Macro-financial imbalances and cyclical systemic risk dynamics: Understanding the factors driving the financial cycle in the presence of non-linearities: Technical Appendix *

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A Priors

The prior distributions for the initial values of the states B_0, α_0 and h_0 are postulated to be normal and are assumed to be independent of one another. The independence assumption also holds for the distribution of the hyperparameters.

A.1 Unobserved components

The initial states of the trend and cycle components of $F_{0/0}$ are set using a bandpass filter with a passband frequency range specified between 8 and 32. In order to reflect the uncertainty surrounding the choice of starting values, a large prior covariance of the states $P_{0/0}$ is assumed.

A.2 Priors on the VAR parameters

Following Bańbura et al. (2010) we introduce natural conjugate prior for the VAR parameters via dummy observations. We choose the prior means μ_n as OLS estimates

*The views expressed in this paper are those of the authors only and not necessarily reflect those of the European Central Bank or of the Central Bank of Ireland.

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of the coefficients of an AR(1) regression estimated for each endogenous variable using a training sample consisting of the 40 first observations. These are removed from the sample afterwards. The scaling factors σ_n are set using the standard deviation of the error terms from these preliminary AR(1) regressions. Here τ reflects the degree of shrinkage which is higher the closer it is to 0. We set the overall prior tightness τ based on the highest models' LS for the target variable. We report the results for the examined grid of τ in Table 3 of the article. We set $c = 1000$ indicating a tight prior on the constant at 0. This is akin to excluding the estimation of the constant parameter. However, we opt for this approach to enhance computational efficiency, leveraging the natural conjugate prior for the VAR model. This prior necessitates uniformity among all right-hand side variables, including the inclusion of a constant in all equations.

Litterman (1986a) and Litterman (1986b) proposed this priors through the application of methods of Bayesian shrinkage. We implement the Normal Inverted Wishart prior through Dummy observations as in equation (1). These are set such that the moments of the Minnesota prior are matched. In that sense the prior variance decreases with increasing lag length, carrying the belief that more recent lags contain more relevant information.

$$Y_{D,1} = \begin{pmatrix} \frac{diag(\sigma_1\mu_1,...,\sigma_N\mu_N)}{\tau} \\ 0_{Nx(P-1)xN} \\ .... \\ diag(\sigma_1....\sigma_N) \\ .... \\ 0_{1xN} \end{pmatrix}, \; and \;\; X_{D,1} = \begin{pmatrix} \frac{J_P \otimes diag(\sigma_1,...,\sigma_N)}{\tau} 0_{NPx1} \\ 0_{NxNP} \; 0_{Nx1} \\ .... \\ 0_{1xNP} \;\; c \end{pmatrix} \tag{1}$$

Additionally, we impose a prior on the sum of coefficients for its shown benefits in improving the models forecasting accuracy, drawing insights from Sims (1992), Robertson and Tallman (1999) and Sims and Zha (1999). This is a modification of the Minnesota prior suggested by Doan et al. (1984) and carries the belief, that the sum of the coefficients of the lags equates to 1 (Robertson and Tallman (1999)). The tightness of the sum of coefficients prior is set as in Bańbura et al. (2010) $\lambda = 10\tau$ and is introduced by adding the following dummy observations:

$$Y_{D,2} = \begin{pmatrix} \frac{diag(\sigma_1\mu_1,...,\sigma_N\mu_N)}{\lambda} \end{pmatrix} \tag{2}$$

$$X_{D,2} = \begin{pmatrix} \frac{(1\ \ 2...p)\otimes diag(\sigma_1\mu_1,...,\sigma_N\mu_N)}{\lambda} 0_{nx1} \end{pmatrix}$$

A.3 Priors on covariance parameters

As to calibrate the prior distribution of α_0 we run a time-invariant VAR including on the auxiliary variables. This is based on the training sample. $\hat{\Sigma_0}$ is the estimated covariance matrix of the residuals ϵ_t. As in Benati and Mumtaz (2007) let C be the lower-triangular Choleski factor of $\hat{\Sigma_0}$ such that C'C= $\hat{\Sigma_0}$ and $C = \hat{\Sigma_0^{\frac{1}{2}}}$. The estimated matrix $\hat{C_0}$ is computed by dividing each column of C by the corresponding element of the diagonal. Through this transformation the elements outside the main diagonal are normalized. After computing the inverse of $\hat{C}$ the elements below the main diagonal of $\hat{C_0^{-1}}$ are collected (i.e.all non-zero and non-one entries). This values will be set as the starting values of α in the vector $\tilde{\alpha_0} \equiv [\alpha_{0,21}, \alpha_{0,31}, \alpha_{0,32}]$.

A normal prior is assumed for the regression coefficients in each equation, as in equation(3). The conditional posterior distribution of α_i in equation (4) is also assumed to be normal. Here Z_i are the left-hand variables and z_i right-hand variables transformed proportional to the variance of the structural shocks for the weighted regressions in section B.2). As in Mumtaz and Theodoridis (2017) $\underline{V_{i0}}$ is assumed to be diagonal with its elements set equal to 10 times the absolute value of the corresponding element of $\underline{\alpha_{i0}}$.

$$\alpha_{i0} \sim N(\underline{\alpha_{i0}}, \underline{V_{i0}}),\ \ i = 2, 3 \tag{3}$$

$$\alpha_i | B, \Lambda_i^T, Y^T \sim N(\overline{\alpha_i}, \overline{V_i}),\ \ i = 2, 3 \tag{4}$$

Where

$$\overline{V_i} = (\underline{V_{i0}^{-1}} + Z_i' Z_i)^{-1}, \tag{5}$$

$$\overline{\alpha_i} = (\underline{V_{i0}}^{-1}\underline{\alpha_{i0}} + Z_i' z_i) \tag{6}$$

A.4 Priors of the idiosyncratic shock volatility transition equation of the cyclical dynamics

Following Ellis et al. (2014) the prior for the diagonal elements Λ_t of the covariance matrix Σ_t , see Equation [**8**] in Section 2 of the article, is assumed to be normal, with μ_0^z set as the logs of diagonal elements of the Cholesky decomposition of $\hat{\Sigma}_0$ and $\sigma_0^z = 10$.

$$ln(\lambda^z) \sim N(\mu_0^z, \sigma_0^z) \tag{7}$$

An inverse Gamma prior is set for σ_ω with $g_0 = 0.01^2$ and $\nu_0 = 1$.

$$p(\sigma_\omega) \sim IG(g_0, \nu_0) \tag{8}$$

A.5 Priors of the idiosyncratic shock volatility transition equation for the trend

The prior for the stochastic volatilities for the trend (9) is normal with μ_0^τ set as the logs of the standard deviation of the first difference of the pre-sample estimate of the trend and $\sigma_0^\tau = 10$. An inverse Gamma prior is set for σ_ρ with $g_0 = 0.01^2$ and $\nu_0 = 1$.

$$ln(\lambda_0^\tau) \sim N(\mu_0^\tau, \sigma_0^\tau) \tag{9}$$

$$p(\sigma_\rho) \sim IG(g_0, \nu_0) \tag{10}$$

B Sampling from the Posterior Density

The model is estimated using a Metropolis-within-Gibbs sampler. This methodology was developed in Cogley and Sargent (2005) for VAR models and by Primiceri (2005) for state space models. The volatilities of the reduced form shocks H_t are drawn using the date by date blocking scheme introduced in Jacquier et al. (2002) which assumes that the stochastic volatilities are independent.

This procedure is reduced into five blocks. The first involves sampling the VAR parameters β^T that relate the cycle and the auxiliary variables. The second block involves the estimation of the covariance parameters α. The third step draws the standard deviation of the volatility innovation. The fourth step draws the stochastic volatilities. The last block draws the unobserved components delivering the estimates for the trend and the cycle.

B.1 VAR Parameters β

We condition on the estimated unobserved components for the trend and the cycle $\{\tau_t, c_t\}$, the history of Λ_t and the stochastic volatility parameters α and draw the non time varying parameters $\{\beta_{q,1}, ..., \beta_{q,pq}\}$ that describe the relationship between the cycle and the auxiliary variables.

The vector of coefficients is sampled from a normal posterior distribution with mean $\overline{M}$ and variance $\overline{\Omega}^{-1}$, based on prior mean M (14) and variance Ω (28) as in Clark (2011). Where $Z_t = \{c_t, AX_{1t}, AX_{2t}\}$ and $X_t = \{c_{t-1}, AX_{1t-1}, AX_{2t-1}, c_{t-2}, AX_{1t-2}, AX_{2t-2},\ 1\}$ for p=2 and k=2.

$$Z_t = X_t B + \upsilon_t,\ \ VAR(\upsilon_t) = \Sigma_t \tag{11}$$

$$\Sigma_t = A^{-1}\Lambda_t(A^{-1})' \tag{12}$$

$$B \sim (\overline{M}, \overline{\Omega}^{-1}) \tag{13}$$

$$\overline{M} = \overline{\Omega}\{vec(\sum_{t=1}^{T} Q_t^{-1} Y_t X_t') + \Omega^{-1}\}^{-1} \tag{14}$$

$$\overline{\Omega}^{-1} = \Omega^{-1} + \sum_{t=1}^{T}(Q_t^{-1} \bigotimes X_t X_t') \tag{15}$$

B.2 Covariance parameters α

The second block involves the estimation of the covariance parameters. In the following, we consider the distribution of α conditional on the data and other parameters. Z_t, X_t and the draw B. This implies knowledge of υ_t . The residuals satisfy the relation described in equation (16). Where ϵ_t is a vector of orthogonalized residuals (structural shocks) with known error variance Λ_t. Λ_t is a diagonal matrix with elements $\lambda_{i,t}$ and $VAR(\epsilon_t) = \Lambda_t$. A is a lower triangular matrix with elements αqj, see equation (17).

As in Cogley and Sargent (2005) the relationship between the residuals and structural shocks will be interpreted as a system of unrelated regressions. The modelling strategy for the law of motion of the variance/covariance matrix is given by the following system of equations: The identity in (19) defines the relation for q=1 . The equations for q=2, 3 can be expressed as transformed regressions with independent standard normals. In this regressions the relation between residuals υ_{it} and structural shocks is transformed proportional to the variance of the structural shocks Λ_t such that $\epsilon^*_{it} \sim N(0,1)$ for i={2, 3}. Therefore, the second equation can be expressed as (20) and the third equation as (21).

$$A\upsilon_t = \epsilon_t \tag{16}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ \alpha_{21} & 1 & 0 \\ \alpha_{31} & \alpha_{32} & 1 \end{bmatrix} \begin{bmatrix} \upsilon_{1,t} \\ \upsilon_{2,t} \\ \upsilon_{3,t} \end{bmatrix} = \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ \alpha_{21} & 1 & 0 \\ \alpha_{31} & \alpha_{32} & 1 \end{bmatrix} \tag{17}$$

$$\Lambda_t = \begin{bmatrix} \lambda_{1t} & 0 & 0 \\ 0 & \lambda_{2t} & 0 \\ 0 & 0 & \lambda_{3t} \end{bmatrix} \tag{18}$$

$$\upsilon_{1t} = \epsilon_{1t} \tag{19}$$

$$\frac{\upsilon_{2t}}{\sqrt{\lambda_{2t}}} = \alpha_{21}(-\frac{\upsilon_{1t}}{\sqrt{\lambda_{2t}}}) + \frac{\epsilon_{2t}}{\sqrt{\lambda_{2t}}} \tag{20}$$

$$\frac{\upsilon_{3t}}{\sqrt{\lambda_{3t}}} = \alpha_{31}(-\frac{\upsilon_{1t}}{\sqrt{\lambda_{3t}}}) + \alpha_{32}(-\frac{\upsilon_{2t}}{\sqrt{\lambda_{3t}}}) + \frac{\epsilon_{3t}}{\sqrt{\lambda_{3t}}} \tag{21}$$

B.3 Stochastic Volatilities, Λ_t

The diagonal elements of Λ_t are independent, univariate stochastic volatilities that evolve as geometric random walks without a drift. Based on the draw of A the contemporaneously uncorrelated structural residuals can be computed as specified in (16). The independence MH algorithm can be applied for each orthogonalized VAR residual(ϵ_{it}) conditional on a draw of σ_i.

$$ln\lambda_{it} = ln\lambda_{it-1} + \sigma_i \eta_{it} \tag{22}$$

B.4 Standard deviations of volatility innovations σ_i

Conditional on a draw for λ_{it}, the standard deviations for the volatility innovations σ_i can be drawn from the inverse Gamma distribution (23). Assuming an inverse-gamma prior with scale parameters γ_0 and v_0 degrees of freedom, the posterior has an inverse-gamma distribution with degrees of freedom $v_1 = v_0 + T$ and scale parameter $\gamma_1 = \gamma_0 + \sum_{t=1}^{T}(\Delta \ln(\lambda_{it}))$.

$$f(\sigma_i^2 | \lambda_i^T, Y^T) = IG(\frac{v_1}{2}, \frac{\gamma_1}{2}) \tag{23}$$

C Time-invariant filter

C.1 Model

The homoskedastic multivariate filter is a special case of the stochastic volatility filter described in Section 2.

$$\boldsymbol{\Theta_t} = \boldsymbol{\tau_t} + \boldsymbol{c_t} \tag{24}$$

$$\tau_t = \tau_{t-1} + \upsilon_t, \;\; \upsilon_t \sim N(0, 1) \tag{25}$$

As before, the trend τ_t evolves as a random walk and the cyclical component c_t displays the stationary variation within the time series. However, in this specification, the local disturbances of the trend υ follow a normal distribution. Also, the BVAR(p) is linear.

$$Z_t = F_t B + \nu_t, \;\; \nu_t = \Omega^{0.5}\epsilon_t, \;\; \epsilon_t \sim N(0, I_N), \tag{26}$$

where $Z_t = (c_t, AX_{1t}, .., AX_{kt})'$ is a matrix of endogenous variables (for $i = 1, .., N$ model variables of which $k = 1, .., K$ are auxiliary variables). $F_t = (Z'_{t-1}, .., Z'_{t-p}, 1)'$ denotes the matrix of regressors and B is the matrix of coefficients $B = (B_1, ..., B_P, \mu)'$. $\mu = (\mu_1, ..., \mu_n)'$ is an N-dimensional vector of constants and $B_1, ..., B_P$ are NxN autoregressive matrices. Also here natural conjugate priors for the VAR parameters are introduced following Bańbura et al. (2010).

C.2 Sampling from the posterior density

We use Gibbs sampling to approximate the posterior density. This procedure can be summarized into three steps.

1. We condition on the estimated unobserved component for the cycle c_t and sample the vector of coefficients B and Ω. Their conditional posterior distributions are defined in equations (27) and (28).

$$G(B|\Omega) \sim N(B^*, \Omega \bigotimes (F^{*'}F^*)^{-1}) \tag{27}$$

$$G(\Omega|B) \sim IW(S^*, T^*) \tag{28}$$

   As in Alessandri and Mumtaz (2017) the posterior means are given by $B^* = (F^{*'}F^*)^{-1}(F^{*'}Z^*)$ and $S^* = (Z^* - F^*)\tilde{B}(Z^* - F^*\tilde{B})$. The dummy priors are incorporated through Z^* and F^* and $\tilde{B}$ is the draw of the VAR coefficients reshaped to meet the size of the matrix of regressors.

2. Conditioning on the unobserved component for the trend τ_t we sample the variance of the trend. σ^2_τ. The variance of the trend innovations is simulated from an

Inverse Gamma 2 (IG_2) distribution. Where the IG_2 is re-parametrised in terms if the mean and variance. Taking the two first moments of the IG_2 distribution as in Bauwens et al. (2000) and solving for the gamma parameters allows to calibrate the scale parameter g_0 and degrees of freedom d_0 of the prior given values for the mean and the variance. The prior (29) and posterior(30) distribution of σ_τ may be represented as

$$\sigma_\tau^2 \sim \ IG_2(\sigma_0^\tau,\ \nu_0^\tau) \tag{29}$$

$$\sigma_\tau \sim \ IG(\frac{g_1}{2}, \frac{d_1}{2}) \tag{30}$$

With g_1 and d_1 defined as

$$g_1 = g_0 + T \tag{31}$$

$$d_1 = d_0 + (\tau_t - \tau_{t-1})'(\tau_t - \tau_{t-1}) \tag{32}$$

We set a loose prior $\sigma_\tau \sim IG_2(0.1, 1)$.

3. Conditional on the draws we apply the Carter and Kohn (1994) algorithm to cast the unobserved components in a state space model as in Mumtaz (2010).

D Estimation using simulated data

In order to assess the efficacy of the algorithm, we undertake a concise simulation exercise. The experiment involves the generation of two synthetic datasets, each comprising a target variable and three auxiliary variables. We generate 220 observations from a data generating process that is divided into 2 steps. First, we model the stationary cycle component, auxiliary variables and trend. In a second step, we generate the target variable as a sum of the cycle and the trend. For k=4 and p=2 we generate four series: the cycle and three auxiliary variables. In line with Rünstler and Vlekke (2018) the financial cycle is characterized as a persistent process. Furthermore, the parameters for both Data Generating Processes (DGPs) are calibrated using macro-financial variables for the United States spanning from the first quarter of 1975 to the fourth quarter of 2015. Specifically, these variables include loans to non-financial corporations, the annual growth rate of the house price index, the unemployment rate, and the Excess Bond Premia .

DGP 1 is generated from the following process:

$$Z_t = X_t B + A^{-1}\Lambda_t^{0.5}\epsilon_t, \;\; \epsilon_t \sim N(0, I_4) \tag{33}$$

The elements of the time-varying diagonal matrix Λ_t evolve as:

$$ln(\lambda_t^z) = ln(\lambda_{t-1}^z) \; + Chol(Q)\, \eta_t^z, \;\; Q = I_4 \; x \; P_t, \;\; \eta_t^z \sim N(0, I_4) \tag{34}$$

where Pt is a vector of standard deviations of volatility innovations.

In parallel, we generate the series for the trend as

$$\tau_t = \tau_{t-1} + \upsilon_t \sqrt{exp(ln\lambda_t^\tau)}, \;\; \upsilon_t \sim N(0, 1) \tag{35}$$

$$ln(\lambda_t^\tau) = ln(\lambda_{t-1}^\tau) \; + \; 0.1\eta_t^\tau, \;\; \eta_t^\tau \sim N(0, 1) \tag{36}$$

DGP 2 imposes a constant variance process. The cycle and auxiliary variables are generated as

$$Z_t = X_t B + \Omega^{0.5}\epsilon_t, \;\; \epsilon_t \sim N(0, I_4) \tag{37}$$

And the series for the trend follow

$$\tau_t = \tau_{t-1} + \upsilon_t, \;\; \upsilon_t \sim N(0, 1) \tag{38}$$

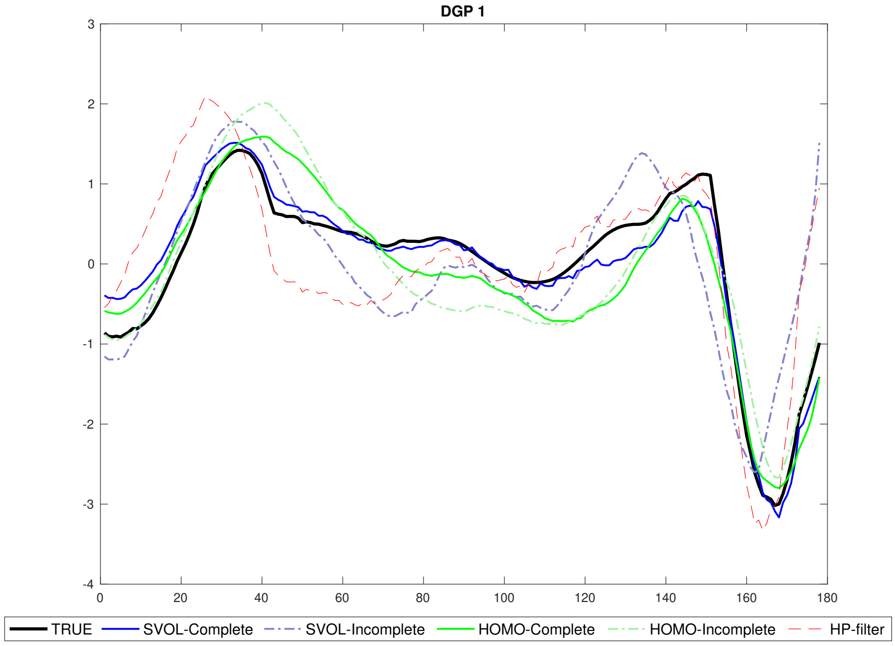
We then generate the target variable Θ_t as the sum of c_t and τ_t. As motivated in the introduction, in our model the financial cycle is approximated by a measure of the interrelation of aggregates reflective of macroeconomic imbalances. Our fictitious dataset replicates this two-sided relationship between the state variable of the cycle and the auxiliary variables.

We estimate the model with loose priors. For the SVOL specifications we use 40 observations as training sample. This leaves 180 observations for the estimation. For both the SVOL and Homoskedastic filter, the model estimation uses 15000 Gibbs iterations with a burn-in of 14500 iterations.

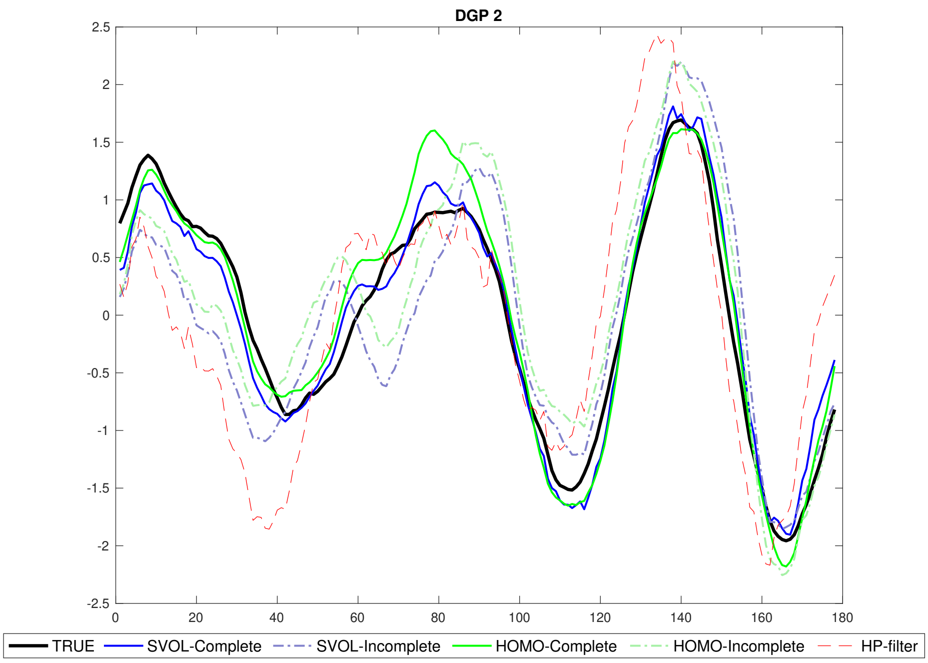
Figure D.1 displays the cycle estimated based on the HP filter, its measurement with the proposed methodology as well as a homoskedastic version of it, against its true value for DGP 1 (LHS) and DGP 2 (RHS). [1] Visual inspection shows that the cyclical component estimated with the HP filter exaggerates the real developments and does not capture the cyclical turns in all occasions. This can be partly explained by the fact that the HP filter does not model the evolution of the volatility of the simulated time series and by its inherent high persistence.

Figure D.1: Monte Carlo simulations: True cycle and cycle estimates

(a) DGP 1: Time-varying variance Data Generating Process



(b) DGP 2: Constant variance Data Generating Process

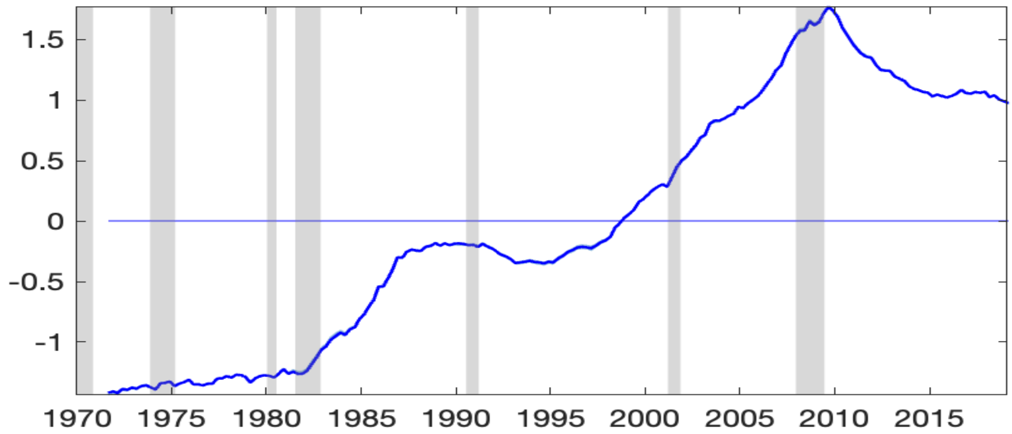


Note: All estimates are standardised.

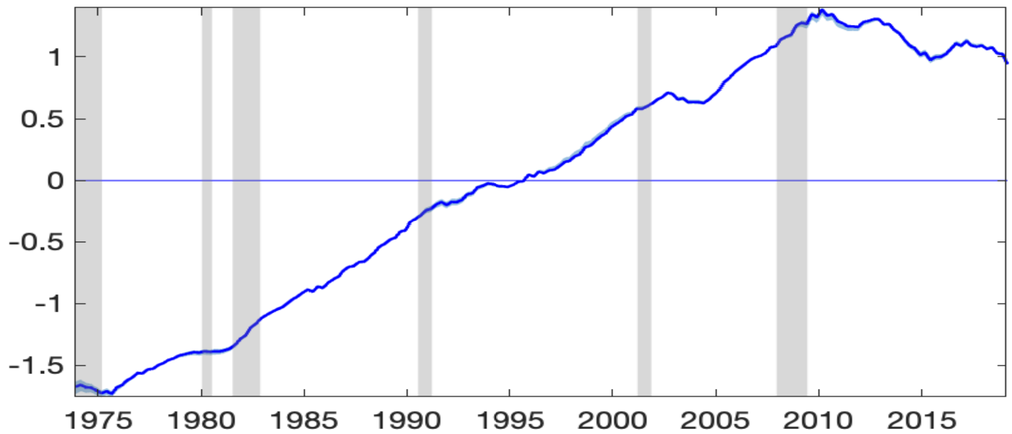
E Empirical results

Figure E.1: Estimated trend

(a) United States 1971:Q3-2019:Q1



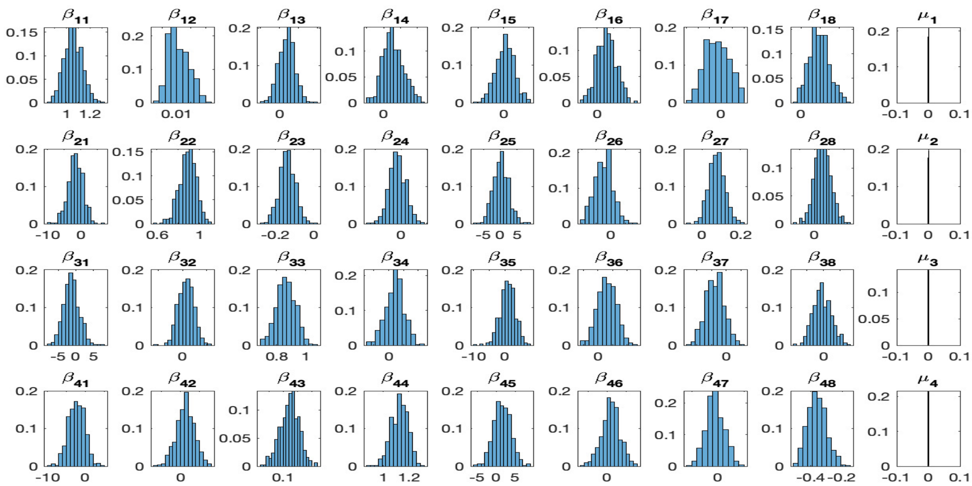
(b) United Kingdom 1973:Q4-2019:Q1



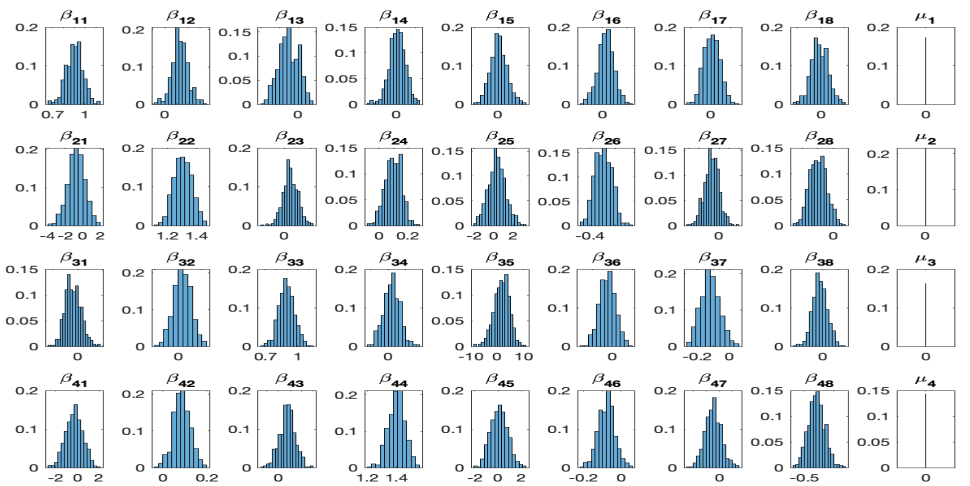
Note: Solid line shows the median of the posterior distribution of the estimated trend, the shaded area represents the 68% error band. Grey vertical areas display the NBER economic recessions.

Figure E.2: Posterior distribution of the estimated autoregressive parameters based on the final estimates (whole sample)

(a) United States 1971:Q3-2019:Q1



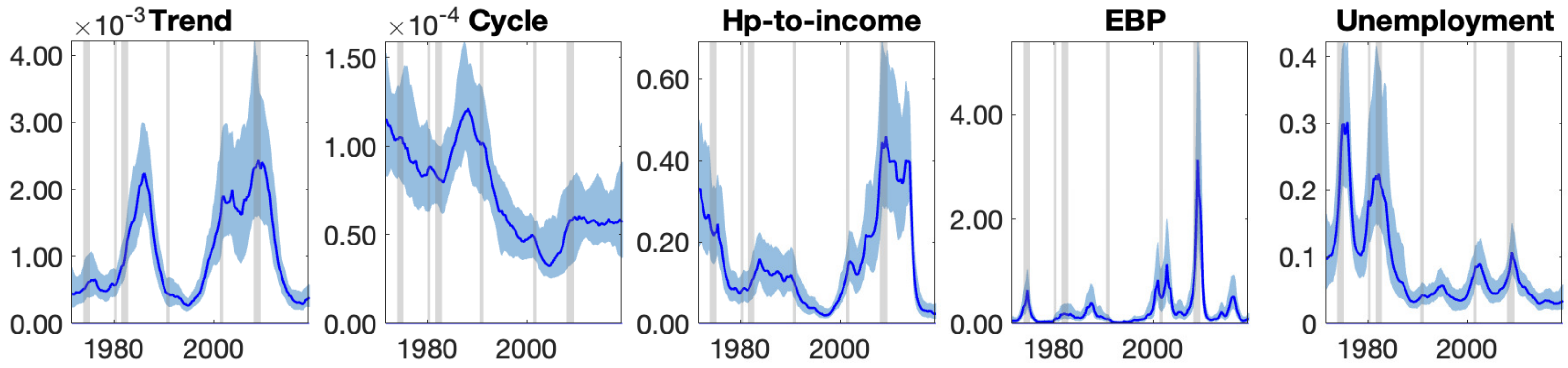
(b) United Kingdom 1973:Q4-2019:Q1



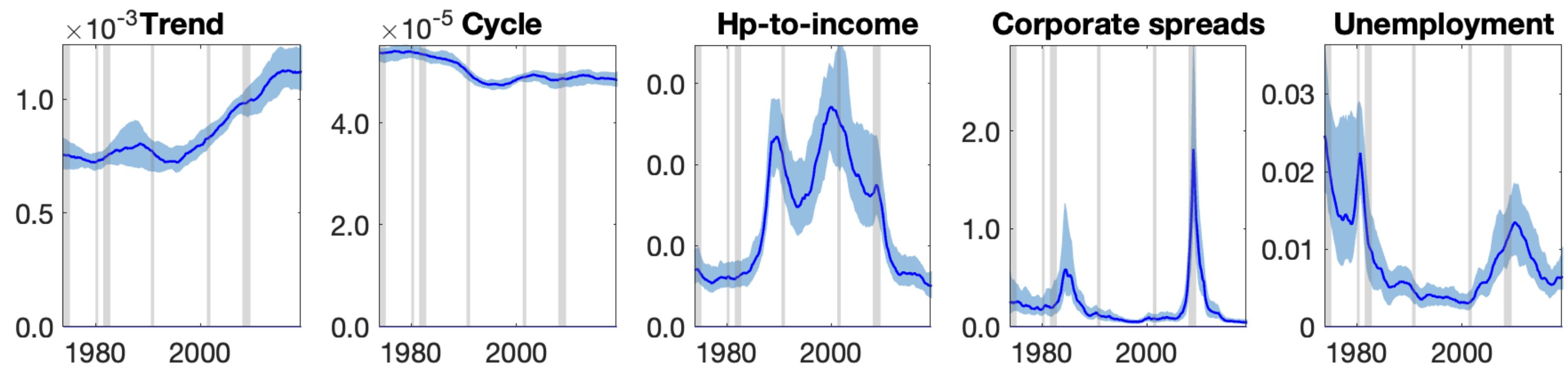
Note: Last column displays the estimates for the intercept. Order of variables consistent with that in the article: Cycle; Household vulnerabilities; Financial conditions; Economic activity.

Figure E.3: Estimated stochastic volatilities

(a) United States 1971:Q3-2019:Q1



(b) United Kingdom 1973:Q4-2019:Q1



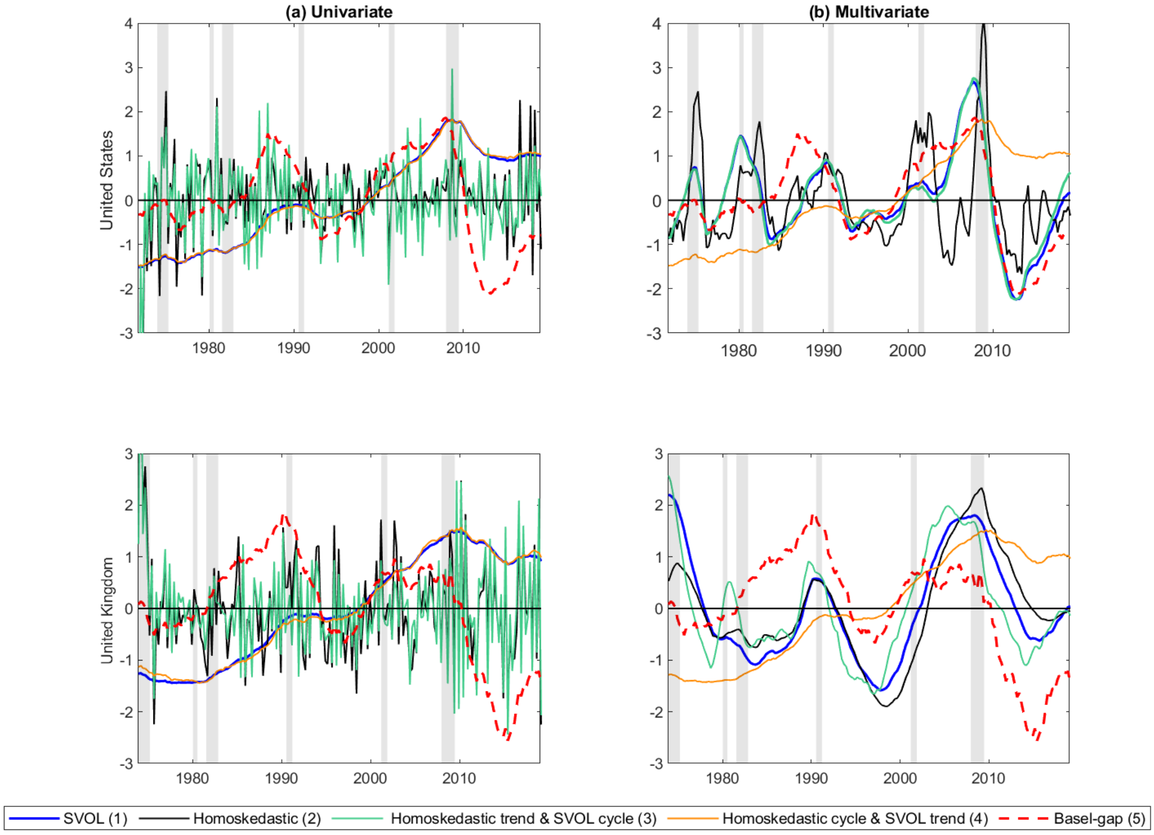
Note: Solid line shows the median of the posterior distribution of the estimated volatilities, the shaded area represents the 68% error band. Grey vertical areas display the NBER economic recessions.

Figure E.4 displays the univariate (column **(a)**) and multivariate estimates (column **(b)**) for the following set-ups: SVOL (1), Homoskedastic (2), Homoskedastic trend & SVOL cycle (3), Homoskedastic cycle & SVOL trend (4) and Basel-gap (5).

None of the model-based estimates of the cycle in the univariate set-ups successfully captures the financial cycle. Nonetheless, upon categorizing the results into two distinct groups, it becomes apparent that introducing stochastic volatility into the cycle, as opposed to solely in the trend or not at all, enhances the cyclical properties of the univariate estimates. On one hand, the cycle derived from the fully Homoskedastic approach (2) and the homoskedastic cycle & SVOL trend (3) demonstrates an almost perfect fit to the target variable, signifying an acyclic nature. On the other hand, the cycles estimated through SVOL (1) and Homoskedastic trend & SVOL cycle (3) exhibit characteristics typically associated with cyclical patterns, including periodicity and zero-centeredness. However, it is noteworthy that even this second group does not manifest the properties of the financial cycle in terms of amplitude or periodicity.

The multivariate estimates are depict in column **(b)** of Figure E.4. Amid the cross-country heterogeneity regarding the Homoskedastic estimates (see discussion in Subsection 4.3 of the article), almost all the cycle estimates trace the fluctuations in macro-financial imbalances with the exception of the Homoskedastic cycle & SVOL trend (4) specification. The comprehensive analysis of the entire set of results depicted in the panel underscores the crucial role of multivariate information in synthesizing macro-financial imbalances and cyclic systemic risk dynamics. Additionally, it becomes apparent that incorporating stochastic volatility into the cycle yields more favorable outcomes compared to its exclusive inclusion in the trend. Furthermore, restricting stochastic volatility solely to the trend, in conjunction with a homoskedastic error structure for the cycle, results in less informative findings when juxtaposed with the comprehensive homoskedastic approach.

Figure E.4: Financial Cycle Estimates

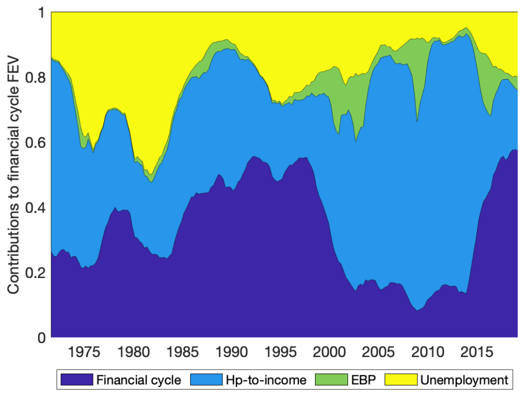


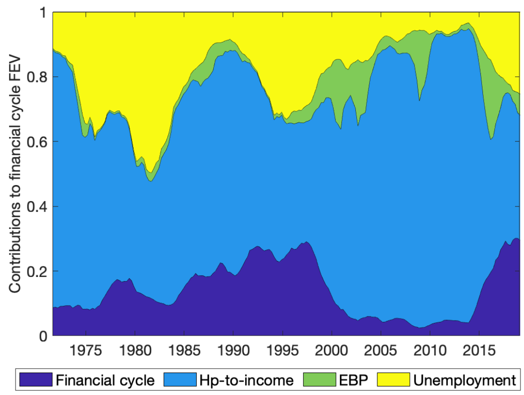
Note: Solid lines shows the median of the posterior distribution of the estimated cycles. All estimates are standardised.

Figure E.5 plots the time varying forecast error variance decomposition (FEVD) of the financial cycle for the US and the UK. Note, that the X-axis represents the time-periods while the Y-axis is the contribution to the forecast error variance at a short term horizon of 2 years in the LHS and a medium term horizon of 5 years in the RHS.[2]

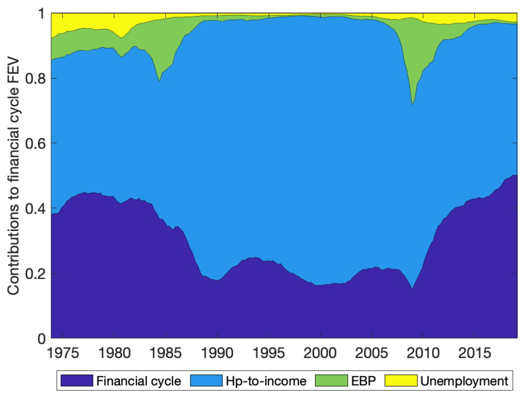
Figure E.5: Forecast error variance decomposition of the financial cycle

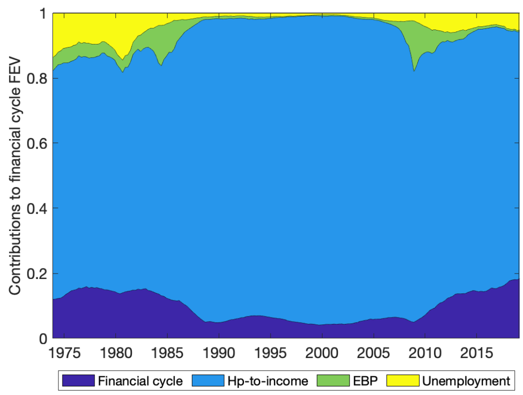
(a) United States 1971:Q3-2019:Q1





(b) United Kingdom 1973:Q4-2019:Q1





Short term horizon of 2 years

Medium term horizon of 5 years

F The particle filter

Following Mertens (2016), we re-estimate the multivariate stochastic volatility state-space model presented in Section 2 with a Rao-Blackwellized particle filter.[3] Conditional on the log-variances of the trend Equation [**5**] and the cycle Equation [**7**], in Section 2 of the article, the state-space model is linear. Therefore, the proposal distribution can be divided into two parts.[4] First the stochastic volatility components of the state vector are drawn and then the partcile filter uses a Rao-Blackwellization that analytically represents the distribution of τ_t and Z_t conditional on observables and the particle draw. Beyond providing estimates for the dynamic components of the trend and the cycle, the particle filter allows to measure the sensitivity of the cycle estimates to different input variables through a weighted average of the Kalman gains for each particle.[5] These provide information about what signals stemming from the auxiliary variables recieve a strong weight from the filter.

Equation (39) displays how for each particle draw *(i)* the cycle estimate is updated with the innovation weighted with the i*th* particle's Kalman gain $K_t^{(i)}$. The innovation is defined as the gap between the observable variables Y_t and its predicted value within the state space $Y_{t|t-1}$, it is hence a measure of the bias of the estimation. The filtered estimates of the cycle are given by equation (40) where w^i represent the particle weights, $K_t = \sum_i w_t^{(i)} K_t^{(i)}$ and $Y_{t|t-1} = \sum_i w_t^{(i)} Y_{t|t-1}^{(i)}$.

$$c_{t|t}^{(i)} = c_{t-1|t-1}^{(i)} + K_t^{(i)}(Y_t - Y_{t|t-1}^{(i)}) \tag{39}$$

$$c_{t|t} = \sum_i w_t^{(i)} c_{t|t}^{(i)} \approx c_{t-1|t-1} + K_t(Y_t - Y_{t|t-1}) \tag{40}$$

As outlined in Mertens (2016) the suitability of (40) will depend on the quality of the approximation of the weighted estimates ($K_t Y_{t|t-1} \approx \sum_i w_t^{(i)} K_t^{(i)} Y_{t|t-1}^{(i)}$). We apply the particle filtering algorithm with I=10,000 particles and systematic resampling. Model parameters $(B, A, \sigma_\rho, \sigma_\omega)$ are kept fixed at their posterior median values by the MCMC estimation.

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