Appendix of "Mergers and Acquisitions and the Aggregate Markup" (For Online Publication Only).

A Data Method and Additional Figures

Estimating Markups We use the price-marginal cost markup, $P_i/MC_i - 1$, to indicate the monopoly power of firm i, and measure it by using methods proposed in De Loecker and Warzynski (2012) and De Loecker et al. (2020). The method starts with the typical cost minimization problem of firm i is

$$\mathcal{L}(V_i, K_i, \lambda_i) = P_i^V V_i + r_i K_i - \lambda_i (F(V_i, K_i) - \bar{Y}),$$

where P_i^V and V_i are the price and quantity of the variable input, r_i and K_i are the user cost and quantity of capital. $F(\cdot)$ is the production function and $\bar{Y}$ is the targeted output level, and λ_i is the Lagrangian multiplier. The advantage of writing in this way is that $\lambda = \frac{\partial \mathcal{L}}{\partial \bar{Y}}$, i.e. λ_i gives the marginal cost of firm i. The first order condition *w.r.t.* V_i reads

$$P_i^V - \lambda_i \frac{\partial Y_i}{\partial V_i} = 0.$$

$Y_i \equiv F(V_i, K_i)$ denotes total output. Equivalently

$$\xi_{V,i} \equiv \frac{\partial Y_i}{\partial V_i} \frac{V_i}{Y_i} = \frac{1}{\lambda_i} \frac{P_i^V V_i}{Y_i},$$

where $\xi_{V,i}$ is the elasticity of output *w.r.t.* the variable input. It follows that the markup, $\mathcal{M}_i = P_i/\lambda_i - 1$, equals to

$$\mathcal{M}_i = \xi_{V,i} \frac{P_i Y_i}{P_i^V V_i} - 1,$$

where $P_i Y_i$ and $P_i^V V_i$ are observed in data, and the elasticity of output *w.r.t.* the variable input can be estimated from data.

Assume the production function is, $Q = F(V, K)\exp(z)$[1]. Take log on both sides and use lowercase letters to denote the natural logarithm of variables

$$q = \xi_v v + \xi_k k + z + \epsilon.$$

[1] We assume the same production function at the 2-digit sector level among Compustat firms.

Demand of the variable input, V, is a function of capital stock, K, and unobserved productivity, Z, $V = g(K, Z)$. We can then represent productivity, $Z(z)$, as a function of $K(k)$ and $V(v)$, i.e., $z = h(k, v)$. In the first stage, run the following regression non-parametrically (or approximate ϕ by a polynomial)

$$q = \phi(v, k) + \epsilon.$$

Further, assume the exogenous productivity follows an $AR(1)$ process

$$z_{t+1} = \rho z_t + \epsilon_z.$$

Obtain $\hat{z} = \hat{\phi}(v, k) - \xi_v v - \xi_k k$ from the first stage, and we then apply the general method of moments to estimate ξ_v, by using the following moment conditions

$$E[(\hat{z}_t - \rho \hat{z}_{t-1}) X_{t-1}] = 0.$$

In the baseline case, we include capital stock in period t, k_t, and variable input in period $t-1$, v_{t-1}, in X_{t-1}.

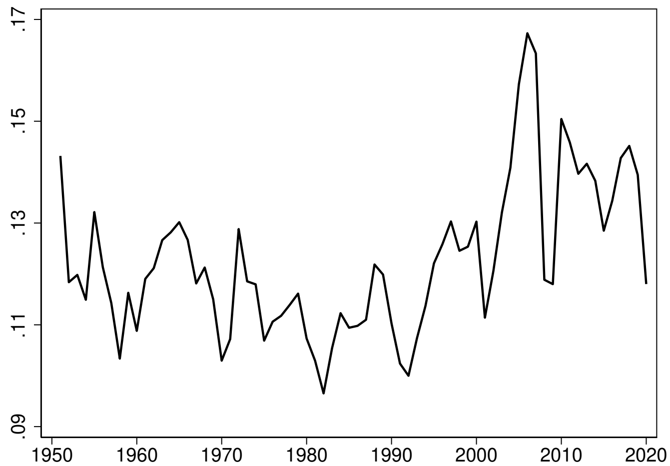
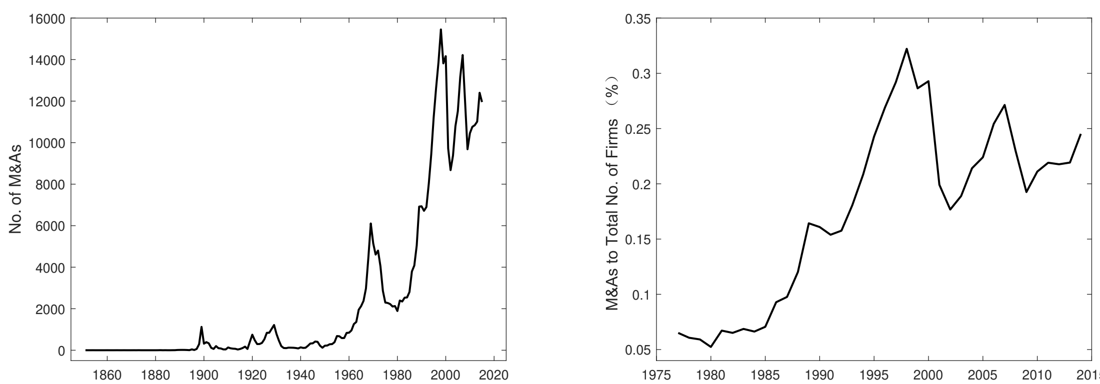


Figure A.1: The Lerner index 1951-2020

Note: This figure plots the aggregate Lerner index, the ratio of operating income after depreciation to sales. *Data source:* Compustat.



(a) Number of M&As, 1851-2015

(b) Ratio of M&As to Total No. of Firms

Figure A.2: The Number and Ratio of M&As in the U.S.

Note: The total number of M&As is from the official website of the Institute for Mergers, Acquisitions, and Alliances, https://imaa-institute.org., while the total number of firms is from Business Dynamics Statistics.

B Proofs for Propositions 1 and 2

We present proofs for Propositions 1 and 2 in this section, which are regarding theoretical properties of the equilibrium relative size function $q^*(z)$ and profit function $\pi(z)$.

Step 1: $q^*(z)$ is strictly increasing.

This statement can be derived directly from the first-order condition in equation (12). One can easily verify that the left-hand side of the equation is strictly decreasing in q, while the right-hand side is strictly decreasing in z. So as z increases, the equilibrium relative size q^* must also increase such that the equation holds. Besides, equation (12) also implies that $q^*(z)$ is continuously differentiable and bounded above by a constant $\beta^{\frac{\beta}{\alpha}}$.

Step 2: $q^*(z)$ is strictly convex on some $(0, \underline{z}_q)$, and strictly concave on some $(\bar{z}_q, \infty)$.

Equation (12) gives us a unique q^* for each level of z. But unfortunately, we can not find an analytical solution for the function $q^*(z)$. So instead, we work with its inverse function, $z(q)$,[2] which has a clear analytical form

$$z(q) = \frac{w\beta}{\beta - 1} \exp\left(\frac{q^{\frac{\alpha}{\beta}} - 1}{\alpha}\right) \left(\frac{\beta}{\beta - q^{\frac{\alpha}{\beta}}}\right).$$

Easy to see that $z(q)$, as well as the original $q^*(z)$, are continuously differentiable to an infinite order. The first and second-order derivatives of $z(q)$ are

$$z'(q) = \frac{w\beta}{\beta - 1} \exp\left(\frac{q^{\frac{\alpha}{\beta}} - 1}{\alpha}\right) \frac{\left(\alpha + \beta - q^{\frac{\alpha}{\beta}}\right) q^{\frac{\alpha - \beta}{\beta}}}{\left(\beta - q^{\frac{\alpha}{\beta}}\right)^2},$$

and

$$z''(q) = \frac{w}{\beta - 1} \exp\left(\frac{q^{\frac{\alpha}{\beta}} - 1}{\alpha}\right) \frac{\left(\alpha + \beta - q^{\frac{\alpha}{\beta}}\right) q^{\frac{2(\alpha - \beta)}{\beta}}}{\left(\beta - q^{\frac{\alpha}{\beta}}\right)^3}$$
$$\times \left[\alpha + 2\beta - q^{\frac{\alpha}{\beta}} - \beta(\beta - \alpha) q^{-\frac{\alpha}{\beta}} - \frac{\left(\beta - q^{\frac{\alpha}{\beta}}\right)/\alpha}{1 + \left(\beta - q^{\frac{\alpha}{\beta}}\right)/\alpha}\right].$$

[2] We drop the superscript * for notation simplicity.

One can verify that $z'(q) > 0,\ \forall q \in \left(0, \beta^{\frac{\beta}{\alpha}}\right)$, which validates our results in step 1. The sign of $z''(q)$ though, is ambiguous on the full support $\left(0, \beta^{\frac{\beta}{\alpha}}\right)$, so we only show limiting properties.

Observe that in our expression of $z''(q)$, the term in the first line is always positive. As q approaches its lower bound 0, the term in the second line approaches $-\infty$. That proves the concavity of $z(q)$, or equivalently, the convexity of $q^*(z)$, around its lower bound of the support. The existence of $\underline{z}_q$ follows immediately since $q^*(z)$ and all of its higher-order derivatives are continuously differentiable.

Using the same logic, one can check that as q approaches its upper bound $\beta^{\frac{\beta}{\alpha}}$, the second line term approaches $2\alpha > 0$. This proves the convexity of $z(q)$, or equivalently, the concavity of $q^*(z)$, around its lower bound of the support, the existence of $\bar{z}_q$ follows immediately.

Step 3: $\pi(z)$ is strictly increasing.

To prove this statement and the next one, we utilize the correlation between the profit π and relative size q, which has an analytical form of

$$\pi(q) = \frac{(\beta-1)DY}{\beta^2} \exp\left(\frac{1-q^{\frac{\alpha}{\beta}}}{\alpha}\right) q^{\frac{\alpha+\beta}{\beta}},$$

where D and Y are positive constants in the aggregate economy.

The first-order derivative of $\pi(q)$ is

$$\pi'(q) = \frac{(\beta-1)DY}{\beta^3} \exp\left(\frac{1-q^{\frac{\alpha}{\beta}}}{\alpha}\right) \left(\alpha+\beta-q^{\frac{\alpha}{\beta}}\right) q^{\frac{\alpha}{\beta}},$$

which is strictly positive on the support of $(0, \beta^{\frac{\beta}{\alpha}})$. Since we have shown that $q^*(z)$ is strictly increasing in z, and bounded above by $\beta^{\frac{\beta}{\alpha}}$, the fact that $\pi(z) \equiv \pi\left(q^*(z)\right)$ is strictly increasing follows immediately.

Step 4: $\pi(z)$ is strictly convex on some $(0, \underline{z}_\pi)$, and strictly concave on some $(\bar{z}_\pi, \infty)$.

Now we need the second-order derivative of $\pi(q)$, which is

$$\pi''(q) = \frac{(\beta-1)DY}{\beta^4} \exp\left(\frac{1-q^{\frac{\alpha}{\beta}}}{\alpha}\right) q^{\frac{\alpha-\beta}{\beta}} \times \left[\left(q^{\frac{\alpha}{\beta}}\right)^2 - (3\alpha+\beta)q^{\frac{\alpha}{\beta}} + \alpha(\alpha+\beta)\right].$$

Using quadratic algebra, one can verify that $\pi''(q) > 0$ on $(0, \hat{q})$ and < 0 on $(\hat{q}, \beta^{\frac{\beta}{\alpha}})$, where $\hat{q} \equiv \left(\frac{3\alpha+\beta-\sqrt{5\alpha^2+2\alpha\beta+\beta^2}}{2}\right)^{\frac{\beta}{\alpha}}$. That is, $\pi(q)$ is convex-concave in q. Consequently, $\pi(z) \equiv \pi(q^*(z))$ is strictly convex (concave) in z when around 0 (approaching ∞). The existence of $\underline{z}_\pi$ and $\bar{z}_\pi$ follows the fact that $\pi(z)$ and all of its higher-order derivatives are continuously differentiable. That concludes our proofs for Propositions 1 and 2. QED

C Quantitative Analysis Appendix

C.1 Numerical Solution Algorithm

We solve the stationary equilibrium of the model economy using an iteration method with four loops. Among them, loop 1 is the outermost, while loop 4 is the innermost. We discretize the state space of z by setting 400 grid points on a support of $[1, 40]$. To improve precision, we set the grid points to be denser on the lower end of the support, where most of the firms reside. The detailed algorithm is described as follows.[3]

- Guess
  - Loop 1: Guess an effective wage rate w, solve for the relative size function $q^*(z)$ from the f.o.c. specified in equation (12).
  - Loop 2: Guess a discretized probability mass vector representing $F(z)$, solve for the aggregate variables M, D, W, Y, and the profit function $\pi(z)$ using the following equations

$$M \int \sigma(q^*) dF(z) = 1;$$

$$D = \left[M \int \sigma'(q^*) q^* dF(z) \right]^{-1};$$

$$W = wD;$$

$$MY \int \frac{q^*}{z} dF(z) = 1;$$

$$\pi(z) = \left[\sigma'(q^*) q^* - \frac{w}{z} q^* \right] DY.$$

  - Loop 3: Guess a value vector representing $V(z)$, construct the $\Delta V(z_A, z_T)$ matrix based on the value vector and the merger technology.
  - Loop 4: Guess a pair of market tightness θ_A and θ_T, solve for the optimal search intensity vectors representing $\mu_A(z)$ and $\mu_T(z)$.

- Update

[3] MATLAB codes are available upon request, please email: caolinyi@mail.shufe.edu.cn.

- Loop 4: Use the newly solved $\mu_A(z)$ and $\mu_T(z)$ vectors to find a new pair of θ_A and θ_T. Define distance $= \max\left\{\left|\theta_A^{\text{new}} - \theta_A^{\text{old}}\right|, \left|\theta_T^{\text{new}} - \theta_T^{\text{old}}\right|\right\}$, iterate the process until the distance is below a tolerance of 10^{-5}.
- Loop 3: Plug $\mu_A(z)$, $\mu_T(z)$, θ_A, and θ_T into the right-hand-side of the value function, to find a new value vector. Define distance $= \max\left\{\left|[V^{\text{new}}(z) - V^{\text{old}}(z)]/V^{\text{old}}(z)\right|\right\}$, iterate the process until the distance is below a tolerance of 10^{-5}.
- Loop 2: First calculate the equilibrium entrant mass $M_E = \eta M + M_T$. Then plug $\Delta V(z_A, z_T)$, $\mu_A(z)$, $\mu_T(z)$, θ_A, θ_T, and M_E/M into the right-hand side of the Kolmogorov forward equation, to find a new probability mass vector. Define distance $= \max\left\{\left|F^{\text{new}}(z) - F^{\text{old}}(z)\right|\right\}$, iterate the process until the distance is below a tolerance of 10^{-5}.
- Loop 1: Check the free entry condition specified by equation (25). If the entrant's expected value $\int V(z)dH(z)$ is larger than the entry cost c_E, raise w; if smaller, reduce w. Define distance $= \left|\int V(z)dH(z) - c_E\right|$, continue the process until the distance or the change in w is below a tolerance of 10^{-5}.

C.2 Identification of the Parameters

To show that the parameters listed in Table 4.3 are well identified, we first check how the total sum of distances changes when each parameter deviates from its benchmark value. Figure C.1 gives the result, and the total sum of distances is indeed minimized at the parameters' benchmark values.

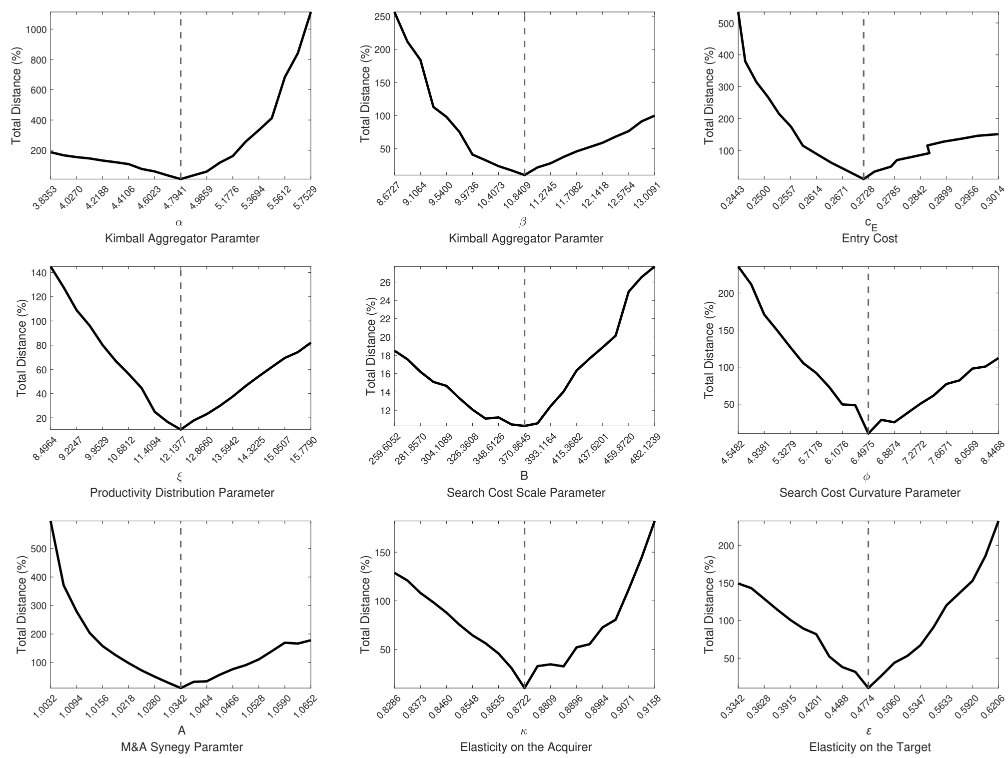


Figure C.1: Total Distance w.r.t. Each Parameter

Note: This figure shows how the total sum of distances (Y-axis) changes as we move each of the 9 parameters (X-axis) away from its benchmark value.

Though estimated jointly by a minimum distance, some moments are particularly informative about the corresponding parameters. To support our argument, Figure C.2 plots each of the 9 moments as a function of the corresponding parameter, keeping all other parameters at their benchmark values. The figure shows significant variation in each moment when the corresponding parameter deviates from its benchmark value.

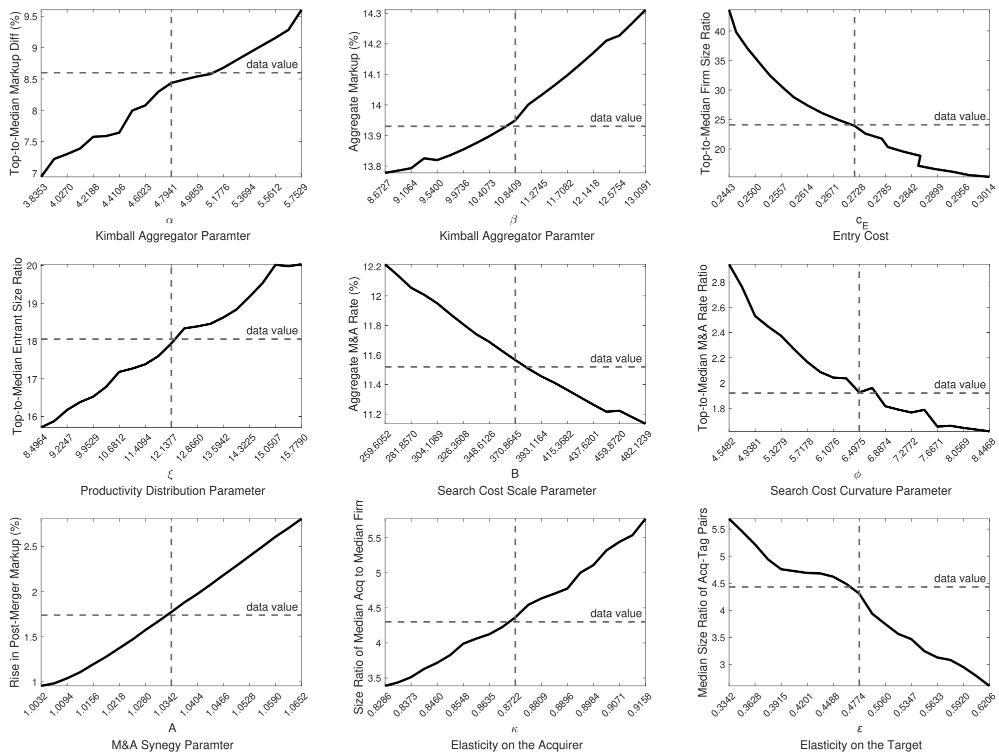


Figure C.2: Informative Moment w.r.t. Each Parameter

Note: This figure checks the sensitivity of each of the 9 model-generated moments (Y-axis) as a function of the corresponding parameter (X-axis).

C.3 Decreasing Return to Scale

Under the linear production technology specified by equation (9), it is possible that firms whose productivity is low choose to produce nothing, i.e., $q^* = 0$, a corner solution. While the impact of this property on the aggregate markup is negligible, as such firms are very small, we can fix the issue by assuming a decreasing return to scale production technology

$$y_t = z\ell_t^{\delta}, \quad \delta \in (0,1).$$

In this setup, it is guaranteed that $q^*(z) > 0, \forall z > 0$. To check how it would change the main results of the paper, we set $\delta = 0.99$ to deviate as little as possible from the baseline case and replicate Table 4.5 in the following Table C.1. As can be seen, the two results are qualitatively and quantitatively similar.

Table C.1: Surging M&As and the Aggregate Markup

	B	$\bar{\mathcal{M}}_{model}$	$\bar{\mathcal{M}}_{data}$	explanation Power
1981-1985	370.86	12.07%	13.93%	—
2013-2017	11.40	18.45%	23.68%	
change	—	6.38p.p.	9.75 p.p.	65.47%

Note: The explanation power is calculated as the model-generated change in the aggregate markup divided by that in the data. All numbers are rounded to two decimal places for better presentation.

C.4 Output and Welfare

In the model, a lower merger cost affects the total output through three channels: (i) it *enhances productivity* of the economy, thus raising total output; (ii) it worsens the *misallocation* of the economy, thus reducing total output; and (iii) it increases firm values and encourages *entry*, thus raising total output. In the quantified model, we find that the two positive effects dominate, causing the total output to rise from 1.51 in the benchmark to 1.86 in the counterfactual economy.

To further decompose the strength of these three channels, we denote $\hat{Y}_i$, $i = bm, cf$ as the hypothetical output level under homogeneous markups, i.e., $\alpha = 0$, for the benchmark and counterfactual cases, respectively. The gap $\hat{Y}_i - Y_i$ thus measures the severity of misallocation caused by the diminishing demand elasticity. Furthermore, we use the term $\hat{Y}_{interm}$ to denote that output when we have the mass of firms in the counterfactual case but keep the productivity distribution in the benchmark case. Then, changes in the total output can be written as

$$Y_{cf} - Y_{bm} = \underbrace{(\hat{Y}_{cf} - \hat{Y}_{interm})}_{\text{(i) productivity-enhancing}} - \underbrace{\left[(\hat{Y}_{cf} - Y_{cf}) - (\hat{Y}_{bm} - Y_{bm})\right]}_{\text{(ii) misallocation}} + \underbrace{(\hat{Y}_{interm} - \hat{Y}_{bm})}_{\text{(iii) entry}}$$

Table C.2 presents the decomposition results. The productivity-enhancing effect contributes to an increase of 0.58 in the output, while the entry effect contributes to an additional 0.04. The worsening misallocation, indicated by the enlarging output gap $\hat{Y}_i - Y_i$, reduces output by -0.27.[4]

[4]Output increases less in an economy with heterogeneous markups compared to one with homoge-

Table C.2: Surging M&As, Total Output, and Consumption

$Y_{cf} - Y_{bm}$	(i) productivity-enhancing	(ii) misallocation	(iii) entry
0.35	0.58	-0.27	0.04
Contribution	165.71%	-77.14%	11.43%

Note: The contribution of each channel is calculated by dividing the corresponding number by the net change in Y. The detailed procedures of decomposition are given in the main text. All numbers are rounded to two decimal places for better presentation.

As for welfare, which is simply measured by consumption in our setup

$$\text{Consumption} = Y - \underbrace{\int \left[C(\mu_A(z)) + C(\mu_T(z))\right] dF(z)}_{\text{search costs}} - \underbrace{M_e c_E}_{\text{entry cost}} .$$

It also rises, but to a smaller extent of 0.24. That is because the rise in total output is partially offset by the increased search and entry costs.

The output and welfare implications of the model analyzed above should be interpreted under two caveats. First, the result that the negative impact of M&As on the markup dispersion and misallocation is dominated follows from the restriction in our framework that the markup is fully determined by productivity. In a framework where markup can be separated from productivity and the impact of M&A on the two variables does not necessarily always occur simultaneously, the dominance might be altered. Second, our model only captures the impact of M&A on productivity and associated markups, but abstracts from other effects. For example, M&A might have a negative impact on R&D intensity (Hall et al., 1990), might kill potential competitors (Cunningham et al., 2021), or affect the market power of rivals (Stiebale and Szücs, 2022). Our model, being silent on all these dimensions, clearly underestimates the negative effects of M&As.

neous markups, as large, productive firms reduce their quantities to raise their markups. This implies that antitrust policies need to be size-dependent.

References

Cunningham, Colleen, Florian Ederer, and Song Ma, "Killer Acquisitions," *Journal of Political Economy*, 2021, *129* (3), –.

De Loecker, Jan and Frederic Warzynski, "Markups and Firm-Level Export Status," *American Economic Review*, 2012, *102* (6), 2437–2471.

_ , Jan Eeckhout, and Gabriel Unger, "The Rise of Market Power and the Macroeconomic Implications," *Quarterly Journal of Economics*, 2020, *135* (2), 561–644.

Hall, Bronwyn H., Ernst Berndt, and Richard C. Levine, "The Impact of Corporate Restructuring on Industrial Research and Development," *Brookings Papers on Economic Activity. Microeconomics*, 1990, *1990*, 85–124.

Stiebale, Joel and Florian Szücs, "Mergers and market power: evidence from rivals' responses in European markets," *RAND Journal of Economics*, 2022, *53* (4), 687–702.