Appendix to "Bank Shocks and the Debt Structure"

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This appendix is divided into five sections. Section 1 derives the full model and lists all equilibrium conditions. Section 2 provides sensitivity tests for the predictions of the modified NK model. Section 3 presents the VAR methodology and the data used in the VAR estimation. Section 5 presents the results from a VAR model where only bank and non-bank shocks are identified. Section 6 gives additional results from the IRF matching estimation.

1. Model Derivation

This section provides the derivation of the model and lists all the equations.

1.1. Households

A representative household decides its optimal level of consumption C_t, capital K_t and deposit D_t in order to maximize utility defined as:

$$E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t^C \left\{ \log(C_t) - \psi_H \frac{H_t^{1+\sigma_H}}{1+\sigma_H} \right\}.$$

The budget constraint writes as:

$$p_t C_t + p_t D_t + q_t^K K_t \leq w_t H_t + p_{t-1} R_t D_{t-1} + \left[q_t^K (1-\delta) + p_t r_t^K \right] K_{t-1} + O_t. \quad (1)$$

The Lagrangian associated to the households' problem can be written as:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} (\beta)^t \zeta_t^C \Bigg\{ \log(C_t) - \psi_H \frac{H_t^{1+\sigma_H}}{1+\sigma_H}$$
$$+ \tilde{\Lambda}_t \left(w_t H_t + p_{t-1} R_t D_{t-1} + \left[q_t^K (1-\delta) + p_t r_t^K \right] K_{t-1} + O_t - p_t C_t - p_t D_t - q_t^K K_t \right) \Bigg\}.$$

The first-order condition with respect to consumption C_t is:

$$\zeta_t^C \tilde{\Lambda}_t p_t = \frac{\zeta_t^C}{C_t}. \tag{2}$$

The first-order condition with respect to labor H_t is:

$$\psi_H H_t^{\sigma_H} = w_t \tilde{\Lambda}_t. \tag{3}$$

The first-order condition with respect to risk-free deposits D_t is:

$$\zeta_t^C \tilde{\Lambda}_t p_t = \beta E_t \zeta_{t+1}^C \tilde{\Lambda}_{t+1} p_{t+1} \frac{R_{t+1}}{\pi_{t+1}}. \tag{4}$$

Households supply capital K_t to entrepreneurs. The first-order condition with respect to capital K_t is:

$$\zeta_t^C \tilde{\Lambda}_t = \beta E_t \zeta_{t+1}^C \tilde{\Lambda}_{t+1} R_{t+1}^K, \tag{5}$$

with,

$$R_{t+1}^K = \frac{q_{t+1}^K (1-\delta) + r_{t+1}^K p_{t+1}}{q_t^K}. \tag{6}$$

1.2. Capital Installer

The capital installer selects its optimal level of investment I_t to maximize the sum of its profits discounted with households' stochastic discount factor:

$$E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t^C \tilde{\Lambda}_t \left\{ q_t^K K_t - p_t I_t \right\},$$

and using the following technology:

$$K_t = (1-\delta)K_{t-1} + \left[1 - S\left(\zeta_t^I \frac{I_t}{I_{t-1}}\right)\right] I_t.$$

The first order condition for profit maximization with respect to I_t writes:

$$\zeta_t^C \tilde{\Lambda}_t q_t^K \left[1 - S\left(\zeta_t^I \frac{I_t}{I_{t-1}}\right) - \zeta_t^I \frac{I_t}{I_{t-1}} S'\left(\zeta_t^I \frac{I_t}{I_{t-1}}\right)\right] - \zeta_t^C \tilde{\Lambda}_t p_t + \beta \zeta_{t+1}^C \tilde{\Lambda}_{t+1} q_{t+1}^K \zeta_{t+1}^I \left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\zeta_{t+1}^I \frac{I_{t+1}}{I_t}\right) = 0. \tag{7}$$

1.3. Firms

I follow Gali (2010) in assuming a three-sector structure for good producers. Firms in the final goods sector produce differentiated goods using entrepreneurs production bought in competitive markets. The former are subject to nominal rigidity introduced via staggered-price contracts à la Calvo.

1.3.1 Entrepreneurs

Entrepreneurs produce intermediate goods using capital and labor obtained from the households. There exists a continuum $e \in [0,1]$ of entrepreneurs operating in competitive markets. An entrepreneur e enters the period with net worth N_{et} pledged to obtain debt X_{et}. Debt is used to fund working capital and is a fixed proportion of the net worth:

$$X_{et} = \xi N_{et}. \tag{8}$$

Here ξ is a parameter that corresponds to entrepreneurs leverage. Entrepreneur e sells production Y_{et}^E at a competitive price p_t^E to retailers, where Y_{et}^E is produced using the following Cobb-Douglas technology:

$$Y_{et}^E = \varepsilon_{et} A_t K_{et}^{\alpha} H_{et}^{1-\alpha}, \tag{9}$$

where K_{et} and H_{et} are capital and labor input used to produce. Variable A_t is a technology shock and ε_{et} is a sequence of idiosyncratic shock realizations. An entrepreneur is constrained on her capital inputs K_{et} and labor inputs H_{et} relative to her debt capacity X_{et} according to the following debt constraint:

$$X_{et} \geq r_t^K K_{et} + \tilde{w}_t H_{et}. \tag{10}$$

An entrepreneur e maximizes her real profits defined as,

$$\frac{p_t^E Y_{et}^E}{p_t} - r_t^K K_{et} - \tilde{w}_t H_{et}, \tag{11}$$

by choosing optimal inputs K_{et} and H_{et} for a given level of debt X_{et} and subject to the debt constraint defined in equation (10). The first order conditions for the optimization problem of the entrepreneur can be written as:

$$\alpha X_{et} = r_t^K K_{et}, \tag{12}$$

$$(1-\alpha) X_{et} = \tilde{w}_t H_{et}. \tag{13}$$

Defining s_t the aggregate component of the marginal cost of production expressed in terms of the final goods implies:

$$s_t = \frac{1}{A_t}\left(\frac{p_t}{p_t^E}\right)\left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t^K}{\alpha}\right)^{\alpha}. \tag{14}$$

For later use, it is also convenient to define $q_t = \frac{1}{s_t}$, where q_t is a measure of the aggregate entrepreneurial markup over input costs.[1]

[1]Here s_t must not be confused with the marginal cost of the intermediate good producer, $\tilde{p}_t^E = \frac{p_t^E}{p_t}$, which is taken as given by entrepreneurs.

Idiosyncratic Shocks.—Each period, an entrepreneur e is hit by a sequence of three idiosyncratic shocks. I summarize here the characteristics of the successive shocks:

Shock $\varepsilon_{1,et}$: Publicly-observed, realizes along aggregate shocks for all entrepreneurs. This shock creates heterogeneity in entrepreneurs' productivity.

Shock $\varepsilon_{2,et}$: Publicly-observed, only observed by bank-financed entrepreneurs. This shock is the rationale for choosing bank finance over the less expensive bond finance.

Shock $\varepsilon_{3,et}$: Privately-observed, can be monitored at a cost by financial intermediaries. This shock creates a rationale for the existence of risky debt contract.

Financial Contracts.—The model assumes a continuum of risk-neutral financial intermediaries of each type, bank b or market c, able to fully diversify risk among entrepreneurs. Both fund using deposits from households remunerated at the nominal rate R_t. After the realization of the first two idiosyncratic shocks, an entrepreneur e and a financial intermediary of type f agree on a standard debt contract conditional on ε^f_{et}, the expected productivity of the contracting entrepreneur, where:

$$\varepsilon^f_{et} = \begin{cases} \varepsilon_{1,et} & \text{, } \textit{if bond financing} \\ \varepsilon_{1,et}\varepsilon_{2,et} & \text{, } \textit{if loan financing.} \end{cases} \tag{15}$$

Given an optimal threshold $\bar{\omega}^f_{et}$ for ω^f_{et} under which monitoring occurs, the expected share of final output accruing to a contracting entrepreneur is:

$$v(\bar{\omega}^f_{et}, \sigma^f_t) = \int_{\bar{\omega}^f_{et}}^{\infty} (\omega - \bar{\omega}^f_{et})\varphi(\omega, \sigma^f_t) d\omega, \tag{16}$$

and the expected share of final output accruing to a lender of type f is:

$$g(\bar{\omega}^f_{et}, \sigma^f_t) = \int_0^{\bar{\omega}^f_{et}} (1 - \mu_f)\omega\varphi(\omega, \sigma) d\omega + \bar{\omega}^f_{et}[1 - \Phi(\bar{\omega}^f_{et}, \sigma^f_t)], \tag{17}$$

where $\varphi(\omega_{et}^f, \sigma_t^f)$ and $\Phi(\omega_{et}^f, \sigma_t^f)$ correspond respectively to the distribution and cumulative density functions of ω_{et}^f implied by the distributional assumptions on idiosyncratic shock distributions. Here the first and second terms on the right hand side correspond respectively to revenues seized from monitored entrepreneurs and payments from non-defaulting entrepreneurs.

The optimal debt contract chosen by entrepreneur e sets a threshold $\bar{\omega}_{et}^f$ under which monitoring occurs and maximizing the expected fixed repayment $\varepsilon_{et}^f \bar{\omega}_{et}^f X_{et} q_t$ paid to the financial intermediary. The problem of the entrepreneur is subject to the debt constraint from equation (10) and,

$$\varepsilon_{et}^f q_t g^f(\bar{\omega}_{et}^f, \sigma_t^f) X_{et} \geq (X_{et} - N_{et}^f) R_t, \tag{18}$$

$$v(\bar{\omega}_{et}^f, \sigma_t^f) + g^f(\bar{\omega}_{et}^f, \sigma_t^f) \leq 1 - G_\omega^f(\bar{\omega}_{et}^f, \sigma_t^f), \tag{19}$$

$$\varepsilon_{et}^f q_t v(\bar{\omega}_{et}^f, \sigma_t^f) X_{et} \geq N_{et}^f, \tag{20}$$

where $G_\omega^f(\bar{\omega}_{et}^f, \sigma_t^f) = \mu_f \int_0^{\bar{\omega}_{et}^f} \omega \varphi(\omega, \sigma_t^f) d\omega$ denotes the share of output lost to monitoring. Equation (18) implies that the financial intermediaries' expected returns must exceed repayment to households, equation (19) ensures the feasibility of the debt contract, and equation (20) guarantees entrepreneur's willingness to borrow from a financial intermediary. Notice that because the problem of the entrepreneur is linear in net worth, the optimal solution implies that each entrepreneur invests all or none of her net worth.

Under optimal contracts and assuming free entry for financial intermediaries such that equation (18) is always binding, optimal thresholds $\bar{\omega}_{et}^f$ are given as the minimal solution to:

$$g^f(\bar{\omega}_{et}^f, \sigma_t^f) = \left(\frac{\xi - 1}{\xi}\right) \frac{R_t}{\varepsilon_{et}^f q_t} \quad for\, f \in \{b, c\}. \tag{21}$$

These equations implicitly define thresholds $\bar{\omega}_{et}^f$ as functions of aggregate variables q_t, R_t, v_t and idiosyncratic expected idiosyncratic productivity ε_{et}^f such that:

$$\bar{\omega}_{et}^f = \begin{cases} \bar{\omega}^c(\varepsilon_{1,et}, q_t, R_t) & \text{, } \textit{if bond financing} \\ \bar{\omega}^b(\varepsilon_{1,et}\varepsilon_{2,et}, q_t, R_t, \nu_t) & \text{, } \textit{if loan financing,} \end{cases} \tag{22}$$

where it can be seen from equation (21) that both thresholds $\bar{\omega}_{et}^f$ for $f \in \{b, c\}$ are increasing in R_t and decreasing in q_t, ν_t and ε_{et}^f.

Funding Choices.—Following De Fiore and Uhlig (2011) it is possible to show the existence and uniqueness of thresholds in the realizations of idiosyncratic productivity shocks to characterize entrepreneurs' funding decisions.

First, consider an entrepreneur e having contracted with a bank in period t. After the second idiosyncratic shock $\varepsilon_{2,et}$ is observed this entrepreneur decides to proceed with a loan only if her expected profit from producing is higher than the opportunity cost of producing, what corresponds to her net worth. The total expected return for a bank-funded entrepreneur is given by $V^d(\varepsilon_{1,et}, \varepsilon_{2,et}, q_t, R_t, \nu_t)N_{et}^b$ where:

$$V^d(\varepsilon_1, \varepsilon_2, q_t, R_t, \nu_t) = \varepsilon_1\varepsilon_2 q v(\bar{\omega}^b(\varepsilon_1\varepsilon_2, q_t, R_t, \nu_t))\xi. \tag{23}$$

Conditional on the realizations of $\varepsilon_{1,et}$ and aggregate variables q_t, R_t and ν_t, entrepreneur e proceeds with bank finance only if the realization of $\varepsilon_{2,et}$ is higher than a threshold $\bar{\varepsilon}_d(\varepsilon_{1,et}, q_t, R_t, \nu_t)$ implicitly defined by:

$$1 = V^d(\varepsilon_{1,et}, \bar{\varepsilon}_{d,et}, q_t, R_t, \nu_t). \tag{24}$$

This equation implies that the threshold $\bar{\varepsilon}_d$ is increasing in $\varepsilon_{1,et}$, q_t and ν_t and decreasing in R_t. The funding decision of an entrepreneur having observed $\varepsilon_{1,et}$ is deduced similarly by comparing her expected payoffs conditional on her funding choice. The expected payoff for an entrepreneur proceeding with bank finance conditional on the realization of $\varepsilon_{1,et}$ is $V^b(\varepsilon_{1,et}, q_t, R_t, \nu_t)N^b_{et}$, where:

$$V^b(\varepsilon_1, q_t, R_t, \nu_t) = \int_{\bar{\varepsilon}_d} V^d(\varepsilon_1, \varepsilon_2, q_t, R_t, \nu_t)\Phi(d\varepsilon_2) + \Phi(\bar{\varepsilon}_d(\varepsilon_1, q_t, R_t, \nu_t)). \tag{25}$$

Here the two terms on the right-hand side correspond respectively to the expected returns for producing and abstaining bank-financed entrepreneurs. Similarly, the expected payoff for an entrepreneur proceeding with bond finance after having observed $\varepsilon_{1,et}$ is $V^c(\varepsilon_{1,et}, q_t, R_t)N^c_{et}$, where:

$$V^c(\varepsilon_1, q, r) = \varepsilon_1 q v(\bar{\omega}^c(\varepsilon_1, q, E))\xi. \tag{26}$$

Finally, the expected total payoff for an entrepreneur abstaining from production is N_{et}. Based on the realization of $\varepsilon_{1,et}$ each entrepreneur selects the funding option delivering the maximum expected payoff $V(\varepsilon_{1,et}, q_t, R_t)N_{et}$ such that:

$$V(\varepsilon_1, q_t, R_t, \nu_t) = max\{1, (1-\tau_b)V^b(\varepsilon_1, q_t, R_t, \nu_t), V^c(\varepsilon_1, q, R)\}. \tag{27}$$

Under the conditions that $\frac{\partial V^b(.)}{\partial \varepsilon_1} \geq 0$ and $\frac{\partial V^c(.)}{\partial \varepsilon_1} > \frac{\partial V^b(.)}{\partial \varepsilon_1}$, it can be shown that there exists a unique threshold $\bar{\varepsilon}^b$ for the first idiosyncratic shock ε_1 implicitly defined by the condition $V^b(\bar{\varepsilon}_{b,t}, q_t, R_t, \nu_t) = 1$ and under which entrepreneurs do not raise external finance. Because this cutoff point depends only on aggregate variables such that $\bar{\varepsilon}_{b,t} = \bar{\varepsilon}_b(q_t, R_t, \nu_t)$, it is identical across all entrepreneurs. Similarly, there exists a unique threshold $\bar{\varepsilon}_c$ for ε_1, implicitly defined by the condition $V^b(\bar{\varepsilon}_{c,t}, q_t, R_t, \nu_t) = V^c(\bar{\varepsilon}_{c,t}, q_t, R_t)$ such that $\bar{\varepsilon}_{c,t} = \bar{\varepsilon}_c(q_t, R_t, \nu_t)$ and above which entrepreneurs prefer to fund from markets.

Financial Variables.—Using the productivity thresholds $\bar{\varepsilon}^b_t$ and $\bar{\varepsilon}^c_t$, it is possible to express entrepreneur average risk premia and default rates conditional on their funding decisions. Denoting respectively ψ^{Mb}_t and ψ^{Mc}_t the default rates for bank-funded and market-funded entrepreneurs gives:

$$\psi^{Mb}_t = \int_{\bar{\varepsilon}_b(q_t,R_t,\nu_t)}^{\bar{\varepsilon}_c(q_t,R_t,\nu_t)} \int_{\bar{\varepsilon}_d(\varepsilon_1,q_t,R_t,\nu_t)} \Phi(\bar{\omega}^b(\varepsilon_1\varepsilon_2, q_t, R_t, \nu_t))\Phi(d\varepsilon_2)\Phi(d\varepsilon_1), \tag{28}$$

$$\psi_t^{Mc} = \int_{\bar{\varepsilon}_c(q_t,R_t,\nu_t)} \Phi(\bar{\omega}^c(\varepsilon_1, q_t, R_t, \nu_t))\Phi(d\varepsilon_1). \tag{29}$$

With the expected fixed repayment for the financial intermediary being $\varepsilon_{et}^f \bar{\omega}_{et}^f q_t$ per unit of fund X_{et}, the credit spread for entrepreneur e writes:

$$\Lambda_{e,t}^f = \frac{\xi}{\xi - 1} \frac{q_t \varepsilon_{e,t}^f \bar{\omega}_{e,t}^f}{R_t} - 1. \tag{30}$$

Denoting ψ_t^{rb} and ψ_t^{rc} the aggregate credit spreads paid respectively by bank-funded and market-funded entrepreneurs yields:

$$\psi_t^{rb} = \int_{\bar{\varepsilon}_b(q_t,R_t,\nu_t)}^{\bar{\varepsilon}_c(q_t,R_t,\nu_t)} \int_{\bar{\varepsilon}_d(\varepsilon_1,q_t,R_t,\nu_t)} \left\{ \frac{\xi}{\xi - 1} \frac{\varepsilon_1 \varepsilon_2 \bar{\omega}_{e,t}^b q_t}{R_t} - 1 \right\} \Phi(d\varepsilon_2)\Phi(d\varepsilon_1), \tag{31}$$

$$\psi_t^{rc} = \int_{\bar{\varepsilon}_c(q_t,R_t,\nu_t)} \left\{ \frac{\xi}{\xi - 1} \frac{\varepsilon_1 \bar{\omega}_{e,t}^c q_t}{R_t} - 1 \right\} \Phi(d\varepsilon_1). \tag{32}$$

Finally, it is possible to express Λ_t^b and Λ_t^c the average spreads for bank-funded and bond-funded entrepreneurs as:

$$\Lambda_t^b = \frac{\psi_t^{rb}(q_t, R_t, \nu_t)}{s_t^{bp}}, \tag{33}$$

$$\Lambda_t^c = \frac{\psi_t^{rc}(q_t, R_t, \nu_t)}{s_t^c}. \tag{34}$$

Aggregate Production.—The expected output for entrepreneur e at the time of contracting with a financial intermediary writes as:

$$Y_{et}^E = \varepsilon_{et}^E K_{et}^\alpha H_{et}^{1-\alpha}. \tag{35}$$

Using the first-order conditions from equations (12) and (13), entrepreneur individual production writes as:

$$\begin{aligned} Y_{et}^{E} &= \varepsilon_{et}^{E}\left(\frac{p_t^E}{p_t}\right)K_{et}^{\alpha}H_{et}^{1-\alpha}, \\ &= \varepsilon_{et}^{E}\left(\frac{p_t^E}{p_t}\right)A_t\left(\alpha\frac{X_{et}}{r_t^K}\right)^{\alpha}\left((1-\alpha)\frac{X_{et}}{\tilde{w}_t}\right)^{1-\alpha}, \\ &= \varepsilon_{et}^{E}\left(\frac{p_t^E}{p_t}\right)A_tX_{et}\left(\frac{\alpha}{r_t^K}\right)^{\alpha}\left(\frac{1-\alpha}{\tilde{w}_t}\right)^{1-\alpha}, \\ &= \frac{\varepsilon_{et}^{E}X_{et}}{s_t}. \end{aligned}$$

Defining $\psi_t^Y = \int_0^1 \varepsilon_{et}^E de$, the aggregate production is obtained as:

$$\begin{aligned} Y_t^E &= \int_0^1 Y_{et}^E, \\ &= \frac{\psi_t^Y \xi N_t}{s_t}. \end{aligned}$$

Where N_t is the aggregate net worth and ψ_t^Y aggregates the realizations of the different idiosyncratic productivity shocks.

1.4. Retailers

Retailers are monopolistically competitive firms indexed by $j \in [0, 1]$. They produce differentiated final good Y_{jt} with the following technology:

$$Y_{jt} = Y_{jt}^E,$$

where Y_{jt}^E is the quantity of intermediate good used by retailers j as an input and purchased to entrepreneurs j in a competitive market at price p_t^E. Assuming price-staggered contracts as in Calvo (1983), $1 - \xi_p$ is defined as the probability for a retailer to be able to reset its price each period. Defining p_{jt} the price of a firm j in period t:

$$p_{jt} = \begin{cases} p_t^* & \text{if adjusts, with probability } 1-\xi_p, \\ p_{jt-1}\bar{\pi}_t & \text{if does not adjust, with probability } \xi_p. \end{cases}$$

Here $\bar{\pi}_t$ is the inflation rate for retailers not adjusting their prices. The model assumes some degree of price indexation expressed as a combination of steady-state inflation π and past period inflation π_{t_1}, hence $\bar{\pi}_t$ can be written as:

$$\bar{\pi}_t = \pi_{t-1}^{\iota_p} \pi^{1-\iota_p}. \tag{36}$$

The nominal flow of profits for a retailer j in period $t+s$ is:

$$p_{jt+s} Y_{jt+s} - (1-\tau_y) p_{t+s}^E Y_{jt+s}^E, \tag{37}$$

with τ_y a subsidy rate. Accordingly the net present value of its profits is:

$$E_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \zeta_t^C \tilde{\Lambda}_{t+s} p_{t+s} \left[\frac{p_{jt+s}}{p_{t+s}} Y_{jt+s} - (1-\tau_y) \frac{p_{t+s}^E}{p_{t+s}} Y_{jt+s}^E \right],$$

where $\zeta_t^C \tilde{\Lambda}_t$ is the multiplier used in the household's budget constraint. Taking into account the demand curve of final goods producer from equation (43), retailer profits rewrite as:

$$E_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \zeta_t^C \tilde{\Lambda}_{t+s} p_{t+s} \left[\left(\frac{p_{jt+s}}{p_{t+s}} \right)^{\frac{1}{1-\lambda_p}} Y_{t+s} - (1-\tau_y) \frac{p_{t+s}^E}{p_{t+s}} \left(\frac{p_{jt+s}}{p_{t+s}} \right)^{\frac{\lambda_p}{1-\lambda_p}} Y_{t+s} \right].$$

Here p_{jt+s} denotes the price of a firm in period $t+s$ that sets $p_{jt} = p_{jt}^*$ in t and does not reoptimize between $t+1, ..., t+s$. Using the indexing rule of non-adjusters,

$$\begin{aligned} p_{jt+s} &= p_{jt+s-1} \bar{\pi}_{t+s} \\ &= p_{jt} \bar{\pi}_{t+1} \bar{\pi}_{t+2} \ldots \bar{\pi}_{t+s}, \end{aligned}$$

similarly,

$$\begin{aligned} p_{t+s} &= p_{t+s-1} \pi_{t+s} \\ &= p_t \pi_{t+1} \pi_{t+2} \ldots \pi_{t+s}. \end{aligned}$$

Accordingly it is possible to write,

$$\frac{p_{jt+s}}{p_{t+s}} = \frac{p_{jt}}{p_t} M_t^s,$$

where,

$$M_t^s = \begin{cases} \frac{\bar{\pi}_{t+s} \ldots \bar{\pi}_{t+1}}{\pi_{t+s} \ldots \pi_{t+1}}, & \text{if } s > 0 \\ 1 & \text{if } s = 0. \end{cases}$$

Finally, the net present value of retailer real profits can be expressed as:

$$E_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \zeta_t^C \tilde{\Lambda}_{t+s} p_{t+s} Y_{t+s} \left[\left(M_t^s \frac{p_{jt}}{p_t} \right)^{\frac{1}{1-\lambda_p}} - (1-\tau_y) \frac{p_{t+s}^E}{p_{t+s}} \left(M_t^s \frac{p_{jt}}{p_t} \right)^{\frac{\lambda_p}{1-\lambda_p}} \right].$$

Because firms able to set their price in period t all face the same problem, they have the same solution and set the same price written p_t^*. Accordingly, the first-order condition for maximizing the net discounted sum of profits is:

$$E_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \Psi_{t+s} p_t^{*\frac{\lambda_p}{1-\lambda_p}} \left[M_t^s \frac{p_t^*}{p_t} - \lambda_p (1-\tau_y) \frac{p_{t+s}^E}{p_{t+s}} \right] = 0,$$

where Ψ_{t+s} is exogenous from the point of view of the firm:

$$\Psi_{t+s} = \zeta_t^C \tilde{\Lambda}_{t+s} p_{t+s} Y_{t+s} \left(M_t^s \right)^{\frac{\lambda_p}{1-\lambda_p}}.$$

Rearranging the previous condition yields the optimal price for a reoptimizing firm:

$$p_t^* = \lambda_p \frac{E_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \Psi_{t+s} (1-\tau_y) \frac{p_{t+s}^E}{p_{t+s}}}{E_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \Psi_{t+s} M_t^s} = \frac{K_{p,t}}{F_{p,t}}.$$

Where auxiliary variables $K_{p,t}$ and $F_{p,t}$ are defined as:

$$K_{p,t} = (1-\tau_y) \lambda_p E_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \Psi_{t+s} \frac{p_{t+s}^E}{p_{t+s}},$$

$$F_{p,t} = E_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \Psi_{t+s} M_t^s.$$

Rewriting the previous definitions:

$$E_t \left[\zeta_t^C \tilde{\Lambda}_t p_t Y_t + \beta \xi_p \left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_p}} F_{p,t+1} - F_{p,t} \right] = 0, \tag{38}$$

$$E_t \left[\lambda_p (1-\tau_y) \frac{p_t^E}{p_t} \zeta_t^C \tilde{\Lambda}_t p_t Y_t + \beta \xi_p \left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_p}{1-\lambda_p}} K_{p,t+1} - K_{p,t} \right] = 0. \tag{39}$$

The aggregate price index writes:

$$\begin{aligned} p_t &= \left[\int_0^1 p_{jt}^{\frac{1}{1-\lambda_p}} dj \right]^{1-\lambda_p}, \\ &= \left[\int_{j \text{ adj}} p_{jt}^{\frac{1}{1-\lambda_p}} dj + \int_{j \text{ dont adj}} p_{jt}^{\frac{1}{1-\lambda_p}} dj \right]^{1-\lambda_p}, \\ &= \left[\int_{j \text{ adj}} {p^*}_{jt}^{\frac{1}{1-\lambda_p}} dj + \bar{\pi}_t^{\frac{1}{1-\lambda_p}} \int_{j \text{ dont adj}} p_{jt-1}^{\frac{1}{1-\lambda_p}} dj \right]^{1-\lambda_p}, \\ &= \left[(1-\xi_p) {p_t^*}^{\frac{1}{1-\lambda_p}} + {\pi_t^*}^{\frac{1}{1-\lambda_p}} \xi_p \int_j p_{jt-1}^{\frac{1}{1-\lambda_p}} dj \right]^{1-\lambda_p}. \end{aligned} \tag{40}$$

Accordingly inflation can be written as:

$$\begin{aligned} \pi_t &= \left[(1-\xi_p) {p_t^*}^{\frac{1}{1-\lambda_p}} \pi_t^{\frac{1}{1-\lambda_p}} + \xi_p \bar{\pi}_t^{\frac{1}{1-\lambda_p}} \right]^{1-\lambda_p}, \\ &= \left[\frac{\xi_p}{1 - (1-\xi_p) {p_t^*}^{\frac{1}{1-\lambda_p}}} \right]^{1-\lambda_p} \bar{\pi}_t, \end{aligned} \tag{41}$$

and the aggregate price index is:

$$p_t^* = \left[\frac{1 - \xi_p \left(\frac{\bar{\pi}_t}{\pi_t} \right)^{\frac{1}{1-\lambda_p}}}{1-\xi_p} \right]^{1-\lambda_p}. \tag{42}$$

1.5. Final Goods Producers

A representative final good producer manufactures homogeneous final goods using technology:

$$Y_t = \int_0^1 \left[Y_{jt}^{\frac{1}{\lambda_p}} \right]^{\lambda_p} dj, \lambda_p > 1.$$

The first order conditions for profit maximization by final good producers are:

$$p_{jt} = p_t \left(\frac{Y_{jt}}{Y_t} \right)^{\frac{\lambda_p}{\lambda_p - 1}}, \quad \text{for} j \in [0, 1]. \tag{43}$$

Finally the price of final goods satisfies the following relation:

$$p_t = \left[\int_0^1 p_{jt}^{\frac{1}{1-\lambda_p}} dj \right]^{1-\lambda_p}. \tag{44}$$

1.6. Adjustment Cost Functions

The investment adjustment cost function is taken from Christiano, Motto, and Rostagno (2014) and writes:

$$S(\eta_t) = \frac{1}{2} \left[\exp\left(\sqrt{S''/2}(\eta_t - \eta) \right) + \exp\left(-\sqrt{S''/2}(\eta_t - \eta) \right) - 2 \right], \tag{45}$$

where $\eta_t = \zeta_t^I I_t / I_{t-1}$. This implies $S(\eta) = S'(\eta) = 0$ and $S''(\eta) = S''$ which is a parameter.

Summary of Equilibrium Conditions

For convenience let us define $\tilde{q}_t^K = \frac{q_t^K}{p_t}$, $\tilde{p}_t^E = \frac{p_t^E}{p_t}$, $\tilde{w}_t = \frac{w_t}{p_t}$ and, $\Lambda_t = \tilde{\Lambda}_t p_t$.

Prices

First-order condition 1 price:

$$E_t \left[\zeta_t^C \Lambda_t Y_t + \beta \xi_p \left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_p}} F_{p,t+1} - F_{p,t} \right] = 0 \tag{1}$$

First-order condition 2 price:

$$E_t \left[(1-\tau_Y) \lambda_p \tilde{p}_t^E \zeta_t^C \Lambda_t Y_t + \beta \xi_p \left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_p}{1-\lambda_p}} K_{p,t+1} - K_{p,t} \right] = 0 \tag{2}$$

Aggregate price index:

$$p_t^* = \left[\frac{1 - \xi_p \left(\frac{\bar{\pi}_t}{\pi_t} \right)^{\frac{1}{1-\lambda_p}}}{1-\xi_p} \right]^{1-\lambda_p} \tag{3}$$

Households

Households' resource constraint:

$$C_t + D_t + \tilde{q}_t^K K_t = \tilde{w}_t H_t + \frac{R_t}{\pi_t} D_{t-1} + \left[\tilde{q}_t^K (1-\delta) + r_t^K \right] K_{t-1} + O_t \tag{4}$$

First-order condition consumption:

$$\zeta_t^C \Lambda_t = \frac{\zeta_t^C}{C_t} \tag{5}$$

First-order condition labor:

$$\psi_H H_t^{\sigma_H} = \tilde{w}_t \Lambda_t . \tag{6}$$

First-order condition deposit:

$$\zeta_t^C \Lambda_t = \beta E_t \zeta_{t+1}^C \Lambda_{t+1} \frac{R_{t+1}}{\pi_{t+1}} \tag{7}$$

Capital returns:

$$R^K_{t+1} = \pi_{t+1} \frac{\tilde{q}^K_{t+1}(1-\delta) + r^k_{t+1}}{\tilde{q}^K_t} \tag{8}$$

First-order condition capital:

$$\zeta^C_t \Lambda_t = \beta E_t \zeta^C_{t+1} \Lambda_{t+1} R^K_{t+1} \tag{9}$$

Capital accumulation:

$$K_t = (1-\delta)K_{t-1} + \left[1 - S\left(\zeta^I_t \frac{I_t}{I_{t-1}}\right)\right] I_t \tag{10}$$

First-order condition investment:

$$\zeta^C_t \Lambda_t \tilde{q}^K_t \left[1 - S\left(\zeta^I_t \frac{I_t}{I_{t-1}}\right) - \zeta^I_t \frac{I_t}{I_{t-1}} S'\left(\zeta^I_t \frac{I_t}{I_{t-1}}\right)\right] - \zeta^C_t \Lambda_t + \beta \zeta^C_{t+1} \Lambda_{t+1} \tilde{q}^K_{t+1} \zeta^I_{t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\zeta^I_{t+1} \frac{I_{t+1}}{I_t}\right) = 0 \tag{11}$$

Entrepreneurs

Aggregate production:

$$Y_t = \frac{\psi^Y_t \xi N_t}{s_t} \tag{12}$$

First-order condition capital:

$$\alpha X_t = r^K_t K_t \tag{13}$$

First-order condition labor

$$(1-\alpha) X_t = \tilde{w}_t H_t \tag{14}$$

Marginal cost:

$$s_t = \frac{1}{A_t \tilde{p}^E_t} \left(\frac{r^K_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha} \tag{15}$$

Entrepreneur dividends:

$$O_t = (1-\gamma)\psi^V_{t-1} n_{t-1} \tag{16}$$

Entrepreneur net worth:

$$N_t = \gamma \psi^V_{t-1} N_{t-1} \tag{17}$$

Aggregates

Aggregate resource constraint:

$$Y_t = C_t + I_t + y_t^M \tag{18}$$

Aggregate profits:

$$\psi_t^V = \int V\left(\varepsilon_1, q_t, R_t, \nu_t\right) \Phi(d\varepsilon_1) \tag{19}$$

$$\psi_t^V = s^a + \int_{\bar{\varepsilon}_b(q_t,R_t,\nu_t)}^{\bar{\varepsilon}_c(q_t,R_t,\nu_t)} V^b(\varepsilon, q_t, R_t, \nu_t)\Phi(d\varepsilon_1) + \int_{\bar{\varepsilon}_c(q_t,R_t,\nu_t)} V^c(\varepsilon_1, q, R)\Phi(d\varepsilon_1) \tag{20}$$

Aggregate productivity:

$$\psi_t^Y = (1-\tau_b)\int_{\bar{\varepsilon}_b(q_t,R_t,\nu_t)}^{\bar{\varepsilon}_c(q_t,R_t,\nu_t)} \varepsilon_1 \int_{\bar{\varepsilon}_d(\varepsilon_1,q_t,R_t,\nu_t)} \varepsilon_2\Phi(d\varepsilon_2)\Phi(d\varepsilon_1) + \int_{\bar{\varepsilon}_c(q_t,R_t,\nu_t)} \varepsilon_1\Phi(d\varepsilon_1) \tag{21}$$

Aggregate default:

$$\psi_t^M = (1-\tau_b)\mu_b\psi_t^{Mb} + \mu_c\psi_t^{Mc} \tag{22}$$

$$\psi_t^{Mb} = \int_{\bar{\varepsilon}_b(q_t,R_t,\nu_t)}^{\bar{\varepsilon}_c(q_t,R_t,\nu_t)} \int_{\bar{\varepsilon}_d(\varepsilon_1,q_t,R_t,\nu_t)} \Phi(\bar{\omega}^b(\varepsilon_1\varepsilon_2, q_t, R_t, \nu_t))\Phi(d\varepsilon_2)\Phi(d\varepsilon_1) \tag{23}$$

$$\psi_t^{Mc} = \int_{\bar{\varepsilon}_c(q_t,R_t,\nu_t)} \Phi(\bar{\omega}^c(\varepsilon_1, q_t, R_t, \nu_t))\Phi(d\varepsilon_1) \tag{24}$$

Monetary Policy

$$R_t - R = \rho_p(R_{t-1} - R) + (1-\rho_p)\left(\alpha_\pi\left(E\pi_{t+1} - \pi\right) + \frac{\alpha_{\Delta Y}}{4} g_{Y,t}\right) + \frac{1}{400}\varepsilon_t^p \tag{25}$$

Miscellaneous

$$S(\eta_t) = \frac{1}{2}\left\{\exp\left[\sqrt{S''/2}(\eta_t - \eta)\right] + \exp\left[-\sqrt{S''/2}(\eta_t - \eta)\right] - 2\right\} \tag{26}$$

Log-Linearised Equations

Prices

First-order condition 1 price:

$$E_t\left[\zeta_t^C\Lambda_t Y_t+\beta\xi_p\left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_p}}F_{p,t+1}-F_{p,t}\right]=0 \tag{1}$$

$$(1-\beta\xi_p)(\hat{\Lambda}_t+\hat{\zeta}_t^C+\hat{Y}_t)+\left[\left(\frac{1}{1-\lambda_p}\right)(\hat{\bar{\pi}}_{t+1}-\hat{\pi}_{t+1})+\hat{F}_{p,t+1}\right]=\hat{F}_{p,t} \tag{2}$$

First-order condition 2 price:

$$E_t\left[(1-\tau_Y)\lambda_p\tilde{p}_t^E\zeta_t^C\Lambda_t Y_t+\beta\xi_p\left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_p}{1-\lambda_p}}K_{p,t+1}-K_{p,t}\right]=0 \tag{3}$$

$$(1-\beta\xi_p)\left[\hat{\tilde{p}}_t^E+\hat{\Lambda}_t+\hat{\zeta}_t^C+\hat{Y}_t\right]+\beta\xi_p\left[\frac{\lambda_p}{1-\lambda_p}(\hat{\bar{\pi}}_{t+1}-\hat{\pi}_{t+1})+\hat{K}_{p,t+1}\right]=\hat{K}_{p,t} \tag{4}$$

Aggregate price index:

$$\frac{\hat{K}_{p,t}}{\hat{F}_{p,t}}=\left[\frac{1-\xi_p\left(\frac{\bar{\pi}_t}{\pi_t}\right)^{\frac{1}{1-\lambda_p}}}{1-\xi_p}\right]^{1-\lambda_p} \tag{5}$$

$$K_{p,t}-F_{p,t}=\frac{\xi_p}{1-\xi_p}\left[\hat{\pi}_t-\hat{\bar{\pi}}_t\right] \tag{6}$$

Households

Households' resource constraint (not required):

$$C_t+D_t+\tilde{q}_t^K K_t=\tilde{w}_t H_t+\frac{R_t}{\pi_t}D_{t-1}+\tilde{q}_t^K\frac{(1+r_t^K-\delta)}{\pi_t}K_{t-1}+O_t \tag{7}$$

First-order condition consumption:

$$\zeta_t^C\Lambda_t=\frac{\zeta_t^C}{C_t} \tag{8}$$

$$\hat{\Lambda}_t=-\hat{C}_t \tag{9}$$

First-order condition labor:

$$\psi_H H_t^{\sigma_H} = \tilde{w}_t \Lambda_t. \tag{10}$$

$$\sigma_H \hat{H}_t = \hat{\tilde{w}}_t + \hat{\Lambda}_t. \tag{11}$$

First-order condition deposit:

$$\zeta_t^C \Lambda_t = \beta E_t \zeta_{t+1}^C \Lambda_{t+1} \frac{R_{t+1}}{\pi_{t+1}} \tag{12}$$

$$\hat{\zeta}_t + \hat{\Lambda}_t = \hat{\zeta}_{t+1}^C + \hat{\Lambda}_{t+1} + \hat{R}_{t+1} - \hat{\pi}_{t+1} \tag{13}$$

Capital returns:

$$\frac{R_{t+1}^K}{\pi_{t+1}} = \frac{\tilde{q}_{t+1}^K (1-\delta)}{\tilde{q}_t^K} \tag{14}$$

$$\hat{R}_{t+1}^K - \hat{\pi}_{t+1} = \frac{\hat{\tilde{q}}_{t+1}^K (1-\delta) + r^K \hat{R}_{t+1}^K}{R^K} - \hat{\tilde{q}}_t^K \tag{15}$$

First-order condition capital:

$$\zeta_t^C \Lambda_t = \beta E_t \zeta_{t+1}^C \Lambda_{t+1} R_{t+1}^K \tag{16}$$

$$\hat{\zeta}_t^C + \hat{\Lambda}_t = \hat{\zeta}_{t+1}^C + \hat{\Lambda}_{t+1} + \hat{R}_{t+1}^K - \hat{\pi}_{t+1} \tag{17}$$

Capital accumulation:

$$K_t = (1-\delta) K_{t-1} + \left[1 - S\left(\zeta_t^I \frac{I_t}{I_{t-1}}\right)\right] I_t \tag{18}$$

$$\hat{K}_t = (1-\delta)\hat{K}_{t-1} + \delta \hat{I}_t \tag{19}$$

First-order condition investment:

$$\zeta_t^C \Lambda_t \tilde{q}_t^K \left[1 - S\left(\zeta_t^I \frac{I_t}{I_{t-1}}\right) - \zeta_t^I \frac{I_t}{I_{t-1}} S'\left(\zeta_t^I \frac{I_t}{I_{t-1}}\right)\right] - \zeta_t^C \Lambda_t + \beta \zeta_{t+1}^C \Lambda_{t+1} \tilde{q}_{t+1}^K \zeta_{t+1}^I \left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\zeta_{t+1}^I \frac{I_{t+1}}{I_t}\right) = 0 \tag{20}$$

$$\hat{\tilde{q}}_t^K = S'' \left[-\hat{I}_{t-1} + (1+\beta)\hat{I}_t + \hat{\zeta}_t^I - \beta \hat{I}_{t+1} - \beta \hat{\zeta}_{t+1}^I\right] \tag{21}$$

Entrepreneurs

Aggregate production:

$$Y_t = \frac{\psi_t^Y \xi N_t}{s_t} \tag{22}$$

$$\hat{Y}_t = \hat{\psi}_t^y + \hat{N}_t - \hat{s}_t \tag{23}$$

First-order condition capital:

$$\alpha X_t = r_t^K K_t \tag{24}$$

$$\hat{X}_t = \hat{r}_t^K + \hat{K}_t \tag{25}$$

First-order condition labor:

$$(1-\alpha) X_t = \tilde{w}_t H_t \tag{26}$$

$$\hat{X}_t = \hat{w}_t + \hat{H}_t \tag{27}$$

Marginal cost:

$$s_t = \frac{1}{A_t \tilde{p}_t^E} \left(\frac{r_t^K}{\alpha} \right)^{\alpha} \left(\frac{\tilde{w}_t}{1-\alpha} \right)^{1-\alpha} \tag{28}$$

$$\hat{s}_t = (1-\alpha)\hat{\tilde{w}}_t + \alpha \hat{r}_t^K - \hat{A}_t - \hat{\tilde{p}}_t^E \tag{29}$$

Entrepreneur dividends:

$$O_t = (1-\gamma)\psi_{t-1}^V N_{t-1} \tag{30}$$

$$\hat{O}_t = \hat{\psi}_{t-1}^V + \hat{N}_{t-1} \tag{31}$$

Entrepreneur networth:

$$N_t = \gamma \psi_{t-1}^V N_{t-1} \tag{32}$$

$$\hat{N}_t = \hat{\psi}_{t-1}^V + \hat{N}_{t-1} \tag{33}$$

Aggregates

Resource constraint:

$$Y_t = C_t + I_t + y_t^M \tag{34}$$

$$\hat{Y}_t = \frac{C}{Y}\hat{C}_t + \frac{I}{Y}\hat{I}_t + \frac{y^M}{Y}\hat{y}_t^M \tag{35}$$

Debt equilibrium:

$$D_t = \left[(1-\tau_b)s_t^{bp} + s_t^c\right](\xi - 1)N_t \tag{36}$$

Entrepreneur's funding:

$$X_t = \left[(1-\tau_b)s_t^{bp} + s_t^c\right]\xi N_t \tag{37}$$

Profits:

$$\psi_t^V = \int V\left(\varepsilon_1, q_t, R_t, \nu_t\right)\Phi(d\varepsilon_1) \tag{38}$$

$$\psi_t^V = s^a + \int_{\bar{\varepsilon}_b(q_t,R_t,\nu_t)}^{\bar{\varepsilon}_c(q_t,R_t,\nu_t)} V^b(\varepsilon, q_t, R_t, \nu_t)\Phi(d\varepsilon_1) + \int_{\bar{\varepsilon}_c(q_t,R_t,\nu_t)} V^c(\varepsilon_1(q_t, R_t, \nu_t))\Phi(d\varepsilon_1) \tag{39}$$

Productivity:

$$\psi_t^Y = (1-\tau^b)\int_{\bar{\varepsilon}_b(q_t,R_t,\nu_t)}^{\bar{\varepsilon}_c(q_t,R_t,\nu_t)} \varepsilon_1 \int_{\bar{\varepsilon}_d(\varepsilon_1,q_t,R_t,\nu_t)} \varepsilon_2\Phi(d\varepsilon_2)\Phi(d\varepsilon_1) + \int_{\bar{\varepsilon}_c(q_t,R_t,\nu_t)} \varepsilon_1\Phi(d\varepsilon_1) \tag{40}$$

Monitoring costs:

$$\psi_t^M = (1-\tau^b)\mu^b\psi_t^{Mb} + \mu^c\psi_t^{Mc} \tag{41}$$

$$\psi_t^{Mb} = \int_{\bar{\varepsilon}_b(q_t,R_t,\nu_t)}^{\bar{\varepsilon}_c(q_t,R_t,\nu_t)} \int_{\bar{\varepsilon}_d(\varepsilon_1,q_t,R_t,\nu_t)} \Phi(\bar{\omega}^b(\varepsilon_1\varepsilon_2, q_t, R_t, \nu_t))\Phi(d\varepsilon_2)\Phi(d\varepsilon_1) \tag{42}$$

$$\psi_t^{Mc} = \int_{\bar{\varepsilon}_c(q_t,R_t,\nu_t)} \Phi(\bar{\omega}^c(\varepsilon_1, q_t, R_t, \nu_t))\Phi(d\varepsilon_1) \tag{43}$$

Monetary Policy

$$R_t - R = \rho_p(R_{t-1} - R) + (1 - \rho_p)\left(\alpha_\pi\left(E\pi_{t+1} - \pi\right) + \frac{\alpha_{\Delta Y}}{4} g_{Y,t}\right) + \frac{1}{400}\varepsilon_t^p \tag{44}$$

Miscellaneous

$$S(\eta_t) = \frac{1}{2}\left\{\exp\left[\sqrt{S''/2}(\eta_t - \eta)\right] + \exp\left[-\sqrt{S''/2}(\eta_t - \eta)\right] - 2\right\} \tag{45}$$

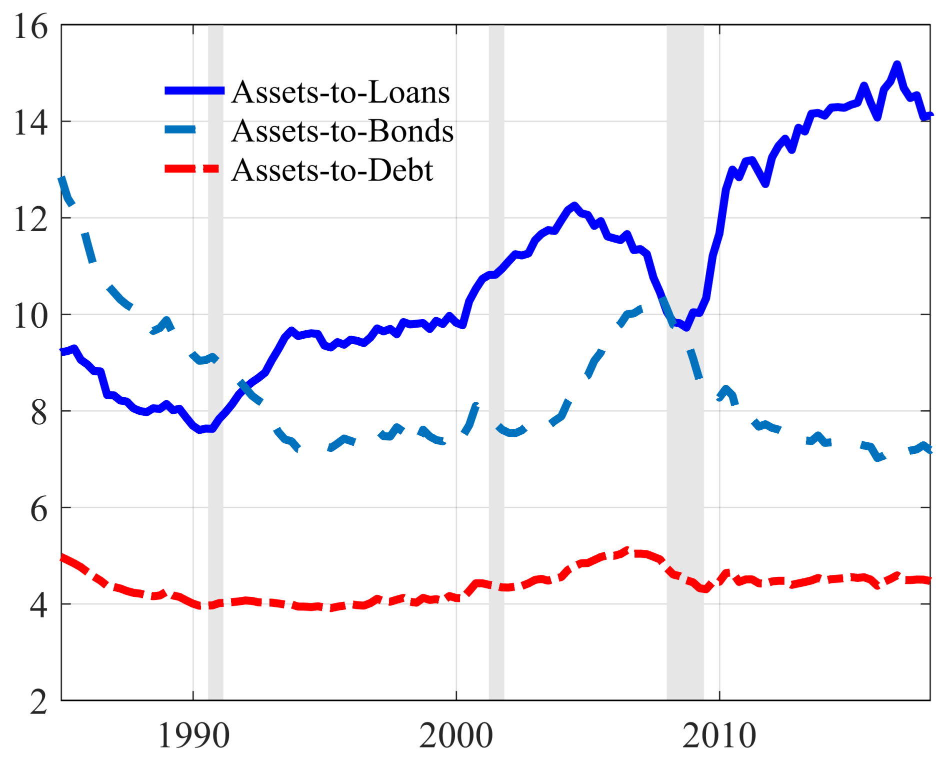

Figure A1: Debt Composition and Firm Leverage

Note: This figure plots the ratios of assets-over-bonds, assets-over-loans and assets-over-total debt for US non-financial corporate firms. The bond series corresponds to the sum of commercial papers and bonds. All series are obtained from FRED.

2. Sensitivity Analysis

This section provides a sensitivity analysis for the results reported in Section 4.2 of the main text. The objective of this section is to demonstrate that the qualitative properties of the NK model presented in the main text, and used to identify the VAR, are robust. Figure A2 on the following page plots various impulse response functions for different shocks using diverse calibrations. The columns correspond to the responses for specific shocks, while the rows correspond to the responses of the different variables. The grey area represents the IRFs for different calibrations. The graph shows that the signs of the response functions for the model's different types of shocks are not dependent on a specific parameterization. Most importantly, only the bank shocks (the first column) can generate opposite movements in loan and bond responses (5th and 6th rows).

Figure A3 shows the parameter ranges used for the sensitivity analysis. The supports of the distributions are given by the x-axis and chosen so that the model can be solved and does not generate explosive solutions. Parameters used for the calibration of Section 4.2 and the estimation of Section 6.1 fall within these ranges. Parameter draws that imply opposite movements in bonds and loans conditional on bank shocks, and comovements in bonds and loans for all other shocks are marked as 'Positive' in blue. Because the model cannot be solved for some parameter combinations, the y-axis shows the proportion of parameter specifications for which the model has a solution.[2]

[2]For instance, the α_π is a sensitive parameter for the determinacy of the NK model. Hence, an α_π closer to one makes the model more unstable and will restrict the number of parameter combinations for which the model is determinate.

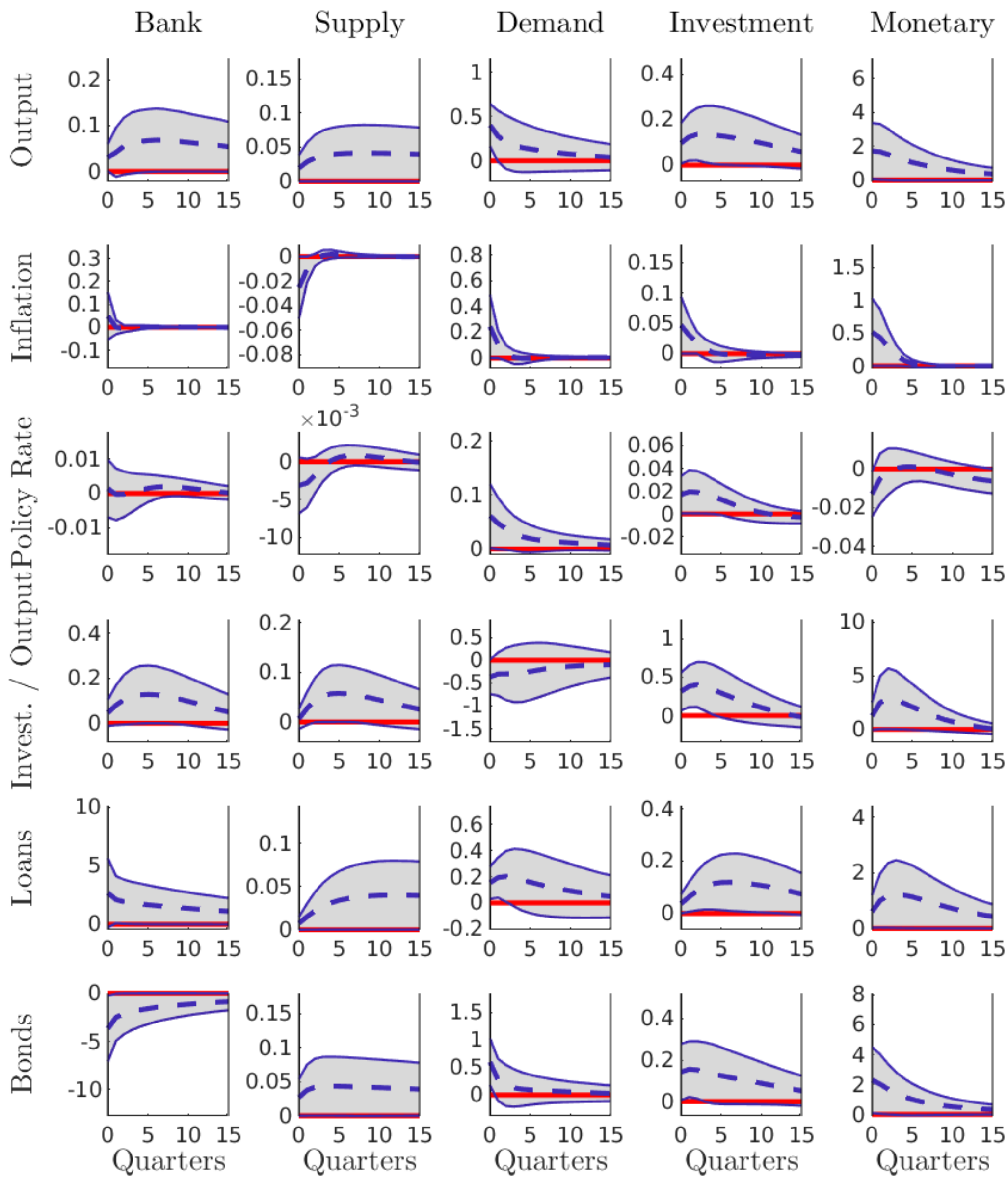

Figure A2: Robustness Test - Impulse Responses

Note: Impulse response functions for the NK model. The grey area corresponds to the IRFs for different calibrations. The highest and lowest two percentiles are trimmed out to remove responses when the model approaches instability. The dashed lines correspond to the means of the total set of IRFs. A total of 100000 sets of parameter are drawn from the uniform distributions displayed in figure A3. Inflation is shown here instead of prices to ease readability.

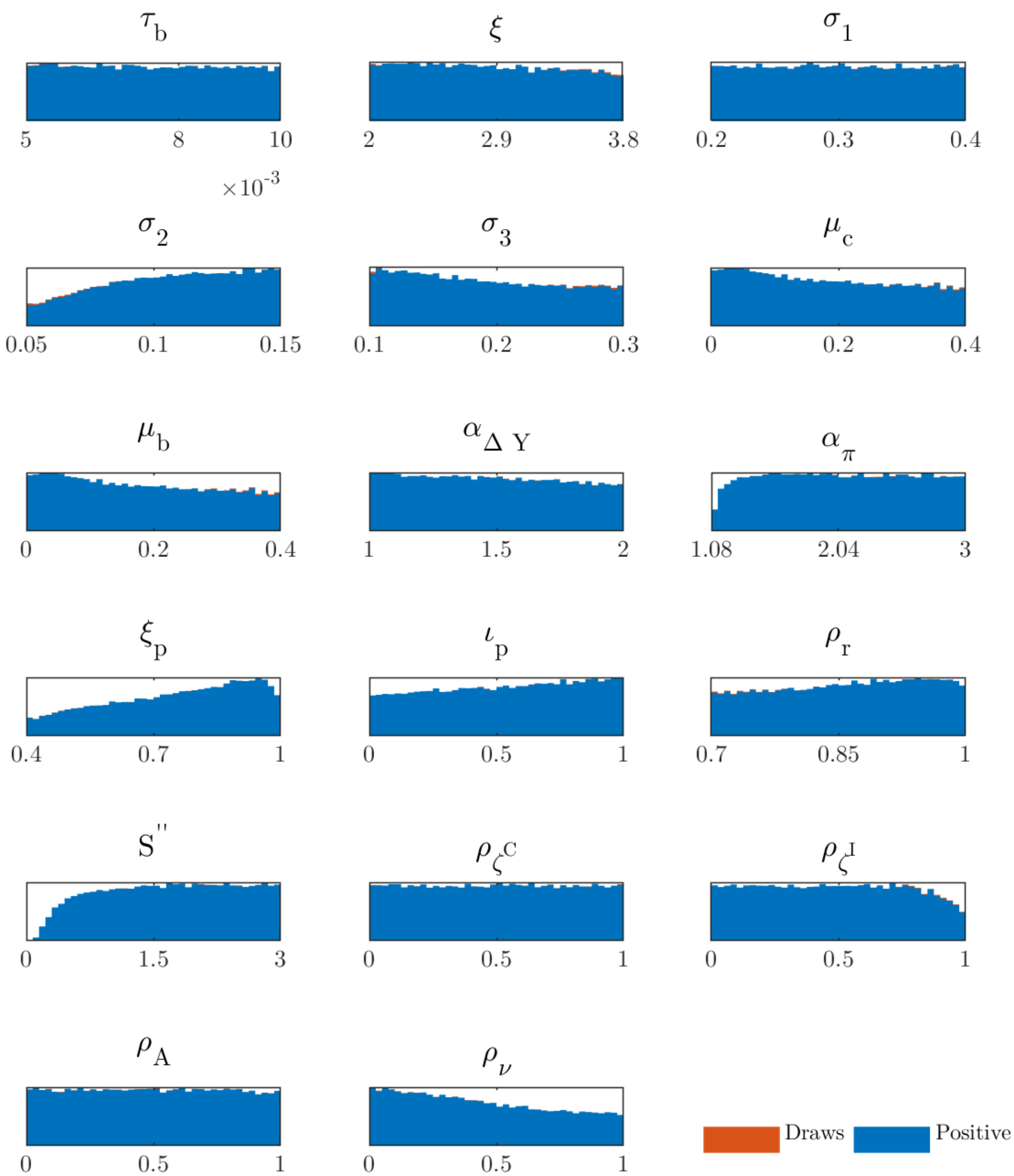

Figure A3: Parameter Acceptance.

Note: This graph plots parameters drawn from uniform distributions and implying model determinacy and a positive response of output. A total of 100000 draws are realized. The supports of the distributions are given by the x-axis. Parameter draws implying opposite movements in bonds and loans for bank shocks and comovements in bonds and loans for all other shocks are marked as 'Positive'.

3. Time-Series Analysis

3.1. Bayesian VAR

This subsection gives an overview of the methods used to compute the reduced form VAR model, a complete description of the Bayesian VAR methodology can be found in Kilian and Lütkepohl (2017).

Consider the following reduced form VAR of order p:

$$y_t = c + \Sigma_{i=1}^{p} B_i y_{t-i} + u_t, \tag{1}$$

where y_t is a $N \times 1$ vector containing the N endogenous variables, c a $N \times 1$ vector of constant, B_i for $i = 1, ..p$ are $N \times N$ parameter matrices. The vector u_t is a $N \times 1$ vector of prediction errors with $u_t \sim N(0, \Sigma)$ and Σ a variance-covariance matrix. Defining matrices Y, B, U and X such that $Y = [y_1 \quad \dots \quad y_T]'$, $B = [c \quad B1 \quad \dots \quad Bp]'$, $U = [u_1 \quad \dots \quad u_T]'$ and,

$$X = \begin{bmatrix} 1 & y'_0 & y'_1 & \dots & y'_{-p} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & y'_{T-1} & y'_1 & \dots & y'_{T-p} \end{bmatrix},$$

the VAR model defined in (1) rewrites as $Y = XB + U$. Vectorising this equation yields:

$$y = (I_N \otimes X)\beta + u, \tag{2}$$

where $y = vec(Y)$, $\beta = vec(B)$ and $u = vec(U)$. Here $vec()$ denotes column wise vectorisation operator. The error term u is assumed to follow a normal distribution with a zero mean and a variance-covariance matrix $\Sigma \otimes I_T$. Accordingly, the likelihood function in B and Σ can be expressed as:

$$L(B, \Sigma) \propto |\Sigma|^{-\frac{T}{2}} exp\left[-\frac{1}{2}\left(\beta - \hat{\beta}\right)'\left(\Sigma^{-1} \otimes X'X\right)\left(\beta - \hat{\beta}\right)\right] exp\left[-\frac{1}{2} tr\left(\Sigma^{-1} S\right)\right], \tag{3}$$

where $S = \left[\left(Y - X\hat{B}\right)'\left(Y - X\hat{B}\right)\right]$ and $\hat{\beta} = vec(\hat{B})$ and $\hat{B} = (X'X)^{-1}X'Y$. I use the Jeffreys' prior distribution for B and Σ which is proportional to $|\Sigma|^{\frac{-(n+1)}{2}}$. Following Kadiyala and Karlsson (1997) the joint posterior density for B and Σ can be written as:

$$p(B, \Sigma|Y, X) \propto |\Sigma|^{-\frac{T+n+1}{2}} exp\left[-\frac{1}{2}\left(\beta - \hat{\beta}\right)'\left(\Sigma^{-1} \otimes X'X\right)\left(\beta - \hat{\beta}\right)\right] exp\left[-\frac{1}{2}tr\left(\Sigma^{-1}S\right)\right]. \quad (4)$$

Where it is possible to draw β conditional on Σ from:

$$\beta|\Sigma, Y, X \sim N(\hat{\beta}, \Sigma \otimes (X'X)^{-1}), \quad (5)$$

and to draw Σ from:

$$\Sigma|Y, X \sim IW(S, z), \quad (6)$$

where $z = (T - N) \times (p - 1)$.

3.2. Sign-Restriction Algorithm

This subsection sketches the method used to characterize the subset of structural VAR models satisfying the imposed sign restrictions and drawn from the previous distribution of models. While various identification schemes are available, the identification of a VAR model with sign restrictions allows to identify structural shocks with a minimal and qualitative set of hypotheses.[3]

The algorithm used in this paper is developed in Arias, Rubio-Ramirez, and Waggoner (2018), the method is as follows. It is possible to express the vector of prediction error u_t as a combination of structural innovations ε_t where $u_t = D\varepsilon_t$ and $\varepsilon_t \sim N(0, I_N)$ with I_N an identity matrix and D a non-singular parameter matrix such that $DD' = \Sigma$. To construct the matrix D, one first draw candidates β and Σ using the posterior distributions given by expressions (5) and (6). The next step involves computing a random orthogonal matrix Q drawn from $N(0, I_N)$. This is achieved by drawing a matrix W from $N(0, I_N)$ further transformed into an orthogonal Q matrix using the QR

[3]Advantages of sign-restriction methods are detailed in Uhlig (2005), see Fry and Pagan (2011) for a more critical treatment.

factorization. The matrix D is computed as the product matrix of P and Q, where P corresponds to the lower-triangular Cholesky decomposition of Σ. The following step is to compute the impulse responses implied by the coefficient matrices β and D for the different structural shocks ε_t. The draws for β, Σ and W that imply impulse responses satisfying the sign restrictions are kept. The same process is repeated until a sufficient number of draws are obtained. The set of structural models gathered allows to characterize the distributions of models derived from the reduced form VAR satisfying the sign restrictions imposed.

3.3. Data

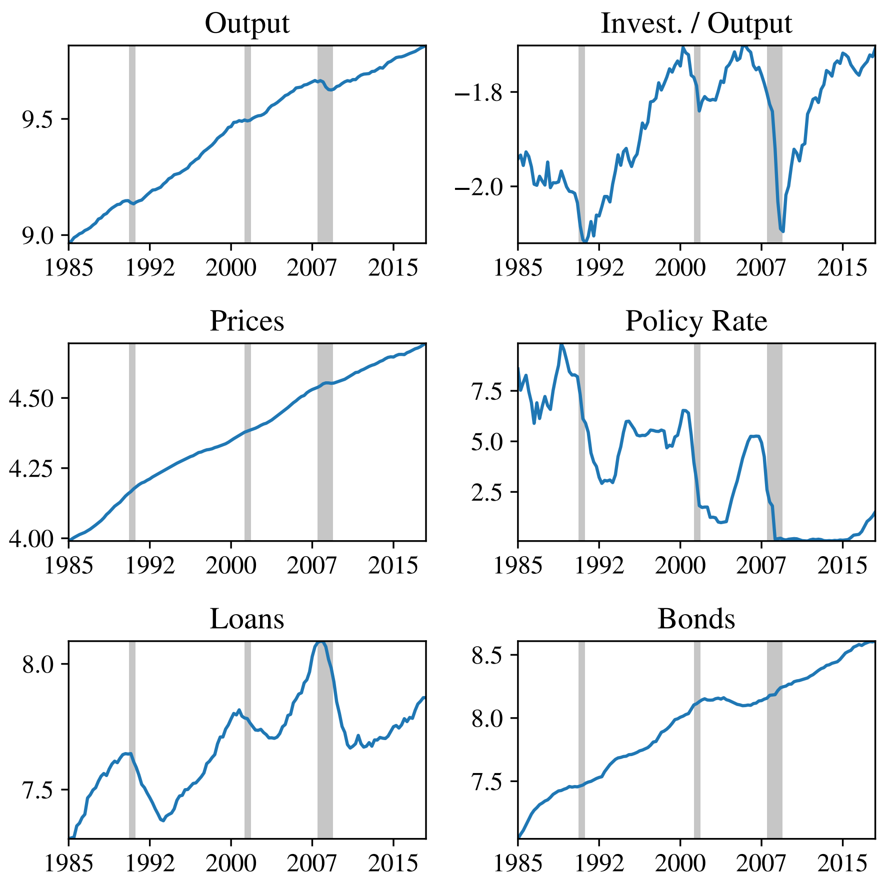

Figure A4: Data used for the SR-VAR estimation

Note: All series are expressed in log-level except the policy rate which is expressed in annual percentage points. GDP, investment, as well loan and bond volumes are expressed in real terms. Prices correspond to the GDP deflator.

Mnemonic	Description	Unit	Source
A. Macroeconomic Series			
GDP	Gross domestic product	$bn	BEA
GDPDEF	Gross domestic product: implicit price deflator	idx	BEA
GPDI	Gross private domestic investment	$bn	BEA
FEDFUNDS	Effective federal funds rate	%	FRSBG
B. Financial Series			
AAAFFM	Moody's Aaa corporate bond yield Minus Federal Funds Rate	%	Moody
NCBDBIQ027S	Nonfinancial corporate business: corporate bonds	$bn	FRSBG
CPLBSNNCB	Nonfinancial corporate business: commercial paper	$bn	FRSBG
FL103165005.Q*	Nonfinancial corporate business; total mortgages; liability	$bn	FRSBG
FL103168005.Q*	Nonfinancial corporate business; depository institution loans n.e.c.	$bn	FRSBG
FL103169005.Q*	Nonfinancial corporate business; other loans and advances; liability	$bn	FRSBG

Notes: BEA: Bureau of Economic Analysis; FRSBG: Federal Reserve System–Board of Governors. The bond series corresponds to the sum of correporate bonds and commercial paper. The loan series corresponds to the sum of depository institution loans, total mortgages and other loans and advances. *For the loan series, mnemonics correspond to the FRSBG codes. The other series can be retrieved directly from Fred using the corresponding code. All series are seasonally adjusted and expressed in log levels except for the federal funds rate which is in levels.

Table A1: Data Sources and Treatments

4. Alternative Dataset

This section presents results for the model estimated using the shadow rate from Wu and Xia (2016) instead of the fed funds rate. Figure A5 presents the impulse response following a bank shock.

Figure A5: Responses to a Bank Shock

Note: Median impulse responses to a one standard deviation bank shock. The grey lines correspond to the 16th and 84th quantiles. All series are expressed in percentage points. The policy rate is annualized.

5. Alternative Identification: Less Restrictions and a Credit Spread

This section presents results from a VAR model identified restricting only the responses of output, loans, and bonds. Only two types of structural shocks are considered here, bank shocks that imply comovements in output and loans and opposite movements in bonds, and other shocks that imply comovements in output, loans and bonds. I also include Moody's Aaa corporate bond yield minus the federal funds rate in the dataset.

Figure A6: Responses to a Bank Shock

Note: Median impulse responses to a one standard deviation bank shock. The grey lines correspond to the 16th and 84th quantiles. All series are expressed in percentage points. Credit spread and the policy rate are annualized.

Figure A6 shows the impulse responses following a one standard deviation bank shock. The purple line corresponds to the model when only bank and non-bank shocks are identified, the blue dashed line corresponds to the full specification. The characteristics of the financial shocks implied by the two different sets of restrictions are very close.

6. Impulse Response Matching

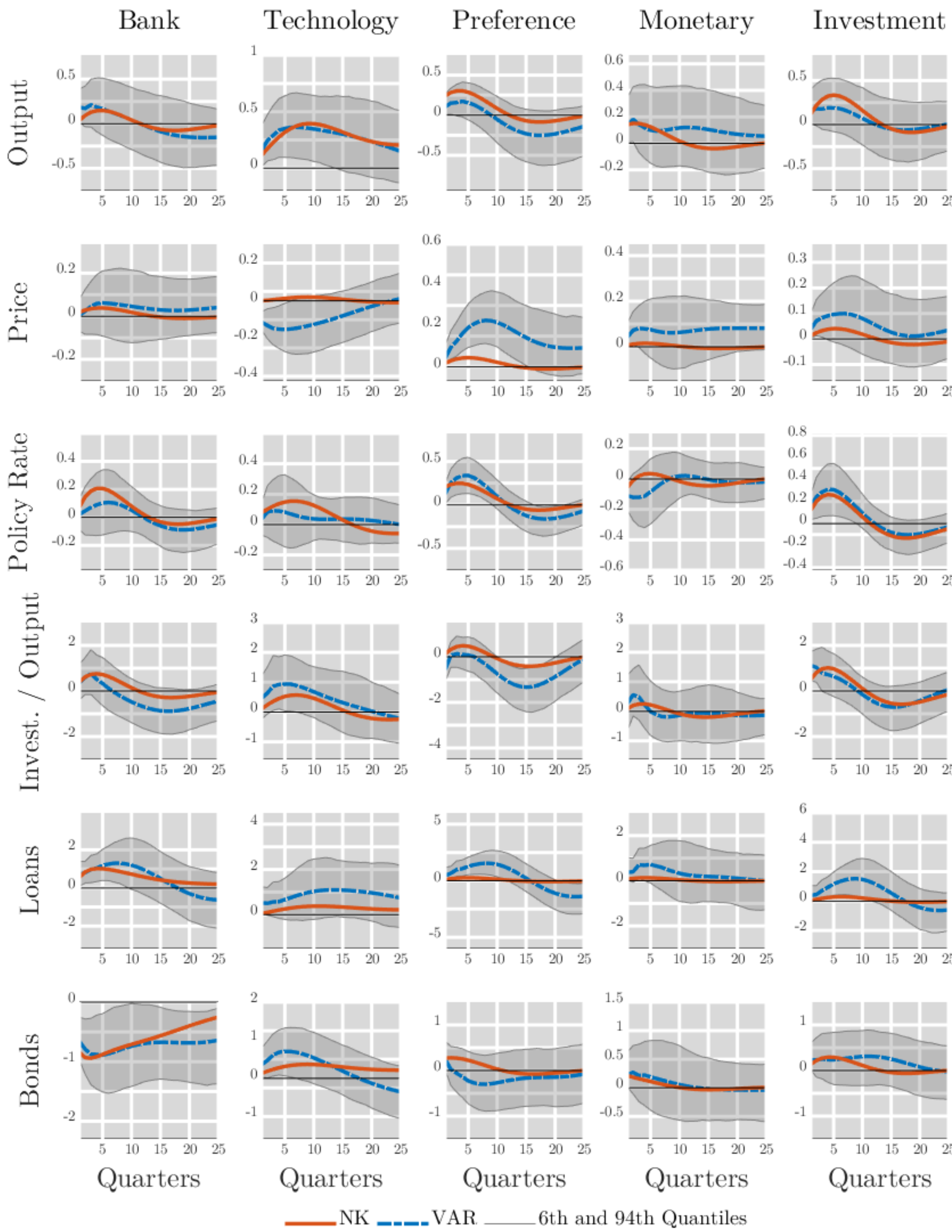

Figure A7: Robust Responses

Note: Median impulse responses to a one standard deviation shock, the grey lines correspond to the 6th and 94th quantiles. All series are expressed in percentage points. The policy rate is annualized. The dash blue lines correspond to the median responses from the VAR model, the orange lines correspond to responses from the NK model.

Param.	Description	Mode
τ_b	Bank intermediation costs	0.005
ξ	Pledgeable fraction of networth	3.7
μ_b	Monitoring cost for loans	0.22
μ_c	Monitoring cost for bonds	0.33
σ_1	Idiosyncratic shock dispersion	0.34
σ_2	Idiosyncratic shock dispersion	0.1
σ_3	Idiosyncratic shock dispersion	0.2
$\alpha_{\Delta Y}$	Taylor rule output coefficient	2.3
α_π	Taylor rule inflation coefficient	2.5
ρ_p	Taylor rule smoothing	0.79
ξ_p	Calvo price stickiness	0.94
ι_p	Price indexation on inflation target	0.035
S''	Invest. adjustment cost curvature	0.2
ρ_{ζ^C}	Autocorr. preference	0.94
ρ_{ζ^I}	Autocorr. MEI	0.57
ρ_A	Autocorr. stationary technology	0.14
ρ_ν	Autocorr. financial	0.93
σ_{ζ^C}	SD preference	0.0079
σ_{ζ^I}	SD MEI	0.0086
σ_A	SD stationary technology	0.083
σ_ν	SD financial	0.07
σ_{ε_p}	SD monetary policy	0.15

Note: This table displays the parameters minimizing the distance between the impulse responses from the NK model and from the median impulse responses implied by the BVAR.

Table A2: Estimated Parameters

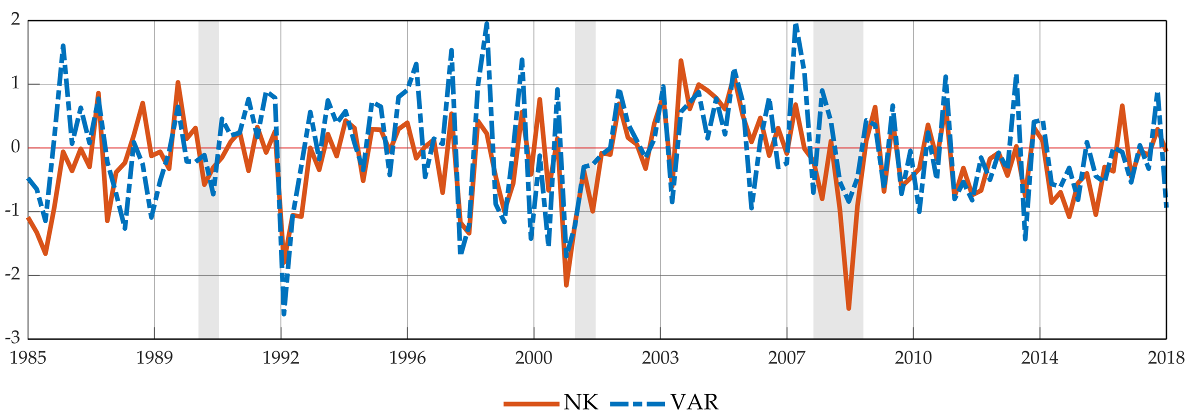

Figure A8: Bank Shocks - NK vs VAR

Note: The orange line corresponds the estimate of the updated bank shocks. The blue line corresponds to the mean of the bank shocks estimated in the VAR model. Grey areas correspond to NBER recession dates. Correlation between the two series is 0.66.

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