Appendix

A.1 Data Appendix

A.1.1 Data description for empirical evidence

For the VXO series, we use the CBOE S&P 100 Volatility Index's daily series accessed from the Federal Reserve Bank of St. Louis database from 1996 to 2019. The series is available with daily frequency which we convert to quarterly series by taking simple quarterly averages. We create a quarterly panel data for 14 economies from 1996:Q1 to 2019:Q4. We consider seven AEs (US, UK, Canada, Japan, Australia, South Korea and Sweden) and seven EMEs (Brazil, Chile, Indonesia, India, Mexico, Russia and South Africa).

The primary source for most of the macroeconomic series is the quarterly national accounts data compiled by the Organization for Economic Cooperation and Development (2021). We consider a seasonally adjusted volume index for the following series: GDP, private consumption, government consumption and private investment (GFCF). The reference year for the all the data series in the dataset is 2015. For India we consider the nominal series data (for GDP, private consumption, government consumption and private investment (GFCF)) at current prices instead of the volume index data because the volume index data for India is available from 2011:Q1. We later adjust the nominal data series with the CPI (consumer price index) for India to get real indices for all the variables mentioned above. We also consider data from Federal State Statistic Service for some series on Russian economy.

We create trade balance (total exports-total imports) series from the quarterly nominal data series on total imports and total exports. To normalize the trade balance series we take the ratio of the trade balance to GDP. We get monthly series on nominal exchange rates (currency per US dollar) from the OECD. We create quarterly nominal exchange rate series by taking quarterly averages of the monthly series. The relative consumer price indices (in terms of US dollars) data is used to capture the real effective exchange rate. Any increase (decrease) in the index would thus mean currency appreciation (depreciation).

We use short term interest rate (per annum) series to approximate the nominal interest rate series (policy rate). We also consider money supply measures including broad money and narrow money as control variables for local projections. We consider seasonally adjusted narrow and broad money quarterly indices and adjust them with CPI series to get real narrow and broad money series.

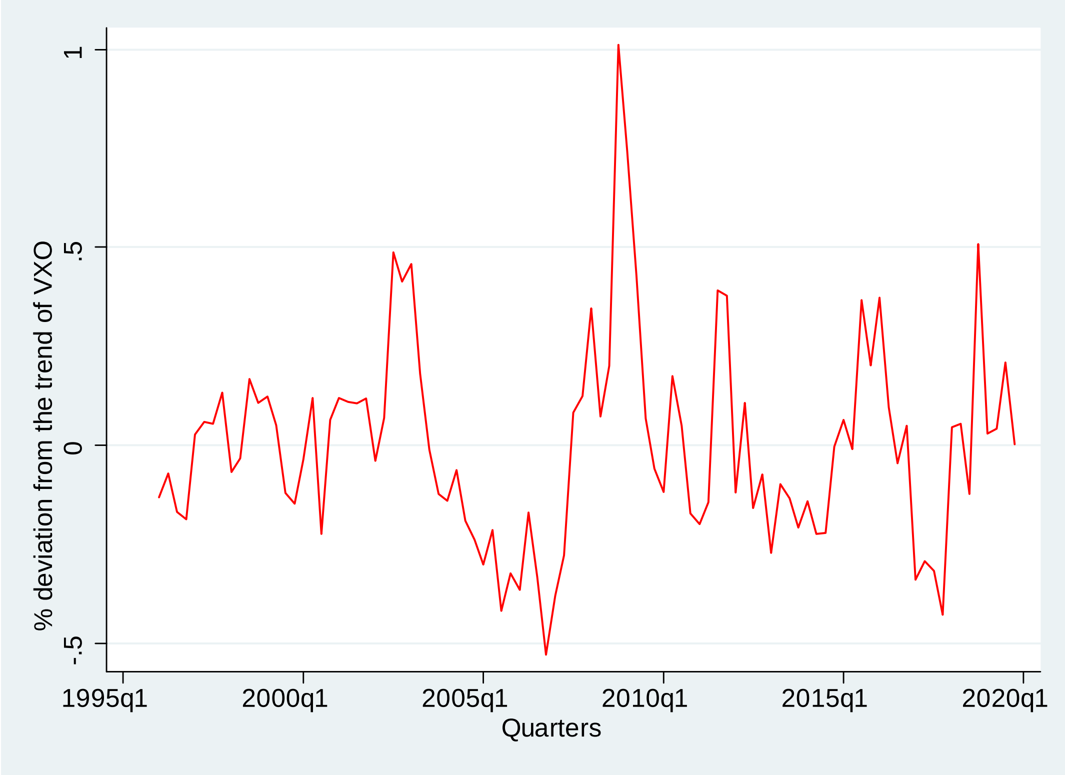

Figure 1: HP filtered log(VXO) series.

We get the country wise quarterly series on net portfolio investment (US dollars) for most countries from OECD (2021) and for Indonesia, Chile and Mexico from International Monetary Fund's International Financial Statistics (2021). We also consider the net financial account (except exceptional financing) series as a control in local projections. To normalize the net portfolio investment and net financial account we do $\frac{X_t - X_{mean}}{X_{\max} - X_{\min}}$ where X_t are the series.

We take log of the following series and then take HP filter for the analysis: VXO, real GDP, real private consumption, real government consumption, real private investment, trade balance ratio to GDP, net portfolio investment, net financial account, real narrow money, real broad money, CPI, nominal exchange rates and relative CPI. This is done to get interpretations of the results in percentage change. Figure A.1 shows the HP filtered log(VXO) series we consider as an estimated uncertainty series in the Local Projection regression.

We run panel data local projections on the above described dataset. To get the impulse response on a single variable, with VXO being an impulse variable, we control for all the variables with lag up to 4 periods over a horizon of 8 periods.

A.1.2 Data description for calibration

We estimate the degree of openness parameter, χ, to be 0.6, as the average trade share to GDP of emerging market economies. To get this we use the World Bank's country level trade data for year 2015 (2018). We take the average for 13 emerging market economies, namely: Argentina, Brazil, Chile, Colombia, Costa Rica, Hungary, India, Indonesia, Mexico, Poland, Russia, South Africa and Turkey to get average value as 0.6. We get the trade share of each country as a ratio of the total value of trade of a country with the world to the value of country's GDP, for year 2015.

The value of the initial parameter in the asset market condition, κ, is estimated to be 3.8. We measure κ as the ratio of ratio of marginal utility of the domestic country to the foreign country adjusted with the real exchange rates. For this exercise we consider a simple utility function and estimate: $Q_0\left(\frac{C_0}{C_0^*}\right)^{-\nu}$, where ν is the inverse of IES. We calculate this using the OECD (2019) database on annual national accounts. First, using the exchange rate and the consumption series at constant prices of 2015, we get real consumption series in US dollars. We then calculate the average for EMEs and AEs from 2005-2015. We consider 13 EMEs namely: Argentina, Brazil, Chile, Colombia, Costa Rica, Hungary, India, Indonesia, Mexico, Poland, Russia, South Africa, Turkey, and 31 AEs namely: Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Iceland, Ireland, Israel, Italy, Japan, Korea, Latvia, Lithuania, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Slovakia, Slovenia, Spain, Sweden, Switzerland, UK and the US. We then calculate the marginal utilities ratio using the utility parameter (inverse of IES) as 1.5. [Calculation: $\kappa = (109293.4/266609)^{-1.5} = 3.8$].

A.2 Technical Appendix

A.2.1 Derivation of the demand functions

Demand for a variety i of domestic good by domestic households

$$\max_{C_{D,t}(i)} C_{D,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n \left(C_{D,t}\left(i\right)\right)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}$$

subject to constraint,

$$\int_0^n P_{D,t}\left(i\right) C_{D,t}\left(i\right) di = Z_{D,t}$$

$$\mathcal{L}_t = \max_{C_{D,t}(i)} \left[\left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n \left(C_{D,t}(i) \right)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} - \lambda_{D,t} \left(\int_0^n P_{D,t}(i)\, C_{D,t}(i)\, di - Z_t \right) \right]$$

First order condition,

$$\frac{\partial \mathcal{L}}{\partial C_{D,t}(i)} = \frac{\sigma}{\sigma-1} \left(C_{D,t} \right)^{\frac{1}{\sigma-1}} \left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \left(C_{D,t}(i) \right)^{\frac{\sigma-1}{\sigma}-1} - \lambda_{D,t} P_{D,t}(i) = 0$$

For any two variety i_1, i_2, we get,

$$\begin{aligned} \frac{\left(C_{D,t}(i_1) \right)^{-\frac{1}{\sigma}}}{\left(C_{D,t}(i_2) \right)^{-\frac{1}{\sigma}}} &= \frac{P_{D,t}(i_1)}{P_{D,t}(i_2)} \\ C_{D,t}(i_1) &= \left(\frac{P_{D,t}(i_1)}{P_{D,t}(i_2)} \right)^{-\sigma} C_{D,t}(i_2) \end{aligned}$$

Substituting the value in $C_{D,t}$,

$$\begin{aligned} C_{D,t} &= \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n \left(C_{D,t}(i_1) \right)^{\frac{\sigma-1}{\sigma}} di_1 \right]^{\frac{\sigma}{\sigma-1}} \\ &= \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n \left(\left(\frac{P_{D,t}(i_1)}{P_{D,t}(i_2)} \right)^{-\sigma} C_{D,t}(i_2) \right)^{\frac{\sigma-1}{\sigma}} di_1 \right]^{\frac{\sigma}{\sigma-1}} \\ &= \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n P_{D,t}(i_1)^{1-\sigma} di_1 \right]^{\frac{\sigma}{\sigma-1}} \frac{C_{D,t}(i_2)}{\left(P_{D,t}(i_2) \right)^{-\sigma}} \end{aligned}$$

let $\left[\left(\frac{1}{n} \right) \int_0^n P_{D,t}(i_2)^{1-\sigma} di_2 \right]^{\frac{1}{1-\sigma}} = P_{D,t}$.

$$C_{D,t} = \left(\frac{1}{n} \right)^{-1} \left(P_{D,t} \right)^{-\sigma} \frac{C_{D,t}(i_2)}{\left(P_{D,t}(i_2) \right)^{-\sigma}}$$

Above equation can be re-arranged for a variety i as,

$$C_{D,t}(i) = \left(\frac{1}{n} \right) \left(\frac{P_{D,t}(i)}{P_{D,t}} \right)^{-\sigma} C_{D,t}$$

where,

$$P_{D,t} = \left[\left(\frac{1}{n} \right) \int_0^n P_{D,t}(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}$$

Substituting the value of $C_{D,t}(i) = \left(\frac{1}{n}\right)\left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\sigma} C_{D,t}$ in the constraint,

$$\begin{aligned}
\int_0^n P_{D,t}(i)\left(\frac{1}{n}\right)\left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\sigma} C_{D,t} di &= Z_{D,t} \\
(P_{D,t})^{1-\sigma}(P_{D,t})^{\sigma} C_{D,t} &= Z_{D,t} \\
P_{D,t} C_{D,t} &= Z_{D,t}
\end{aligned}$$

Similarly it can be shown,

$$\begin{aligned}
C_{F,t}(i) &= \left(\frac{1}{1-n}\right)\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\sigma} C_{F,t}\ ;\text{ where } P_{F,t} = \left[\left(\frac{1}{1-n}\right)\int_n^1 P_{F,t}(i)^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} \\
C^*_{D,t}(i) &= \left(\frac{1}{n}\right)\left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\sigma} C^*_{D,t};\text{ where } P^*_{D,t} = \left[\left(\frac{1}{n}\right)\int_0^n P^*_{D,t}(i)^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} \\
C^*_{F,t}(i) &= \left(\frac{1}{1-n}\right)\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\sigma} C^*_{F,t};\text{ where } P^*_{F,t} = \left[\left(\frac{1}{1-n}\right)\int_n^1 P^*_{F,t}(i)^{1-\sigma} di\right]^{\frac{1}{1-\sigma}}
\end{aligned}$$

by maximizing $C_{F,t} = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}}\int_n^1 (C_{F,t}(i))^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}$ subject to $\int_n^1 P_{F,t}(i) C_{F,t}(i) di = Z_{F,t}$, $C^*_{D,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}}\int_0^n \left(C^*_{D,t}(i)\right)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}$ subject to $\int_0^n P^*_{D,t}(i) C^*_{D,t}(i) di = Z^*_{D,t}$ and $C^*_{F,t} = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}}\int_n^1 \left(C^*_{F,t}(i)\right)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}$ subject to $\int_n^1 P^*_{F,t}(i) C^*_{F,t}(i) di = Z^*_{D,t}$, respectively. It can also be shown that expenditure $Z_{F,t} = P_{F,t}C_{F,t}$, $Z^*_{D,t} = P^*_{D,t}C^*_{D,t}$, $Z^*_{F,t} = P^*_{F,t}C^*_{F,t}$.

For the domestic and foreign goods in the total consumption basket

$$\max_{C_{D,t},C_{F,t}} C_t = \left[(\mu_D)^{1/\xi_D}(C_{D,t})^{\frac{\xi_D-1}{\xi_D}} + (1-\mu_D)^{1/\xi_D}(C_{F,t})^{\frac{\xi_D-1}{\xi_D}}\right]^{\frac{\xi_D}{\xi_D-1}}$$

subject to,

$$P_{D,t}C_{D,t} + P_{F,t}C_{F,t} = Z_t$$

$$\mathcal{L}_t = \left[(\mu_D)^{1/\xi_D}(C_{D,t})^{\frac{\xi_D-1}{\xi_D}} + (1-\mu_D)^{1/\xi_D}(C_{F,t})^{\frac{\xi_D-1}{\xi_D}}\right]^{\frac{\xi_D}{\xi_D-1}} - \lambda_{D,t}\left[P_{D,t}C_{D,t} + P_{F,t}C_{F,t} - Z_t\right]$$

The first order conditions are,

$$\frac{\partial \mathcal{L}}{\partial C_{D,t}} = (C_t)^{\frac{1}{\xi_D - 1}} (\mu_D)^{1/\xi_D} (C_{D,t})^{\frac{\xi_D - 1}{\xi_D} - 1} - \lambda_{D,t} P_{D,t} = 0$$

$$\frac{\partial \mathcal{L}}{\partial C_{F,t}} = (C_t)^{\frac{1}{\xi_D - 1}} (1 - \mu_D)^{1/\xi_D} (C_{F,t})^{\frac{\xi_D - 1}{\xi_D} - 1} - \lambda_{D,t} P_{F,t} = 0$$

Combining the above two conditions we get,

$$C_{F,t} = \frac{(1 - \mu_D)}{\mu_D} \left(\frac{P_{F,t}}{P_{D,t}} \right)^{-\xi_D} C_{D,t}$$

Substituting this value in the consumption bundle, we get

$$\begin{aligned}
C_t &= \left[(\mu_D)^{1/\xi_D} (C_{D,t})^{\frac{\xi_D - 1}{\xi_D}} + (1 - \mu_D)^{1/\xi_D} \left(\frac{(1 - \mu_D)}{\mu_D} \left(\frac{P_{F,t}}{P_{D,t}} \right)^{-\xi_D} C_{D,t} \right)^{\frac{\xi_D - 1}{\xi_D}} \right]^{\frac{\xi_D}{\xi_D - 1}} \\
&= \left[\frac{(\mu_D)^{\frac{\xi_D - 1}{\xi_D}} (\mu_D)^{1/\xi_D} (P_{D,t})^{1-\xi_D} + (1 - \mu_D)^{1/\xi_D} (1 - \mu_D)^{\frac{\xi_D - 1}{\xi_D}} (P_{F,t})^{1-\xi_D}}{(\mu_D)^{\frac{\xi_D - 1}{\xi_D}} (P_{D,t})^{1-\xi_D}} \right]^{\frac{\xi_D}{\xi_D - 1}} C_{D,t} \\
&= (\mu_D)^{-1} (P_{D,t})^{\xi_D} C_{D,t} \left[\mu_D (P_{D,t})^{1-\xi_D} + (1 - \mu_D) (P_{F,t})^{1-\xi_D} \right]^{\frac{\xi_D}{\xi_D - 1}}
\end{aligned}$$

Assuming,

$$P_t = \left[\mu_D (P_{D,t})^{1-\xi_D} + (1 - \mu_D) (P_{F,t})^{1-\xi_D} \right]^{\frac{1}{1-\xi_D}}$$

$$C_t = (\mu_D)^{-1} (P_{D,t})^{\xi_D} C_{D,t} (P_t)^{-\xi_D}$$

$$\therefore \quad C_{D,t} = \mu_D (T_{D,t})^{-\xi_D} C_t$$

Similarly substituting,

$$C_{D,t} = \frac{\mu_D}{(1 - \mu_D)} \left(\frac{P_{D,t}}{P_{F,t}} \right)^{-\xi_D} C_{F,t}$$

in C_t we get,

$$\begin{aligned}
C_t &= \left[(\mu_D)^{1/\xi_D} \left(\frac{\mu_D}{(1-\mu_D)} \left(\frac{P_{D,t}}{P_{F,t}} \right)^{-\xi_D} C_{F,t} \right)^{\frac{\xi_D - 1}{\xi_D}} + (1-\mu_D)^{1/\xi_D} \left(C_{F,t} \right)^{\frac{\xi_D - 1}{\xi_D}} \right]^{\frac{\xi_D}{\xi_D - 1}} \\
&= (P_t)^{-\xi_D} \frac{C_{F,t} \left(P_{F,t} \right)^{\xi_D}}{(1-\mu_D)}
\end{aligned}$$

Re-arranging the above equation,

$$C_{F,t} = (1-\mu_D) \left(\frac{P_{F,t}}{P_t} \right)^{-\xi_D} C_t$$

$$C_{F,t} = (1-\mu_D) \left(T_{F,t} \right)^{-\xi_D} C_t$$

Substituting the demand functions in the constraint,

$$\begin{aligned}
P_{D,t} \mu_D \left(\frac{P_{D,t}}{P_t} \right)^{-\xi_D} C_t + P_{F,t} (1-\mu_D) \left(\frac{P_{F,t}}{P_t} \right)^{-\xi_D} C_t &= Z_t \\
\left[\frac{\mu_D \left(P_{D,t} \right)^{1-\xi_D} + (1-\mu_D) \left(P_{F,t} \right)^{1-\xi_D}}{(P_t)^{-\xi_D}} \right] C_t &= Z_t \\
P_t C_t &= Z_t
\end{aligned}$$

Similarly, maximizing the aggregate consumption bundle C_t^* subject to the expenditure on the bundle:

$$\max_{C_{D,t}^8, C_{F,t}^*} C_t^* = \left[(\mu_F)^{1/\xi_F} \left(C_{D,t}^* \right)^{\frac{\xi_F - 1}{\xi_F}} + (1-\mu_F)^{1/\xi_F} \left(C_{F,t}^* \right)^{\frac{\xi_F - 1}{\xi_F}} \right]^{\frac{\xi_F}{\xi_F - 1}}$$

subject to,

$$P_{D,t}^* C_{D,t}^* + P_{F,t}^* C_{F,t}^* = Z_t^*.$$

We get the following,

$$
\begin{aligned}
C^*_{D,t} &= \mu_F \left(\frac{P^*_{D,t}}{P^*_t}\right)^{-\xi_F} C^*_t \\
&\text{and} \\
C^*_{F,t} &= (1-\mu_F)\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\xi_F} C^*_t \\
\text{where } P^*_t &= \left[\mu_F \left(P^*_{D,t}\right)^{1-\xi_F} + (1-\mu_F)\left(P^*_{F,t}\right)^{1-\xi_F}\right]^{\frac{1}{1-\xi_F}}
\end{aligned}
$$

It can also be shown that total expenditure $Z^*_t = P^*_t C^*_t$.

A.2.2 Derivation of Euler's equation and labour supply equation

For domestic households,

$$
\max_{\{C_t, H_{D,t}, b_{D,t+1}\}} V_{D,t} = \left(U(C_t, H_{D,t})^{1-\eta} + \beta\left(E_t\left(V_{D,t+1}\right)^{1-\gamma}\right)^{\frac{(1-\eta)}{(1-\gamma)}}\right)^{\frac{1}{(1-\eta)}}
$$

subject to the constraint,

$$
W_{D,t}H_{D,t} + profit_{D,t} = P_tC_t - B_{D,t} + E_t\left\{B_{D,t+1}M_{t,t+1}\right\}
$$

Writing the above constraints in real terms implies,

$$
\begin{aligned}
\frac{W_{D,t}H_{D,t} + profit_t}{P_t} &= \frac{P_tC_t}{P_t} - \frac{B_{D,t}}{P_t} + \frac{E_t\left\{B_{D,t+1}M_{t,t+1}\right\}}{P_t} \\
w_{D,t}T_{D,t}H_{D,t} + \ \Omega_{D,t} &= C_t - \frac{B_{D,t}}{P_t} + \frac{E_t\left\{B_{D,t+1}M_{t,t+1}\right\}}{P_t} \\
w_{D,t}T_{D,t}H_{D,t} + \ \Omega_{D,t} &= C_t - b_{D,t} + E_t\left\{\pi_{t+1}b_{D,t+1}M_{t,t+1}\right\}
\end{aligned}
$$

where $w_{D,t} = \frac{W_{D,t}}{P_{D,t}}$, $T_{D,t} = \frac{P_{D,t}}{P_t}$ and $\Omega_{D,t}$ are real profits.

Maximizing the utility subject to constraint, we get

$$\frac{\partial \mathcal{L}_t}{\partial C_t} = \frac{\partial V_t}{\partial C_t} - \lambda_{D,t}$$

$$\frac{\partial \mathcal{L}_t}{\partial H_{D,t}} = \frac{\partial V_t}{\partial H_{D,t}} + w_{D,t} T_{D,t} \lambda_{D,t}$$

$$\frac{\partial \mathcal{L}_t}{\partial b_{D,t+1}} = -\lambda_{D,t} E_t \{\pi_{t+1} M_{t,t+1}\} + V_t^{\eta} \beta \left(E_t (V_{t+1})^{1-\gamma} \right)^{\frac{(\gamma-\eta)}{(1-\gamma)}} (V_{t+1})^{-\gamma} \lambda_{D,t+1}$$

Using optimality conditions, $\frac{\partial \mathcal{L}_t}{\partial C_t} = \frac{\partial \mathcal{L}_t}{\partial H_{D,t}} = \frac{\partial \mathcal{L}_t}{\partial b_{D,t+1}} = 0$, and $\frac{\partial V_{D,t}}{\partial C_t} = V_{D,t}^{\eta} U_{D,t}^{-\eta} U'_{C_t}$; $\frac{\partial V_{D,t}}{\partial H_{D,t}} = V_{D,t}^{\eta} U_t^{-\eta} U'_{H_{D,t}}$, the above expressions can be simplified to

$$E_t \{M_{t,t+1}\} = \beta \left(\frac{E_t (V_{t+1})^{1-\gamma}}{(V_{t+1})^{1-\gamma}} \right)^{\frac{(\gamma-\eta)}{(1-\gamma)}} \left(\frac{U_{D,t+1}}{U_{D,t}} \right)^{1-\eta} \left(\frac{C_t}{C_{t+1}} \right) \left(\frac{1}{E_t \{\pi_{t+1}\}} \right)$$

$$w_{D,t} T_{D,t} = \frac{1-\nu}{\nu} \frac{C_t}{1 - H_{D,t}}$$

where $E_t \{M_{t,t+1}\} = \frac{1}{(1+R_t)}$.

Similarly for foreign households,

$$\max_{\{C_t, H_{F,t}, b_{F,,t+1}\}} V_{F,t} = \left(\Gamma_{F,t} U(C_t^*, H_{F,t})^{1-\eta} + \beta \left(E_t (V_{F,t+1})^{1-\gamma} \right)^{\frac{(1-\eta)}{(1-\gamma)}} \right)^{\frac{1}{(1-\eta)}}$$

subject to the constraint,

$$W_{F,t} H_{F,t} + profit_{F,t} = P_t^* C_t^* - B_{F,t} + B_{F,t+1} E_t \left\{ M_{t,t+1}^* \right\}$$

Writing the above constraints in real terms,

$$\begin{aligned} \frac{W_{F,t} H_{F,t} + profit_t^*}{P_t^*} &= \frac{P_t^* C_t^*}{P_t^*} - \frac{B_{F,t}}{P_t^*} + \frac{E_t \left\{ M_{t,t+1}^* B_{F,t+1} \right\}}{P_t^*} \\ w_{F,t} \frac{T_{F,t}}{Q_t} H_{F,t} + \ \Omega_{F,t} &= C_t^* - \frac{B_{F,t}}{P_t^*} + \frac{E_t \left\{ M_{t,t+1}^* B_{F,t+1} \right\}}{P_t^*} \\ w_{F,t} \frac{T_{F,t}}{Q_t} H_{F,t} + \ \Omega_{F,t} &= C_t^* - b_{F,t} + E_t \left\{ \pi_{t+1}^* M_{t,t+1}^* b_{F,t+1} \right\} \end{aligned}$$

where $w_{F,t} = \frac{W_{F,t}}{P_{F,t}^*}$, $T_{F,t} = \frac{P_{F,t}}{P_t}$, $Q_t = \frac{X_t P_t^*}{P_t}$, $\pi_{t+1}^* = \frac{P_{t+1}^*}{P_t^*}$ and Ω_t^* are real profits. Simplifying the optimal conditions like above would give us,

$$w_{F,t}\frac{T_{F,t}}{Q_t}=\frac{1-\nu}{\nu}\frac{C_t^*}{1-H_{F,t}}$$

$$E_t\left\{M_{t,t+1}^*\right\}=\beta\left(\frac{\Gamma_{F,t+1}}{\Gamma_{F,t}}\right)\left(\frac{E_t\left(V_{t+1}^*\right)^{1-\gamma}}{\left(V_{t+1}^*\right)^{1-\gamma}}\right)^{\frac{(\gamma-\eta)}{(1-\gamma)}}\left(\frac{U_{F,t+1}}{U_{F,t}}\right)^{1-\eta}\left(\frac{C_t^*}{C_{t+1}^*}\right)\left(\frac{1}{E_t\left\{\pi_{t+1}^*\right\}}\right)$$

where $E_t\left\{M_{t,t+1}^*\right\}=\frac{1}{(1+R_t^*)}$.

A.2.3 Derivation of price-setting equations

For domestic firms: since the domestic sector is a sticky price sector, $(1-\alpha_D)$ firms which can optimize, maximize the following profit function,

$$\max_{\overline{P}_{D,t}(i)}\sum_{k=0}^{\infty}\alpha_D^k M_{t,t+k}\left(\overline{P}_{D,t}(i)Y_{D,t_{+k}}(i)-MC_{D,t+k}Y_{D,t_{+k}}(i)\right)$$

$$\text{where } Y_{D,t+k}(i)=\left(\frac{\overline{P}_{D,t}(i)}{P_{D,t+k}}\right)^{-\sigma}Y_{D,t+k}$$

$$\begin{aligned}\frac{\partial\mathcal{L}_t}{\partial\overline{P}_{D,t}(i)} &= \sum_{k=0}^{\infty}\alpha_D^k M_{t,t+k}\left(Y_{D,t+k}(i)+\overline{P}_{D,t}(i)\frac{\partial Y_{D,t+k}(i)}{\partial\overline{P}_{D,t}(i)}-MC_{D,t+k}\frac{\partial Y_{D,t+k}(i)}{\partial\overline{P}_{D,t}(i)}\right)=0\\ where\ \frac{\partial Y_{D,t+k}(i)}{\partial\overline{P}_{D,t}(i)} &= -\sigma\left(\frac{\overline{P}_{D,t}(i)}{P_{D,t+k}}\right)^{-\sigma}\frac{1}{\overline{P}_{D,t}(i)}Y_{D,t+k}\\ &= -\sigma\frac{Y_{D,t+k}(i)}{\overline{P}_{D,t}(i)}\end{aligned}$$

Therefore,

$$\sum_{k=0}^{\infty}\alpha_D^k M_{t,t+k}\left(Y_{D,t+k}(i)+\overline{P}_{D,t}(i)\left(-\sigma\frac{Y_{D,t+k}(i)}{\overline{P}_{D,t}(i)}\right)-MC_{D,t+k}\left(-\sigma\frac{Y_{D,t+k}(i)}{\overline{P}_{D,t}(i)}\right)\right)=0$$

$$\overline{P}_{D,t}(i)=\frac{\sigma}{\sigma-1}\frac{\sum_{k=0}^{\infty}\alpha_D^k M_{t,t+k}MC_{D,t+k}Y_{D,t+k}(i)}{\sum_{k=0}^{\infty}\alpha_D^k M_{t,t+k}Y_{D,t+k}(i)}$$

The remaining α_D share of the firms keep their price the same as the aggregate of last year prices, such that the aggregate price in the manufacturing sector is

$$(P_{D,t}(i))^{-\sigma} = \alpha_D \left(P_{D,t-1}(i)\right)^{-\sigma} + (1-\alpha_D)\left(\overline{P}_{D,t}(i)\right)^{-\sigma}$$

Writing the price equation recursively, note that the stochastic discount factor, $M_{t,t+k}$, is given by

$$M_{t,t+k} = \frac{1}{(1+R_t)}$$

Now from the household's optimization,

$$M_{t,t+k} = \beta^k \frac{\lambda_{D,t+k}}{\lambda_{D,t}} \frac{P_t}{P_{t+k}}$$

$$\begin{aligned}
\overline{P}_{D,t}(i) &= \frac{\sigma}{\sigma-1} \frac{\sum_{k=0}^{\infty} \alpha_D^k M_{t,t+k} \frac{MC_{D,t+k}}{P_{D,t+k}} P_{D,t+k} \left(\frac{\overline{P}_{D,t}(i)}{P_{D,t+k}}\right)^{-\sigma} Y_{D,t+k}}{\sum_{k=0}^{\infty} \alpha_D^k M_{t,t+k} \left(\frac{\overline{P}_{D,t}(i)}{P_{D,t+k}}\right)^{-\sigma} Y_{D,t+k}} \\
&= \frac{\sigma}{\sigma-1} \frac{\sum_{k=0}^{\infty} \alpha_D^k \beta^k \left(\frac{E_t\left(V_{D,t}\right)}{\left(V_{D,t+k}\right)^{1-\gamma}}^{1-\gamma}\right)^{\frac{(\gamma-\eta)}{(1-\gamma)}} \left(\frac{V_{D,t}}{V_{D,t+k}}\right)^{\eta} \frac{\lambda_{D,t+k}}{\lambda_{D,t}} \frac{P_t}{P_{t+k}} mc_{D,t+k} \left(P_{D,t+k}\right)^{\sigma+1} Y_{D,t+k}}{\sum_{k=0}^{\infty} \alpha_D^k \beta^k \left(\frac{E_t\left(V_{D,t+k}\right)}{\left(V_{D,t+k}\right)^{1-\gamma}}^{1-\gamma}\right)^{\frac{(\gamma-\eta)}{(1-\gamma)}} \left(\frac{V_{D,t}}{V_{D,t+k}}\right)^{\eta} \frac{\lambda_{D,t+k}}{\lambda_{D,t}} \frac{P_t}{P_{t+k}} \left(P_{D,t+k}\right)^{\sigma} Y_{D,t+k}} \\
&= \frac{\sigma}{\sigma-1} \frac{\sum_{k=0}^{\infty} (\alpha_D\beta)^k \left(\frac{E_t\left(V_{D,t+k}\right)}{\left(V_{D,t+k}\right)^{1-\gamma}}^{1-\gamma}\right)^{\frac{(\gamma-\eta)}{(1-\gamma)}} \left(\frac{1}{V_{D,t+k}}\right)^{\eta} \lambda_{D,t+k} T_{D,t+k} mc_{D,t+k} \left(P_{D,t+k}\right)^{\sigma} Y_{D,t+k}}{\sum_{k=0}^{\infty} (\alpha_D\beta)^k \left(\frac{E_t\left(V_{D,t+k}\right)}{\left(V_{D,t+k}\right)^{1-\gamma}}^{1-\gamma}\right)^{\frac{(\gamma-\eta)}{(1-\gamma)}} \left(\frac{1}{V_{D,t+k}}\right)^{\eta} \lambda_{D,t+k} T_{D,t+k} \left(P_{D,t+k}\right)^{\sigma-1} Y_{D,t+k}}
\end{aligned}$$

$$\frac{P^{\#}_{D,t}}{P_{D,t-1}} = \frac{\sigma}{\sigma-1} \frac{\sum_{k=0}^{\infty} (\alpha_D\beta)^k \left(\frac{E_t\left(V_{D,t+k}\right)}{\left(V_{D,t+k}\right)^{1-\gamma}}^{1-\gamma}\right)^{\frac{(\gamma-\eta)}{(1-\gamma)}} \left(\frac{1}{V_{D,t+k}}\right)^{\eta} \lambda_{D,t+k} T_{D,t+k} mc_{D,t+k} \left(\pi_{D,t} \times \pi_{D,t+1} \times \pi_{D,t+2} \times \ldots\right.}{\sum_{k=0}^{\infty} (\alpha_D\beta)^k \left(\frac{E_t\left(V_{D,t+k}\right)}{\left(V_{D,t+k}\right)^{1-\gamma}}^{1-\gamma}\right)^{\frac{(\gamma-\eta)}{(1-\gamma)}} \left(\frac{1}{V_{D,t+k}}\right)^{\eta} \lambda_{D,t+k} T_{D,t+k} \left(\pi_{D,t} \times \pi_{D,t+1} \times \pi_{D,t+2} \times \ldots\ldots\pi_{D,t}\right.}$$

We can write $\pi^*_{D,t}$ in recursive form,

$$\pi^{\#}_{D,t} = \frac{\sigma}{\sigma-1} \pi_{D,t} \frac{X_{D,t}}{Z_{D,t}} \tag{1}$$

where,

$$X_{D,t} = \lambda_{D,t} Y_{D,t} mc_{D,t} T_{D,t} + \alpha_D \beta \left(\pi_{D,t+1}\right)^{\sigma} \left(\frac{E_t \left(V_{D,t+1}\right)^{1-\gamma}}{\left(V_{D,t+1}\right)^{1-\gamma}} \right)^{\frac{(\gamma-\eta)}{(1-\gamma)}} \left(\frac{1}{V_{D,t+1}} \right)^{\eta} E_t \left\{ X_{D,t+1} \right\} \quad (2)$$

$$Z_{D,t} = \lambda_{D,t} Y_{D,t} T_{D,t+k} + \alpha_D \beta \left(\pi_{D,t+1}\right)^{\sigma-1} \left(\frac{E_t \left(V_{D,t+k}\right)^{1-\gamma}}{\left(V_{D,t+1}\right)^{1-\gamma}} \right)^{\frac{(\gamma-\eta)}{(1-\gamma)}} \left(\frac{1}{V_{D,t+1}} \right)^{\eta} E_t \left\{ Z_{D,t+1} \right\} \quad (3)$$

Aggregate prices for domestically produced goods is given by,

$$P_{D,t} = \left[\left(\frac{1}{n}\right) \int_0^n P_{D,t}\left(i\right)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}$$

i is a variety here.

$$\left(P_{D,t}\right)^{(1-\sigma)} = \left(\frac{1}{n}\right) \left[\int_0^{n\alpha_D} P_{D,t-1}(i)^{1-\sigma} di + \int_{n\alpha_D}^{n} \overline{P}_{D,t}(i)^{1-\sigma} di \right]$$

$$= \left(\frac{1}{n}\right) \left[n\alpha_D \left(P_{D,t-1}\left(i\right)\right)^{1-\sigma} + n\left(1-\alpha_D\right) \left(\overline{P}_{D,t}\left(i\right)\right)^{1-\sigma} \right]$$

dropping i due to symmetry,

$$\left(P_{D,t}\right)^{(1-\sigma)} = \alpha_D \left(P_{D,t-1}\right)^{1-\sigma} + \left(1-\alpha_D\right) \left(\overline{P}_{D,t}\right)^{1-\sigma}$$

$$\therefore \quad P_{D,t} = \left[\alpha_D \left(P_{D,t-1}\right)^{1-\sigma} + \left(1-\alpha_D\right) \left(\overline{P}_{D,t}\right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

Re-writing this in recursive form yields,

$$\frac{P_{D,t}}{P_{D,t-1}} = \left[\alpha_D \left(\frac{P_{D,t-1}}{P_{D,t-1}} \right)^{1-\sigma} + \left(1-\alpha_D\right) \left(\frac{\overline{P}_{D,t}}{P_{D,t-1}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

$$\pi_{D,t} = \left[\alpha_D + \left(1-\alpha_D\right) \left(\overline{\pi}_{D,t}\right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

Similarly for foreign firms, where $(1-\alpha_F)$ firms can optimize, they maximize the following profit function,

$$\max_{\overline{P}_{F,t}(i)} \sum_{k=0}^{\infty} \alpha_F^k M^*_{t,t+k} \left(\overline{P}_{F,t}(i) Y_{F,t_{+k}}(i) - MC_{F,t+k} Y_{F,t_{+k}}(i) \right)$$

$$\text{where } Y_{F,t+k}(i) = \left(\frac{\overline{P}_{F,t}(i)}{P^*_{F,t+k}} \right)^{-\sigma} Y_{F,t+k}$$

To get the price setting equation,

$$\overline{P}_{F,t}(i) = \frac{\sigma}{\sigma - 1} \frac{\sum_{k=0}^{\infty} \alpha_F^k M_{t,t+k}^* MC_{F,t+k} Y_{F,t+k}(i)}{\sum_{k=0}^{\infty} \alpha_F^k M_{t,t+k}^* Y_{F,t+k}(i)}$$

which can written recursively as,

$$\overline{\pi}_{F,t} = \frac{\sigma}{\sigma - 1} \pi_{F,t}^* \frac{X_{F,t}}{Z_{F,t}}$$

where,

$$X_{F,t} = \lambda_{F,t} Y_{F,t} mc_{F,t} T_{F,t} + \alpha_F \beta \left(\pi_{F,t+1}^*\right)^{\sigma} \left(\frac{E_t \left(V_{F,t+1}\right)^{1-\gamma}}{\left(V_{F,t+1}\right)^{1-\gamma}}\right)^{\frac{(\gamma-\eta)}{(1-\gamma)}} \left(\frac{1}{V_{F,t+1}}\right)^{\eta} E_t \left\{X_{F,t+1}\right\} \quad (4)$$

$$Z_{F,t} = \lambda_{F,t} Y_{F,t} T_{F,t} + \alpha_F \beta \left(\pi_{F,t+1}^*\right)^{\sigma-1} \left(\frac{E_t \left(V_{F,t+k}\right)^{1-\gamma}}{\left(V_{F,t+1}\right)^{1-\gamma}}\right)^{\frac{(\gamma-\eta)}{(1-\gamma)}} \left(\frac{1}{V_{F,t+1}}\right)^{\eta} E_t \left\{Z_{F,t+1}\right\} \quad (5)$$

The aggregate foreign producer's price inflation is given by,

$$\pi_{F,t}^* = \left[\alpha_F + (1 - \alpha_F) \left(\overline{\pi}_{F,t}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

A.2.4 Equilibrium

Aggregate demand functions for the domestic and foreign produce Total demand for each variety i of the output produced by domestic firms,

$$Y_{D,t}(i) = C_{D,t}(i) = nC_{D,t}(i) + (1-n) C_{D,t}^*(i)$$

$$= n \left(\frac{1}{n}\right) \left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\sigma} C_{D,t} + (1-n) \left(\frac{1}{n}\right) \left(\frac{P_{D,t}^*(i)}{P_{D,t}^*}\right)^{-\sigma} C_{D,t}^*$$

Note, $P_{D,t}(i) = X_t P^*_{D,t}(i)$, $P_{D,t} = X_t P^*_{D,t}$, where X_t is the nominal exchange rate. Real exchange rate $Q_t = \frac{X_t P^*_t}{P_t}$, $T_t = \frac{P_{F,t}}{P_{D,t}}$ Thus,

$$
\begin{aligned}
Y_{D,t}(i) &= n\left(\frac{1}{n}\right)\left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\sigma} C_{D,t} + (1-n)\left(\frac{1}{n}\right)\left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\sigma} C^*_{D,t} \\
&= \left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\sigma}\left[C_{D,t} + \left(\frac{1-n}{n}\right) C^*_{D,t}\right] \\
&= \left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\sigma} Y_{D,t}
\end{aligned}
$$

Total demand for agricultural produce, $Y_{D,t} = C_{D,t} + \left(\frac{1-n}{n}\right) C^*_{D,t}$. Aggregate demand, $Y_{D,t}$, can be re-written as,

$$
\begin{aligned}
Y_{D,t} &= C_{D,t} + \left(\frac{1-n}{n}\right) C^*_{D,t} \\
&= \mu_D \left(\frac{P_{D,t}}{P_t}\right)^{-\xi_D} C_t + \left(\frac{1-n}{n}\right)\mu_F \left(\frac{P^*_{D,t}}{P^*_t}\right)^{-\xi_F} C^*_t \\
&= \left(\frac{P_{D,t}}{P_t}\right)^{-\xi_D}\left[\mu_D C_t + \left(\frac{1-n}{n}\right)\mu_F \left(\frac{P^*_{D,t}}{P^*_t}\right)^{-\xi_F}\left(\frac{P_t}{P_{D,t}}\right)^{-\xi_D} C^*_t\right] \\
&= \left(\frac{P_{D,t}}{P_t}\right)^{-\xi_D}\left[\mu_D C_t + \left(\frac{1-n}{n}\right)\mu_F \left(\frac{X_t P_{D,t}}{X_t P_t Q_t}\right)^{-\xi_F}\left(\frac{P_{D,t}}{P_t}\right)^{\xi_D} C^*_t\right] \\
&= \left(\frac{P_{D,t}}{P_t}\right)^{-\xi_D}\left[\mu_D C_t + \left(\frac{1-n}{n}\right)\mu_F Q_t^{\xi_F}\left(\frac{P_{D,t}}{P_t}\right)^{\xi_D-\xi_F} C^*_t\right] \\
&= (T_{D,t})^{-\xi_D}\left[\mu_D C_t + \left(\frac{1-n}{n}\right)\mu_F Q_t^{\xi_F}(T_{D,t})^{\xi_D-\xi_F} C^*_t\right]
\end{aligned}
$$

Similarly, total demand for each variety i of the output produced by foreign firms can be written as,

$$\begin{aligned}
Y_{F,t}(i) &= C_{F,t}(i) = nC_{F,t}(i) + (1-n)\,C^{*}_{F,t}(i) \\
&= n\left(\frac{1}{1-n}\right)\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\sigma} C_{F,t} + \left(\frac{1-n}{1-n}\right)\left(\frac{P^{*}_{F,t}(i)}{P^{*}_{F,t}}\right)^{-\sigma} C^{*}_{F,t} \\
&= \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\sigma} Y_{F,t}
\end{aligned}$$

where total demand for agricultural produce, $Y_{F,t} = \frac{n}{(1-n)} C_{F,t} + C^{*}_{F,t}$. Aggregate demand, $Y_{F,t}$, can be re-written as,

$$\begin{aligned}
Y_{F,t} &= \frac{n}{(1-n)} C_{F,t} + C^{*}_{F,t} \\
&= \frac{n}{(1-n)}(1-\mu_D)\left(\frac{P_{F,t}}{P_t}\right)^{-\xi_D} C_t + (1-\mu_F)\left(\frac{P^{*}_{F,t}}{P^{*}_t}\right)^{-\xi_F} C^{*}_t \\
&= \frac{n}{(1-n)}(1-\mu_D)\left(\frac{P_{F,t}}{P_t}\right)^{-\xi_D} C_t + (1-\mu_F)\left(\frac{X_t P_{F,t}}{X_t Q_t P_t}\right)^{-\xi_F} C^{*}_t \\
&= \left(\frac{P_{F,t}}{P_t}\right)^{-\xi_D}\left[\frac{n}{(1-n)}(1-\mu_D)\,C_t + (1-\mu_F)\,Q_t^{\xi_F}\left(\frac{P_{F,t}}{P_t}\right)^{-\xi_F}\left(\frac{P_{F,t}}{P_t}\right)^{\xi_D} C^{*}_t\right] \\
&= (T_{F,t})^{-\xi_D}\left[\frac{n}{(1-n)}(1-\mu_D)\,C_t + (1-\mu_F)\,Q_t^{\xi_F}(T_{F,t})^{\xi_D-\xi_F} C^{*}_t\right]
\end{aligned}$$

Labour market equilibrium For the domestic country, aggregate labour supply would equalize aggregate labour demand in equilibrium,

$$\begin{aligned}
H_{D,t} &= \frac{1}{n}\int_0^n H_{D,t}(i)\, di \\
&= \frac{1}{n}\int_0^n \frac{Y_{D,t}(i)}{A_{D,t}} di \\
&= \frac{Y_{D,t}}{A_{D,t}} Disp_{D,t} \\
\text{where } Disp_{D,t} &= \frac{1}{n}\int_0^n \left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\sigma} di
\end{aligned}$$

Re-writing $Disp_{D,t}$ in recursive form,

$$\begin{aligned}
Disp_{D,t} &= \frac{1}{n}\int_0^n \frac{\alpha_D \left(P_{D,t-1}(i)\right)^{-\sigma} + (1-\alpha_D)\left(\overline{P}_{D,t}(i)\right)^{-\sigma}}{\left(P_{D,t}\right)^{-\sigma}} di \\
&= \alpha_D \frac{1}{n}\int_0^n \left(\frac{P_{D,t-1}(i)}{P_{D,t}}\right)^{-\sigma} di + (1-\alpha_D)\frac{1}{n}\int_0^n \left(\frac{\overline{P}_{D,t}(i)}{P_{D,t}}\right)^{-\sigma} di \\
&= \alpha_D \frac{1}{n}\int_0^n \left(\frac{P_{D,t-1}(i)}{P_{D,t}}\frac{P_{D,t-1}}{P_{D,t-1}}\right)^{-\sigma} di + (1-\alpha_D)\left(\frac{\overline{P}_{D,t}}{P_{D,t}}\right)^{-\sigma} \\
&= \alpha_D \left(\frac{P_{D,t-1}}{P_{D,t}}\right)^{-\sigma} Disp_{D,t-1} + (1-\alpha_D)\left(\frac{\overline{P}_{D,t}}{P_{D,t}}\frac{P_{D,t-1}}{P_{D,t-1}}\right)^{-\sigma}
\end{aligned}$$

$$\begin{aligned}
Disp_{D,t} &= \alpha_D \left(\pi_{D,t}\right)^{\sigma} Disp_{D,t-1} + (1-\alpha_D)\left(\overline{\pi}_{D,t}\right)^{-\sigma}\left(\pi_{D,t}\right)^{\sigma} \\
&= \left(\pi_{D,t}\right)^{\sigma}\left[\alpha_D Disp_{D,t-1} + (1-\alpha_D)\left(\overline{\pi}_{D,t}\right)^{-\sigma}\right]
\end{aligned}$$

where $Disp_{D,t-1} = \frac{1}{n}\int_0^n \left(\frac{P_{D,t-1}(i)}{P_{D,t-1}}\right)^{-\sigma} di.$

Similarly, in the foreign country labour supply in equilibrium would be,

$$\begin{aligned}
H_{F,t} &= \frac{Y_{F,t}}{A_{F,t}} Disp_{F,t} \\
\text{where } Disp_{F,t} &= \left(\frac{1}{1-n}\right)\int_n^1 \left(\frac{P^*_{F,t}(i)}{P^*_{F,t}}\right)^{-\sigma} di
\end{aligned}$$

and $Disp_{F,t}$ can be written recursively as,

$$Disp_{F,t} = \left(\pi^*_{F,t}\right)^{\sigma} \left[\alpha_F Disp_{F,t-1} + (1-\alpha_F)\left(\overline{\pi}_{F,t}\right)^{-\sigma}\right]$$

where $Disp_{F,t-1} = \left(\frac{1}{1-n}\right)\int_n^1 \left(\frac{P^*_{F,t-1}(i)}{P^*_{F,t-1}}\right)^{-\sigma} di.$