Appendix
Solutions to the Elemental Model

From Section 2.2 of the text, we here express the equilibrium condition, (11), for the real yield on the private portfolio, r^P, in terms of real (lower case) variables:

$$a_0^S + a_1^S\left(y_0 + \Delta y\right) - a_2^S\, y^e - a_3^S \Delta w^L + a_4^S\, r^P - a_0^i - a_1^i\, r^{Ke} + a_2^i\, r^P = g - t - \Delta m^B + \left(\gamma r^P + \pi^e\right) d_0^{GP} \;,$$

from which we can derive the financial market's portfolio yield as:

$$r^P = \frac{\left(a_0^i - a_0^S - a_1^S y_0\right) - \rho^S + \left(g - t\right) - \Delta m^B + \pi^e d_0^{GP} + -a_1^S\, \Delta y + a_2^S\, y^e + a_3^S \Delta w^L + a_1^i\, r^{Ke}}{\left(a_4^S + a_2^i - \gamma d_0^{GP}\right)} . \quad \text{(A.1)}$$

Signing the coefficients depends on the likely event that $a_4^S + a_2^i > \gamma d_0^{GP}$.[1] As in the text, however, the endogeneity of Δm^B suggests moving on to a reformulation of the expression, before we interpret the signs.

The money market equilibrium condition, from (14) in the text, yields the following expression for the change in the monetary base:

$$\Delta m^B = \left(a_0^P - a_0^M\right) + \left(a_1^P + a_1^M\right) v + a_2^P \Delta w^H - a_3^P\left(r^P + \pi^e\right) + \phi \Delta y \,. \quad \text{(A.2)}$$

Substituting this in (A1.1), after some manipulation, we obtain:

$$\begin{aligned} r^P = {} & \frac{a_0^i - a_0^S - a_0^P + a_0^M - a_1^S y_0}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P} - \rho^S \frac{1}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P} \\ & - \Delta y \frac{\left(a_1^S + \phi\right)}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P} + \left(g - t\right) \frac{1}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P} \\ & + \Delta w^L \frac{a_3^S}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P} - \Delta w^H \frac{a_2^P}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P} \\ & - v \frac{\left(a_1^P + a_1^M\right)}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P} + \pi^e \frac{\left(d_0^{GP} + a_3^P\right)}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P} \\ & + y^e \frac{a_2^S}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P} + r^{Ke} \frac{a_1^i}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P} \end{aligned} \quad \text{(A.3)}$$

[1] This depends on our ability to sign the expression: $a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P$ as >0. In any calibration, a_4^S and a_2^i are linear coefficients of yields, expressed proportionally. Necessarily they are large numbers. For feasible levels of initial government debt held by the private sector we can be assured of the expression's positivity. Even were this less certain, the assumption of its positivity yields results that are consistent with standard theory, for example the crowding out effect of a rise in $g - t$.

We can also evaluate the period change in the real monetary base, Δm^B, that is chosen by the private sector, as indicated in the text. By substituting (A.3) into (A.2) we have:

$$\Delta m^B = \left(a_0^P - a_0^M\right) + \left(a_1^P + a_1^M\right)v + a_2^P \Delta w^H - a_3^P \pi^e + \phi \Delta y$$
$$-\frac{a_3^P}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P}\begin{bmatrix} \left(a_0^i - a_0^S - a_0^P + a_0^M - a_1^S y_0\right) - \rho^S - \Delta y\left(a_1^S + \phi\right) \\ +\left(g - t\right) + \Delta w^L a_3^S - \Delta w^H a_2^P - v\left(a_1^P + a_1^M\right) \\ +\pi^e\left(d_0^{GP} + a_3^P\right) + y^e a_2^S + r^{Ke} a_1^i \end{bmatrix}. \quad \text{(A.4)}$$

Collecting terms yields:

$$\Delta m^B = \left(a_0^P - a_0^M\right) - \frac{a_3^P\left(a_0^i - a_0^S - a_0^P + a_0^M - a_1^S y_0\right)}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P}$$
$$+\Delta y \frac{a_3^P\left(a_1^S + \phi\right) + \phi\left[a_4^S + a_2^i - \gamma d_0^{GP} + a_3^P\right]}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P} + \rho^S \frac{a_3^P}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P}$$
$$-\left(g - t\right)\frac{a_3^P}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P} + \Delta w^H\left[a_2^P + \frac{a_2^P a_3^P}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P}\right]$$
$$-\Delta w^L \frac{a_2^P a_3^S}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P} + v\left[\left(a_1^P + a_1^M\right) + \frac{a_3^P\left(a_1^P + a_1^M\right)}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P}\right].$$
$$-\pi^e a_3^P\left[1 + \frac{d_0^{GP} + a_3^P}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P}\right] - y^e \frac{a_3^P a_2^S}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P}$$
$$-r^{Ke}\frac{a_3^P a_1^i}{a_4^S + a_2^i - \gamma d_0^{GP} - a_3^P}$$

(A.5)

The rate of inflation depends on the expansion of the monetary base, and any associated change in the money multiplier, as they influence the period change in real money balances. Critically, it depends on the difference between the proportional changes in the money multiplier, μ, and real money balances:

$$\pi = \frac{\Delta P}{P_0} = \frac{\Delta M^B}{M_0^B} + \frac{\Delta \mu}{\mu_0} - \frac{\Delta m^S}{m_0^S} \quad \text{(A.6)}$$

We can sign the determinants of the final two terms from our workings above, but there is a more direct route, since we have a formulation for the period change in the real monetary base. Since inflation affects the nominal to real ratio of the monetary base by the same proportion as that of overall money balances, it is simpler to write:

$$\pi = \frac{\Delta P}{P_0} = \frac{\Delta M^B}{M_0^B} - \frac{\Delta m^B}{m_0^B} \tag{A.7}$$

This yields the signage offered for the determinants of inflation in the text.