

Supplementary Materials

A Note on the Size Distribution of Government Debt

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Table A1: Lognormal estimation results.

Year	LL	AIC	BIC	μ	σ	KS Stat.	KS P-val.	AD Stat.	AD P-val.
1980	-538.636	1081.271	1086.796	3.409	0.799	0.074	0.535	0.841	0.452
1981	-555.273	1114.546	1120.104	3.480	0.792	0.083	0.381	0.916	0.404
1982	-590.580	1185.160	1190.768	3.616	0.824	0.112	0.095	1.505	0.175
1983	-606.821	1217.642	1223.250	3.776	0.801	0.063	0.721	0.970	0.373
1984	-626.709	1257.417	1263.058	3.791	0.856	0.091	0.261	1.561	0.163
1985	-634.969	1273.938	1279.594	3.908	0.781	0.079	0.409	0.979	0.368
1986	-644.933	1293.865	1299.554	3.979	0.726	0.066	0.641	1.096	0.311
1987	-637.654	1279.308	1284.949	4.057	0.716	0.071	0.556	1.050	0.332
1988	-644.208	1292.415	1298.072	4.026	0.748	0.076	0.461	1.426	0.195
1989	-644.763	1293.525	1299.182	4.038	0.742	0.086	0.315	1.170	0.279
1990	-683.793	1371.586	1377.306	4.037	0.856	0.107	0.107	2.656	0.041
1991	-684.073	1372.147	1377.897	4.023	0.803	0.107	0.099	2.263	0.066
1992	-723.393	1450.786	1456.640	4.082	0.772	0.094	0.171	1.416	0.198
1993	-761.484	1526.969	1532.936	4.066	0.764	0.076	0.372	1.093	0.312
1994	-786.618	1577.235	1583.257	4.040	0.806	0.072	0.410	1.111	0.304
1995	-810.119	1624.238	1630.350	3.989	0.781	0.078	0.294	1.186	0.273
1996	-821.813	1647.627	1653.789	3.941	0.775	0.094	0.115	1.822	0.116
1997	-823.378	1650.757	1656.956	3.905	0.738	0.100	0.076	1.837	0.113
1998	-862.421	1728.841	1735.101	3.999	0.730	0.079	0.242	1.134	0.294
1999	-867.321	1738.643	1744.902	4.035	0.725	0.073	0.322	0.989	0.363
2000	-885.016	1774.031	1780.326	4.016	0.749	0.069	0.394	1.180	0.275
2001	-901.053	1806.106	1812.424	3.980	0.802	0.096	0.082	1.969	0.096
2002	-906.426	1816.852	1823.181	3.989	0.797	0.110	0.028	2.289	0.064
2003	-906.348	1816.696	1823.025	3.943	0.833	0.121	0.012	2.916	0.030
2004	-891.901	1787.802	1794.109	3.859	0.884	0.115	0.021	3.161	0.023
2005	-888.904	1781.808	1788.138	3.744	0.920	0.121	0.012	3.804	0.011
2006	-857.125	1718.250	1724.568	3.582	0.928	0.087	0.142	1.953	0.098
2007	-838.344	1680.688	1687.006	3.465	0.936	0.084	0.169	1.986	0.094
2008	-834.941	1673.881	1680.200	3.446	0.936	0.087	0.147	2.116	0.080
2009	-843.352	1690.703	1697.033	3.561	0.851	0.087	0.144	1.987	0.093
2010	-836.936	1677.871	1684.201	3.564	0.819	0.107	0.037	2.786	0.035
2011	-832.712	1669.424	1675.742	3.576	0.811	0.103	0.051	2.309	0.063
2012	-838.342	1680.685	1687.003	3.603	0.815	0.099	0.064	2.351	0.059
2013	-848.778	1701.557	1707.875	3.635	0.838	0.122	0.011	2.956	0.029
2014	-864.575	1733.150	1739.468	3.664	0.892	0.129	0.006	4.246	0.007
2015	-870.606	1745.212	1751.530	3.759	0.840	0.134	0.004	4.905	0.003
2016	-875.581	1755.163	1761.481	3.822	0.811	0.150	0.001	5.601	0.002
2017	-873.764	1751.527	1757.846	3.854	0.778	0.144	0.001	5.447	0.002
2018	-880.786	1765.571	1771.889	3.872	0.795	0.157	0.000	6.195	0.001
2019	-869.613	1743.226	1749.544	3.898	0.727	0.136	0.003	4.375	0.006
2020	-893.832	1791.665	1797.972	4.058	0.733	0.133	0.004	5.175	0.002

Notes: The reported are log-likelihood (LL), Akaike information criterion (AIC), and Bayesian information criterion (BIC), and the Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) tests. The KS and AD p-values formally test the null hypothesis that the sample is drawn from the fitted distribution.

Table A2: Double Pareto-lognormal estimation results.

Year	LL	AIC	BIC	μ	σ	α	β	KS Stat.	KS P-val.	AD Stat.	AD P-val.
1980	-531.511	1071.021	1082.070	3.538	0.405	2.533	1.910	0.085	0.360	145.826	0.944
1981	-550.503	1109.006	1120.122	3.661	0.344	2.457	1.704	0.101	0.178	144.244	0.939
1982	-583.006	1174.012	1185.228	3.815	0.284	2.312	1.583	0.074	0.520	180.167	0.882
1983	-601.911	1211.821	1223.037	3.985	0.318	2.434	1.615	0.074	0.520	209.462	0.830
1984	-616.822	1241.645	1252.926	4.028	0.235	2.257	1.471	0.081	0.396	195.182	0.856
1985	-630.664	1269.328	1280.641	4.140	0.317	2.591	1.618	0.104	0.134	216.541	0.821
1986	-638.504	1285.008	1296.385	4.180	0.302	2.822	1.803	0.087	0.296	172.753	0.903
1987	-631.134	1270.267	1281.548	4.237	0.328	2.912	1.910	0.065	0.680	118.510	0.981
1988	-637.028	1282.056	1293.369	4.194	0.276	2.534	1.776	0.072	0.536	156.197	0.939
1989	-638.207	1284.414	1295.727	4.162	0.264	2.377	1.836	0.080	0.401	199.991	0.858
1990	-667.415	1342.831	1354.270	4.183	0.114	2.042	1.571	0.101	0.145	309.397	0.646
1991	-672.083	1352.167	1363.668	4.183	0.113	2.143	1.596	0.084	0.314	157.288	0.950
1992	-716.076	1440.152	1451.861	4.182	0.278	2.218	1.813	0.072	0.463	284.836	0.725
1993	-756.346	1520.692	1532.627	4.219	0.258	2.295	1.700	0.062	0.636	159.879	0.966
1994	-781.959	1571.919	1583.961	4.277	0.327	2.484	1.564	0.047	0.900	170.301	0.948
1995	-806.602	1621.204	1633.429	4.288	0.291	2.711	1.497	0.089	0.165	299.352	0.781
1996	-814.970	1637.941	1650.266	4.292	0.216	2.834	1.428	0.062	0.564	211.132	0.914
1997	-815.081	1638.162	1650.562	4.226	0.140	2.807	1.477	0.073	0.344	147.195	0.986
1998	-857.186	1722.372	1734.892	4.145	0.000	2.153	1.637	0.065	0.471	328.784	0.742
1999	-862.128	1732.256	1744.776	4.217	0.272	2.586	1.758	0.065	0.471	171.074	0.969
2000	-877.472	1762.943	1775.533	4.171	0.287	2.460	1.782	0.058	0.606	179.958	0.970
2001	-888.602	1785.203	1797.839	4.180	0.296	2.472	1.653	0.046	0.856	163.365	0.975
2002	-892.648	1793.296	1805.955	4.216	0.257	2.522	1.602	0.051	0.744	166.457	0.982
2003	-888.456	1784.912	1797.571	4.131	0.226	2.271	1.592	0.080	0.213	310.934	0.819
2004	-872.189	1752.378	1764.991	4.130	0.287	2.461	1.477	0.075	0.283	388.327	0.684
2005	-866.221	1740.442	1753.101	4.119	0.257	2.640	1.326	0.074	0.289	386.553	0.715
2006	-845.447	1698.895	1711.531	3.954	0.352	2.541	1.305	0.057	0.614	205.284	0.932
2007	-825.531	1659.063	1671.699	3.842	0.375	2.613	1.317	0.063	0.490	237.763	0.904
2008	-821.334	1650.669	1663.305	3.823	0.344	2.547	1.299	0.075	0.286	229.575	0.909
2009	-829.062	1666.123	1678.782	3.972	0.435	3.969	1.509	0.051	0.744	188.143	0.969
2010	-819.293	1646.585	1659.244	3.831	0.222	2.558	1.519	0.051	0.744	137.719	0.995
2011	-818.145	1644.290	1656.926	3.843	0.292	2.700	1.569	0.052	0.741	200.719	0.939
2012	-822.951	1653.902	1666.538	3.941	0.417	3.724	1.648	0.069	0.380	320.395	0.776
2013	-830.564	1669.128	1681.764	3.993	0.372	3.430	1.540	0.063	0.490	197.362	0.945
2014	-833.081	1674.162	1686.798	4.100	0.372	4.234	1.487	0.069	0.380	331.475	0.759
2015	-833.895	1675.791	1688.427	4.127	0.307	4.004	1.619	0.086	0.150	174.974	0.967
2016	-834.686	1677.372	1690.008	4.122	0.263	3.619	1.736	0.052	0.741	140.506	0.986
2017	-832.227	1672.455	1685.091	4.062	0.236	3.201	1.924	0.092	0.105	397.437	0.662
2018	-836.281	1680.562	1693.198	4.081	0.225	3.145	1.896	0.098	0.072	442.495	0.610
2019	-839.086	1686.172	1698.808	4.058	0.222	2.945	1.999	0.092	0.105	464.325	0.587
2020	-858.605	1725.210	1737.823	4.273	0.260	3.501	2.000	0.121	0.012	603.898	0.421

Notes: The reported are log-likelihood (LL), Akaike information criterion (AIC), and Bayesian information criterion (BIC), and the Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) tests. The KS and AD p-values formally test the null hypothesis that the sample is drawn from the fitted distribution.

Table A3: Generalized beta II estimation results.

Year	LL	AIC	BIC	a	b	p	q	KS Stat.	KS P-val.	AD Stat.	AD P-val.
1980	-538.531	1085.061	1096.110	0.656	13.895	10.140	6.236	0.103	0.171	216.384	0.799
1981	-555.417	1118.834	1129.951	0.572	18.159	12.070	8.791	0.126	0.046	319.240	0.591
1982	-592.173	1192.345	1203.561	0.634	11.860	10.823	5.486	0.180	0.001	1073.453	0.071
1983	-607.309	1222.618	1233.834	0.540	17.481	14.319	8.928	0.139	0.018	638.521	0.236
1984	-629.260	1266.519	1277.800	0.634	11.188	10.969	4.792	0.194	0.000	1151.365	0.062
1985	-637.461	1282.922	1294.235	0.693	11.746	11.914	4.657	0.160	0.003	824.226	0.140
1986	-647.490	1302.980	1314.357	0.716	13.020	13.091	5.034	0.142	0.012	961.854	0.104
1987	-640.602	1289.203	1300.484	0.718	13.511	13.842	5.179	0.137	0.019	518.646	0.327
1988	-645.162	1298.324	1309.637	0.578	17.343	15.760	8.235	0.104	0.134	478.666	0.365
1989	-647.346	1302.692	1314.005	0.712	12.639	12.941	4.727	0.120	0.055	468.150	0.376
1990	-685.659	1379.318	1390.757	0.638	11.985	11.779	4.650	0.194	0.000	1322.612	0.046
1991	-685.816	1379.632	1391.133	0.563	15.906	14.948	7.726	0.191	0.000	1309.940	0.045
1992	-725.631	1459.262	1470.971	0.665	13.359	13.695	5.380	0.159	0.002	1106.463	0.084
1993	-764.280	1536.560	1548.495	0.685	12.855	13.248	4.977	0.164	0.001	1032.445	0.122
1994	-789.736	1587.472	1599.515	0.658	12.295	12.728	4.952	0.140	0.006	1206.875	0.090
1995	-813.056	1634.112	1646.337	0.642	12.803	13.112	5.489	0.115	0.032	609.266	0.368
1996	-823.837	1655.674	1668.000	0.545	16.349	15.663	8.608	0.118	0.023	823.494	0.237
1997	-824.193	1656.385	1668.785	0.464	24.840	19.734	14.296	0.098	0.088	540.501	0.440
1998	-864.647	1737.293	1749.813	0.719	13.256	13.207	5.062	0.136	0.004	936.612	0.194
1999	-870.264	1748.528	1761.048	0.707	13.924	13.837	5.380	0.077	0.270	444.058	0.579
2000	-888.426	1784.852	1797.442	0.710	12.882	12.994	4.865	0.093	0.102	677.905	0.364
2001	-905.605	1819.210	1831.846	0.650	11.959	12.388	4.939	0.115	0.020	971.128	0.187
2002	-911.345	1830.690	1843.349	0.645	12.146	12.563	5.058	0.137	0.003	1372.109	0.089
2003	-909.117	1826.234	1838.893	0.560	14.399	13.053	6.664	0.200	0.000	2707.947	0.009
2004	-892.233	1792.467	1805.080	0.479	20.089	13.967	9.519	0.191	0.000	2332.189	0.015
2005	-890.811	1789.623	1802.282	0.520	13.862	11.956	6.885	0.183	0.000	2505.889	0.012
2006	-857.600	1723.200	1735.836	0.478	18.539	12.905	9.491	0.161	0.000	1585.558	0.055
2007	-838.335	1684.670	1697.306	0.487	18.446	11.590	8.990	0.144	0.002	1402.845	0.074
2008	-834.904	1677.809	1690.445	0.467	20.616	11.078	9.204	0.132	0.005	1564.209	0.057
2009	-845.677	1699.354	1712.013	0.611	12.896	10.986	6.121	0.069	0.383	264.902	0.875
2010	-836.755	1681.509	1694.168	0.535	20.026	12.587	9.438	0.091	0.107	532.838	0.526
2011	-834.565	1677.130	1689.766	0.604	14.315	11.360	6.574	0.115	0.020	905.703	0.216
2012	-840.911	1689.822	1702.458	0.629	12.738	11.100	5.851	0.132	0.005	1092.257	0.146
2013	-850.221	1708.443	1721.079	0.564	15.482	11.864	7.332	0.092	0.105	555.110	0.475
2014	-858.221	1724.441	1737.077	0.811	28.535	5.515	4.762	0.167	0.000	1865.600	0.035
2015	-858.643	1725.287	1737.923	0.925	31.682	4.964	3.802	0.075	0.286	617.722	0.413
2016	-865.237	1738.473	1751.110	0.744	30.620	7.683	5.906	0.109	0.032	1014.475	0.173
2017	-868.829	1745.658	1758.295	0.710	18.649	11.219	6.046	0.086	0.150	638.165	0.399
2018	-876.846	1761.692	1774.328	0.704	17.094	11.123	5.644	0.115	0.020	829.331	0.254
2019	-860.760	1729.521	1742.157	0.909	39.781	6.726	5.628	0.109	0.032	636.308	0.401
2020	-891.689	1791.377	1803.991	0.560	30.737	14.530	10.029	0.110	0.031	1148.930	0.131

Notes: The reported are log-likelihood (LL), Akaike information criterion (AIC), and Bayesian information criterion (BIC), and the Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) tests. The KS and AD p-values formally test the null hypothesis that the sample is drawn from the fitted distribution.

Table A4: Pareto-tails lognormal estimation results.

Year	LL	AIC	BIC	μ	σ	τ_u	α_u	τ_l	α_l	KS Stat.	KS P-val.	AD Stat.	AD P-val.
1980	-530.995	1073.991	1090.564	1.951	3.420	50.468	2.219	20.788	1.970	0.043	0.983	55.720	1.000
1981	-550.394	1112.787	1129.462	4.255	4.514	50.151	2.158	25.409	1.593	0.050	0.923	73.089	0.999
1982	-583.043	1178.086	1194.910	7.191	5.635	52.370	1.971	30.209	1.622	0.060	0.779	91.504	0.991
1983	-600.592	1213.184	1230.008	8.786	2.445	58.479	2.098	32.184	1.409	0.066	0.670	72.361	0.999
1984	-616.091	1244.183	1261.104	8.661	2.181	59.785	2.154	32.225	1.433	0.040	0.988	53.615	1.000
1985	-632.096	1276.192	1293.162	9.020	1.933	57.599	1.914	32.926	1.441	0.112	0.087	162.600	0.925
1986	-639.058	1290.116	1307.181	9.543	2.031	63.587	2.207	37.855	1.580	0.074	0.498	127.879	0.969
1987	-633.186	1278.372	1295.294	9.146	1.661	59.419	1.930	38.731	1.763	0.073	0.531	135.402	0.959
1988	-639.003	1290.007	1306.977	9.719	1.989	58.639	1.802	38.950	1.750	0.080	0.401	187.281	0.875
1989	-639.171	1290.343	1307.313	8.800	1.674	58.639	1.891	38.431	1.707	0.064	0.685	126.595	0.971
1990	-667.044	1346.088	1363.246	10.022	1.585	59.539	1.910	39.036	1.409	0.047	0.943	59.052	1.000
1991	-670.621	1353.242	1370.494	9.396	1.492	58.699	1.985	34.040	1.254	0.046	0.946	70.537	1.000
1992	-715.037	1442.074	1459.638	8.648	1.344	54.798	1.743	31.365	1.200	0.072	0.463	174.741	0.922
1993	-755.431	1522.862	1540.764	9.890	1.622	58.349	1.811	34.031	1.172	0.068	0.500	214.220	0.879
1994	-781.821	1575.643	1593.707	9.113	1.608	58.329	1.731	30.439	0.967	0.084	0.235	311.612	0.720
1995	-805.301	1622.603	1640.940	8.955	1.835	65.048	2.004	28.747	0.946	0.089	0.165	319.970	0.735
1996	-815.926	1643.852	1662.340	9.092	1.659	58.979	2.012	29.719	1.030	0.099	0.083	307.028	0.789
1997	-814.933	1641.866	1660.465	9.387	1.937	62.815	2.354	29.007	1.565	0.049	0.830	131.957	0.991
1998	-855.281	1722.562	1741.341	8.730	1.821	63.805	2.120	27.522	1.466	0.047	0.843	129.847	0.996
1999	-862.864	1737.728	1756.507	9.568	1.751	59.830	1.967	33.080	1.585	0.061	0.554	207.873	0.935
2000	-878.327	1768.654	1787.539	8.525	1.613	58.449	1.935	25.829	1.536	0.064	0.483	161.845	0.978
2001	-889.479	1790.958	1809.912	8.984	1.648	57.149	1.885	33.878	1.481	0.063	0.490	189.079	0.962
2002	-892.502	1797.003	1815.992	8.889	1.514	57.509	1.961	31.518	1.314	0.051	0.744	162.867	0.985
2003	-888.649	1789.297	1808.286	9.203	1.635	56.679	1.952	33.419	1.410	0.057	0.617	181.969	0.973
2004	-872.517	1757.035	1775.955	11.328	3.019	59.599	1.955	42.346	1.485	0.057	0.634	180.765	0.962
2005	-865.945	1743.889	1762.878	11.213	4.601	66.610	2.336	43.688	1.318	0.034	0.986	106.800	0.999
2006	-844.218	1700.437	1719.391	9.270	12.347	66.999	2.339	37.110	1.169	0.034	0.986	59.183	1.000
2007	-825.549	1663.097	1682.051	8.708	5.640	56.499	2.208	30.919	1.211	0.034	0.986	76.427	1.000
2008	-820.906	1653.813	1672.767	8.477	3.490	56.142	2.331	24.088	1.276	0.029	0.999	55.257	1.000
2009	-829.736	1671.472	1690.461	10.441	6.346	53.259	2.242	35.505	1.391	0.029	0.999	81.367	1.000
2010	-819.405	1650.809	1669.798	7.611	6.747	53.647	2.441	36.760	1.466	0.034	0.986	84.750	1.000
2011	-818.468	1648.935	1667.889	7.782	6.337	52.021	2.275	35.699	1.505	0.039	0.951	92.573	1.000
2012	-823.883	1659.767	1678.721	3.589	7.981	56.382	2.233	33.659	1.579	0.046	0.856	151.966	0.987
2013	-831.083	1674.166	1693.120	7.444	5.043	57.800	2.332	35.819	1.507	0.046	0.856	154.239	0.987
2014	-835.278	1682.556	1701.511	8.448	3.249	57.469	2.349	35.908	1.535	0.052	0.741	167.613	0.976
2015	-835.815	1683.630	1702.585	9.092	2.744	58.749	2.517	38.277	1.648	0.057	0.614	214.351	0.932
2016	-836.711	1685.423	1704.377	8.802	1.809	56.970	2.571	34.754	1.700	0.057	0.614	226.092	0.918
2017	-832.000	1676.001	1694.955	9.923	3.827	64.066	2.798	39.925	1.998	0.057	0.614	180.671	0.970
2018	-836.079	1684.158	1703.113	11.843	4.882	67.999	2.867	40.558	1.979	0.063	0.490	209.780	0.939
2019	-839.188	1690.375	1709.330	9.243	2.749	59.520	2.481	42.421	2.045	0.059	0.580	218.015	0.917
2020	-858.897	1729.795	1748.714	14.138	3.117	70.940	2.649	48.331	2.006	0.058	0.610	279.569	0.850

Notes: The reported are log-likelihood (LL), Akaike information criterion (AIC), and Bayesian information criterion (BIC), and the Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) tests. The KS and AD p-values formally test the null hypothesis that the sample is drawn from the fitted distribution.