Appendix: Mitigation or Adaptation to Climate Change? The Role of Fiscal Policy

Mouez Fodha* and Hiroaki Yamagami†

A Proof of Proposition 1

We have the following characteristics regarding the dynamics of capital stock accumulation:

$$\lim_{k_t \to 0} \Gamma(k_t) = \frac{\beta(1-v)\bar{h}}{(1-\beta)\phi - \beta(1-v)} \equiv \breve{k}, \quad (76)$$

$$\Gamma'(k_t) = \frac{(1-\beta)\phi(1-\delta_E) - (1-v)\left\{1 - \alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma)\right\}\alpha\beta A k_t^{\alpha-1}}{(1-\beta)\phi - \beta(1-v)}, \quad (77)$$

$$\Gamma''(k_t) = \frac{(1-v)\left\{1 - \alpha\frac{1-\tau^y}{1+\tau^c}(1+\epsilon/\gamma)\right\}\alpha(1-\alpha)\beta A k_t^{\alpha-2}}{(1-\beta)\phi - \beta(1-v)}. \quad (78)$$

From these derivations, the mitigation subsidy plays an important role in determining the steady state and stability. We will discuss each case below.

A.1 Cases with the high mitigation subsidy: $v > 1 - \phi\frac{1-\beta}{\beta} \equiv \underline{v}$

In this case, from (76) – (78), we have $\lim_{k_t \to 0} \Gamma(k_t) = \breve{k} > 0$ and $\Gamma'(k_t) \gtreqless 0$ but $\lim_{k_t \to 0} \Gamma'(k_t) = -\infty$ and $\lim_{k_t \to +\infty} \Gamma'(k_t) = \frac{(1-\beta)\phi(1-\delta_E)}{(1-\beta)\phi-\beta(1-v)} > 0$, and $\Gamma''(k_t) > 0$. These properties imply that the dynamics of k exhibit a U curve, with a minimum value of k_{t+1} at $\bar{k}$ satisfying $\Gamma'(\bar{k}) = 0$. Note that $\lim_{k_t \to +\infty} \Gamma'(k_t)$ is greater than unity if $v > \bar{v}$. This condition also plays a role in determining the dynamics.

Case (i) and (ii): $\Gamma(\overline{k}) \leq \overline{k}$

Suppose $\Gamma(\overline{k}) \leq \overline{k}$ and $v \geq \bar{v}$. In this case, because $\lim_{k_t \to 0} \Gamma(k_t) = \breve{k} > 0$ and $\Gamma(\overline{k}) \leq \overline{k}$, there is a unique steady state in which capital stock, k_{ss}, is in the range of $(0, \overline{k})$. The stability depends on k_{ss} compared with $\underline{k}$, which is defined in (19). If $k_{ss} > \underline{k}$, then the equilibrium sequence of $\{k_t\}_{t=0}^{\infty}$ is on an oscillatory convergence path towards the steady-state level k_{ss} because $\Gamma'(k_{ss}) \in (-1, 0)$. Subsequently, the steady-state is stable. By contrast, it is unstable if $k_{ss} \leq \underline{k}$. In this case, unless k_t remains at k_{ss} from $t = 0$, the sequence $\{k_t\}_{t=0}^{\infty}$ does not remain at k_{ss} through an oscillatory divergence path or a cycle path because $\Gamma'(k_{ss}) \leq -1$.

Next, suppose $\Gamma(\overline{k}) \leq \overline{k}$ and $\underline{v} < v \leq \bar{v}$. In this case, there are two steady states in which the capital stock is given by k_{ss}^1 and k_{ss}^2. Then, k_{ss}^1 is in the range of $(0, \overline{k}]$ as in case (i), because $\lim_{k_t \to 0} \Gamma(k_t) = \breve{k} > 0$ and $\Gamma(\overline{k}) \leq \overline{k}$. In addition, because $v \leq \bar{v}$, there is another steady state in which k_{ss}^2 is greater than $\overline{k}$ because $\lim_{k_t \to +\infty} \Gamma'(k_t) = \frac{(1-\beta)\phi(1-\delta_E)}{(1-\beta)\phi-\beta(1-v)} > 1$. The steady state

*Paris School of Economics and University Paris 1 Panthéon-Sorbonne. PjSE, 48 Boulevard Jourdan, 75014 Paris, France. E-mail: mouez.fodha@univ-paris1.fr.

†Faculty of Economics, Seikei University. 3-3-1 Kichijoji-kitamachi, Musashino-shi, 180-0001, Tokyo, Japan. E-mail: yamagami@econ.seikei.ac.jp.

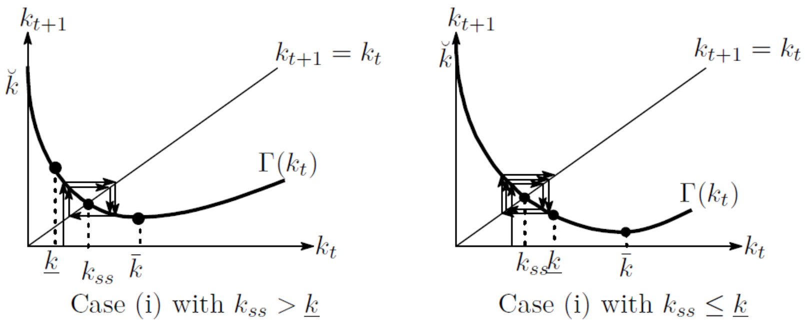


Figure 2: Capital stock dynamics in case (i): $\Gamma(\bar{k}) \leq \bar{k}$ and $v > \bar{v}$

with k_{ss}^2 is unstable because $\Gamma'(k_{ss}^2) > 1$. The stability of the steady state with k_{ss}^1 depends on the location of the steady-state capital stock level, as in case (i). The steady state is stable if $k_{ss}^1 > \underline{k}$ via an oscillatory convergence path. By contrast, it becomes unstable if $k_{ss}^1 \leq \underline{k}$ through an oscillatory divergence path or a cycle path. The stability of these two cases is shown in Figure 3 and is summarized in case (ii) of Proposition 1.

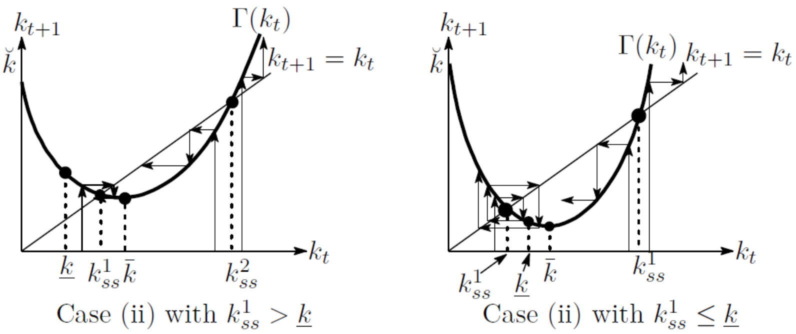


Figure 3: Capital stock dynamics in Case (ii): $\Gamma(\overline{k}) \leq \overline{k}$ and $\underline{v} < v \leq \bar{v}$

Case (iii), (iv), (v), and (vi): $\Gamma(\bar{k}) > \bar{k}$

Suppose $\Gamma(\overline{k}) > \overline{k}$. In this case, steady states exist only for the range of $k_t > \overline{k}$. Moreover, by supposing $\lim_{k_t \to +\infty} \Gamma'(k_t) \leq 1$, equivalently, $v \geq \bar{v}$, we have a unique steady state in which the capital stock, k_{ss}, is greater than $\overline{k}$. This corresponds to case (iii) in Proposition 1 and is shown in Figure 4. Then, k_{ss} is stable because $\Gamma'(k_{ss}) < 1$ as long as k_{ss} is finite.

If $\Gamma(\overline{k}) > \overline{k}$ and $\underline{v} < v < \bar{v}$, there are three more cases classified with $\hat{k}$ that satisfy $\Gamma'(\hat{k}) = 1$ in (20). First, when $\Gamma(\hat{k}) < \hat{k}$, there are two steady states with k_{ss}^1 and k_{ss}^2. This is depicted in case (iv) in Figure 5. Because the lower steady state k_{ss}^1 is between $\overline{k}$ and $\hat{k}$, we have $\Gamma'(k_{ss}^1) < 1$. Then, the steady state with k_{ss}^1 is stable. By contrast, the other steady state with k_{ss}^2 is unstable because $k_{ss}^2 > \hat{k}$ and thus, $\Gamma'(k_{ss}^2) > 1$. Second, when $\Gamma(\hat{k}) = \hat{k}$, there is a unique steady state with k_{ss}. The dynamics are depicted in case (v) in Figure 5. In this case, the U curve touches the 45° line at $\hat{k}$. This dynamic curve is above the 45° line in all other ranges. As a result, capital stock converges to k_{ss} from $k_t \leq k_{ss}$. Once capital stock exceeds k_{ss}, the economy diverges to, where k_t approaches infinity. Because the capital stock cannot stay in the steady state because of fluctuations, the steady state is unstable. Finally, when $\Gamma(\hat{k}) > \hat{k}$, it is obvious that there is no steady state.

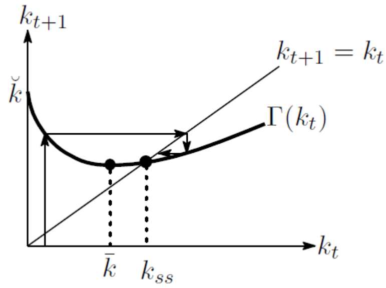
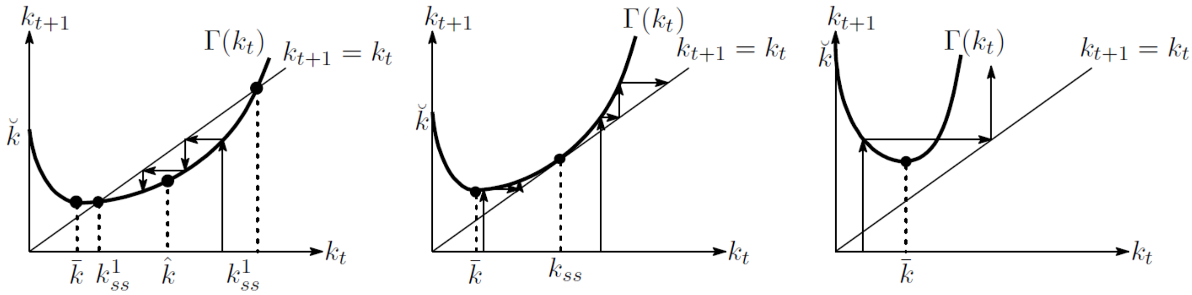


Figure 4: Capital stock dynamics in Case (iii): $\Gamma(\bar{k}) > \bar{k}$ and $v \geq \bar{v}$



Case (iv) with $\Gamma(\hat{k}) < \hat{k}$ Case (v) with $\Gamma(\hat{k}) = \hat{k}$ Case (vi) with $\Gamma(\hat{k}) > \hat{k}$

Figure 5: Capital stock dynamics in cases (iv), (v), and (vi): $\Gamma(\bar{k}) > \bar{k}$ and $\underline{v} < v < \bar{v}$

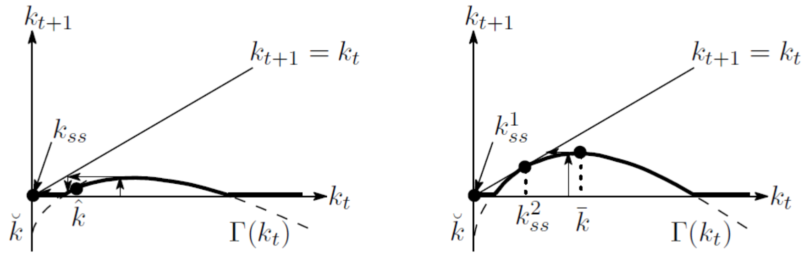
A.2 Cases with the low mitigation subsidy: $v < 1 - \phi\frac{1-\beta}{\beta} \equiv \underline{v}$

In this case, from (76) – (78), we have $\lim_{k_t \to 0} \Gamma(k_t) = \breve{k} < 0$ and $\Gamma'(k_t) \gtreqless 0$ but $\lim_{k_t \to 0} \Gamma'(k_t) = +\infty$ and $\lim_{k_t \to +\infty} \Gamma'(k_t) = \frac{(1-\beta)\phi(1-\delta_E)}{(1-\beta)\phi - \beta(1-v)} < 0$, and $\Gamma''(k_t) < 0$. These properties imply that the dynamics of k exhibit an inverted-U curve with a maximum value of k_{t+1} at $\bar{k}$ satisfying $\Gamma'(\bar{k}) = 0$. As the dynamics draws the inverted-U curve, Γ goes negative for some k_t. When $\Gamma < 0$, the capital stock converges to $k_{t+1} = 0$. As $\bar{v} > \underline{v}$, we do not consider cases with $v \geq \bar{v}$.

Cases (vii), (viii), and (ix)

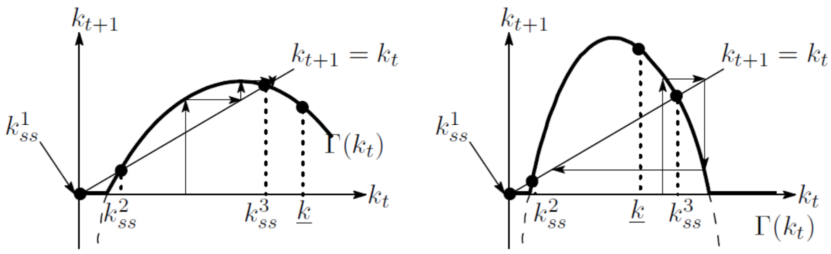
There are three cases, depending on $\hat{k}$ that satisfy $\Gamma'(\hat{k}) = 1$. First, if $\Gamma(\hat{k}) < \hat{k}$, then the inverted U-curve is below the 45° line for all ranges of k_t. This is shown in case (vii) in Figure 6. Because $k_{t+1} = 0$ when $\Gamma(k_t) < 0$, the steady state is unique and stable at $k_{ss} = 0$. Second, if $\Gamma(\hat{k}) = \hat{k}$, then there are two steady states with k_{ss}^1 and k_{ss}^2. This is depicted in case (viii) in Figure 6. On the one hand, k_{ss}^1 is zero, and the steady state is stable, as in case (vii). On the other hand, as in case (v), the inverted U curve touches the 45° line at $k_t = \hat{k}$ and is below it for the other range. Capital stock converges to k_{ss}^2 only for $k_t \in [k_{ss}^2, \overline{k}]$, whereas it converges to k_{ss}^1 otherwise. However, even after $k_t = k_{ss}^2$ holds, once the capital stock turns below k_{ss}^2 by fluctuation, the capital stock converges to $k_{ss}^1 = 0$. Therefore, the steady state with k_{ss}^2 is unstable.

Finally, if $\Gamma(\hat{k}) > \hat{k}$, then there are three steady states with k_{ss}^1, k_{ss}^2, and k_{ss}^3. This case is depicted in case (ix) in Figure 7. k_{ss}^1 is zero and stable, and k_{ss} in case (vii) and k_{ss}^1 in case (viii). The steady state with k_{ss}^2 is clearly unstable. By contrast, the stability of k_{ss}^3 depends on whether k_{ss}^3 is greater than $\underline{k}$. If k_{ss}^3 is strictly lower than $\underline{k}$ satisfying $\Gamma'(\underline{k}) = -1$ as shown in (19), the steady state with k_{ss}^3 is stable. By contrast, if k_{ss}^3 is higher than $\underline{k}$, it is unstable.



Case (vii) with $\Gamma(\hat{k}) < \hat{k}$ Case (viii) with $\Gamma(\hat{k}) = \hat{k}$

Figure 6: Capital stock dynamics in cases (vii) and (viii): $\Gamma(\hat{k}) \leq \hat{k}$ and $v < \underline{v}$



Case (ix) with $\Gamma(\underline{k}) < \underline{k}$ Case (ix) with $\Gamma(\underline{k}) \geq \underline{k}$

Figure 7: Capital stock dynamics in case (ix): $\Gamma(\hat{k}) > \hat{k}$ and $v < \underline{v}$

A.3 Summary

Proposition 1 shows nine cases for the capital accumulation dynamics. Assumption 1 (a) ensures that the sensitivity of consumption is greater than that of pollution, $\beta > \phi(1-\beta)$. However, the subsidy level also imposes constraints on these sensitivities. So we have two cases in terms of capital stock dynamics: an inverted-U or a U curve.

First, the denominator of (16) is positive when the subsidy rate is low enough. When the subsidy is small enough, savings lead to a well-known inverted-U curve with respect to the capital stock. That is, as the capital stock grows from very low level, households income increases and so do savings. For higher levels of capital stock, because the interest rates are lowered enough, savings decrease.

Second, if the level of subsidy is sufficiently high, the dynamics of the capital stock will change and take the form of a U-shaped curve. As the net cost of mitigation is low enough, and because the households income increases with the capital stock, the mitigation is prioritised over consumption, and savings are reduced. For sufficiently high levels of capital stock, the marginal (positive) effect of mitigation on utility decreases as the capital stock increases. Instead, the consumption and thereby the savings increase.

In addition to the subsidy rate, the capital stock itself affects the dynamics of capital stock accumulation. Consequently, the mitigation subsidy generates three cases, while the capital stock also adds three sub-cases.

B Proof of Proposition 3

Mitigation and adaptation are complements if $\frac{dm_{ss}}{dh} > 0$ in (26). This condition corresponds to $\Delta m \frac{dk_{ss}}{dh} > 1$. We examine the sign of each term with respect to v and k_{ss}. The key thresholds are $\hat{v}$, $\bar{v}$, k^n, and k^m as shown in (21), (24), and (27).

First, we suppose $v \leq \hat{v} < \bar{v}$. Then, $0 < k^n \leq k^m$. In this case, if k_{ss} is sufficiently small or large such that $k_{ss} \leq k^m \leq k^n$ or $k^m \leq k^n \leq k_{ss}$, we have $\Delta m \frac{dk_{ss}}{dh} \geq 0$. If this term is strictly greater than unity, m and h are complements. Otherwise, they are neutral or substitutes. By contrast, if k_{ss} is in the middle range, such that $k^m \leq k_{ss} \leq k^n$, we have $\Delta m \frac{dk_{ss}}{dh} \leq 0$. Thus, m and h are substitutes.

Second, we suppose $\hat{v} < v < \bar{v}$. Then, $0 < k^m < k^n$. In this case, if k_{ss} is sufficiently small or large such that $k_{ss} \leq k^n < k^m$ or $k^m < k^n < k_{ss}$, then $\Delta m \frac{dk_{ss}}{dh} > 0$. Therefore, if this term is strictly greater than unity, m and h complements. Otherwise, they are neutral or substitutes. By contrast, if k_{ss} is in the middle range, such that $k^m < k_{ss} < k^n$, we have $\Delta m \frac{dk_{ss}}{dh} < 0$. Thus, m and h are substitutes.

Finally, we suppose $\hat{v} < \bar{v} \leq v$. Then, $k^m \leq 0 < k^n$. In this case, if k_{ss} is sufficiently large, such that $k^m \leq 0 < k^n < k_{ss}$, we have $\Delta m \frac{dk_{ss}}{dh} \geq 0$. Therefore, if $\Delta m \frac{dk_{ss}}{dh} > 1$, m and h are complements. Otherwise, they are neutral or substitutes. If k_{ss} is in the middle range, such that $k^m \leq 0 < k_{ss} < k^n$, we have $\Delta m \frac{dk_{ss}}{dh} \leq 0$. Thus, m and h are substitutes.

C Proof of Proposition 5

From (39), $\Gamma_z(\cdot)$ satisfies $\lim_{k_t \to 0} \Gamma_z(k_t) = 0$ and $\lim_{k_t \to +\infty} \Gamma_z(k_t) \gtreqless 0$: $\Gamma_z(\cdot)$ also exhibits the following characteristics:

$$\Gamma'_z(k_t) = \frac{(1-v)\left\{1 - \alpha \frac{1-\tau^y}{1+\tau^c}(1 + \epsilon/\gamma + \epsilon z)\right\} \alpha\beta A k_t^{\alpha-1} - (1-\beta)(1-\delta_E)\phi}{(1-v)\beta - (1-\beta)\phi}, \tag{79}$$

$$\Gamma''_z(k_t) = -\frac{(1-v)\left\{1 - \alpha \frac{1-\tau^y}{1+\tau^c}(1 + \epsilon/\gamma + \epsilon z)\right\} (1-\alpha)\alpha\beta A k_t^{\alpha-2}}{(1-v)\beta - (1-\beta)\phi}. \tag{80}$$

From these derivatives, the mitigation subsidy and adaptation provision rule play important roles in determining the steady state and its stability. We will look at each case.

C.1 Case with $v > \bar{v}$ and $z > \bar{z}$

Because $\bar{v} > \underline{v}$, we have $v > \bar{v} > \underline{v}$. Then, $\lim_{k_t \to 0} \Gamma'(k_t) = +\infty$, $\lim_{k_t \to +\infty} \Gamma'(k_t) = -\frac{(1-\beta)(1-\delta_E)\phi}{(1-v)\beta - (1-\beta)\phi} > 0$, and $\Gamma''(k_t) < 0$. By rearranging $\lim_{k_t \to +\infty} \Gamma'(k_t)$, we obtain $\lim_{k_t \to +\infty} \Gamma'(k_t) = -\frac{(1-\beta)(1-\delta_E)\phi}{(1-\beta)(1-\delta_E)\phi + \beta\left\{v - \left(1 - \frac{1-\beta}{\beta}\phi\delta_E\right)\right\}}$. As $v > \bar{v}$, we can see $\lim_{k_t \to +\infty} \Gamma'(k_t) \in (0, 1)$. The dynamics shown by $\Gamma_z(k_t)$ exhibit an increasing curve. This curve has two intersections with a 45° line at $k_t = k_{ss}^1$ and k_{ss}^2, as given by (40). This case is depicted in Figure 8 and corresponds to case (i) in Proposition 5. The steady state with k_{ss}^1 is unstable because $\Gamma'_z(0) \to +\infty$, whereas that with k_{ss}^2 is stable because $\Gamma'_z(k_{ss}^2) \in (0, 1)$.

C.2 Cases with $v < \bar{v}$ and $z < \bar{z}$

We divide these cases by either $v < \underline{v}$ or $\underline{v} < v < \bar{v}$.[1]

[1] From Assumption 3, we do not consider the case with $v = \underline{v}$.

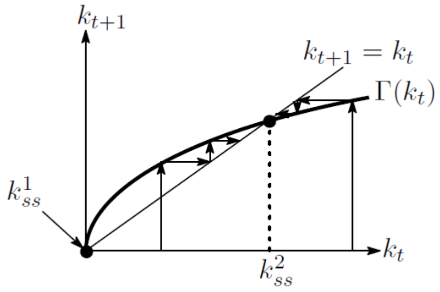


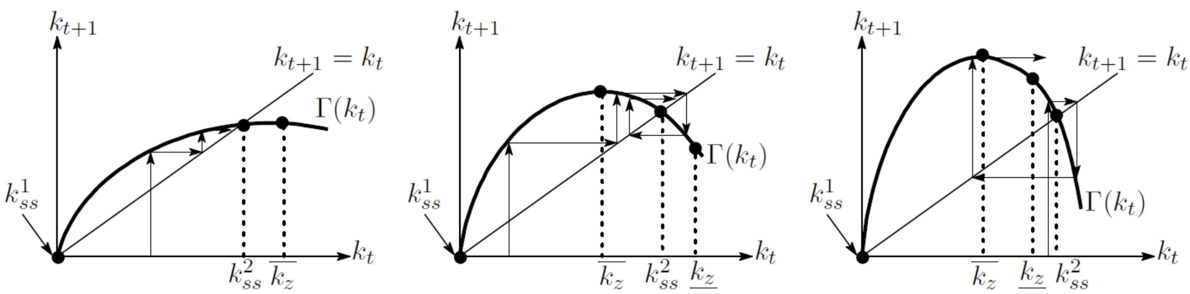
Figure 8: Capital stock dynamics with a proportional adaptation in case (i): $v > \bar{v}$ and $z > \bar{z}$

Cases (ii) and (iii): $v < \underline{v}$ and $z < \bar{z}$

If the mitigation subsidy is sufficiently small, such that $v < \underline{v}$, we have $\lim_{k_t \to 0} \Gamma'_z(k_t) = +\infty$ and $\lim_{k_t \to +\infty} \Gamma'_z(k_t) = -\frac{(1-\beta)(1-\delta_E)\phi}{(1-\beta)(1-\delta_E)\phi + \beta\left\{v - \left(1 - \frac{1-\beta}{\beta}\phi\delta_E\right)\right\}} < -1$. The dynamics of capital stock then illustrate an inverted U curve; thus, we have $\hat{k_z} < \overline{k_z} < \underline{k_z}$. This curve has two intersections, with a 45 ° line at k_{ss}^1 and k_{ss}^2 as given in (40). Although the stability of the steady state with k_{ss}^1 is clearly unstable because $\Gamma'_z(0) = +\infty$, that with k_{ss}^2 is unclear. Therefore, we further divide this case into two depending on where k_{ss}^2 is placed.

First, we suppose $k_{ss}^2 < \underline{k_z}$. As the policy mix of $v < \underline{v}$ and $z < \bar{z}$ ensures the presence of a nontrivial steady state with $k_{ss}^2 > 0$, we have $\Gamma'_z(k_{ss}^2) \in (-1, 1)$. The two figures in Figure 9 depict the two cases summarized in case (ii) of Proposition 5. If $\Gamma'_z(\overline{k_z}) < \overline{k_z}$, then the capital stock converges via a monotone path from $k_t < k_{ss}^2$ and via an oscillatory path from $k_t > k_{ss}^2$. Additionally, if $\Gamma_z(\overline{k_z}) \geq \overline{k_z}$, the capital stock converges to k_{ss}^2 via an oscillatory path. This case is depicted in Figure 9 and corresponds to case (ii) in Proposition 5.

Second, we suppose $k_{ss}^2 \geq \underline{k_z}$. Then, because of the policy mix of $v < \bar{v}$ and $z < \bar{z}$, there is a steady state with $k_{ss}^2 > 0$. However, the capital stock does not converge to a steady state k_{ss}^2 because $\Gamma'_z(k_{ss}^2) \leq -1$. This case is depicted in Figure 9 and corresponds to case (iii) in Proposition 5.



Case (ii) with $k_{ss}^2 < \underline{k_z}$ and $\Gamma_z(\overline{k_z}) > \overline{k_z}$ — Case (ii) with $k_{ss}^2 < \underline{k_z}$ and $\Gamma_z(\overline{k_z}) \leq \overline{k_z}$ — Case (iii) with $k_{ss}^2 \geq \underline{k_z}$

Figure 9: Capital stock dynamics with a proportional adaptation in cases (ii) and (iii): $v < \underline{v}$ and $z < \bar{z}$

Case (iv): $\underline{v} < v < \bar{v}$ and $z < \bar{z}$

If the mitigation subsidy is not as small as in cases (ii) and (iii), such that $\underline{v} < v < \bar{v}$, we have $\lim_{k_t \to 0} \Gamma'_z(k_t) = -\infty$ and $\lim_{k_t \to +\infty} \Gamma'_z(k_t) = -\frac{(1-\beta)(1-\delta_E)\phi}{(1-\beta)(1-\delta_E)\phi+\beta\left\{v-\left(1-\frac{1-\beta}{\beta}\phi\delta_E\right)\right\}} > 1$. The dynamics of capital stock then follows a U curve. In addition to the cases above, this curve also has two intersections with a 45° line at k_{ss}^1 and k_{ss}^2 as given in (40). The steady state with k_{ss}^1 is stable because $\Gamma'_z(0) = -\infty$. That with k_{ss}^2 is unstable. This case is depicted in Figure 10 and corresponds to case (iv) of Proposition 5.

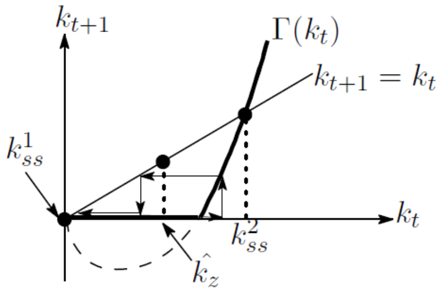


Figure 10: Capital stock dynamics with a proportional adaptation in case (iv): $\underline{v} < v < \bar{v}$ and $z < \bar{z}$

D Proof of Proposition 6

First, we suppose $z \leq \underline{z}$. Then, the denominator of (57) is positive and the square brackets of the numerator are positive. Because k_{ss}^2 is also a factor that determines the sign of (57), we rearrange the numerator with respect to k_{ss}^2. As a result, if $k_{ss}^2 < k^\ell$, we have $dm_{ss}/dh_{ss} > 0$; that is, the two instruments are complements. This case is referred to as case (i) in Proposition 6. By contrast, if $k_{ss}^2 > k^\ell$, they are substitutes. If $k_{ss}^2 = k^\ell$, they are neutral.

Second, we suppose $\underline{z} < z < \hat{z}$. Then, the denominator is negative and the square brackets of the numerator are positive. Additionally, if $k_{ss}^2 < k^\ell$, the two instruments are substitutes. Conversely, if $k_{ss}^2 > k^\ell$, then they are complements. This case is referred to as case (ii) in Proposition 6. Moreover, if $k_{ss}^2 = k^\ell$, then they are neutral.

Third, we suppose $z \geq \hat{z}$. Then, the denominator and numerator are both negative. Regardless of k_{ss}^2, the two instruments are complements. This case is referred to as case (iii) in Proposition 6.

E Proof of Proposition 7

From (62), (63), and (64), we can characterize the steady states and their stability using the capital stock and mitigation subsidy. As Assumption 3 rules out $v = \underline{v}$ which leads to a trivial result, we suppose that either $v < \underline{v}$ or $v > \underline{v}$.

E.1 Cases with $v < \underline{v}$

In this case, $\lim_{k_t \to 0} \Gamma_B(k_t) = \lim_{k_t \to +\infty} \Gamma_B(k_t) = -\infty$, $\lim_{k_t \to 0} \Gamma'_B(k_t) = +\infty$, $\lim_{k_t \to +\infty} \Gamma'_B(k_t) = -\frac{(1-\delta_E)(1-\beta)\phi}{(1-v)\beta-(1-\beta)\phi} < 0$, and $\Gamma''_B(k_t) < 0$. This implies that $\Gamma_B(k_t)$ exhibits an inverted U curve with respect to k_t.

The economy then has steady states other than $k_{ss} = 0$ if there is a range of k_t that satisfies $\Gamma_B(k_t) \geq k_t$. This condition corresponds to $\Gamma_B(\hat{k_B}) > \hat{k_B}$, where $\hat{k_B}$ satisfies $\Gamma'_B(\hat{k_B}) = 1$. Using $\hat{k_B}$, we can characterize the steady state in the following three cases:

First, if $\Gamma_B(\hat{k_B}) < \hat{k_B}$, the inverted U-curve describing the dynamics of capital stock is below the 45° line. Therefore, a unique steady state is characterized by $k_{ss} = 0$. As capital stock converges to k_{ss}, it is stable. This case corresponds to case (i) of Proposition 7 and is depicted in case (i) of Figure 11. Second, if $\Gamma_B(\hat{k_B}) = \hat{k_B}$, then there are two steady states with $k^1_{ss} = 0$ and $k^2_{ss} > 0$. The inverted U-curve touches the 45° line at $k_t = \hat{k_B}$ and is below it for the other range. Then, capital stock converges to k^2_{ss} only for $k_t \in [k^2_{ss}, \overline{k_B}]$ and converges to k^1_{ss} otherwise. Even if capital stock reaches k^2_{ss}, once it is below k^2_{ss} by a fluctuation, it converges to $k^1_{ss} = 0$. Therefore, the steady state with k^2_{ss} is unstable. This is presented as case (ii) in Proposition 7 and depicted in case (ii) in Figure 11. Finally, if $\Gamma_B(\hat{k_B}) > \hat{k_B}$, then there are three steady states with k^1_{ss}, k^2_{ss}, and k^3_{ss}. One steady state is characterized as $k^1_{ss} = 0$ and is stable, as in the previous cases. The steady state with k^2_{ss} is clearly unstable. By contrast, because the stability of k^3_{ss} is still ambiguous, we have two more cases, depending on whether k^3_{ss} is greater than $\underline{k_B}$. If k^3_{ss} is strictly lower than $\underline{k_B}$ satisfying $\Gamma'_B(\underline{k_B}) = -1$, capital stock converges to k^3_{ss} through an oscillatory path. However, if capital stock is higher than k^4 such that $\Gamma_B(k^4) = \Gamma_B(k^2_{ss})$, it converges to k^1_{ss}. Therefore, the steady state with k^3_{ss} is locally stable. This case corresponds to the former case (iii) of Proposition 7 and is depicted in case (iii–a) of Figure 11. By contrast, if k^3_{ss} is higher than $\underline{k_B}$, the steady state with k^3_{ss} is unstable. This is the latter case (iii) of Proposition 7 and is depicted in case (iii–b) of Figure 11.

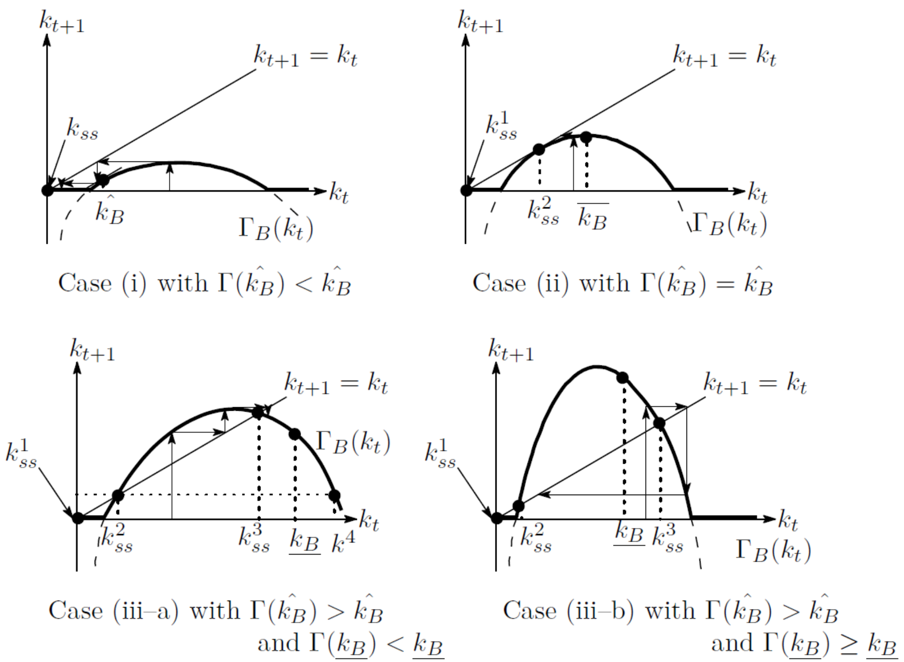


Figure 11: Capital stock dynamics with debts in cases (i) – (iii): $v < \underline{v}$

E.2 Cases with $v > \underline{v}$

In this case, $\lim_{k_t \to 0} \Gamma_B(k_t) = \lim_{k_t \to +\infty} \Gamma_B(k_t) = +\infty$, $\lim_{k_t \to 0} \Gamma'_B(k_t) = -\infty$, $\lim_{k_t \to +\infty} \Gamma'_B(k_t) = -\frac{(1-\delta_E)(1-\beta)\phi}{(1-v)\beta-(1-\beta)\phi} > 0$, and $\Gamma''_B(k_t) > 0$. This implies that $\Gamma_B(k_t)$ exhibits a U curve with respect to k_t. Thus, steady states exist in a finite range if $\Gamma_B(\hat{k_B}) \leq \hat{k_B}$. Using $\hat{k_B}$, we can characterize the steady state in the following cases:

First, if $\Gamma_B(\hat{k_B}) > k_B$, the dynamics of capital stock lie above the 45° line and do not intersect with it. Then, capital stock diverges to $+\infty$. The steady state is undefined at a finite level. This is presented as case (iv) of Proposition 7 and is depicted in Figure 12. Second, if $\Gamma_B(\hat{k_B}) = k_B$, the dynamics touch the 45° line only when $k_{ss} = \hat{k_B}$. Capital stock converges to k_{ss} for $k_t \in [\overline{k_B}, k_{ss}]$, whereas it diverges to infinity for $k_t > k_{ss}$. Therefore, the steady state is unstable. This case corresponds to case (v) in Proposition 7 and is depicted in case (v) in Figure 12. Third, if $\Gamma_B(\hat{k_B}) < \hat{k_B}$, then there are two steady states: k^1_{ss} and k^2_{ss}. The steady state with k^2_{ss} is clearly unstable because $\Gamma'_B(k^2_{ss}) > 1$. By contrast, the stability of the steady state with k^1_{ss} depends on the location of k^1_{ss}. In this case, k^1_{ss} is less than $\hat{k_B}$. Therefore, stability depends on whether the absolute value of $\Gamma'_B(k^1_{ss})$ is less than unity. This is equivalent to determining whether k^1_{ss} is lower than $\underline{k_B}$. If $\underline{k_B} < k^1_{ss} < \hat{k_B}$, capital stock converges to k^1_{ss} via a monotone or an oscillatory path for $k^3 < k_t < k^2_{ss}$, where k^3 satisfies $\Gamma_B(k^3) = \Gamma_B(k^2_{ss})$. The monotone path is depicted in case (vi–a) in Figure 12 and the oscillatory path is depicted in case (vi–b). By contrast, if $k^1_{ss} \le \underline{k_B}$, the steady states with k^1_{ss} are unstable because $\Gamma'_B(k^1_{ss}) \le -1$. This case is depicted in cases (vi–c) in Figure 12. As a result, these are summarized in case (vi) of Proposition 7.

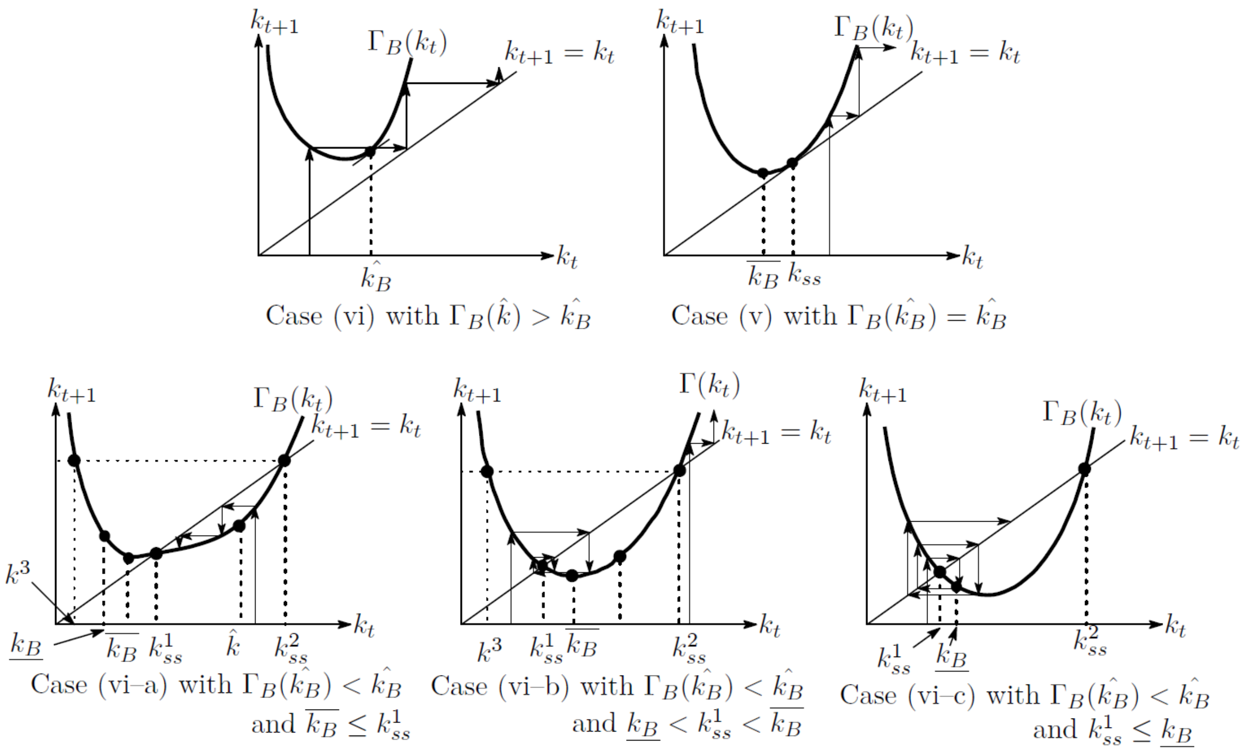


Figure 12: Capital stock dynamics with debts in cases (iv) – (vi): $v > \underline{v}$

F Proof of Proposition 8

The dynamics of the capital accumulation has the following characteristics: $\lim_{k_t \to +\infty} \Gamma_H(k_t) = +\infty$,

$$\lim_{k_t \to 0} \Gamma_H(k_t) = \frac{\beta(1-v)}{\mu(1-\beta)+\beta(1-v)}(\bar{H}-\bar{m}), \tag{81}$$

$$\Gamma'_H(k_t) = \frac{\mu(1-\beta)(1-\delta_H)}{\mu(1-\beta)+\beta(1-v)} + \frac{\beta(1-v)\left(1-\frac{\alpha(1-\tau^y)}{1+\tau^c}\right)}{\mu(1-\beta)+\beta(1-v)}\alpha A k_t^{\alpha-1} \geq 0, \tag{82}$$

$$\Gamma''_H(k_t) = -\frac{\beta(1-v)\left(1-\frac{\alpha(1-\tau^y)}{1+\tau^c}\right)}{\mu(1-\beta)+\beta(1-v)}\alpha(1-\alpha) A k_t^{\alpha-2} \leq 0. \tag{83}$$

From (82), the dynamics is monotonically increasing and is characterised by $\lim_{k_t \to 0} \Gamma'_H(k_t) = +\infty$ and $\lim_{k_t \to +\infty} \Gamma'_H(k_t) = \frac{\mu(1-\beta)(1-\delta_H)}{\mu(1-\beta)+\beta(1-v)} \in (0,1)$.

First, suppose that the natural adaptation is large enough and greater than the mitigation, $\bar{H} > \bar{m}$. Then, we have $\Gamma_H(k_t) > 0 \; \forall k_t \geq 0$. And there is a stable and unique steady state at $k_{ss} > 0$ because $\lim_{k_t \to +\infty} \Gamma'_H(k_t) \in (0,1)$. This case is shown as case (i) of Proposition 8 and in Figure 13.

Second, if natural adaptation equals mitigation such that $\bar{H} = \bar{m}$, the dynamics becomes $k_{t+1} = k_t = 0$, which is an unstable steady state. As $\Gamma'_H(k_t) > 0$, $\Gamma''_H(k_t) \leq 0$, and $\lim_{k_t \to +\infty} \Gamma'_H(k_t) \in (0,1)$, there is the other steady state, which is positive and stable, $k^1_{ss} > 0$. This case is shown as case (ii).

Third, if the natural adaptation is small enough such that $\bar{H} < \bar{m}$, $\Gamma_H(k_t)$ turns negative for some low k_t. In the range of $\Gamma_H(k_t) < 0$, k_{t+1} goes to zero. Thus, in this case, there is a steady state at $k^1_{ss} = 0$. Furthermore, letting $\hat{k}_H$ be a capital stock satisfying $\Gamma'_H(\hat{k}_H) = 1$, we have three more cases, case (iii) of $\Gamma_H(\hat{k}_H) > \hat{k}_H$, case (iv) of $\Gamma_H(\hat{k}_H) = \hat{k}_H$, and case (v) of $\Gamma_H(\hat{k}_H) < \hat{k}_H$. In case (iii), there exist two more steady states k^2_{ss} and k^3_{ss} with $k^2_{ss} < k^3_{ss}$, of which k^2_{ss} is unstable and k^3_{ss} is stable. In case (iv), there is one more steady state, k^2_{ss}, which is unstable. Finally, in case (v), there are no other steady states than $k^1_{ss} = 0$.

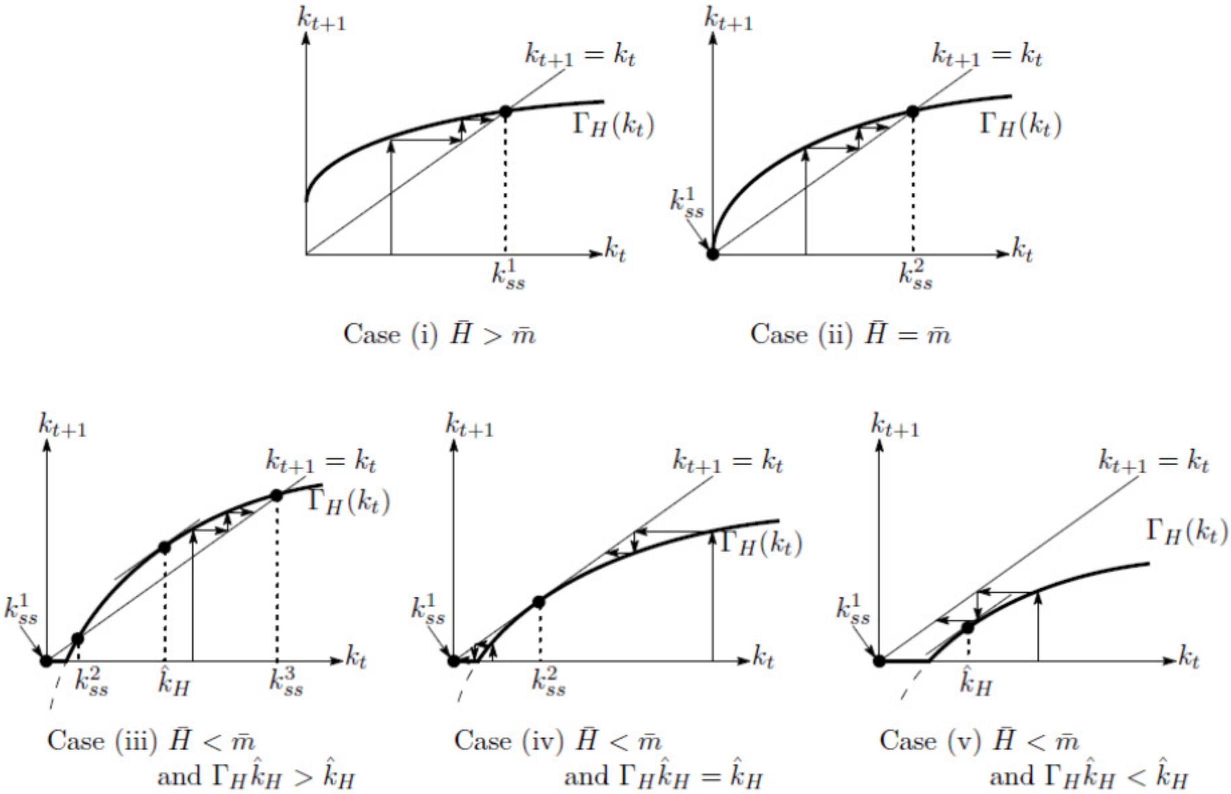


Figure 13: Capital stock dynamics with private adaptation and public mitigation