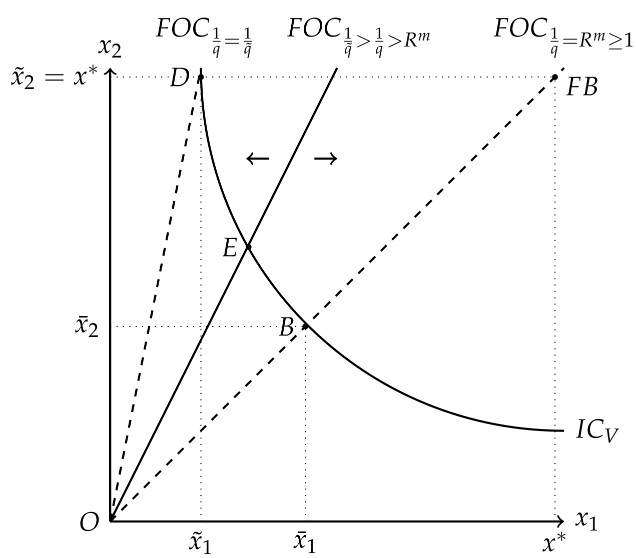
Online Appendix

A Policy Implementation Details

A.1 Channel System

In Figure A.1, the point E describes the equilibrium allocation (x_1, x_2) that intersects FOC curve from (17)-(18) and IC curve (19). Given $R^m \geq 1$ the equilibrium allocations (x_1, x_2) are feasible on the IC curve between the points B and D associated with $\frac{1}{q} \in [R^m, \frac{1}{\tilde{q}}]$ where $\tilde{q}$ is defined as the lower bound of q at the point D. Since δ is increasing in x_1 in (20), each $q \in [\tilde{q}, \frac{1}{R^m}]$ is implemented by a corresponding $\delta \in [\tilde{\delta}, \bar{\delta}]$, where $\tilde{\delta}$ and $\bar{\delta}$ are the lower and upper bound of δ: Given the utility function, $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$, $q = \left(\frac{1-\rho}{\rho}\frac{\delta}{1-\delta}\right)^{\frac{\gamma}{1-\gamma}}$ can derived from (17)-(20). As shown in Figure A.1, when $(\tilde{x}_1, \tilde{x}_2)$ and $(\bar{x}_1, \bar{x}_2)$ are defined as the allocations at $q = \tilde{q}$ and $q = \frac{1}{R^m}$, respectively, each $\tilde{\delta}$ and $\bar{\delta}$ satisfies (20) with $\tilde{x}_1$ and $\bar{x}_1$, respectively.[1]



[Figure A.1 Equilibrium in Channel System]

In a channel system, monetary policy is implemented only by open-market operations, because the level of the interest rate on reserves, R^m, is irrelevant to determine the equilibrium allo-

[1] Note that in case of the lower bound, $\frac{1}{q} = R^m$, $\delta > \bar{\delta}$ can also support the same equilibrium allocation with $\delta = \bar{\delta}$, because the excess reserves can be held at $\frac{1}{q} = R^m$.

cations. For example, suppose that the central bank tries to raise the nominal interest rate, $\frac{1}{q} - 1$, by reducing δ, i.e. open-market sales. Then, the currency is absorbed and the government bonds are injected into the market, so the liquidity premium on currency goes up whereas the liquidity premium on the government bonds falls. Thus, the consumption in the currency trade, x_1, falls while the consumption in the collateral transaction, x_2, rises.[2] Since the real rate of return on currency decreases, the inflation rate rises. Meanwhile, the real rate of return on government bonds increases. Therefore, the nominal interest rate also rises by the Fisher equation.

In channel system, open-market operations can be ineffective only at the lower bound, $\frac{1}{q} = R^m$.[3] Since the rates of return on reserves and government bonds are equal at $\frac{1}{q} = R^m$, we can have excess reserves, $m \geq 0$ and $x_2^m \geq 0$, in this case. The allocation (x_1, x_2) is determined from (17)-(18) with $\frac{1}{q} = R^m$, but reserves and government bonds are indeterminate in equilibrium from (13)-(14) and (19)-(20), $m + b = V - \rho x_1 u'(x_1)$. Thus, if $\delta > \bar{\delta}$, the OMOs is no longer effective.

A.2 Floor System without Large Excess Reserves

In this case reserves are used as a perfect substitute for government bonds and the rates of return on both assets are equal. As shown in the lower bound case in the channel system, $R^m = 1$, OMOs are no longer effective because reserves and government bonds are indeterminate in equilibrium as $m + b = R^m\{V - \rho x_1 u'(x_1)\}$ from (13)-(14) and (19)-(20). However, the nominal interest rate target, $\frac{1}{q} - 1$, can be always achieved by adjusting the level of the interest rate on reserves, R^m. So, the interest on reserves is effective as a policy tool and we can obtain the same allocation by raising R^m in the floor system instead of reducing δ in the channel system. Therefore, the impacts of monetary policy in both systems on the real interest rate and/or the inflation rate are equivalent.

Note that although the equilibrium allocations are identical, the implementation mechanism in the floor system is different from the one in the channel system. In the channel system, the real quantities of currency/CBDC and government bonds pin down the rates of return on currency/CBDC and government bonds, whereas in the floor system the relative price between currency/CBDC and reserves, that is, the interest rate on reserves, determines the demands for currency/CBDC and reserves, respectively.

[2] In Figure A.1, the *IC* curve remains, while the *FOC* curve shifts to the left.

[3] In this respect the equilibrium feature at the lower bound is similar to that in the floor system.

B Liquidity Market

In the model a banking contract provides a liquidity insurance to maximize the agent's expected utility ex ante, but the incentive problem with the truth-telling constraint arises when the large excess reserves exist. In order to know whether the incentive problem is created by the banking contract or not, in this subsection we introduce a type of liquidity market as shown in **?**. Then, a representative buyer solves the following problem of in the CM of period t:

$$\begin{aligned} \underset{c_t,d_t,m_t,b_t,x_{1t},x_{2t}\geq 0}{Max} \quad & -c_t - d_t - m_t - q_t b_t + \tfrac{\beta\phi_{t+1}}{\phi_t}\{c_t + R_t^d d_t + R_t^d R_t^m m_t + b_t\} \\ & +\rho\{u(x_{1t}) - x_{1t}\} + (1-\rho)\{u(x_{2t}) - x_{2t}\} \end{aligned} \tag{B.1}$$

subject to the assets constraints,

$$\frac{\beta\phi_{t+1}}{\phi_t}\{\frac{c_t + R_t^d d_t}{\rho} + R_t^d m_t\} \geq x_{1t}, \tag{B.2}$$

$$\frac{\beta\phi_{t+1}}{\phi_t}\{R_t^d R_t^m m_t + \frac{b_t}{1-\rho}\} \geq x_{2t}, \tag{B.3}$$

and the trading incentive constraints,

$$\frac{c_t + R_t^d d_t}{\rho} + R_t^d m_t \geq c_t + R_t^d d_t + R_t^d m_t, \tag{B.4}$$

$$R_t^d R_t^m m_t + \frac{b_t}{1-\rho} \geq R_t^m c_t + R_t^d R_t^m d_t + R_t^d R_t^m m_t + b_t. \tag{B.5}$$

Given the trading opportunity in the liquidity market, the buyers maximize the expected utility in (B.1) by purchasing cash, CBDC, reserves and bonds and trading each other after their types are revealed. Since CBDC and reserves are convertible with the interest rate on reserves, R^m, without loss of generality we can assume that the buyers will hold CBDC, d_t, to trade with bonds and hold reserves, m_t, for using either type 1 or type 2 transactions directly as shown in (B.2)-(B.3). (B.4)-(B.5) represent the trading incentives for type 1 and 2 buyers, respectively, after their types are revealed.

The liquidity market clears with

$$(1-\rho)\{c_t + R_t^d d_t\} = \rho b_t p_t \tag{B.6}$$

where p_t is the price of bonds in terms of currency/CBDC.

From the maximization problem, the first-order conditions for d_t, m_t, b_t can be derived as

$$\frac{\mu}{\beta R^d} = u'(x_1), \tag{B.7}$$

$$\frac{\mu}{\beta R^d} = \rho u'(x_1) + (1-\rho)u'(x_2)R^m, \tag{B.8}$$

$$q\frac{\mu}{\beta} = u'(x_2), \tag{B.9}$$

and the asset markets clear in the CM with (12).

Note that the trading incentive constraints do not bind if $p_t \in (0, \frac{1}{R_t^m}]$ holds in equilibrium.[4] Thus, given the price $p_t \in (0, \frac{1}{R_t^m}]$, they always trade in the liquidity market and they will decide how much each asset to hold in advance in the CM.

In a channel system there are no excess reserves, $m_t = 0$, in equilibrium. Then, since the first-order conditions (B.7) and (B.9) are exactly the same as (17)-(18), and the feasibility condition is also maintained as (19), the equilibrium allocation is the same as the one with the banking contract. This result also holds in a floor system without large excess reserves. Given $m_t > 0$, $\frac{1}{q} = R^m$ holds from the first-order conditions (B.7)-(B.9). Thus, the equilibrium allocation is also kept with the first-order condition $qu'(x_1) = u'(x_2)$(or $u'(x_1) = u'(x_2)R^m$).

However, if the asset portfolio is filled with the large excess reserves, the first-order condition cannot be supported as long as reserves can be converted into CBDC. For example, if the utility function is simply assumed as $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$, then the first-order condition is $\frac{x_2}{x_1} = (R^m)^{\frac{1}{\gamma}}$, so the ratio between type 1 and type 2 buyer's consumption is greater than R^m. However, as the proportion of m_t increases in the assets constraints (B.2)-(B.3), the ratio between type 1 and type 2 buyer's consumption $\frac{x_2}{x_1}$ must approaches to R^m. In order to raise the ratio $\frac{x_2}{x_1}$ as much as the level at the first-order condition, the price must be close to zero. Even in this case, if $\alpha = \frac{x_2^m}{x_2^m + x_2^b} > \hat{\alpha} := (R^m)^{1-\frac{1}{\gamma}}$ holds, then the first-order condition is not sustainable any longer, which is exactly the same result from the Lemma 2.[5]

Thus, it does not seem to matter whether the banking contract or the liquidity market is provided. This inefficient liquidity distribution result can occur when the buyers can convert an

[4]Since the bonds are useless for type 1 buyers, they will sell even if the price approaches to zero.

[5]Note that $x_1 = c_t + R_t^d d_t + R_t^d m_t$, $x_2^m = R_t^m c_t + R_t^d R_t^m d_t + R_t^d R_t^m m_t$, $x_2^b = \frac{b_t}{1-\rho}$ and $x_2 = x_2^m + x_2^b$.

amount of less-liquid asset into a liquid asset at a given price, i.e. $R^m = \frac{x_2}{x_1}$, and it is not aligned with the relative marginal utility from these assets, $R^m = \frac{u'(x_1)}{u'(x_2)}$.

C Proofs

Lemma 1. *In channel systems the truth-telling constraints (6)-(7) do not bind.*

Proof. *Given the utility function, $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$ where $\gamma < 1$, the first-order conditions (17)-(18) can be rewritten as $x_2 = \frac{1}{q}^{\frac{1}{\gamma}} x_1$. Since $\frac{1}{q} > R^m \geq 1$ in channel systems, $x_2 = \frac{1}{q}^{\frac{1}{\gamma}} x_1 > R^m x_1$ always holds, so the truth-telling constraint (7) does not bind in equilibrium. The truth-telling constraint for type 1 buyers (6) does not bind in channel systems because there are no excess reserves to use, $x_2^m = 0$. QED*

Lemma 2. *In the floor system the truth-telling constraint (7) never binds, but the truth-telling constraint (6) can bind when the excess reserves are sufficiently large as $\delta > \hat{\delta}$ where $\hat{\delta}$ satisfies $\frac{\hat{\delta}-\bar{\delta}}{1-\bar{\delta}} = (R^m)^{1-\frac{1}{\gamma}}$.*

Proof. *When $\lambda_1 = \lambda_2 = 0$, the equilibrium conditions (17)-(19) still hold given $\frac{1}{q} = R^m > 1$, so we have $x_1 = (R^m)^{-\frac{1}{\gamma}} x_2$ in equilibrium. The truth-telling constraint for type 2 buyers (7) does not bind because $\frac{1}{q}^{\frac{1}{\gamma}} > R^m$ at $\frac{1}{q} = R^m > 1$. However, the truth-telling constraint for type 1 buyers (6) can bind when the excess reserves are sufficiently large. Given $R^m > 1$ there exists a threshold $\hat{\alpha} \in (0,1)$ which satisfies with $\hat{\alpha} = (R^m)^{1-\frac{1}{\gamma}}$ from (21) and the first-order condition, $x_1 = (R^m)^{-\frac{1}{\gamma}} x_2$. Since $\hat{\alpha}$ is associated with $\hat{\delta}$ as $\hat{\alpha} = \frac{\hat{\delta}-\bar{\delta}}{1-\bar{\delta}}$, we know that (21) binds at $\delta \in (\hat{\delta}, 1]$, which is equivalent with $\alpha \in (\hat{\alpha}, 1]$. Note that $\hat{\delta} > \bar{\delta}$ because (21) does not bind at $\alpha = 0$. QED*

Proposition 1. *Given R^m, the inflation rate decreases in δ while the real interest rate on government bonds increases in δ in the floor system with LER.*

Proof. *Given R^m, if δ decreases, then the equilibrium allocation (x_1^f, x_2^f) moves toward point C in Figure 3. We can check the changes in the inflation rate and the real interest rate by reducing x_1^f along the curve (19) given the same R^m.*

$$
\begin{aligned}
\frac{\partial \mu}{\partial x_1^f} &= \beta\{\rho u''(x_1^f) + (1-\rho) R^m u''(x_2^f) \frac{\partial x_2^f}{\partial x_1^f}|_V\} = \beta\rho\gamma u'(x_1^f)\{-\frac{1}{x_1^f} + \frac{R^m}{x_2^f}\} < 0, \\
\frac{\partial q\mu}{\partial x_1^f} &= \beta u''(x_2^f) \frac{\partial x_2^f}{\partial x_1^f}|_V > 0.
\end{aligned}
\tag{C.1}
$$

In (C.1) we use $\frac{\partial x_2^f}{\partial x_1^f}|_V = -\frac{\rho u'(x_1)}{(1-\rho)u'(x_2)}$ *from (19), given* $-\frac{xu''(x)}{u'(x)} = \gamma$. *Thus, the inflation rate goes up while the real interest rate on government bonds decreases when* x_1^f *decreases by reducing* δ. *QED*

Proposition 2. *Given* δ, *both the inflation rate and the real interest rate on government bonds increase in the interest of reserves,* R^m, *in the floor system with LER.*

Proof. *By plugging (19) and (21) into (22)-(23), we can have*

$$\begin{aligned} \frac{\mu^f}{\beta} &= \rho u'(x_1^f) + (1-\rho)R^m u'(x_2^f) = (1-\alpha)\rho u'(x_1^f) + \frac{\alpha V}{x_1^f}, \\ \frac{1}{\beta r_b^f} &= u'(x_2^f). \end{aligned} \tag{C.2}$$

Given δ, *if* R^m *is raised then* x_1^f *decreases and* x_2^f *increases along the IC curve (19), so both* μ^f *and* r_b^f *increase in (C.2). QED*

Proposition 3. *At the same level of* δ, *when the interest on reserves is raised, the inflation rate increases less and the real interest rate on the government bonds decreases more in the floor system with LER compared to the floor system without LER.*

Proof. *By plugging (17)-(18) and (21) into (19), for each* $i = \{c, f\}$, *we can have*

$$\begin{aligned} & x_1^c u'(x_1^c)\{\rho + (1-\rho)(R^m)^{\frac{1-\gamma}{\gamma}}\} = V, \\ & x_1^f u'(x_1^f)\left[\rho + (1-\rho)\alpha^{\gamma-1}(R^m)^{1-\gamma}\right] = V. \end{aligned} \tag{C.3}$$

By using (C.3) we can rewrite (17) and (22) as

$$\begin{aligned} \frac{\mu^c}{\beta} &= \left(\frac{\rho+(1-\rho)(R^m)^{\frac{1-\gamma}{\gamma}}}{V}\right)^{\frac{\gamma}{1-\gamma}}, \\ \frac{\mu^f}{\beta} &= \left(\frac{\rho+(1-\rho)\alpha^{\gamma-1}(R^m)^{1-\gamma}}{V}\right)^{\frac{\gamma}{1-\gamma}}\{\rho + (1-\rho)\alpha^{\gamma}(R^m)^{1-\gamma}\}, \end{aligned} \tag{C.4}$$

respectively. From (C.4) we can have

$$\frac{\partial \mu^f}{\partial R^m} = \frac{\partial \mu^c}{\partial R^m}\left(\rho\alpha_\kappa^{-\gamma}(R^m)^{1-\gamma} + (1-\rho)\right)^{\frac{\gamma}{1-\gamma}}\left[\gamma + (1-\gamma)\frac{\rho\alpha + (1-\rho)\alpha^{\gamma}(R^m)^{1-\gamma}}{\rho + (1-\rho)\alpha^{\gamma}(R^m)^{1-\gamma}}\right]. \tag{C.5}$$

Given the same level of R^m, *since* $x_1^c < x_1^f$, $(R^m)^{\frac{1-\gamma}{\gamma}} > \alpha^{\gamma-1}(R^m)^{1-\gamma}$ *holds in (C.3).*[6] *By using this*

[6]This inequality, $(R^m)^{-\frac{1}{\gamma}} < \frac{\alpha}{R^m}$, implies that the slope of the first-order condition (17)-(18) in the channel system(or

inequality, we can find out that $|\frac{\partial \mu^f}{\partial R^m}| < |\frac{\partial \mu^c}{\partial R^m}|$ *holds in (C.5). Since the Fisher equation holds in the model, the effect on the real interest rate in the floor system with LER is greater than one in the floor system without LER. QED*

Corollary 1. *When* δ *is raised, the feasibility equilibrium condition (24) moves towards the origin in the floor system with LER.*

Proof. *By Proposition 2, the real interest rate on private assets goes up when* δ *increases in the floor system with LER. Therefore, the asset price decreases and the feasibility condition moves toward the origin. QED*

Corollary 2. *When the interest on reserves,* R^m*, is raised, the feasibility equilibrium condition (24) moves towards the origin in the floor system with LER.*

Proof. *By Proposition 3, when the interest on reserves,* R^m*, increases, the real interest rate on private assets goes up further in the floor system with LER comparing to the floor system without LER. Therefore, the asset price decreases further and the feasibility condition moves toward the origin. QED*

Proposition 4. *If* ω *is sufficiently large, then the equilibrium allocations in the floor system with LER are suboptimal.*

Proof. *For given a monetary policy* (R_0^m, δ_0) *in the floor system with LER, we can find out a policy set* (R_1^m, δ_0) *which is located on the borderline between the floor system without LER and the floor system with LER in Figure 2. Let* $(x_1, x_2) = (\hat{x}_1, \hat{x}_2)$ *solve (21) and (24) with* $\psi = \frac{\beta y u'(x_2)}{1-\beta u'(x_2)}$ *at* $R^m = R_1^m$*. The slopes of the adjusted welfare function (26) and the IC curve (24) at this allocation* $(\hat{x}_1, \hat{x}_2)$ *are*

$$\frac{\partial x_2}{\partial x_1}\,|_W = -\frac{\rho\{(1-\omega)u'(\hat{x}_1) - 1\}}{(1-\rho)\{u'(\hat{x}_2) - 1\}} \tag{C.6}$$

and

$$\frac{\partial x_2}{\partial x_1}\,|_V = -\frac{\rho(1-\gamma)u'(\hat{x}_1)}{(1-\rho)(1-\gamma)u'(\hat{x}_2) - K'(\hat{x}_2)}, \tag{C.7}$$

respectively, where $K(x_2) := \frac{\beta y A u'(x_2)}{1-\beta u'(x_2)}$ *and* $K'(x_2) < 0$*. Note that there exists a threshold* $\omega^*(R_1^m) \in (0,1)$ *at which the two slopes are equal. If* $\omega > \omega^*(R_1^m)$*, then both* x_1 *and* x_2 *can increase by choosing the original* $R^m = R_0^m$ *and reducing* $\delta < \delta_0$*. QED*

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the floor system without LER) is greater than the slope of the first-order condition (21) in the floor system with LER in Figure 3.