f\right) - U_t = p_t\left(\varepsilon_t^f\right) - p_t\left(\varepsilon_t^e\right)$$

Now using the bargaining solution (13) and rearranging, we have

$$p_t\left(\varepsilon_t^f\right) - p_t\left(\varepsilon_t^e\right) = \frac{1}{1-\eta}\left[\left(1-\eta\right)\Omega_t^w - \eta\Omega_t^f\right]$$

Hence, the result (i) follows by putting $p_t\left(\varepsilon_t\right) = y_t\varepsilon_t$. Similarly, by putting $\varepsilon_t = \varepsilon_t^w$ into (2) again, we have

$$P_t\left(\varepsilon_t^w\right) = p_t\left(\varepsilon_t^w\right) + \beta\left(1-s\right)\mathbb{E}_t\left(1-\chi_{t+1}\left(\varepsilon_{t+1}\right)\right)P_{t+1}\left(\varepsilon_{t+1}\right)$$

Now using (A.15) and the definition of ε_t^w, we have

$$\begin{aligned} W_t\left(\varepsilon_t^w\right) - P_t\left(\varepsilon_t^w\right) &= U_t - V_t - P_t\left(\varepsilon_t^w\right) \\ &= p_t\left(\varepsilon_t^e\right) - p_t\left(\varepsilon_t^w\right) \end{aligned}$$

Finally, using (13) yields

$$p_t\left(\varepsilon_t^e\right) - p_t\left(\varepsilon_t^w\right) = \frac{1}{\eta}\left[\left(1-\eta\right)\Omega_t^w - \eta\Omega_t^f\right]$$

Hence, the result (ii) follows by noting $p_t\left(\varepsilon_t\right) = y_t\varepsilon_t$.

A.5 AOB model

Solving (28) and (29), the unique SPNE wage is given by

$$W_t\left(\varepsilon_t\right) = \frac{1}{2-\delta}\left[\left(1-\delta\right)P_t\left(\varepsilon_t\right) + U_t - V_t\left(\varepsilon_t\right) + \frac{\left(1-\delta\right)^2}{\delta}\gamma_t + \frac{1-\delta}{\delta}z_t\right]$$

Rearranging, we have

$$\frac{1-\delta}{2-\delta}\left[P_t\left(\varepsilon_t\right) - W_t\left(\varepsilon_t\right) - \left(-\frac{1-\delta}{\delta}\gamma_t\right)\right] = \frac{1}{2-\delta}\left[V_t\left(\varepsilon_t\right) + W_t\left(\varepsilon_t\right) - U_t - \frac{1-\delta}{\delta}z_t\right]$$

Hence, Proposition 2 follows by comparing with (13).

From Proposition 2 and the expression (16), we arrive at

$$\varepsilon_t^f - \varepsilon_t^e = \frac{1}{y_t}\left[\frac{1-\delta}{\delta} z_t + \frac{(1-\delta)^2}{\delta}\gamma_t\right]$$

Differentiating, we have

$$\begin{aligned}
\frac{\partial\left(\varepsilon_t^f - \varepsilon_t^e\right)}{\partial z_t} &= \frac{1}{y_t}\left[\frac{1-\delta}{\delta}\right] \\
\frac{\partial\left(\varepsilon_t^f - \varepsilon_t^e\right)}{\partial \gamma_t} &= \frac{1}{y_t}\left[\frac{(1-\delta)^2}{\delta}\right] \\
\frac{\partial\left(\varepsilon_t^f - \varepsilon_t^e\right)}{\partial \delta} &= -\frac{1}{y_t}\left[\frac{z_t + (1-\delta^2)\gamma_t}{\delta^2}\right]
\end{aligned}$$

A.6 Fundamental surplus in Ljungqvist and Sargent (2017)

Consider the steady state MP model with exogenous separation and homogeneous productivity in Ljungqvist and Sargent (2017). After setting $p(\varepsilon) = y$, $b = z$ and $\beta = (1+r)^{-1}$, the homogeneous version of the wage function (A.9) is

$$w = \eta(y + \theta c) + (1-\eta) z + \frac{r + s + \theta q(\theta)}{1+r}\Omega^{net} \tag{A.16}$$

Also, the homogeneous version of equation (A.11) is

$$\frac{c}{q(\theta)} = \frac{y - w}{r + s} \tag{A.17}$$

or

$$w = y - \frac{c(r+s)}{q(\theta)} \tag{A.18}$$

Equating (A.16) and (A.18) and rearranging, we have

$$\frac{1-\eta}{c}\left[y - z - \frac{\Omega^{net}(r+s)}{(1-\eta)(1+r)}\right] = \frac{r+s}{q(\theta)} + \eta\theta + \frac{\theta q(\theta)}{c(1+r)}\Omega^{net}$$

Hence, implicitly differentiating and rearranging, we get

$$\begin{aligned}\frac{d\theta}{dy} &= \frac{r+s+\eta\theta q(\theta)\left(1+\beta q(\theta)\frac{\Omega^{net}}{c\eta}\right)}{\alpha(r+s)+\eta\theta q(\theta)\left(1+(1-\alpha)\beta q(\theta)\frac{\Omega^{net}}{c\eta}\right)}\frac{\theta}{y-z-\frac{1-\beta(1-s)}{(1-\eta)}\Omega^{net}} \\ &= \Upsilon\left(\Omega^{net}\right)\frac{\theta}{y-z-\frac{1-\beta(1-s)}{(1-\eta)}\Omega^{net}}\end{aligned}$$

where $\alpha = \frac{-\theta q'(\theta)}{q(\theta)}$ is the elasticity of the matching function. Hence, the result follows by noting that $\epsilon_{\theta,y} = \frac{y}{\theta}\frac{d\theta}{dy}$.

B Business cycle statistics

In this appendix, I present the business cycle statistics in the data and in the model. I consider US labor market data from 1976 to 2016. The monthly unemployment rate series is taken from the official rates from the Labor Force Statistics of the Current Population Survey (CPS). The EU and UE transition rates are also taken from the same survey. For the period from January, 1976 to December, 2005, the separation rates and job finding rates are taken from Fujita and Ramey (2009). For the remaining years, I calculate the rates by $SR_{t+1} = \frac{EU_{t+1}}{e_t}$ and $FR_t = \frac{UE_{t+1}}{u_t}$, and adjust them for time aggregation error as in Fujita and Ramey (2009). Labor productivity p_t is defined by real GDP per workers, where the real GDP data is from Bureau of Economic Analysis and the number of employed workers is from CPS. Finally, the vacancy data is from the updated series by Barnichon (2010).

A summary table of the business cycle statistics of the US labor market is given in panel (a) of Table B.1. When calculating the business cycle statistics, all series are converted to quarterly data with simple averaging, as well as logged and HP filtered with smoothing parameter 1,600. In the table, σ_x^c denotes the cyclical volatility of the variable x, and $\epsilon_{x,p}$ denotes the elasticity of the variable x with respect to the labor productivity, as defined by

$$\epsilon_{x,p} = \frac{cov\left(x_t, p_t\right)}{var\left(p_t\right)} \tag{B.19}$$

It is worth noting that while the job finding rate is procyclical with an elasticity of 3.07 with respect to changes in labor productivity, the job separation rate is countercyclical with a comparable elasticity of -4.27. This shows that the separation margin of the labor market is indeed important at the business cycle frequency.

The business cycle statistics of the baseline calibration is given in panel (b) of Table B.1. Comparing with the data, we can see that the model matches well for the correlation matrix and the autocorrelation of the variables. A few significant differences can be observed. First, the volatility of job finding rate and the market tightness in the model are much lower than those in the data. This is related to Shimer (2005)'s finding that the MP model fails to generate enough volatility of the market tightness.[38] This problem can most likely be solved by relaxing the free entry condition, as shown in Coles and Moghaddasi Kelishomi (2018). Second, the model fails to produce the Beveridge curve (i.e. negative relationship between unemployment and vacancy). This is due to the fact that the volatility of separation rate in the model is too high compared with the data. As demonstrated in Fujita and Ramey

[38]It is worth noting, however, that the elasticity of job finding rate with respect to productivity in the model matches well with that in the data.

(a) Data								
	p_t	u_t	FR_t	SR_t	UE_t	EU_t	v_t	θ_t
σ_x^c	0.008	0.109	0.085	0.064	0.044	0.057	0.128	0.233
$\epsilon_{x,p}$	1	-4.267	3.072	-4.266	-1.293	-4.059	7.884	12.130
$corr\,(x_t, x_{t-1})$	0.698	0.939	0.838	0.703	0.444	0.651	0.931	0.940
Correlation matrix								
	p_t	u_t	FR_t	SR_t	UE_t	EU_t	v_t	θ_t
p_t	1	-0.295	0.272	-0.507	-0.221	-0.534	0.465	0.393
u_t		1	-0.921	0.750	0.646	0.663	-0.934	-0.981
FR_t			1	-0.612	-0.300	-0.524	0.887	0.918
SR_t				1	0.652	0.991	-0.779	-0.778
UE_t					1	0.617	-0.559	-0.609
EU_t						1	-0.708	-0.698
v_t							1	0.986
θ_t								1
(b) Simulated model								
	p_t	u_t	FR_t	SR_t	UE_t	EU_t	v_t	θ_t
σ_x^c	0.009	0.109	0.027	0.103	0.084	0.097	0.069	0.044
$\epsilon_{x,p}$	1	-11.714	3.049	-10.865	-8.516	-10.167	-6.756	4.961
$corr\,(x_t, x_{t-1})$	0.812	0.863	0.824	0.703	0.861	0.681	0.852	0.823
Correlation matrix								
	p_t	u_t	FR_t	SR_t	UE_t	EU_t	v_t	θ_t
p_t	1	-0.946	0.986	-0.933	-0.897	-0.922	-0.869	0.993
u_t		1	-0.959	0.887	0.990	0.870	0.981	-0.952
FR_t			1	-0.912	-0.914	-0.897	-0.887	0.998
SR_t				1	0.826	0.999	0.813	-0.929
UE_t					1	0.807	0.997	-0.902
EU_t						1	0.794	-0.916
v_t							1	-0.873
θ_t								1

Table B.1: Business cycle statistics of the labor market

Table B.2: Wage moments: data vs. model

Moment	Data	Model
Cyclical volatility	0.0105	0.0098
Autocorrelation	0.6891	0.8162
Elasticity w.r.t. productivity	0.1713	1.1083
Elasticity w.r.t. unemployment	0.0118	0.0895

(2012), this likely can be solved by introducing on-the-job search. Instead of matching all the summary statistics, I abstract from these complications by investigating how inefficient separations affect the labor market dynamics.

Table B.2 shows that the model does well in matching the overall size of wage fluctuations. The cyclical volatility of wages in the model is very close to that in the data. At the same time, wage fluctuations in the model are only slightly more persistent than in the data. However, the model makes wages too responsive to macroeconomic conditions. The wage elasticity with respect to productivity is much larger in the model than in the data, and the reported elasticity with respect to unemployment is also larger. In other words, the model captures the volatility of wages but overstates the systematic transmission of aggregate conditions into wages. It is well-documented that there exists some degree of wage rigidity in the data which is not captured by the standard model. Richer wage-setting environments of the kind emphasized by Gertler and Trigari (2009) could plausibly reduce the wage elasticities further, but introducing those additional sources of wage smoothing would move the paper away from its main goal of isolating the separation channel.

C Computation strategy

First, the stochastic processes of the aggregate and idiosyncratic productivity are discretized as follows. The AR(1) stochastic process (42) is approximated by a finite state Markov chain with N states and a transition probability matrix $[\pi_{i,j}]$ by using the Tauchen (1986) procedure. Also, I approximate the truncated log-normal distribution with K points of support $\{\varepsilon_1, ..., \varepsilon_K\}$, with $G(\varepsilon_K) = 1$. Let $g(\cdot)$ be the probability mass function of the discretized distribution. In the computation exercise, I set $N = 10$ and $K = 400$.

Next, I solve the model by using a fixed point algorithm. Since only the net bargaining wedge matters, I assume $\Omega^w = -\Omega^f \equiv \Omega$. In this case, we have $\Omega^{net} = \Omega$. Then I use a standard iteration procedure to find a fixed point. Specifically, given initial guess of $\{P_0, W_0, V_0, U_0, \theta_0, \chi_0\}$, I update the value functions and the market tightness as follows (derived from equations (1) - (5) and (14)).

$$
\begin{aligned}
\theta_{n+1}(y_i) &= \left\{ \begin{array}{c} \left(\frac{\beta A}{c}\right) \sum_{j=1}^{N} \sum_{k=1}^{K} g(\varepsilon_k) \pi_{i,j} \\ [1 - \chi_n(y_j, \varepsilon_k)] [P_n(y_j, \varepsilon_k) - W_n(y_j, \varepsilon_k) - k] \end{array} \right\}^{\frac{1}{\alpha}} \\
P_{n+1}(y_i, \varepsilon_l) &= y_i \varepsilon_l + \beta(1-s) \sum_{j=1}^{N} \pi_{i,j} \left[\begin{array}{c} \lambda \sum_{k=1}^{K} g(\varepsilon_k) [1 - \chi_n(y_j, \varepsilon_k)] P_n(y_j, \varepsilon_k) \\ + (1-\lambda) [1 - \chi_n(y_j, \varepsilon_l)] P_n(y_j, \varepsilon_l) \end{array} \right] \\
V_{n+1}(y_i, \varepsilon_l) &= \beta \sum_{j=1}^{N} \pi_{i,j} \left\{ \begin{array}{c} \left[s + (1-s) \left[\begin{array}{c} \lambda \sum_{k=1}^{K} g(\varepsilon_k) \chi_n(y_j, \varepsilon_k) \\ + (1-\lambda) \chi_n(y_j, \varepsilon_l) \end{array} \right] \right] U_n(y_j) \\ + (1-s) \left[\begin{array}{c} \lambda \sum_{k=1}^{K} g(\varepsilon_k) [1 - \chi_n(y_j, \varepsilon_k)] V_n(y_j, \varepsilon_k) \\ + (1-\lambda) [1 - \chi_n(y_j, \varepsilon_l)] V_n(y_j, \varepsilon_l) \end{array} \right] \end{array} \right\} \\
U_{n+1}(y_i) &= b + \beta \sum_{j=1}^{N} \sum_{k=1}^{K} g(\varepsilon_k) \pi_{i,j} \\
&\quad \left[\begin{array}{c} A \theta_n(y_i)^{1-\alpha} [1 - \chi_n(y_j, \varepsilon_k)] [W_n(y_j, \varepsilon_k) + V_n(y_j, \varepsilon_k)] \\ + \left(1 - A \theta_n(y_i)^{1-\alpha} + A \theta_n(y_i)^{1-\alpha} \chi_n(y_j, \varepsilon_k)\right) U_n(y_j) \end{array} \right] \\
W_{n+1}(y_i, \varepsilon_l) &= \eta P_{n+1}(y_i, \varepsilon_l) + (1-\eta)(U_{n+1}(y_i) - V_{n+1}(y_i, \varepsilon_l)) + \Omega \\
\chi_{n+1}(y_i, \varepsilon_l) &= \left\{ \begin{array}{cc} 1 & P_{n+1}(y_i, \varepsilon_l) \leq W_{n+1}(y_i, \varepsilon_l) \text{ or} \\ & W_{n+1}(y_i, \varepsilon_l) + V_{n+1}(y_i, \varepsilon_l) \leq U_{n+1}(y_i) \\ 0 & \text{otherwise} \end{array} \right.
\end{aligned}
$$

for all $i = 1, ..., N$ and $l = 1, ..., K$. The error of iteration is defined by

$$\max \{ \|P_{n+1} - P_n\|, \|V_{n+1} - V_n\|, \|U_{n+1} - U_n\|, \|\theta_{n+1} - \theta_n\| \}$$

where $\|\cdot\|$ is the entrywise matrix norm. The iteration continues until the error is less than the tolerance level. In the computation exercise, I set the tolerance level to be 10^{-6}. From the solution of $\{P, W, V, U\}$ from above, I then find the cutoff productivities $\left\{\varepsilon^w, \varepsilon^f, \varepsilon^e\right\}$ from the definitions (9) - (11).

To compute the unemployment volatility, I simulate a time series of unemployment by using the flow equations (18) - (20). I choose the sample size to be $12,000$ weeks, which is equivalent to 1000 quarters. The weekly series is transformed to quarterly frequency by calculating the simple average in a quarter. Finally, the log deviation of the quarterly series computed by using a hp-filter with a coefficient of 1600, and the cyclical part is used to measure the unemployment volatility.

D Additional quantitative results

D.1 Alternative aggregate productivity shock process

Here I estimate the aggregate productivity shock process by using the cyclical component of the quarterly labor productivity time series $\{\tilde{y}_t\}$. I first estimate an AR(1) process:

$$\tilde{y}_{t+1} = \rho \tilde{y}_t + \varepsilon_t$$

where

$$\varepsilon_t \sim N\left(0, \sigma^2\right)$$

Then I convert the quarterly process into weekly process as follows:

$$\begin{aligned} \hat{\alpha} &= \hat{\rho}^{\frac{1}{13}} \\ \hat{\gamma} &= \frac{\hat{\sigma}^2}{\sum_{i=0}^{12} \hat{\alpha}^{2i}} \end{aligned}$$

where $\hat{\alpha}$ and $\hat{\gamma}$ are the estimates for the persistence and standard deviation of the weekly process. I got $\hat{\alpha} = 0.9889$ and $\hat{\gamma} = 0.00397$, which is slightly more volatile than the baseline calibration. The quantitative results are shown in Figure D.1. As a result of the more volatile productivity shocks, the unemployment volatility is higher for each value of Ω^{net}. Therefore, the value of Ω^{net} to produce a realistic level of unemployment volatility is a bit lower at about 0.17. The contribution of inefficient unemployment is almost the same at 29% (vs. 29.1% for the baseline). I conclude that neither qualitative nor quantitative results have changed due to the more volatile process.

D.2 Alternative distribution for the idiosyncratic shocks

In the baseline calibration, the new drawing of the idiosyncratic shocks follows a truncated log-normal distribution. Here I consider an alternative specification where it is gamma distributed. Specifically, I calibrate the shape parameter κ_1 and the scale parameter κ_2 to target the average productivity and average job separation rate. The results are shown in Figure D.2.[39] While we also observe an increasing unemployment volatility as Ω^{net} increases as in the baseline, the curvature is different. In fact, for large values of Ω^{net}, unemployment volatility seems to be less sensitive to the bargaining friction. It is likely because the

[39] The estimated parameters are $\hat{\kappa}_1 = 50$ and $\hat{\kappa}_2 = 0.02$.

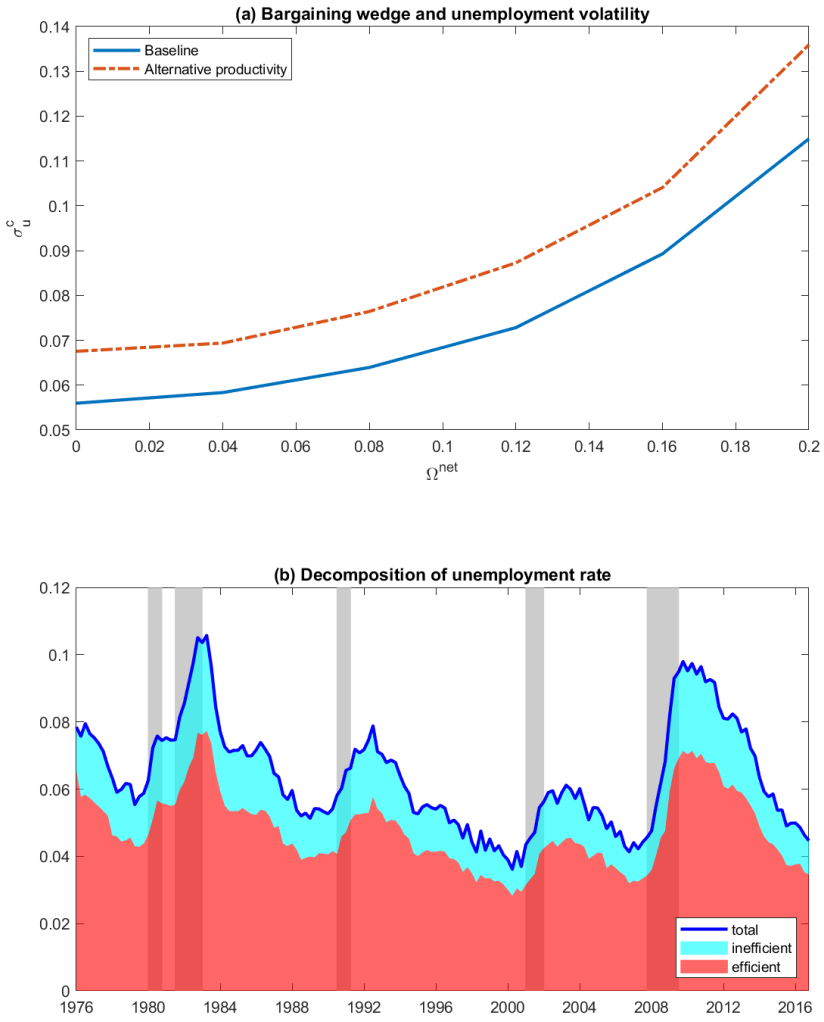

Figure D.1: Robustness: alternative aggregate productivity process

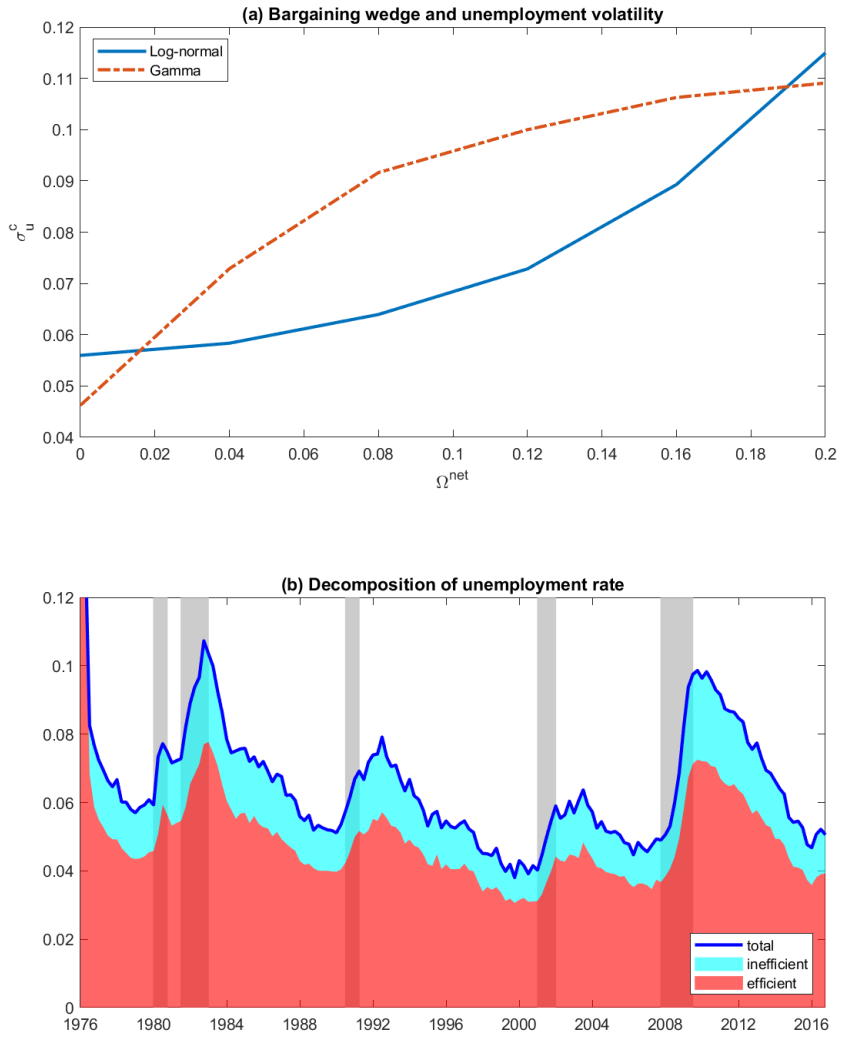

Figure D.2: Robustness: alternative idiosyncratic shocks

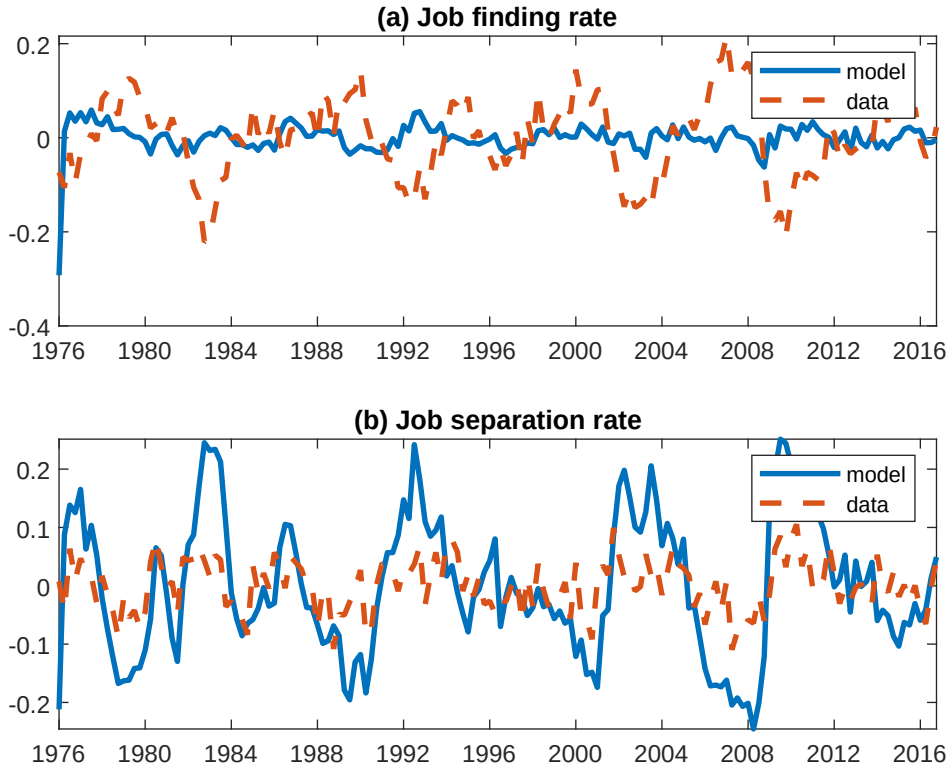

Figure D.3: Job finding and separation rates in the model vs. data

calibrated gamma distribution has a smaller right tail, hence the separations in the model, which mostly happen on the right tail, is less sensitive to changes in the specification of wages. The resulting estimated Ω^{net} is higher at around 0.20, and the importance of inefficient unemployment is slightly higher at 30%. Overall, neither qualitative nor quantitative results have changed for this alternative distribution.

D.3 Job finding and separation rates

Figure D.3 shows the job finding and separation rates in the model and in the data, respectively. As seen in the comparison of business cycle statistics, while the cyclicality of the transition rates in the model tends to follow that in the data, the model produces a less volatile job finding rate and more volatile job separation rate than in the data.

D.4 Constant bargaining wedge

Here I consider another specification of the matching process by assuming that the bargaining wedge is constant over the business cycle. As mentioned before, in the data, the average

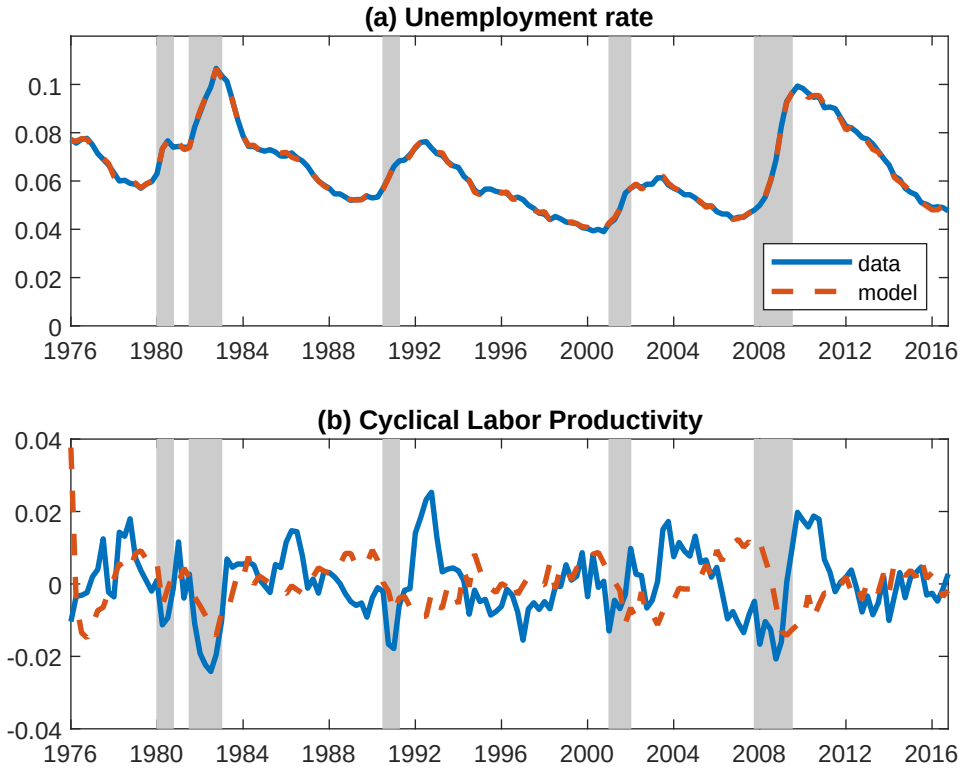

Figure D.4:
Unemployment rate and cyclical productivity (constant bargaining wedge)

unemployment volatility is about 10.9%, entailing a bargaining wedge of 0.188. I then perform the same procedure to decompose the unemployment rate. Figures D.4 and D.5 show the results.

We can see a similar decomposition with the case of cyclical bargaining wedge. In this case, we have $\gamma = 0.22$. Hence, an acyclical bargaining wedge alone can explain about 22% of the unemployment volatility in the data. Adding cyclicality to the bargaining wedge increases the explanatory power by another 7%.

D.5 Sensitivity analysis

Sensitivity analysis for various parameters is shown in Table D.1. Panel (a) varies the flow value of unemployment b. As b increases, cyclical unemployment volatility rises. The economic intuition is consistent with the small-surplus literature. At the same time, the share of volatility explained by inefficient unemployment remains fairly stable. Thus, changing b mainly alters the overall amount of amplification rather than overturning the importance of inefficient separations.

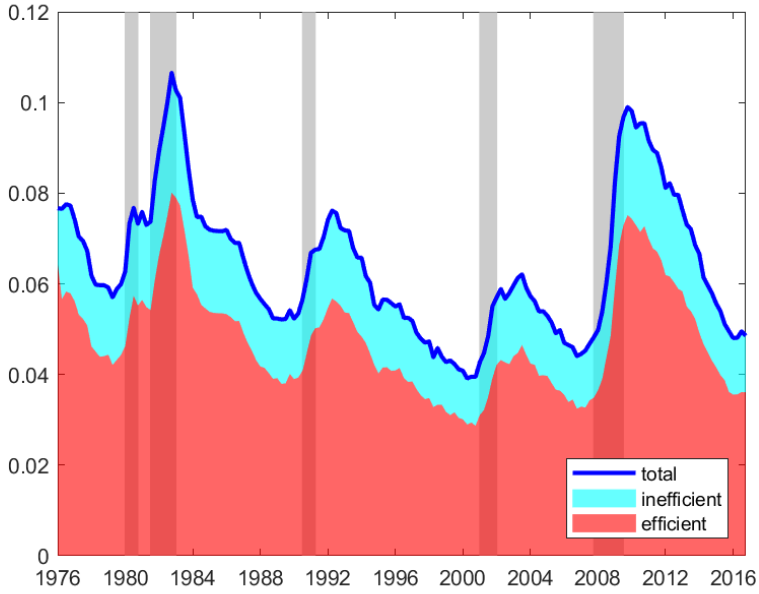

Figure D.5: Decomposition of unemployment rate (constant bargaining wedge)

Table D.1: Sensitivity analysis

Case	Cyclical unemployment volatility, σ_u^c	Contribution of inefficient unemployment, γ
Baseline	0.109	0.291
(a) Varying b		
$b = 0.60$	0.063	0.278
$b = 0.65$	0.090	0.304
$b = 0.75$	0.131	0.279
$b = 0.80$	0.202	0.256
(b) Varying η		
$\eta = 0.40$	0.102	0.321
$\eta = 0.45$	0.107	0.310
$\eta = 0.55$	0.113	0.274
$\eta = 0.60$	0.211	0.254
(c) Varying λ		
$\lambda = 0.08$	0.074	0.280
$\lambda = 0.09$	0.085	0.289
$\lambda = 0.11$	0.112	0.289
$\lambda = 0.12$	0.112	0.226
(d) Varying α		
$\alpha = 0.40$	0.104	0.276
$\alpha = 0.45$	0.103	0.285
$\alpha = 0.55$	0.118	0.297
$\alpha = 0.60$	0.127	0.302

Notes. The baseline calibration sets $b = 0.71$, $\eta = 0.5$, $\lambda = 0.104$, and $\alpha = 0.5$. I vary one parameter at a time while holding the baseline net bargaining wedge fixed at $\Omega^{net} = 0.1881$ and all other parameters at their baseline values. The column σ_u^c reports the standard deviation of quarterly cyclical unemployment, and $\gamma \equiv \text{cov}(u_t, u_t^i)/\text{var}(u_t)$ is the fraction of unemployment volatility explained by inefficient unemployment.

Panel (b) varies the worker's bargaining power η. Moving η away from the baseline changes both wage determination and the size of the effective separation distortion. A larger η shifts the wage bargain further in favor of the worker and tends to compress the firm's surplus, which weakens job creation and raises unemployment volatility. This effect is modest as η increases. The contribution of inefficient unemployment, however, declines from 0.321 at $\eta = 0.40$ to 0.254 at $\eta = 0.60$.

Panel (c) varies the switching rate of match-specific productivity λ. Raising λ increases unemployment volatility from 0.074 to 0.112. Intuitively, more frequent redraws make both efficient and inefficient separations more responsive to aggregate conditions. The contribution of inefficient unemployment is remarkably stable around $0.28 - 0.29$ for $\lambda \in [0.08, 0.11]$, which suggests that the decomposition is quite robust over economically plausible values.

Panel (d) varies the matching elasticity α. Increasing α reduces the elasticity of the job-finding rate with respect to market tightness and gives vacancy creation less direct influence. The table shows that cyclical unemployment volatility rises from 0.104 when $\alpha = 0.40$ to 0.127 when $\alpha = 0.60$. The economic intuition is that when the job-finding rate is less responsive to changes in tightness, fluctuations in separations translate more persistently into unemployment. Also, the contribution of inefficient unemployment increases from 0.276 at $\alpha = 0.40$ to 0.302 at $\alpha = 0.60$. Hence, a higher matching elasticity strengthens the relative importance of the separation channel by making unemployment dynamics less buffered by the job-creation margin.

Overall, Table D.1 shows that the main quantitative conclusions are robust. The exact level of unemployment volatility depends on the calibration, as expected, but inefficient unemployment remains a quantitatively meaningful source of business-cycle fluctuations across all parameter values considered here.