Supplementary Appendix for "Sectoral Inflation under Fragmentation of Information"

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A Tankan survey

Summary statistics. Table A1 presents the summary statistics of our dataset. As of 2023/2Q, the number of sample firms in each industry ranges from 53 to 937, with service sectors typically having a larger number of samples. Regarding the historical averages of DIs in each sector, changes in output prices and domestic supply and demand exhibit negative averages, while averages of changes in input prices are positive across all industries. In all three types of DIs, the first-order autocorrelation, or persistence, tends to be high, around 0.9.

Correlation between DIs and inflation. In the main text, we analyze sectoral inflation dynamics using the output price DI as a proxy. This appendix validates the DI as a proxy by demonstrating the historical correlation between inflation rates, measured by year-on-year changes in producer price indices, and DIs in each industry. Specifically, we employ the Corporate Goods Price Index (CGPI) and Services Producer Price Index (SPPI). Figure A1 displays the developments in both series in each industry along with their correlations. The correlations typically fall within the range of high correlation (0.5-0.8), confirming the relevance of sectoral DIs to sectoral inflation dynamics.

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Table A1: Summary statistics

	Changes in output price				Changes in input price				Domestic supply and demand condition			
	Number of samples in each sector in 2023Q2 wave	Historical average (1974Q2-2023Q2)	Historical standard deviation (1974Q2-2023Q2)	First-order auto correlation (1974Q2-2023Q2)	Number of samples in each sector in 2023Q2 wave	Historical average (1974Q2-2023Q2)	Historical standard deviation (1974Q2-2023Q2)	First-order auto correlation (1974Q2-2023Q2)	Number of samples in each sector in 2023Q2 wave	Historical average (1974Q2-2023Q2)	Historical standard deviation (1974Q2-2023Q2)	First-order auto correlation (1974Q2-2023Q2)
Textiles	194	-11.49	17.24	0.91	193	17.89	22.25	0.93	194	-54.38	15.58	0.93
Lumber & Wood products	105	-10.77	22.24	0.76	105	19.02	26.41	0.78	103	-46.73	23.67	0.90
Pulp & Paper	115	-8.98	25.59	0.85	115	18.40	30.08	0.84	114	-42.29	17.58	0.90
Chemicals	321	-6.36	19.44	0.89	321	17.40	26.32	0.89	321	-28.73	18.24	0.95
Petroleum & Coal products	90	2.09	35.32	0.69	90	22.22	37.76	0.70	90	-31.54	18.84	0.75
Ceramics, Stone & Clay	170	-9.11	21.13	0.94	170	21.48	24.71	0.92	170	-42.21	18.65	0.94
Iron & Steel	207	-7.80	30.50	0.88	206	17.05	30.59	0.83	206	-36.50	32.46	0.94
Nonferrous metals	170	-10.31	25.01	0.82	169	11.70	30.47	0.78	170	-30.08	26.49	0.93
Food & Beverages	362	-1.22	14.78	0.91	362	21.16	21.44	0.92	362	-31.95	10.07	0.94
Processed metals	232	-12.74	20.40	0.94	232	18.82	26.22	0.93	230	-31.06	20.63	0.95
General-purpose, Production & Business oriented machinery	611	-8.11	15.56	0.98	609	24.87	20.43	0.96	605	-14.02	12.10	0.92
Electrical machinery	448	-23.47	16.55	0.94	448	9.46	21.06	0.93	447	-19.59	19.27	0.89
Transportation machinery	397	-15.44	16.25	0.94	395	14.10	22.01	0.94	392	-21.22	19.31	0.94
Other manufacturing	274	-11.55	17.14	0.93	273	18.33	23.88	0.89	268	-38.78	12.31	0.91
Construction	937	-13.54	18.84	0.98	943	17.46	21.47	0.94	916	-28.36	22.34	0.99
Real estate	365	-12.41	19.76	0.96	340	8.66	19.58	0.95	373	-36.69	15.32	0.95
Wholesaling	898	-4.13	17.01	0.91	897	10.51	17.29	0.91	892	-28.14	12.99	0.96
Retailing	588	-1.12	15.42	0.92	586	9.34	13.41	0.90	584	-24.91	10.33	0.92
Transport & Postal activities	566	-5.69	14.56	0.96	562	19.53	20.17	0.85	563	-29.98	16.62	0.96
Information communication	448	-9.21	8.41	0.96	441	7.49	10.02	0.95	446	-12.27	9.05	0.94
Electric & Gas utilities	158	7.01	23.17	0.73	159	21.55	33.59	0.81	160	-1.87	2.56	0.74
Services for businesses	412	-12.73	13.83	0.98	407	11.67	10.55	0.94	412	-14.09	12.39	0.95
Services for individuals	321	-10.00	11.18	0.96	320	15.27	14.71	0.94	325	-30.59	7.68	0.92
Accomodations, Eating & Drinking services	234	-3.19	19.27	0.93	234	33.23	23.56	0.92	229	-45.19	13.56	0.88
Mining & Quarrying of stone and gravel	53	-5.26	18.88	0.84	52	22.65	16.97	0.80	50	-23.53	14.48	0.84

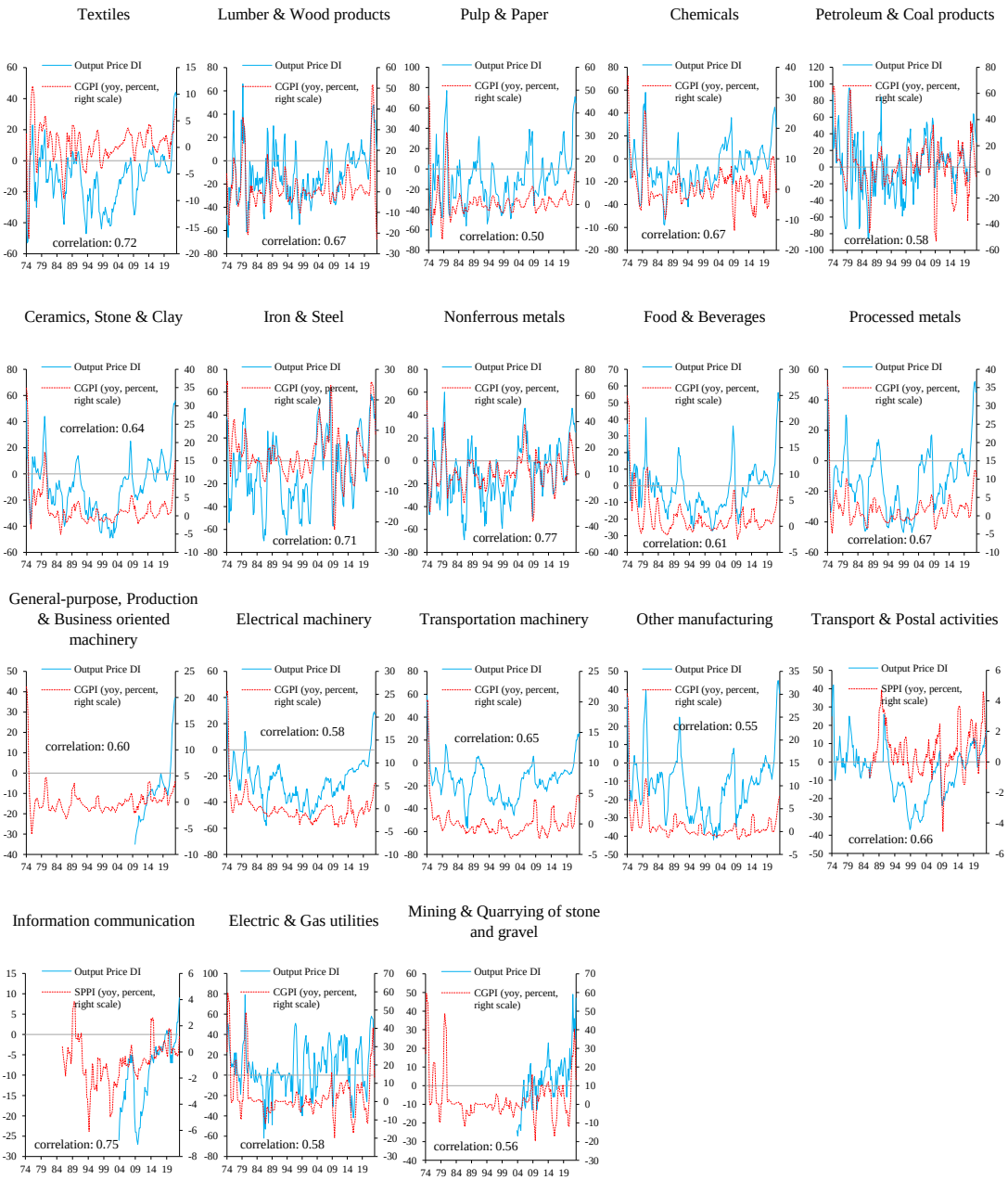

Figure A1: Output price DIs and price indices

B Dispersion of costs

This appendix presents the summary statistics of three types of measures of dispersion of firms' costs in each industry. Firstly, Table C1 indicates the historical averages, standard deviations, and first-order autocorrelations of the dispersion measure of changes in input costs ($d_{s,t}$). The averages range from 0.1 to 0.3, while the standard deviations are around 0.05 to 0.1. The first-order autocorrelation is starkly dispersed, ranging from 0.2 to 0.9. Secondly, Table C2 presents the same series regarding the relative dispersion measure of changes in input costs ($\hat{d}_{s,t}$). Moderate dispersion is observed across all averages, standard deviations, and first-order autocorrelations. Thirdly, Table C3 indicates the same series regarding the dummy variable for the relative dispersion of changes in input costs ($D_{s,t}$). All of the averages, standard deviations, and first-order autocorrelations exhibit significant dispersion.

Table C1: Dispersion of costs by industry

	Historical average (1974Q2-2023Q2)	Historical standard deviation (1974Q2-2023Q2)	First-order auto correlation (1974Q2-2023Q2)
Textiles	0.21	0.06	0.83
Lumber & Wood products	0.25	0.07	0.63
Pulp & Paper	0.24	0.07	0.54
Chemicals	0.22	0.06	0.68
Petroleum & Coal products	0.30	0.12	0.18
Ceramics, Stone & Clay	0.18	0.07	0.81
Iron & Steel	0.29	0.09	0.38
Nonferrous metals	0.28	0.08	0.41
Food & Beverages	0.23	0.06	0.78
Processed metals	0.19	0.06	0.78
General-purpose, Production & Business oriented machinery	0.18	0.04	0.78
Electrical machinery	0.17	0.06	0.87
Transportation machinery	0.17	0.06	0.87
Other manufacturing	0.20	0.07	0.81
Construction	0.17	0.07	0.90
Real estate	0.18	0.06	0.84
Wholesaling	0.22	0.06	0.86
Retailing	0.16	0.05	0.80
Transport & Postal activities	0.19	0.07	0.80
Information communication	0.12	0.04	0.90
Electric & Gas utilities	0.28	0.11	0.63
Services for businesses	0.16	0.04	0.83
Services for individuals	0.16	0.04	0.83
Accomodations, Eating & Drinking services	0.20	0.05	0.83
Mining & Quarrying of stone and gravel	0.19	0.06	0.68

Table C2: Relative dispersion of costs by industry

	Historical average (1974Q2-2023Q2)	Historical standard deviation (1974Q2-2023Q2)	First-order auto correlation (1974Q2-2023Q2)
Textiles	0.01	0.06	0.75
Lumber & Wood products	0.04	0.07	0.53
Pulp & Paper	0.03	0.07	0.56
Chemicals	0.01	0.05	0.55
Petroleum & Coal products	0.07	0.12	0.18
Ceramics, Stone & Clay	-0.02	0.04	0.66
Iron & Steel	0.08	0.09	0.33
Nonferrous metals	0.07	0.09	0.37
Food & Beverages	0.02	0.04	0.54
Processed metals	-0.01	0.04	0.65
General-purpose, Production & Business oriented machinery	-0.03	0.02	0.64
Electrical machinery	-0.02	0.04	0.74
Transportation machinery	-0.02	0.04	0.71
Other manufacturing	0.00	0.05	0.70
Construction	-0.03	0.04	0.76
Real estate	-0.02	0.05	0.72
Wholesaling	0.03	0.05	0.77
Retailing	-0.01	0.04	0.64
Transport & Postal activities	-0.01	0.05	0.61
Information communication	-0.05	0.02	0.60
Electric & Gas utilities	0.06	0.11	0.58
Services for businesses	-0.02	0.02	0.65
Services for individuals	-0.03	0.02	0.46
Accomodations, Eating & Drinking services	-0.02	0.04	0.72
Mining & Quarrying of stone and gravel	-0.02	0.04	0.53

Table C3: Dummy on relative dispersion of costs by industry

	Historical average (1974Q2-2023Q2)	Historical standard deviation (1974Q2-2023Q2)	First-order auto correlation (1974Q2-2023Q2)
Textiles	0.54	0.50	0.37
Lumber & Wood products	0.75	0.43	0.32
Pulp & Paper	0.60	0.49	0.49
Chemicals	0.55	0.50	0.25
Petroleum & Coal products	0.75	0.43	0.19
Ceramics, Stone & Clay	0.29	0.46	0.39
Iron & Steel	0.83	0.38	0.37
Nonferrous metals	0.86	0.35	0.04
Food & Beverages	0.70	0.46	0.31
Processed metals	0.36	0.48	0.48
General-purpose, Production & Business oriented machinery	0.06	0.23	0.29
Electrical machinery	0.28	0.45	0.52
Transportation machinery	0.26	0.44	0.40
Other manufacturing	0.44	0.50	0.52
Construction	0.18	0.39	0.59
Real estate	0.28	0.45	0.47
Wholesaling	0.78	0.42	0.41
Retailing	0.36	0.48	0.53
Transport & Postal activities	0.39	0.49	0.52
Information communication	0.06	0.24	-0.06
Electric & Gas utilities	0.67	0.47	0.47
Services for businesses	0.17	0.37	0.67
Services for individuals	0.06	0.24	0.14
Accomodations, Eating & Drinking services	0.27	0.44	0.54
Mining & Quarrying of stone and gravel	0.35	0.48	0.34

C Systematic relationship between DIs and dispersion

Since the Tankan survey solicits qualitative answers to the questions from firms, variables based on the survey may display some systematic relationship. For instance, during periods of high inflation when input prices increase substantially for many firms, the dispersion of changes in input prices should decrease as the sectoral average of input prices rises, which may not occur if the survey is quantitative. In this appendix, we investigate the extent of such a relationship in our dataset and develop measures to mitigate it.

Figure B1 compares the trends in sectoral input price DI ($\Delta Cost_t(s)$) and the dispersion of changes in input price within each industry ($d_{s,t}$). It also illustrates the correlations between them, with some industries showing high correlation. The correlation in all samples is also modestly positive, or 0.24. Moreover, to visually examine the relationship between input price DI and the dispersion, Figure B2 plots the former in x-axis and the latter in y-axis. The figure implies that there exists an M-shaped relationship between them. Although this does not necessarily imply the influence of systematic relationships, it may warrant robustness checks to mitigate the potential influence of such relationships. This is the motivation for constructing $\hat{d}_{s,t}$ and $D_{s,t}$ and estimation in Tables 2 and 3.

Given the construction of $\hat{d}_{s,t}$ and $D_{s,t}$, this appendix shows the calculation in each step. First, Table B1 shows the 10th percentiles of input price DIs, which are used to split all samples into ten groups. Next, Table B2 shows the median dispersion of changes in input prices in each group. The table implies a slightly positive relationship between the sectoral input price DI and the dispersion of changes in input prices while it is not entirely monotonic. Then, $\hat{d}_{s,t}$ is calculated by subtracting the median dispersion from the dispersion, and $D_{s,t}$ is calculated as it takes one if the dispersion of the changes in input price is above the median in the same group and zero otherwise. This transformation makes relative dispersion measures conditional on the level of sectoral input price DIs, which are expected to mitigate systematic relationships with sectoral input price DIs.

Figures B3 and B4 display the generated relative dispersion measures and their correlations with sectoral DIs. It reveals that the correlation in each industry is significantly lower compared to Figure B1. Additionally, the correlations in all samples are close to zero (-0.09 and -0.02), respectively, confirming that the relative dispersion measures are not influenced by systematic relationships with sectoral input price DIs. Figures B5 and B6 plot the input price DI in the x-axis and the relative dispersion measures in the y-axis, similar to Figure B2.[1] Unlike Figure B2, we do not observe a M-shaped relationship between them.

[1]Note that the dummy variable takes binary (0 or 1).

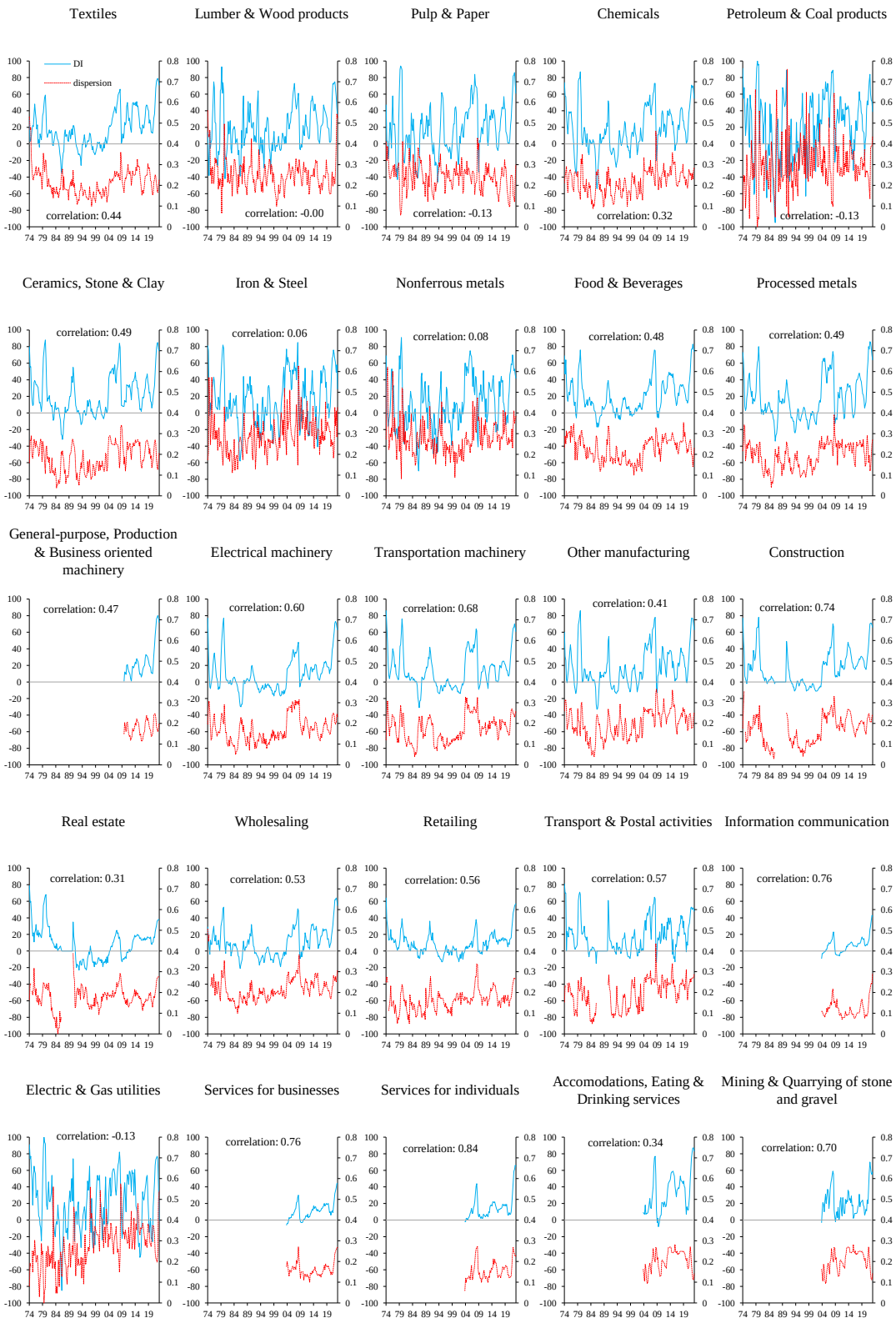

Figure B1: DI and dispersion

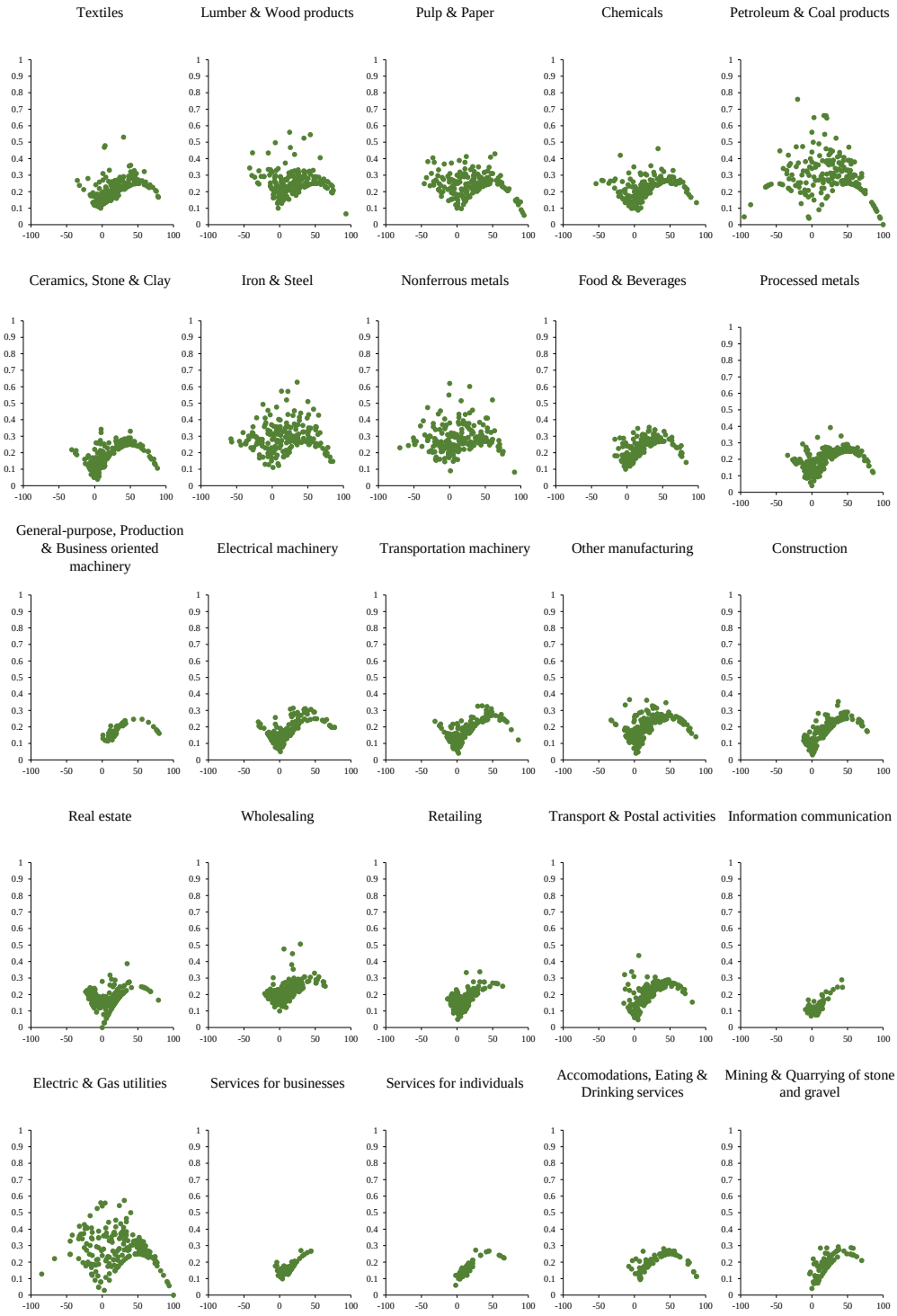

Figure B2: Relationship between DI and dispersion

Table B1: Tenth percentiles of input price DIs

	Changes in input price
min	-95
10 percentile	-10
20 percentile	-2
30 percentile	3
40 percentile	8
50 percentile	13
60 percentile	19
70 percentile	26
80 percentile	37
90 percentile	52
max	100

Table B2: Median dispersion in each tenth percentile of cost price DIs

	Median: dispersion of changes in input price
0-10 percentile	0.23
10-20 percentile	0.16
20-30 percentile	0.15
30-40 percentile	0.15
40-50 percentile	0.17
50-60 percentile	0.19
60-70 percentile	0.22
70-80 percentile	0.25
80-90 percentile	0.27
90-100 percentile	0.24

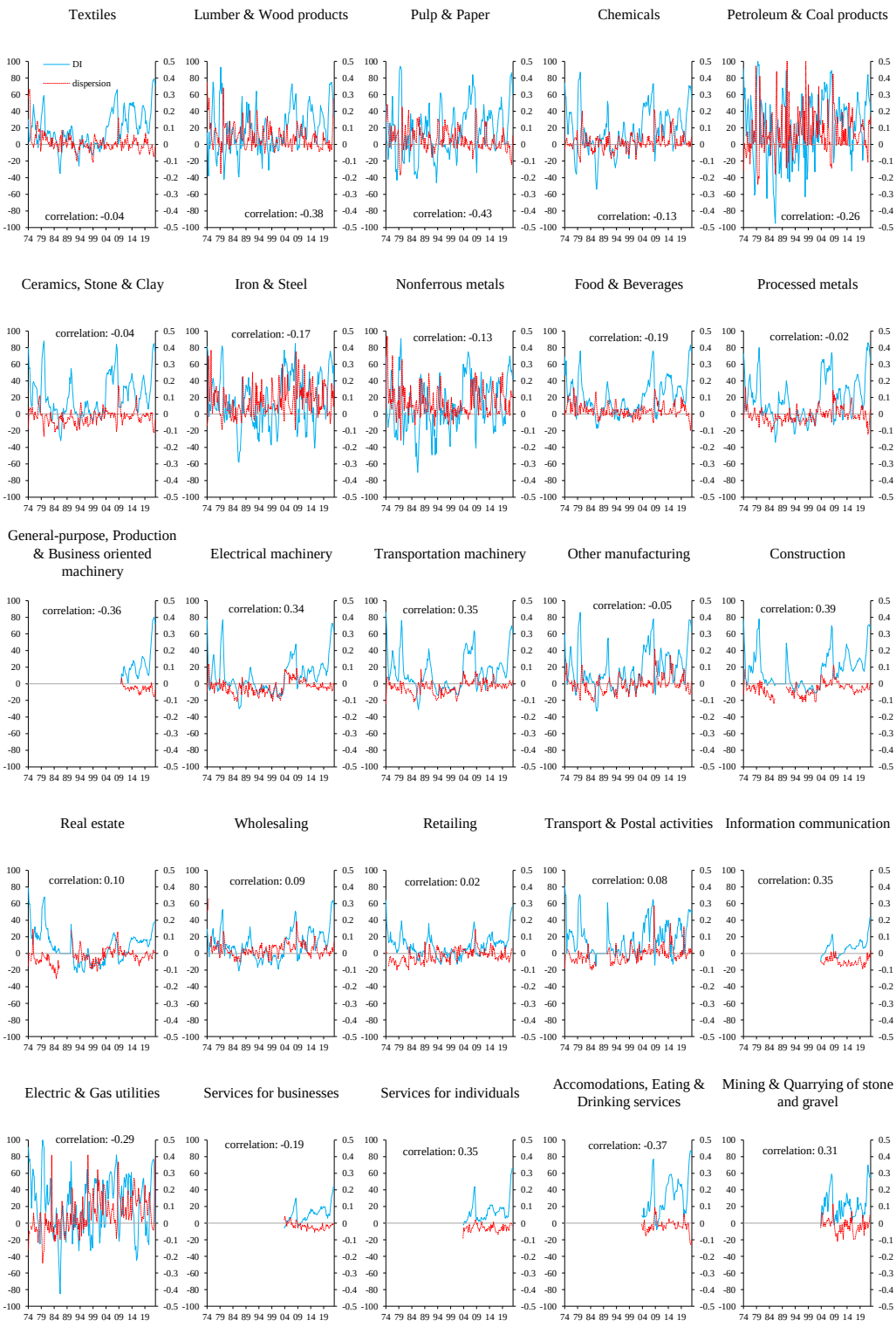

Figure B3: DI and relative dispersion

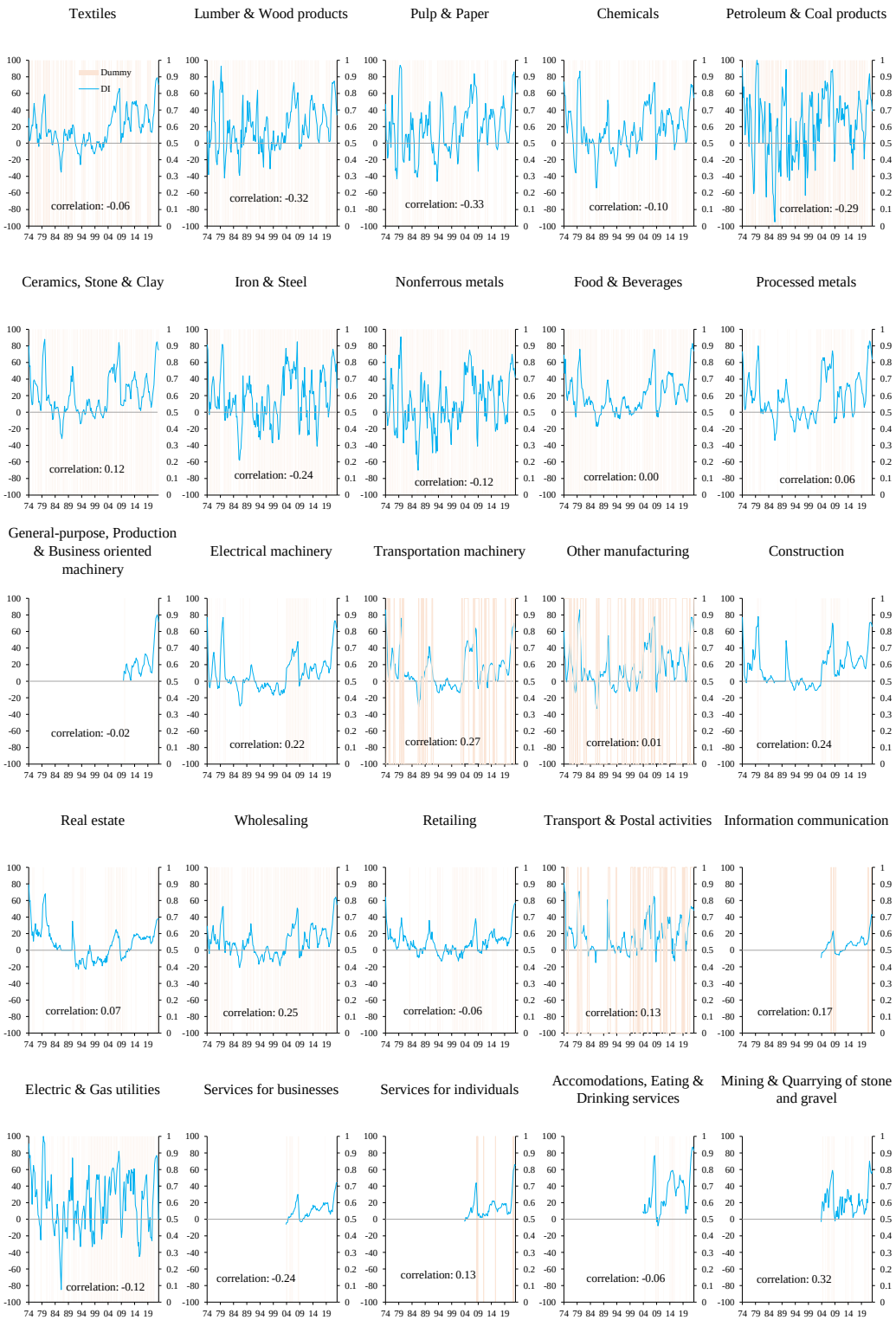

Figure B4: DI and dummy

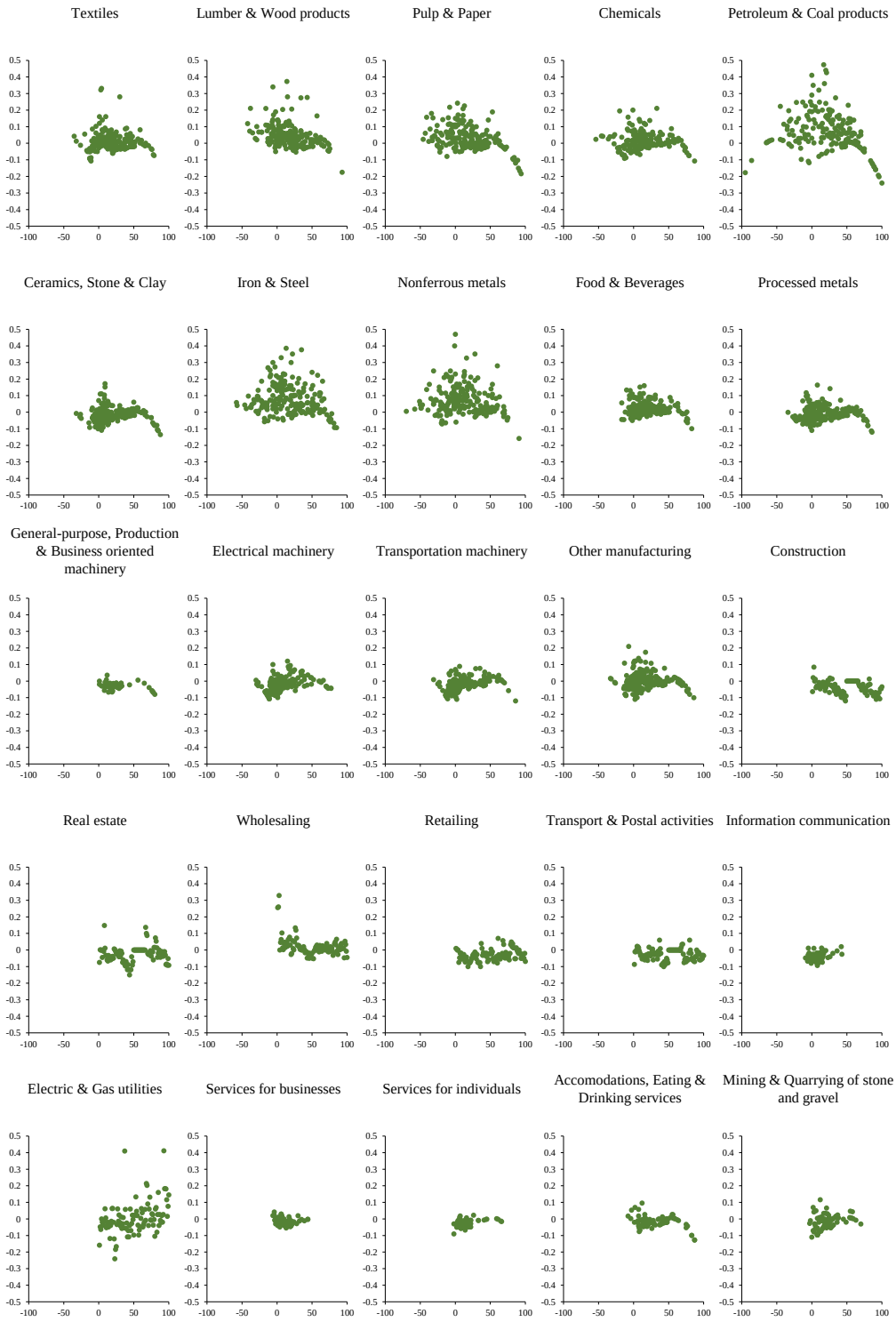

Figure B5: Relationship between DI and relative dispersion

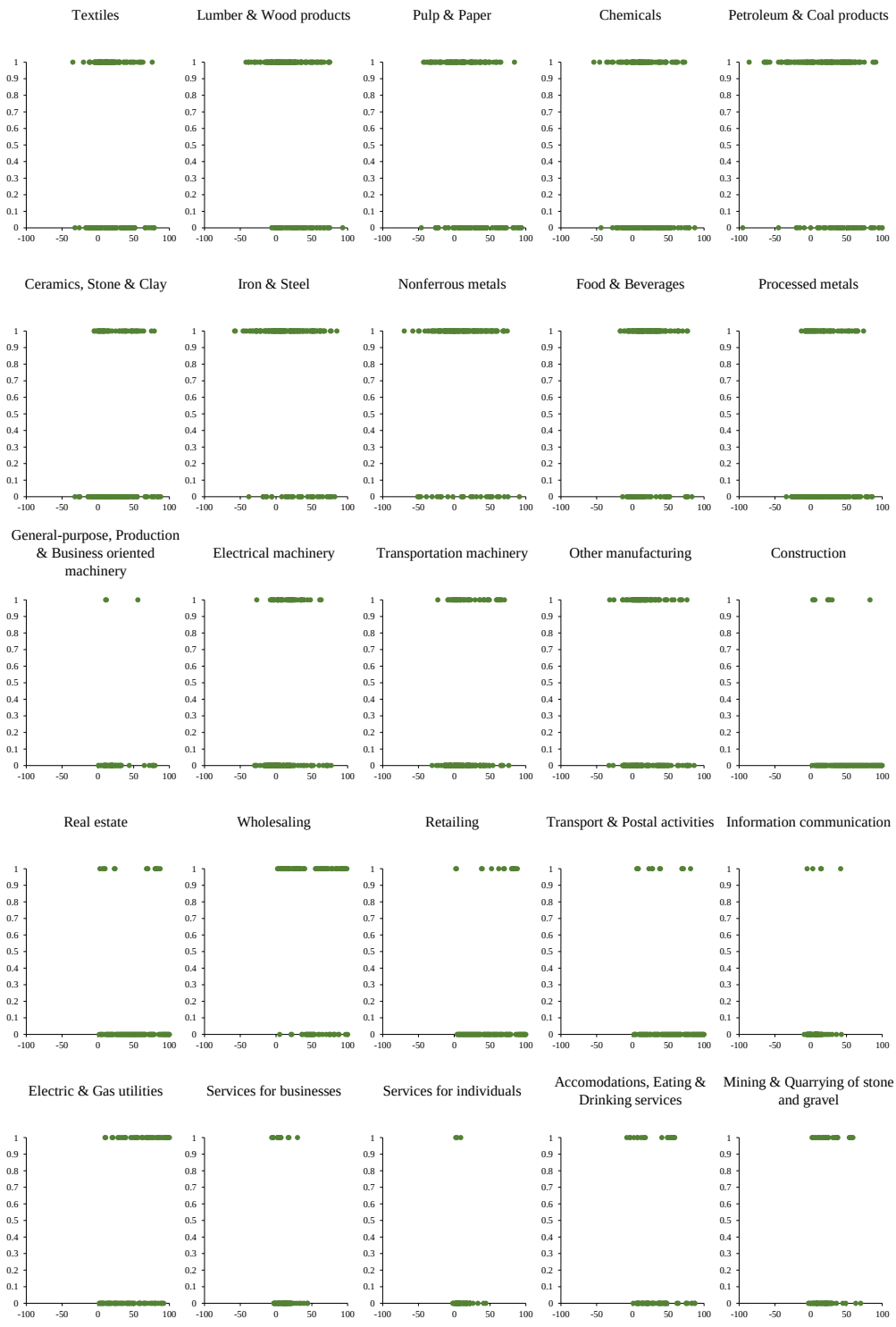

Figure B6: Relationship between DI and Dummy

D Robustness of empirical results

This appendix shows the results of the regressions which support the robustness of the results in Section 2.3. First, Table D1 shows the results based on the Weighted Least Squares, taking the inverse of the variance as the weight. Specifications of columns (1)-(3) correspond to those in columns of (1)-(3) in Table 1, those of columns (4)-(6) correspond to those in columns of (1)-(3) in Table 2, and those of columns (7)-(9) correspond to those in columns of (1)-(3) in Table 3. Note that in Weighted Least Squares, we include only industry-level fixed effects. All estimates for the interaction terms of changes in input prices and current and lagged dispersion of changes in input prices are negative and statistically significant at least at the five percent level, confirming the robustness of the results in the main text.

Table D1: Regression results: weighted least squares

Dependent variable: Changes in output price (1974/2Q-2023/2Q)									
	Weighted Least Squares								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Changes in input price	-0.96 ***		-0.67 ***	-0.80 ***		-0.50 ***	-0.07 ***		-0.06 ***
× Dispersion of changes in input price	(0.23)		(0.17)	(0.22)		(0.16)	(0.01)		(0.01)
Changes in input price		-0.92 ***	-0.60 ***		-0.94 ***	-0.74 ***		-0.06 ***	-0.04 **
× Lagged dispersion of changes in input price		(0.23)	(0.16)		(0.25)	(0.21)		(0.02)	(0.01)
Industry-level fixed effect	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Period fixed effect	No	No	No	No	No	No	No	No	No
Number of observations	3,766	3,766	3,766	3,766	3,766	3,766	3,766	3,766	3,766
Adjusted R-squared	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86

Note: ***, ** and * indicate statistically significant at 1%, 5%, and 10% levels, respectively.
Standard errors are cross-section cluster robust standard errors.

Next, Tables D2, D3, and D4 present the estimation results with the inclusion of first-to-eighth period lags of changes in sectoral costs (sectoral averages of changes in input prices) as control variables to address potential omitted variable biases mentioned in Section 4. Apart from this inclusion, all specifications are identical with those in Tables 1, 2, and 3, respectively. Even though first-to-right period lags of changes in sectoral costs are very tight controls, the estimates still broadly confirm the robustness of the results in these tables.

Specifically, Table D2 indicates that all estimates for the interaction terms of changes in input prices and current and lagged dispersion of changes in input prices are negative and in most entries they are statistically significant at one percent level.

Similarly, Table D3 indicates that all estimates for the interaction terms of changes in input prices and current and lagged relative dispersion of changes in input prices exhibit negative values and most of them are statistically significant at five percent level.

Table D4 further illustrates that all estimates for the interaction terms of changes in input prices and current and lagged dummy variables in input prices display negative values, with the majority being statistically significant at the five percent level.

Finally, Table D5 confirms that the estimated coefficients remain negative and statistically significant when the regressions are conducted with Weighted Least Squares. These results confirm the robustness of the estimation results in the baseline regression models presented in Tables 1, 2, and 3 in terms of potential omitted variable biases.

Table D2: Regression results: OLS with dispersion and lagged changes in costs

Dependent variable: Changes in output price (1974/2Q-2023/2Q)						
	Ordinary Least Squares					
	(1)	(2)	(3)	(4)	(5)	(6)
Changes in input price	-0.90 ***		-0.74 ***	-0.53 ***		-0.49 ***
× Dispersion of changes in input price	(0.19)		(0.17)	(0.13)		(0.14)
Changes in input price		-0.68 ***	-0.35 *		-0.30 ***	-0.10
× Lagged dispersion of changes in input price		(0.24)	(0.20)		(0.10)	(0.08)
Industry-level fixed effect	Yes	Yes	Yes	Yes	Yes	Yes
Period fixed effect	No	No	No	Yes	Yes	Yes
Number of observations	3,612	3,612	3,612	3,612	3,612	3,612
Adjusted R-squared	0.85	0.85	0.85	0.91	0.92	0.92

Note: ***, ** and * indicate statistically significant at 1%, 5%, and 10% levels, respectively.
Standard errors are cross-section cluster robust standard errors.

Table D3: Regression results: OLS with relative dispersion and lagged changes in costs

Dependent variable: Changes in output price (1974/2Q-2023/2Q)						
	Ordinary Least Squares					
	(1)	(2)	(3)	(4)	(5)	(6)
Changes in input price	-0.82 ***		-0.69 ***	-0.47 ***		-0.44 ***
× Dispersion of changes in input price	(0.18)		(0.17)	(0.13)		(0.14)
Changes in input price		-0.60 **	-0.30		-0.23 ***	-0.06
× Lagged dispersion of changes in input price		(0.22)	(0.19)		(0.07)	(0.06)
Industry-level fixed effect	Yes	Yes	Yes	Yes	Yes	Yes
Period fixed effect	No	No	No	Yes	Yes	Yes
Number of observations	3,612	3,612	3,612	3,612	3,612	3,612
Adjusted R-squared	0.85	0.85	0.85	0.91	0.92	0.92

Note: ***, ** and * indicate statistically significant at 1%, 5%, and 10% levels, respectively.
Standard errors are cross-section cluster robust standard errors.

Table D4: Regression results: OLS with dummy and lagged changes in costs

Dependent variable: Changes in output price (1974/2Q-2023/2Q)						
	Ordinary Least Squares					
	(1)	(2)	(3)	(4)	(5)	(6)
Changes in input price	-0.09 ***		-0.07 ***	-0.04 **		-0.03 **
× Dispersion of changes in input price	(0.02)		(0.01)	(0.01)		(0.01)
Changes in input price		-0.07 ***	-0.05 ***		-0.02 *	-0.01
× Lagged dispersion of changes in input price		(0.01)	(0.01)		(0.01)	(0.01)
Industry-level fixed effect	Yes	Yes	Yes	Yes	Yes	Yes
Period fixed effect	No	No	No	Yes	Yes	Yes
Number of observations	3,612	3,612	3,612	3,612	3,612	3,612
Adjusted R-squared	0.85	0.84	0.85	0.91	0.91	0.91

Note: ***, ** and * indicate statistically significant at 1%, 5%, and 10% levels, respectively.
Standard errors are cross-section cluster robust standard errors.

Table D5: Regression results: weighted least squares with lagged changes in costs

Dependent variable: Changes in output price (1974/2Q-2023/2Q)									
	Weighted Least Squares								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Changes in input price	-1.12 ***		-0.87 ***	-0.98 ***		-0.77 ***	-0.08 ***		-0.07 ***
× Dispersion of changes in input price	(0.25)		(0.19)	(0.25)		(0.20)	(0.02)		(0.01)
Changes in input price		-0.94 ***	-0.53 ***		-0.82 ***	-0.49 ***		-0.06 ***	-0.04 **
× Lagged dispersion of changes in input price		(0.23)	(0.15)		(0.24)	(0.17)		(0.02)	(0.02)
Industry-level fixed effect	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Period fixed effect	No	No	No	No	No	No	No	No	No
Number of observations	3,612	3,612	3,612	3,612	3,612	3,612	3,612	3,612	3,612
Adjusted R-squared	0.89	0.88	0.89	0.88	0.88	0.88	0.88	0.88	0.88

Note: ***, ** and * indicate statistically significant at 1%, 5%, and 10% levels, respectively.
Standard errors are cross-section cluster robust standard errors.

E Derivations

E.1 Demand function

A representative household first determines the allocation of consumption across sectors and goods within each sector independently.

Define the expenditure level by $Z_t(s) \equiv \int_0^1 P_t(i;s)C_t(i;s)di$. We then set the Lagrangean as follows.

$$L = \left[\int_0^1 \left(C_t(i;s)\right)^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}} - \lambda\left(\int_0^1 P_t(i;s)C_t(i;s)di - Z_t(s)\right).$$

The first-order conditions are,

$$C_t(i;s)^{-\frac{1}{\eta}} C_t^{\frac{1}{\eta}}(s) = \lambda P_t(i;s).$$

Thus, for any two goods, the following relationship holds.

$$C_t(i;s) = C_t(j;s)\left(\frac{P_t(i;s)}{P_t(j;s)}\right)^{-\eta}.$$

By substituting the equations into the expression for consumption expenditures, we have

$$\int_0^1 P_t(i;s)\left[C_t(j;s)\left(\frac{P_t(i;s)}{P_t(j;s)}\right)^{-\eta}\right]di = Z_t(s)$$

$$\Leftrightarrow \quad C_t(j;s) = P_t^{-\eta}(j;s)Z_t(s)\frac{1}{\int_0^1 P_t^{1-\eta}(i;s)di}.$$

Finally, using

$$\int_0^1 P_t(i;s)C_t(i;s)di = Z_t(s) = P_t(s)C_t(s),$$

we have

$$C_t(i;s) = \left(\frac{P_t(i;s)}{P_t(s)}\right)^{-\eta} C_t(s)\frac{P_t^{1-\eta}(s)}{\int_0^1 P_t^{1-\eta}(i;s)di}.$$

Define $P_t(s) \equiv \left[\int_0^1 P_t^{1-\eta}(i;s)di\right]^{\frac{1}{1-\eta}}$. We then have,

$$C_t(i;s) = \left(\frac{P_t(i;s)}{P_t(s)}\right)^{-\eta} C_t(s).$$

By doing the same calculation with $C_t = \left[\int_0^1 \left(C_t(s)\right)^{\frac{\widetilde{\eta}-1}{\widetilde{\eta}}} ds\right]^{\frac{\widetilde{\eta}}{\widetilde{\eta}-1}}$, we obtain

$$C_t(s) = \left(\frac{P_t(s)}{P_t}\right)^{-\widetilde{\eta}} C_t.$$

By combining these demand functions, we obtain the demand for the good $(i;s)$ as follows.

$$C_t(i;s) = \left(\frac{P_t(i;s)}{P_t(s)}\right)^{-\eta}\left(\frac{P_t(s)}{P_t}\right)^{-\widetilde{\eta}} C_t.$$

E.2 Price setting rule

From

$$
\begin{aligned}
p_t(i;s) &= \mu + mc_t(i;s), \\
c_t(i;s) &= -\eta\left(p_t(i;s) - p_t(s)\right) - \widetilde{\eta}\left(p_t(s) - p_t\right) + c_t, \\
mc_t(i;s) &= w_t + \frac{1-\epsilon}{\epsilon} y_t(i,j) - \frac{1}{\epsilon} a(i;s) - \log(\epsilon), \\
w_t - p_t &= c_t,
\end{aligned}
$$

the market clearing conditions ($y_t = c_t$) and the cash-in-advance constraint ($y_t = q_t - p_t$), $p_t(i;s)$ is given by,

$$
\begin{aligned}
p_t(i;s) &= \mu + y_t + p_t - \frac{1}{\epsilon} a(i;s) - \log(\epsilon) \\
&\quad + \frac{1-\epsilon}{\epsilon}\left[-\eta\left(p_t(i;s) - p_t(s)\right) - \widetilde{\eta}\left(p_t(s) - p_t\right) + c_t\right] \\
&= -\frac{1-\epsilon}{\epsilon}\eta p_t(i;s) + \frac{1-\epsilon}{\epsilon}\left(\eta - \widetilde{\eta}\right) p_t(s) + \left(1 + \frac{1-\epsilon}{\epsilon}\widetilde{\eta}\right) p_t \\
&\quad + \left(\mu - \frac{1}{\epsilon} a(i;s) - \log(\epsilon)\right) + \left(1 + \frac{1-\epsilon}{\epsilon}\right) y_t \\
&= -\frac{1-\epsilon}{\epsilon}\eta p_t(i;s) + \frac{1-\epsilon}{\epsilon}\left(\eta - \widetilde{\eta}\right) p_t(s) + \left(\mu - \frac{1}{\epsilon} a(i;s) - \log(\epsilon)\right) \\
&\quad + \left(1 + \frac{1-\epsilon}{\epsilon}\right) q_t + \frac{1-\epsilon}{\epsilon}\left(\widetilde{\eta} - 1\right) p_t
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow p_t(i;s) &= \frac{\frac{1-\epsilon}{\epsilon}\left(\eta - \widetilde{\eta}\right)}{1 + \frac{1-\epsilon}{\epsilon}\eta} p_t(s) + \frac{1}{1 + \frac{1-\epsilon}{\epsilon}\eta}\left(\mu - \frac{1}{\epsilon} a(i;s) - \log(\epsilon)\right) \\
&\quad + \frac{1 + \frac{1-\epsilon}{\epsilon}}{1 + \frac{1-\epsilon}{\epsilon}\,\eta} q_t + \frac{\frac{1-\epsilon}{\epsilon}\left(\widetilde{\eta} - 1\right)}{1 + \frac{1-\epsilon}{\epsilon}\eta} p_t \\
&= \frac{\left(\eta - \widetilde{\eta}\right)(1-\epsilon)}{\epsilon + \eta\left(1-\epsilon\right)} p_t(s) + \frac{\epsilon}{\epsilon + \eta\left(1-\epsilon\right)}\left(\mu - \log(\epsilon)\right) \\
&\quad + \frac{1}{\epsilon + \eta\left(1-\epsilon\right)}(q_t - a(i;s)) + \frac{(1-\epsilon)\left(\widetilde{\eta} - 1\right)}{\epsilon + \eta\left(1-\epsilon\right)} p_t.
\end{aligned}
$$

E.3 Dispersion of firms' costs ($d_{s,t}$)

$$
\begin{aligned}
d_{s,t} &\equiv \int_0^1 \left(\Delta Cost_t(i;s) - \Delta Cost_t(s)\right)^2 di \\
&= \mathbb{E}\left[v_t(n) - \frac{1}{N_t}\sum_{n'=1}^{N_t} v_t(n')\right]^2 + \mathbb{E}\left[v_{t-1}(n) - \frac{1}{N_{t-1}}\sum_{n'=1}^{N_{t-1}} v_{t-1}(n')\right]^2 \\
&= \mathbb{E}\left[\frac{N_t - 1}{N_t} v_t(n) - \frac{1}{N_t}\sum_{n'=1, n'\neq n}^{N_t} v_t(n')\right]^2 + \mathbb{E}\left[\frac{N_{t-1} - 1}{N_{t-1}} v_{t-1}(n) - \frac{1}{N_{t-1}}\sum_{n'=1, n'\neq n}^{N_{t-1}} v_{t-1}(n')\right]^2 \\
&= \left[\frac{N_t - 1}{N_t}\right]\tau_t^2 + \left[\frac{N_{t-1} - 1}{N_{t-1}}\right]\tau_{t-1}^2.
\end{aligned}
$$

$$
\begin{aligned}
d_{s,t-1} &\equiv \int_0^1 \left(\Delta Cost_{t-1}(i;s) - \Delta Cost_{t-1}(s)\right)^2 di \\
&= \mathbb{E}\left[v_{t-1}(n) - \frac{1}{N_{t-1}}\sum_{n'=1}^{N_{t-1}} v_{t-1}(n')\right]^2 + \mathbb{E}\left[v_{t-2}(n) - \frac{1}{N_{t-2}}\sum_{n'=1}^{N_{t-2}} v_{t-2}(n')\right]^2 \\
&= \mathbb{E}\left[\frac{N_{t-1} - 1}{N_{t-1}} v_{t-1}(n) - \frac{1}{N_{t-1}}\sum_{n'=1, n'\neq n}^{N_{t-1}} v_t(n')\right]^2 + \mathbb{E}\left[\frac{N_{t-2} - 1}{N_{t-2}} v_{t-2}(n) - \frac{1}{N_{t-2}}\sum_{n'=1, n'\neq n}^{N_{t-2}} v_{t-2}(n')\right]^2 \\
&= \left[\frac{N_{t-1} - 1}{N_{t-1}}\right]\tau_{t-1}^2 + \left[\frac{N_{t-2} - 1}{N_{t-2}}\right]\tau_{t-2}^2.
\end{aligned}
$$