

Environmental enforcement federalism

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ONLINE APPENDIX

Appendix A. Proof of Lemma 1

Firm i 's profit maximization problem reads:

$$\begin{aligned} \max_{e_i, z_i} \quad & \mu_i e_i^{1/2} - tz_i - \alpha \theta_i (t + r)(e_i - z_i) \\ \text{subject to} \quad & z_i \geq 0, \\ & z_i \leq e_i. \end{aligned}$$

The Lagrangian is

$$L_i = \mu_i e_i^{1/2} - tz_i - \alpha \theta_i (t + r)(e_i - z_i) + \lambda_1 (z_i - 0) + \lambda_2 (e_i - z_i).$$

The first-order conditions are given by:

$$\frac{\partial L_i}{\partial e_i} = \frac{1}{2} \mu_i e_i^{-1/2} - \alpha \theta_i (t + r) + \lambda_2 = 0,$$

$$\frac{\partial L_i}{\partial z_i} = -t + \alpha \theta_i (t + r) + \lambda_1 = 0.$$

With the assumption $0 \leq \theta_i < \frac{t}{t+r}$ for $i = 1, 2$, $\lambda_1 > 0$, therefore, $z_i^* = 0$. We then obtain $e_i^* = \left(\frac{\mu_i}{2\alpha\theta(t+r)} \right)^2$, resulting in firm profit $\pi_i^* = \frac{\mu_i^2}{4\alpha\theta(t+r)}$. ■

Appendix B. Proof of Proposition 1

Using α^{C*} obtained in Lemma 2 and α_i^{D*} obtained in Lemma 3, we construct the following:

$$\frac{\alpha^{C*}}{\alpha_i^{D*}} = \left(\frac{(\lambda_1 + \lambda_2) \left(\frac{\mu_i^2}{(\theta_i^C)^2} + \frac{\mu_j^2}{(\theta_j^C)^2} \right)^2}{\frac{\mu_i^2}{\theta_i^C} + \frac{\mu_j^2}{\theta_j^C}} \right)^{1/3} \frac{\theta_i^D}{\lambda_i^{1/3} (\mu_i^2 + \frac{\mu_j^2 \lambda_i^2}{\lambda_j^2})^{1/3}} \text{ for } i, j = 1, 2, \text{ and } i \neq j. \quad (\text{A1})$$

We start with the case in which there is full symmetry across regions and agencies – i.e., $\lambda_1 = \lambda_2$, $\mu_1 = \mu_2$, and $\theta_i^C = \theta_i^D = \theta_j^C = \theta_j^D$, where $i, j = 1, 2$ and $i \neq j$. Equation (A1) becomes $\frac{\alpha^{C*}}{\alpha_i^{D*}} = \sqrt[3]{2} > 1$, where $i = 1, 2$.

Then, we consider the case in which there is symmetry across regions and agencies except for λ – i.e., $\lambda_1 \neq \lambda_2$, $\mu_1 = \mu_2$, and $\theta_i^C = \theta_i^D = \theta_j^C = \theta_j^D$, where $i, j = 1, 2$ and $i \neq j$. Equation (A1) can be written as

$$\frac{\alpha^{C*}}{\alpha_i^{D*}} = \left[\frac{2(1 + \frac{\lambda_j}{\lambda_i})}{\frac{\lambda_j^2}{\lambda_i^2} + 1} \right]^{\frac{1}{3}}.$$

When $\frac{\lambda_i}{\lambda_j} > 1.5$, $\alpha_i^{D*} > \alpha^{C*} > \alpha_j^{D*}$, where $i, j = 1, 2$ and $i \neq j$. And if $\frac{\lambda_i}{\lambda_j} < 1.5$, $\alpha_i^{D*} < \alpha^{C*}$, $i = 1, 2$.

We then turn to the case in which there is symmetry across regions and agencies except for μ – i.e., $\lambda_1 = \lambda_2$, $\mu_1 \neq \mu_2$, and $\theta_i^C = \theta_i^D = \theta_j^C = \theta_j^D$, where $i, j = 1, 2$ and $i \neq j$. Equation (A1) is reduced to $\frac{\alpha^{C*}}{\alpha_i^{D*}} = \sqrt[3]{2} > 1$, where $i = 1, 2$.

At last, we consider the case in which there is asymmetry in inspection effectiveness θ , either across regions or across agencies. If there is asymmetry in inspection effectiveness across regions – i.e., $\lambda_1 = \lambda_2$, $\mu_1 \neq \mu_2$, $\theta_i^C = \theta_i^D$, and $\theta_i \neq \theta_j$, where $i, j = 1, 2$ and $i \neq j$. Equation (A1) can be written as

$$\frac{\alpha^{C*}}{\alpha_i^{D*}} = \left(\frac{(\theta_i^2 + \theta_j^2)^2}{(\theta_i + \theta_j)\theta_j^3} \right)^{\frac{1}{3}}.$$

When $\frac{\theta_j}{\theta_i} > 2.2$, $\alpha_j^{D*} < \alpha^{C*} < \alpha_i^{D*}$. Otherwise, when $\frac{\theta_j}{\theta_i} < 2.2$, $\frac{\alpha^{C*}}{\alpha_i^{D*}} > 1$, $i = 1, 2$.

If there is asymmetry in inspection effectiveness across agencies – i.e., $\lambda_1 = \lambda_2$, $\mu_1 \neq \mu_2$, $\theta_i^C = \theta_j^C$, $\theta_i^D = \theta_j^D$, and $\theta^C \neq \theta^D$, where $i, j = 1, 2$ and $i \neq j$. Equation (A1) becomes

$$\frac{\alpha^{C*}}{\alpha_i^{D*}} = \sqrt[3]{2} \frac{\theta_i^D}{\theta_i^C}.$$

When $\frac{\theta^C}{\theta^D} < \sqrt[3]{2}$, $\frac{\alpha^{C*}}{\alpha_i^{D*}} > 1$; when $\frac{\theta^C}{\theta^D} > \sqrt[3]{2}$, $\frac{\alpha^{C*}}{\alpha_i^{D*}} < 1$. ■

Appendix C. Proof of Proposition 2

Based on Lemma 3, the total surplus of both regions is given by

$$W_1^{D*} + W_2^{D*} = \frac{3 \left(\frac{\mu_1^2}{\lambda_1} + \frac{\mu_2^2}{\lambda_2} \right) - \frac{\lambda_2 \mu_1^2}{\lambda_1^2} - \frac{\lambda_1 \mu_2^2}{\lambda_2^2}}{16 \left(\frac{\mu_1^2}{\lambda_1} + \frac{\mu_2^2}{\lambda_2} \right)^{1/3}}.$$

Using the total surplus W^{C*} obtained in Lemma 2 and $W_1^{D*} + W_2^{D*}$ obtained above, we construct the following ratio:

$$\frac{W^{C*}}{W_1^{D*} + W_2^{D*}} = \frac{\frac{3}{16} \left(\frac{\left(\frac{\mu_1^2}{\theta_1^C} + \frac{\mu_2^2}{\theta_2^C} \right)^4}{\lambda^C \left(\frac{\mu_1^2}{(\theta_1^C)^2} + \frac{\mu_2^2}{(\theta_2^C)^2} \right)^2} \right)^{1/3}}{\frac{3 \left(\frac{\mu_1^2}{\lambda_1} + \frac{\mu_2^2}{\lambda_2} \right) - \frac{\lambda_2 \mu_1^2}{\lambda_1^2} - \frac{\lambda_1 \mu_2^2}{\lambda_2^2}}{16 \left(\frac{\mu_1^2}{\lambda_1} + \frac{\mu_2^2}{\lambda_2} \right)^{1/3}}}. \quad (\text{A2})$$

We start with the case in which there is full symmetry across regions and agencies – i.e., $\lambda_1 = \lambda_2$, $\mu_1 = \mu_2$, and $\theta_i^C = \theta_i^D = \theta_j^C = \theta_j^D$, where $i, j = 1, 2$ and $i \neq j$. The total surplus under the centralized enforcement policy is at least as high as that of the decentralized framework simply because $\alpha^{C*} = \argmax_{\alpha^C} W^C(\alpha^C)$. Thus, $\frac{W^{C*}}{W_1^{D*} + W_2^{D*}} \geq 1$.

Then, we consider the case in which there is symmetry across agencies and regions except for λ – i.e., $\lambda_1 \neq \lambda_2$, $\mu_1 = \mu_2$, and $\theta_i^C = \theta_i^D = \theta_j^C = \theta_j^D$, where $i, j = 1, 2$ and $i \neq j$. Equation (A2) can be written as

$$\frac{W^{C*}}{W_1^{D*} + W_2^{D*}} = \frac{3}{\left(1 + \frac{\lambda_2}{\lambda_1}\right)^{1/3}} \frac{2^{2/3} \left(1 + \frac{\lambda_1^2}{\lambda_2^2}\right)^{1/3}}{\left(3 + \frac{3\lambda_1}{\lambda_2} - \frac{\lambda_2}{\lambda_1} - \frac{\lambda_1^2}{\lambda_2^2}\right)}.$$

It turns out that $\frac{W^{C*}}{W_1^{D*} + W_2^{D*}} > 1$ always holds, with the maximum achieved at $\lambda_1 = \lambda_2$.

We then turn to the case in which there is symmetry across agencies and regions except for μ – i.e., $\lambda_1 = \lambda_2$, $\mu_1 \neq \mu_2$, and $\theta_i^C = \theta_i^D = \theta_j^C = \theta_j^D$, where $i, j = 1, 2$ and $i \neq j$. Equation

(A2) is reduced to

$$\frac{W^{C*}}{W_1^{D*} + W_2^{D*}} = \frac{3}{2\sqrt[3]{2}} > 1.$$

Centralized enforcement is always preferred.

Finally, we consider the scenario in which there is only heterogeneity in θ across regions – i.e., $\lambda_1 = \lambda_2$, $\mu_1 = \mu_2$, and $\theta_i^C \neq \theta_j^C$, where $i, j = 1, 2$ and $i \neq j$. Equation (A2) becomes

$$\frac{W^{C*}}{W_1^{D*} + W_2^{D*}} = \frac{3 \left(1 + \frac{\theta_1}{\theta_2}\right)^{4/3}}{4 \left(1 + \frac{\theta_1^2}{\theta_2^2}\right)^{2/3}}.$$

$\frac{W^{C*}}{W_1^{D*} + W_2^{D*}} < 1$ iff $\frac{\theta_i}{\theta_j} > 3.4$, where $i, j = 1, 2$ and $i \neq j$. ■

Appendix D. Proof of Proposition 3

The total surplus of region i under centralized enforcement is given by

$$W_i^C = \pi_i(\alpha^C) - \lambda_i(e_1(\alpha^C) + e_2(\alpha^C))^2,$$

where $i = 1, 2$. Given α^{C*} obtained in Lemma 2, we obtain the following:

$$W_i^{C*} = \left[\frac{\frac{\mu_i^2}{\theta_i^C} + \frac{\mu_j^2}{\theta_j^C}}{(\lambda_1 + \lambda_2) \left(\frac{\mu_i^2}{(\theta_i^C)^2} + \frac{\mu_j^2}{(\theta_j^C)^2} \right)^2} \right]^{1/3} \left(\frac{\mu_i^2}{4\theta_i^C} - \frac{\lambda_i \left(\frac{\mu_i^2}{\theta_i^C} + \frac{\mu_j^2}{\theta_j^C} \right)}{16(\lambda_1 + \lambda_2)} \right),$$

where $i, j = 1, 2$ and $i \neq j$. The surplus of region i under decentralized enforcement is given in Lemma 3.

We compare the total surplus of region i under centralized and decentralized enforcement by starting with the scenario in which $\lambda_1 \neq \lambda_2$, $\mu_1 = \mu_2$, and $\theta_i^C = \theta_i^D = \theta_j^C = \theta_j^D$, where

$i, j = 1, 2$ and $i \neq j$. We construct the following ratio:

$$\frac{W_i^{D*}}{W_i^{C*}} = \frac{\left(3 - \frac{\lambda_i^2}{\lambda_j^2}\right) \left(1 + \frac{\lambda_i}{\lambda_j}\right)^{4/3}}{2^{2/3} \left(2 + \frac{\lambda_i}{\lambda_j}\right) \left[\frac{\lambda_i}{\lambda_j} \left(1 + \frac{\lambda_i^2}{\lambda_j^2}\right)\right]^{1/3}}.$$

$$\frac{W_i^{D*}}{W_i^{C*}} > 1 \text{ iff } \frac{\lambda_j}{\lambda_i} > 1.18.$$

Then, we consider the scenario in which $\lambda_1 = \lambda_2$, $\mu_1 \neq \mu_2$, and $\theta_i^C = \theta_i^D = \theta_j^C = \theta_j^D$, where $i, j = 1, 2$ and $i \neq j$. $\frac{W_i^{D*}}{W_i^{C*}} > 1$ iff $\frac{\mu_i}{\mu_j} > 2.04$.

Finally, we consider the scenario in which $\lambda_1 = \lambda_2$, $\mu_1 = \mu_2$, and $\theta_i^C \neq \theta_j^C$, where $i, j = 1, 2$ and $i \neq j$. The ratio becomes:

$$\frac{W_i^{D*}}{W_i^{C*}} = \frac{\left(1 + \frac{\theta_i^2}{\theta_j^2}\right)^{2/3}}{\left(1 + \frac{\theta_i}{\theta_j}\right)^{1/3} \left(\frac{9}{4} + \frac{\theta_i}{4\theta_j}\right)}.$$

$$\frac{W_i^{D*}}{W_i^{C*}} > 1 \text{ iff } \frac{\theta_i}{\theta_j} > 3.1.$$

To sum up, if there is high heterogeneity across regions, the total surplus of the region that has a lower λ , a higher μ , or a higher θ increases relative to its total surplus under centralized enforcement. Meanwhile, the total surplus of the other region decreases, since the aforementioned conditions cannot be satisfied for both regions at the same time.

Appendix E. Proof of Proposition 4

To evaluate enforcement structures from the perspective of the central authority, we compare W^{C*} obtained in Lemma 2 with $W^D(\lambda^C)$, which is given by

$$W^D(\lambda^C) = \sum_{i=1}^2 \pi_i(\alpha_i^D) - \lambda^C(e_1(\alpha_1^D) + e_2(\alpha_2^D))^2.$$

Given $\lambda_1 = \lambda_2 = \lambda_0$, $\mu_1 = \mu_2$, $\theta_1 \neq \theta_2$, and $\theta_i^C = \theta_i^D$. For $\frac{W^D(\lambda^C)}{W^{C*}} > 1$ to be satisfied, the following inequality should hold:

$$\left(\frac{8 - 2 \left(\frac{\lambda_0}{\lambda^C} \right)^{-1}}{3 \left(\frac{2\lambda_0}{\lambda^C} \right)^{1/3}} \right)^{3/2} > 1 + \frac{2 \frac{\theta_1}{\theta_2}}{1 + \left(\frac{\theta_1}{\theta_2} \right)^2}. \quad (\text{A3})$$

The left-hand side of Inequality (A3) is inverted U-shaped in $\frac{\lambda_0}{\lambda^C}$, and it achieves its maximum of 1.36 at $\frac{\lambda_0}{\lambda^C} = 1$. The right-hand side of Inequality (A3) reaches its maximum value of 2 when $\frac{\theta_1}{\theta_2} = 1$. Therefore, for this inequality to hold, the value of $\frac{\theta_i}{\theta_j}$ should not be close to 1. To further support this inequality, it is better that $2\lambda_0$ be close to λ^C . To be more specific, $\frac{\lambda_0}{\lambda^C} > 0.38$ is required to ensure the left-hand side larger than 1, meaning that regional levels of environmental concern should be close to that of the central authority.

To evaluate enforcement structures from the perspective of regional authorities, we compare $\sum_{i=1}^2 W_i^D$ obtained in Lemma 3 with $\sum_{i=1}^2 W_i^C(\lambda_i)$, which is given by

$$\sum_{i=1}^2 W_i^C(\lambda_i) = \sum_{i=1}^2 \pi_i(\alpha^C) - \sum_{i=1}^2 \lambda_i (e_1(\alpha^C) + e_2(\alpha^C))^2.$$

Given $\lambda_1 = \lambda_2 = \lambda_0$, $\mu_1 = \mu_2$, $\theta_1 \neq \theta_2$, and $\theta_i^C = \theta_i^D$. For $\frac{\sum_{i=1}^2 W_i^C(\lambda_i)}{\sum_{i=1}^2 W_i^D} < 1$ to be satisfied, the following inequality should hold:

$$\left(\frac{2^{2/3}}{\left(\frac{\lambda_0}{\lambda^C} \right)^{1/3} \left(2 - \frac{\lambda_0}{\lambda^C} \right)} \right)^{3/2} > 1 + \frac{2 \frac{\theta_1}{\theta_2}}{1 + \left(\frac{\theta_1}{\theta_2} \right)^2} \quad (\text{A4})$$

The right-hand side of Inequality (A4) is the same as that of Inequality (A3), and the left-hand side is U-shaped in $\frac{\lambda_0}{\lambda^C}$ and is always larger than 1. Thus, for Inequality (A4) to hold, either of the following conditions needs to be satisfied: 1. $\frac{\lambda_0}{\lambda^C}$ is small, suggesting that λ_0 is low enough relative to λ^C ; 2. the value of $\frac{\theta_i}{\theta_j}$ is not close to 1. Notice that if either $\frac{\lambda_0}{\lambda^C}$ or $\frac{\theta_i}{\theta_j}$ is small enough, Inequality (A4) holds.

Appendix F. Proof of Proposition 5

In stage 2, firm i 's decisions remain unchanged. In stage 1, responsible authorities choose the inspection intensity to maximize social welfare. Under centralized enforcement, the central authority chooses the inspection intensity α^C to maximize the total surplus of both regions.

$$\max_{\alpha^C} -\lambda^C(e_1(\alpha^C) + e_2(\alpha^C))^2 - 2\phi^C \alpha^C.$$

Substituting the firm's reaction function obtained in Lemma 1 into the welfare maximization problem of the central authority, the central authority's problem becomes:

$$\max_{\alpha^C} -\frac{\lambda^C}{[2\alpha^C(t+r)]^4} \left(\frac{\mu_1^2}{(\theta_1^C)^2} + \frac{\mu_2^2}{(\theta_2^C)^2} \right)^2 - 2\phi^C \alpha^C$$

Taking the first-order condition and solving for α^C , we obtain the equilibrium inspection intensity under centralized enforcement:

$$\begin{aligned} \alpha^{C*} &= \left[\frac{\lambda^C}{8(t+r)^4 \phi^C} \left(\frac{\mu_1^2}{(\theta_1^C)^2} + \frac{\mu_2^2}{(\theta_2^C)^2} \right)^2 \right]^{1/5}, \\ W^{C*} &= -\frac{5}{2} \left[\frac{\lambda^C (\phi^C)^4}{8(t+r)^4} \left(\frac{\mu_1^2}{(\theta_1^C)^2} + \frac{\mu_2^2}{(\theta_2^C)^2} \right)^2 \right]^{1/5}. \end{aligned} \tag{A5}$$

Note that the total surplus of region i under centralized enforcement is given by

$$\begin{aligned} W_i^{C*} &= -\lambda_i(e_1(\alpha^{C*}) + e_2(\alpha^{C*}))^2 - \phi^C \alpha^{C*} \\ &= -\left[\frac{\lambda^C (\phi^C)^4}{8(t+r)^4} \left(\frac{\mu_1^2}{(\theta_1^C)^2} + \frac{\mu_2^2}{(\theta_2^C)^2} \right)^2 \right]^{1/5} \left(1 + \frac{\lambda_i}{2\lambda^C} \right). \end{aligned}$$

Under decentralized enforcement, the local authority of region i chooses the inspection intensity α_i^D to maximize the total surplus of its region. Region i 's welfare maximization problem

reads:

$$\max_{\alpha_i^D} -\lambda_i(e_1(\alpha_1^D) + e_2(\alpha_2^D))^2 - \phi_i \alpha_i^D,$$

which is then transformed to the following after the substitution of equilibrium firm emissions from Lemma 1:

$$\max_{\alpha_i^D} -\frac{\lambda_i}{[2(t+r)]^4} \left(\frac{\mu_1^2}{\alpha_1^2(\theta_1^D)^2} + \frac{\mu_2^2}{\alpha_2^2(\theta_2^D)^2} \right)^2 - \phi_i \alpha_i^D.$$

Taking the first-order condition to obtain the best response function $\alpha_i^D(\alpha_j^D)$, where $i, j = 1, 2$ and $i \neq j$. Solving the system of best response functions, we obtain the following equilibrium outcomes:

$$\alpha_i^{D*} = \left(\frac{\lambda_i \mu_i^4}{4(t+r)^4(\theta_i^D)^4 \phi_i} \left[1 + \left(\frac{\lambda_i \theta_i^D \mu_j \phi_j}{\lambda_j \theta_j^D \mu_i \phi_i} \right)^{2/3} \right] \right)^{1/5},$$

$$W_i^{D*} = -\frac{1}{4} \left[5 + \left(\frac{\lambda_i \theta_i^D \mu_j \phi_j}{\lambda_j \theta_j^D \mu_i \phi_i} \right)^{2/3} \right] \left(\frac{\lambda_i \mu_i^4 \phi_i^4}{4(t+r)^4(\theta_i^D)^4} \left[1 + \left(\frac{\lambda_i \theta_i^D \mu_j \phi_j}{\lambda_j \theta_j^D \mu_i \phi_i} \right)^{2/3} \right] \right)^{1/5}.$$

Comparing the aggregate welfare of both regions under centralized and decentralized enforcement, we observe both the heterogeneity effect across regions and the efficiency effect across governmental levels.

$$W^C < W^D \Leftrightarrow \frac{\mu_i}{\mu_j} > 4.1 \text{ or } \frac{\theta_i}{\theta_j} > 4.1 \text{ or } \frac{\theta^D}{\theta^C} > 1.25 \text{ or } \frac{\phi^C}{\phi_i} > 1.11$$

We then consider whether shifting from centralized to decentralized enforcement constitutes Pareto improvement. If there is heterogeneity across regions in λ , or μ , or θ ($\theta_i \neq \theta_j$), one region is better off and the other is worse off when the following conditions hold:

$$W_j^D < W_i^C < W_i^D \Leftrightarrow \frac{\lambda_j}{\lambda_i} > 1.18 \text{ or } \frac{\mu_i}{\mu_j} > 4.1 \text{ or } \frac{\theta_i}{\theta_j} > 1.2;$$

otherwise, $W_i^C > W_i^D$ for $i, j = 1, 2$. We also consider whether shifting from centralized to decentralized enforcement constitutes Pareto improvement when there is only heterogeneity

across regions in θ ($\theta^C \neq \theta^D$) or ϕ :

$$W_i^C < W_i^D \Leftrightarrow \frac{\theta^D}{\theta^C} > 1.05 \text{ or } \frac{\phi^C}{\phi_i} > 1.05 \text{ for } i, j = 1, 2.$$

Appendix G. Proof of Proposition 6

We compare the impact of centralized and decentralized enforcement on welfare from the perspective of both the central and regional authorities.

First, we consider the comparison from the perspective of the central authority. W^{C*} is given by Equation (A5), and the total surplus under decentralized enforcement evaluated by environmental concerns of the central authority is given by

$$\begin{aligned} W^D(\lambda^C) &= -\lambda^C(e_1(\alpha_1^D) + e_2(\alpha_2^D))^2 - \phi_1\alpha_1^D - \phi_2\alpha_2^D \\ &= -\left(\frac{\lambda_1\mu_1^4\phi_1^4}{4(t+r)^4(\theta_1^D)^4} \left[1 + \left(\frac{\lambda_1\theta_1^D\mu_2\phi_2}{\lambda_2\theta_2^D\mu_1\phi_1}\right)^{2/3}\right]\right)^{1/5} \left(1 + \frac{\lambda^C}{4\lambda_2} \left[1 + \left(\frac{\lambda_1\theta_1^D\mu_2\phi_2}{\lambda_2\theta_2^D\mu_1\phi_1}\right)^{2/3}\right]\right) \\ &\quad - \left(\frac{\lambda_2\mu_2^4\phi_2^4}{4(t+r)^4(\theta_2^D)^4} \left[1 + \left(\frac{\lambda_2\theta_2^D\mu_1\phi_1}{\lambda_1\theta_1^D\mu_2\phi_2}\right)^{2/3}\right]\right)^{1/5}. \end{aligned}$$

Note that μ_i and $1/\theta_i$ always appear together, hence varying μ is the same as varying $1/\theta$ ($\theta_i \neq \theta_j$) for the welfare comparison, where $i, j = 1, 2$. The same logic applies to θ ($\theta^C \neq \theta^D$ and $\theta_i = \theta_j$) and $1/\phi$; varying θ across tiers of government ($\theta_1 = \theta_2$ and $\theta_i^D \neq \theta^C$) is exactly the same as varying $1/\phi$ ($\phi_1 = \phi_2$ and $\phi_i \neq \phi^C$). Hence, we conduct the welfare comparison in two dimensions: 1. heterogeneity across regions, and 2. heterogeneity across agencies.

First, we consider heterogeneity across regions in μ – i.e., $\mu_1 \neq \mu_2$, $\lambda_1 = \lambda_2 = \lambda_0$, $\theta_1 = \theta_2$ and $\phi_1 = \phi_2 = \phi^C$. For $W^D(\lambda^C) > W^{C*}$ to hold, the following condition must be satisfied:

$$5 \left(1 + \frac{\mu_2^2}{\mu_1^2}\right)^{2/5} - 2^{6/5} \left(\frac{\lambda_0}{\lambda^C}\right)^{1/5} \left[1 + \left(\frac{\mu_2}{\mu_1}\right)^{2/3}\right]^{1/5} \left(1 + \frac{\lambda^C}{4\lambda_0} \left[1 + \left(\frac{\mu_2}{\mu_1}\right)^{2/3}\right] + \left(\frac{\mu_2}{\mu_1}\right)^{4/5}\right) > 0.$$

Note that this left-hand side of the inequation increases in $\frac{\lambda_0}{\lambda^C}$ and is U-shaped in $\frac{\mu_2}{\mu_1}$. If

$\frac{\lambda_0}{\lambda^C} = \frac{1}{2}$, then $W^D(\lambda^C) > W^{C*} \Leftrightarrow \frac{\mu_i}{\mu_j} > 1.82$. We conclude that when $\frac{\lambda_0}{\lambda^C}$ grows large and high heterogeneity in μ exists, or when high heterogeneity in θ across regions exists, the central authority prefers decentralized enforcement.

We then consider heterogeneity between the two tiers of government in θ – i.e., $\theta^C \neq \theta^D$, $\lambda_1 = \lambda_2 = \lambda_0$, $\mu_1 = \mu_2$, $\theta_1 = \theta_2$, and $\phi_1 = \phi_2 = \phi^C$. For $W^D(\lambda^C) > W^{C*}$ to hold, it must be that:

$$\frac{5}{2} - \left(\frac{\theta^C}{\theta^D}\right)^{4/5} \left(\frac{\lambda_0}{\lambda^C}\right)^{1/5} \left(2 + \frac{\lambda^C}{2\lambda_0}\right) > 0.$$

Obviously, the left-hand side of the inequation decreases in $\frac{\theta^C}{\theta^D}$ and increases in $\frac{\lambda_0}{\lambda^C}$ when $\lambda_0 < \lambda^C$. Thus, for decentralized enforcement to be preferred, it must be that regions provide more effective inspections and local levels of environmental concern are also high.

Then we conduct the welfare comparison from the perspective of regional authorities. The total surplus of both regions under centralized enforcement evaluated by regional environmental concerns is given by

$$\sum_{i=1}^2 W_i^C(\lambda_i) = - \left(2 + \frac{\lambda_1 + \lambda_2}{2\lambda^C}\right) \left[\frac{\lambda^C(\phi^C)^4}{8(t+r)^4} \left(\frac{\mu_1^2}{(\theta_1^D)^2} + \frac{\mu_2^2}{(\theta_2^D)^2} \right)^2 \right]^{1/5}.$$

The total surplus of both regions under decentralized enforcement is given by

$$\begin{aligned} \sum_{i=1}^2 W_i^D &= -\frac{1}{4^{6/5}} \left(\frac{\lambda_1 \mu_1^4 \phi_1^4}{(t+r)^4 (\theta_1^D)^4} \left[1 + \left(\frac{\lambda_1 \theta_1^D \mu_2 \phi_2}{\lambda_2 \theta_2^D \mu_1 \phi_1} \right)^{2/3} \right] \right)^{1/5} \left[5 + \left(\frac{\lambda_1 \theta_1^D \mu_2 \phi_2}{\lambda_2 \theta_2^D \mu_1 \phi_1} \right)^{2/3} \right] \\ &\quad - \frac{1}{4^{6/5}} \left(\frac{\lambda_2 \mu_2^4 \phi_2^4}{(t+r)^4 (\theta_2^D)^4} \left[1 + \left(\frac{\lambda_2 \theta_2^D \mu_1 \phi_1}{\lambda_1 \theta_1^D \mu_2 \phi_2} \right)^{2/3} \right] \right)^{1/5} \left[5 + \left(\frac{\lambda_2 \theta_2^D \mu_1 \phi_1}{\lambda_1 \theta_1^D \mu_2 \phi_2} \right)^{2/3} \right] \end{aligned}$$

From the perspective of regional authorities, if there is only heterogeneity across regions in μ , then for $\sum_{i=1}^2 W_i^C(\lambda_i) < \sum_{i=1}^2 W_i^D$ to hold, the following condition must hold:

$$2^{9/5} \left(1 + \frac{\mu_1^2}{\mu_2^2} \right)^{2/5} \frac{2 + \frac{\lambda_0}{\lambda^C}}{\left(\frac{\lambda_0}{\lambda^C}\right)^{1/5}} - \left[5 + \left(\frac{\mu_2}{\mu_1} \right)^{2/3} \right] \left(\frac{\mu_1}{\mu_2} \right)^{4/5} \left[1 + \left(\frac{\mu_2}{\mu_1} \right)^{2/3} \right]^{1/5} - \left[5 + \left(\frac{\mu_1}{\mu_2} \right)^{2/3} \right] \left[1 + \left(\frac{\mu_1}{\mu_2} \right)^{2/3} \right]^{1/5} < 0.$$

It turns out that this inequality holds when $\frac{\mu_i}{\mu_j}$ is large or $\frac{\theta_i}{\theta_j}$ is large.

We then consider the scenario in which there is only heterogeneity between the two tiers of agencies in θ (or ϕ)—that is, $\theta^C \neq \theta^D$. Then,

$$\sum_{i=1}^2 W_i^C(\lambda_i) < \sum_{i=1}^2 W_i^D \Leftrightarrow 2 + \frac{\lambda_0}{\lambda^C} - 3 \left(\frac{\theta^C}{\theta^D} \right)^{4/5} \left(\frac{\lambda_0}{\lambda^C} \right)^{1/5} > 0.$$

Therefore, either the value of $\frac{\lambda_0}{\lambda^C}$ goes to the extreme value, the value of $\frac{\theta^C}{\theta^D}$ is small, or the value of $\frac{\phi_i}{\phi^C}$ is small. ■