Unpublished Appendices of
"Optimal Mix of Monetary and Fiscal Policies in an Innovation-Driven Economy"

Tatsuro Iwaisako

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A Online Appendix A: Labor-income-tax-reducing case

As an alternative distortionary tax regime, we can consider the case in which money growth reduces the labor income tax instead of the corporate income tax. In this appendix , we consider this case. We assume that the corporate income tax rate is constant and the labor income tax rate is endogenously determined to satisfy the budget constraint (17). By substituting (10) into (17) and dividing both sides by $c(t)$, we obtain $\mu\xi + \tau_\Pi \frac{\Pi(t)n(t)}{c(t)} + \tau_w \frac{n(t)^\nu \ell(t)}{\beta c(t)} = \frac{n(t)^\nu \ell_G}{\beta c(t)}$. Using $z(t)$, we can rewrite the equation above as follows: $\mu\xi + \tau_\Pi \left(1 - \frac{1}{\beta}\right) + \tau_w \frac{\ell(t)}{\beta z(t)} = \frac{\ell_G}{\beta z(t)}$. In order to make the analysis simpler, we assume no corporate income tax, that is, $\tau_\Pi = 0$. Subsequently, the labor income tax rate is endogenously determined as

$$\tau_w(t)\ell(t) = \ell_G - \mu\xi\beta z(t). \tag{A.1}$$

By substituting (19) into (23) and using $\dot{z} = 0$, we obtain z^* as follows:

$$\beta z^* = \ell^* - l_G + a\rho. \tag{A.2}$$

From (19), z^* and ℓ^* satisfy

$$(\ell^* - \tau_w \ell^*)(\bar{\ell} - \ell^*) = \gamma \left[1 + \xi(\rho + \mu)\right] \beta z^* \ell^*. \tag{A.3}$$

From (A.1), in the steady state, $\tau_w \ell^* = \ell_G - \mu\xi\beta z^*$. Substituting this into (A.3), we obtain

$$(\ell^* - \ell_G + \mu\xi\beta z^*)(\bar{\ell} - \ell^*) = \gamma \left[1 + \xi(\rho + \mu)\right] \beta z^* \ell^*. \tag{A.4}$$

Dviding both sides by $\beta z^*(\bar{\ell} - \ell^*)$ yields

$$\frac{\ell^* - \ell_G}{\beta z^*} + \mu\xi = \gamma \left[1 + \xi(\rho + \mu)\right] \frac{\ell^*}{\bar{\ell} - \ell^*}. \tag{A.5}$$

Substituting (A.2) into this, we obtain the equation of ℓ^* as follows.

$$1 + \mu\xi - \frac{a\rho}{\ell^* - l_G + a\rho} = \gamma \left[1 + \xi(\rho + \mu)\right] \frac{\ell^*}{\bar{\ell} - \ell^*}, \tag{A.6}$$

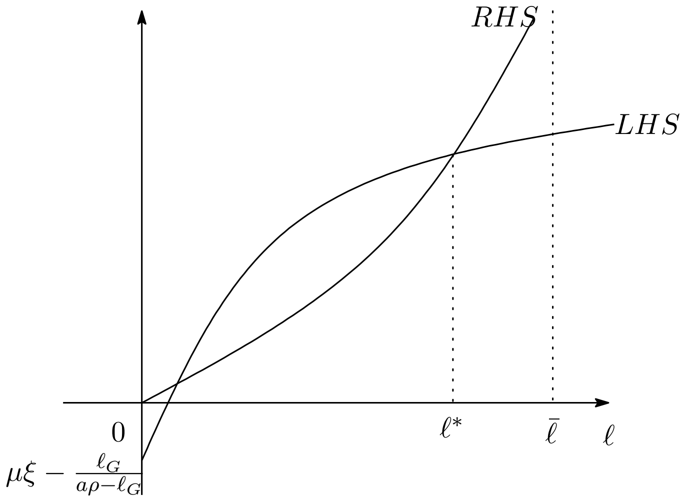

Figure 5: The equilibrium of ℓ^*

which determines the equilibrium value of ℓ. The left-hand side (LHS) and right-hand side (RHS) of the equation are depicted as in Fig. 5. As in this figure, we assume that the LHS intersects with the RHS in the region of positive ℓ. In contrast with the corporate-income-tax-reducing case, the labor-income-tax-reducing case is complicated, and there can be two equilibria of ℓ as in Fig. 5. From this multiplicity of ℓ^*, there are two steady states of z^*. One steady state of z is unstable, while the other is stable. Probably, the steady state corresponding to higher ℓ^* is unstable, and there is a unique equilibrium path. Thus, we focus our analysis on the higher ℓ^*.

We let ℓ^* denote the higher value of ℓ that is determined by (A.6). Differentiating both sides of this with respect to μ, we obtain

$$\left\{ \frac{a\rho}{(\ell^* - l_G + a\rho)^2} - \gamma \left[1 + \xi(\rho + \mu)\right] \frac{\bar{\ell}}{(\bar{\ell} - \ell^*)^2} \right\} \frac{d\ell^*}{d\mu} = \left(\gamma \frac{\ell^*}{\bar{\ell} - \ell^*} - 1 \right) \xi. \tag{A.7}$$

In Fig.5, at the higher equilibrium of ℓ^*, the slope of the RHS is larger than that of the LHS. The first and second terms in the content of the brace of the LHS of (A.7) represents the slopes of the LHS and the RHS of (A.6) respectively. Therefore, the LHS of (A.6) is negative. Rewriting (A.6) yields

$$\gamma \frac{\ell^*}{\bar{\ell} - \ell^*} = \frac{1 + \mu\xi - \frac{a\rho}{\ell^* - l_G + a\rho}}{1 + \xi(\rho + \mu)} < 1. \tag{A.8}$$

This means that the RHS of (A.7) is negative. Hence, we obtain $\frac{d\ell^*}{d\mu} > 0$.

Next, we examine the effect on the growth rate of $n(t)$, g_n^*. From (20), $g_n^* = \frac{1}{a}(\ell^* - \ell_G - z^*)$.

Differentiating g_n^* with respect to μ yields

$$\begin{aligned}\frac{\partial g_n^*}{\partial \mu} &= \frac{1}{a}\left(\frac{d\ell^*}{d\mu} - \frac{dz^*}{d\mu}\right) \\ &= \frac{1}{a}\left(1 - \frac{1}{\beta}\right)\frac{d\ell^*}{d\mu} > 0, \qquad \text{(A.9)}\end{aligned}$$

where we use $\beta\frac{dz^*}{d\mu} = \frac{d\ell^*}{d\mu}$ from (A.2) and $\frac{d\ell^*}{d\mu} > 0$. Therefore, an increase in money growth rate raises the growth rate of output in the labor-income-tax-reducing case also.

Finally, we derive the welfare-maximizing money growth rate. In order to derive the welfare-maximizing money growth rate, we differentiate (28) with respect to μ. In the same way as (29), the derivative of U is given by:

$$\frac{\partial U}{\partial \mu} = \frac{1}{\rho}\Big[\frac{1}{z^*}\frac{\partial z^*}{\partial \mu} \quad + \quad \gamma\frac{1}{\bar{\ell} - \ell^*}\frac{\partial(\bar{\ell} - \ell^*)}{\partial \mu} \quad + \quad \frac{1}{\rho}\nu\frac{\partial g_n^*}{\partial \mu}\Big].$$

Substituting (A.9) and (A.2) into this, we obtain

$$\frac{\partial U}{\partial \mu} = \frac{1}{\rho}\Big[\frac{1}{\ell^* - \ell_G + a\rho} \quad - \quad \gamma\frac{1}{\bar{\ell} - \ell^*} \quad + \quad \frac{1}{\rho}\nu\frac{1}{a}\left(1 - \frac{1}{\beta}\right)\Big]\frac{d\ell^*}{d\mu}. \qquad \text{(A.10)}$$

Therefore, the welfare-maximizing money growth rate satisfies

$$\frac{1}{\ell^* - \ell_G + a\rho} + \frac{1}{\rho}\nu\frac{1}{a}\left(1 - \frac{1}{\beta}\right) = \gamma\frac{1}{\bar{\ell} - \ell^*}. \qquad \text{(A.11)}$$

Since ℓ^* is an increasing function of μ, the LHS and RHS are a decreasing function and increasing function of μ t

hrough ℓ^*, respectively. The intersection of the LHS and RHS determines the welfare-maximizing money growth rate, μ^*. A stricter protection of patents (an increase in β) shifts the LHS upward, and thus, raises the welfare-maximizing money growth rate μ^* in contrast with the corporate-income-tax-reducing case.

Table 1: Effects of the strength of patent protection on the optimal inflation rate

Regression Results For the Dependent Variable	$Inflation_i$	
Constant	18.646**	(3.468)
$LNGDP_i$	-0.793	(-0.976)
$PATENT_i$	-1.354	(-1.229)
Observations	115	
Adjusted R^2	0.073	

Notes: Student's t-test values are in parentheses. *Statistically significant at 10%. **Statistically significant at 5%

B Online Appendix B: Regression results

In this appendix, we reexamine whether the negative correlation between inflation and the strength of patent protection still remains after controlling for per capita GDP. We consider the following empirical specification.

$$inflation_i = a + b_1\, lnGDP_i + b_2\, patentindex_i, \tag{B.1}$$

where a denotes the constant term, and b_1 and b_2 denote the coefficients on $\ln GDP$ and the patent index, respectively. As shown in Table 1, the coefficient on the patent index remains negative, although it is not statistically significant at 10%.