Online Appendix

Analysis of sustainable development: an overlapping generations model of agricultural households in developing countries'

Numerical Illustration of Policy Scenarios

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This online appendix complements the paper by providing a calibrated numerical illustration of the analytical results derived in Section 4 of the main text. All cross-references of the form "equation (n)" or "Section n" below refer to the main text unless otherwise stated.

1 Numerical illustration of policy scenarios

To provide a concrete illustration of the analytical results derived in Sections 4.3–4.5 of the main text, we specialize the general production function and examine the model in a calibrated numerical setting. This section compares four policy scenarios to show how relaxing the credit constraint moves the economy from the non-separable regime to the separable one. The exercise is not intended to generate welfare rankings across steady states – since, as shown below, the government quota fixes the long-run allocation – but rather to

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quantify the price distortion created by the credit friction and identify the policy threshold at which it disappears.

Functional forms. We adopt three standard specifications. Production takes the Cobb-Douglas form

$$f(k_t, e_t) = A\, k_t^{\theta}\, e_t^{\zeta}, \tag{A1}$$

where θ is the capital share, ζ the resource share, and $1 - \theta - \zeta$ the residual labor share. The renewable resource follows the logistic growth function (Verhulst, 1838; Clark, 2010)

$$G(X_t) = \gamma\, X_t \left(1 - \frac{X_t}{M}\right). \tag{A2}$$

Preferences are logarithmic:

$$V = \underbrace{\ln c_t + \eta\, \ln \ell_t + \mu\, \ln X_t}_{\text{young}} + \beta\, \underbrace{\left[\ln d_{t+1} + \mu\, \ln X_{t+1}\right]}_{\text{retired}} \tag{A3}$$

where $\beta \in (0, 1)$ is the subjective discount factor defined in Equation (1) of the main text. The biomass stock X_t is inherited from the previous generation and enters utility as a public good; X_{t+1} is determined endogenously through Equation (41) of the main text. The parameter μ captures the intrinsic value assigned to environmental quality in both periods of life.

Calibration and identification of parameters. The calibration in Table A1 is stylized and intended to reproduce plausible magnitudes for a smallholder agricultural economy rather than to estimate country-specific structural parameters. The capital share $\theta = 0.30$ is consistent with aggregate income-share evidence for developing economies (Gollin, 2002), while the resource share $\zeta = 0.20$ is set to reflect the quantitative role of natural-resource inputs in agricultural production (Barbier, 1999). The discount factor $\beta = 0.50$ corresponds to a 30-year generational horizon (De La Croix and Michel, 2002), and the leisure weight

$\eta = 0.50$ is chosen to deliver an interior labor supply close to one-half of the time endowment, in line with agricultural household evidence (Singh et al., 1986; Benjamin, 1992). The environmental preference $\mu = 0.10$ is calibrated in line with the renewable-resource literature (Jouvet and Rotillon, 2005) and assigns a positive but non-dominant role to environmental quality. The intrinsic growth rate $\gamma = 0.50$ lies within standard values used in applications to renewable resources (Schaefer, 1957; Clark, 2010).

Table A1. Calibration parameters, status, and references

Parameter	Symbol	Value	Status	Reference
Capital share	θ	0.30	Literature-anchored	Gollin (2002)
Resource share	ζ	0.20	Stylized calibration	Barbier (1999)
Discount factor	β	0.50	Generational target	De La Croix and Michel (2002)
Leisure weight	η	0.50	Stylized calibration	Singh et al. (1986); Benjamin (1992)
Environmental pref.	μ	0.10	Stylized calibration	Jouvet and Rotillon (2005)
Growth rate	γ	0.50	Literature-consistent	Clark (2010); Schaefer (1957)
Carrying capacity	M	100	Normalization	–
Titled land share	v	0.03	Conservative benchmark	Deininger et al. (2017); Honig (2022)
Landholding	A_f	2.0 ha	Literature-consistent	Lowder et al. (2016)
Quota endowment	$\bar{e}$	1.0	Normalization	–
Population	N	10	Normalization	–

The titled land share $v = 0.03$ is a conservative benchmark meant to represent a low-collateral rural environment rather than a country-specific estimate (Deininger et al., 2017; Honig, 2022). The landholding size $A_f = 2\,\text{ha}$ is consistent with the prevalence of small farms in low-income agriculture (Lowder et al., 2016). The carrying capacity $M = 100$ is a normalization, and the quota endowment $\bar{e} = 1$ together with the population $N = 10$ are selected to satisfy the sustainability condition and ensure an internally consistent numerical illustration.

Since several parameters are calibrated rather than estimated, the results should be interpreted as comparative statics across policy regimes. The purpose of the exercise is to isolate the price wedge induced by the credit constraint and to assess how financial relaxation affects the implementation of the ITQ mechanism.

Policy scenarios and threshold. The critical borrowing capacity at which the constraint ceases to bind is

$$B^{**} = \bar{e} \cdot f_e(k^*, \bar{e}). \tag{A4}$$

When $B_{\text{eff}} < B^{**}$, the shadow price satisfies $\lambda^* > 0$ and the resource price $\phi^* = B_{\text{eff}}/\bar{e}$ falls below the marginal product $f_e(\cdot)$. When $B_{\text{eff}} \geq B^{**}$, $\lambda^* = 0$ and the ITQ result of Jouvet and Rotillon (2005) is restored. Under the baseline calibration, $B^{**} \approx 0.119$. We compare four scenarios:

S1. *Baseline* ($v = 0.03$, $S = 0$, $B_{\text{eff}} = 0.06$): severely credit-constrained environment with very low formal land titling.

S2. *Credit subsidy* ($v = 0.03$, $S = 0.05$, $B_{\text{eff}} = 0.11$): targeted guarantee without land reform.

S3. *Land titling* ($v = 0.15$, $S = 0$, $B_{\text{eff}} = 0.30$): post-reform environment where land certification has substantially improved collateral access.

S4. *Full relaxation* ($v = 0.03$, $S = 0.20$, $B_{\text{eff}} = 0.26$): $\lambda^* = 0$.

Steady-state results. Table A2 reports the equilibrium outcomes. The main finding is a sharp variation in the *efficiency ratio* $\phi^*/f_e(\cdot)$. In the baseline (S1), the resource price reflects only half of its marginal product ($\phi^*/f_e(\cdot) = 0.50$), and the shadow price is large ($\lambda^* = 3.55$). A modest subsidy (S2) raises the ratio to 0.92. Land titling (S3) or full relaxation (S4) push B_{eff} above the threshold, restoring $\phi^*/f_e(\cdot) = 1$.

Table A2. Steady-state equilibrium across policy scenarios

	S1 Baseline	S2 Subsidy	S3 Titling	S4 Full
B_{eff}	0.060	0.110	0.300	0.260
Constraint binds	Yes	Yes	No	No
λ^*	3.553	0.306	0	0
ϕ^*	0.060	0.110	0.119	0.119
$f_e(k^*, \bar{e})$	0.119	0.119	0.119	0.119
$\phi^*/f_e(\cdot)$	0.503	0.921	1.000	1.000
q^*	0.060	0.110	0.119	0.119
k^*	0.179	0.179	0.179	0.179
y^*	0.597	0.597	0.597	0.597
c^*	0.279	0.279	0.279	0.279
d^*	0.139	0.139	0.139	0.139
X^*	72.36	72.36	72.36	72.36

By contrast, the real allocation variables (k^*, y^*, c^*, d^*, X^*) are identical across all scenarios. This is a structural consequence of the government quota: at steady state, $\tilde{E}^* = N\bar{e}$ from Equation (46) of the main text, so the biomass stock X^* is determined by $G(X^*) = N\bar{e}$ independently of the credit regime. The credit constraint therefore manifests entirely as a price distortion, not as a change in the long-run quantity extracted.

Figure A1 displays these results graphically. Panel (a) shows the decline of λ^* as B_{eff} rises, reaching zero at the threshold $B^{**} \approx 0.119$. Panel (b) plots ϕ^* against the constant marginal product $f_e(\cdot)$; the shaded region represents the efficiency wedge. Panel (c) traces the efficiency ratio $\phi^*/f_e(\cdot)$ and marks B^{**}. Panel (d) shows the quota price q^*, which tracks ϕ^* with a gap $\phi^* - q^* = \frac{U_{X_{t+1}}(d^*, X^*)}{U_{d_{t+1}}(d^*, X^*)}$ reflecting the internalization of the environmental externality as in Equation (40) of the main text. Under the logarithmic specification, this gap reduces to $\mu\, d^*/X^*$.

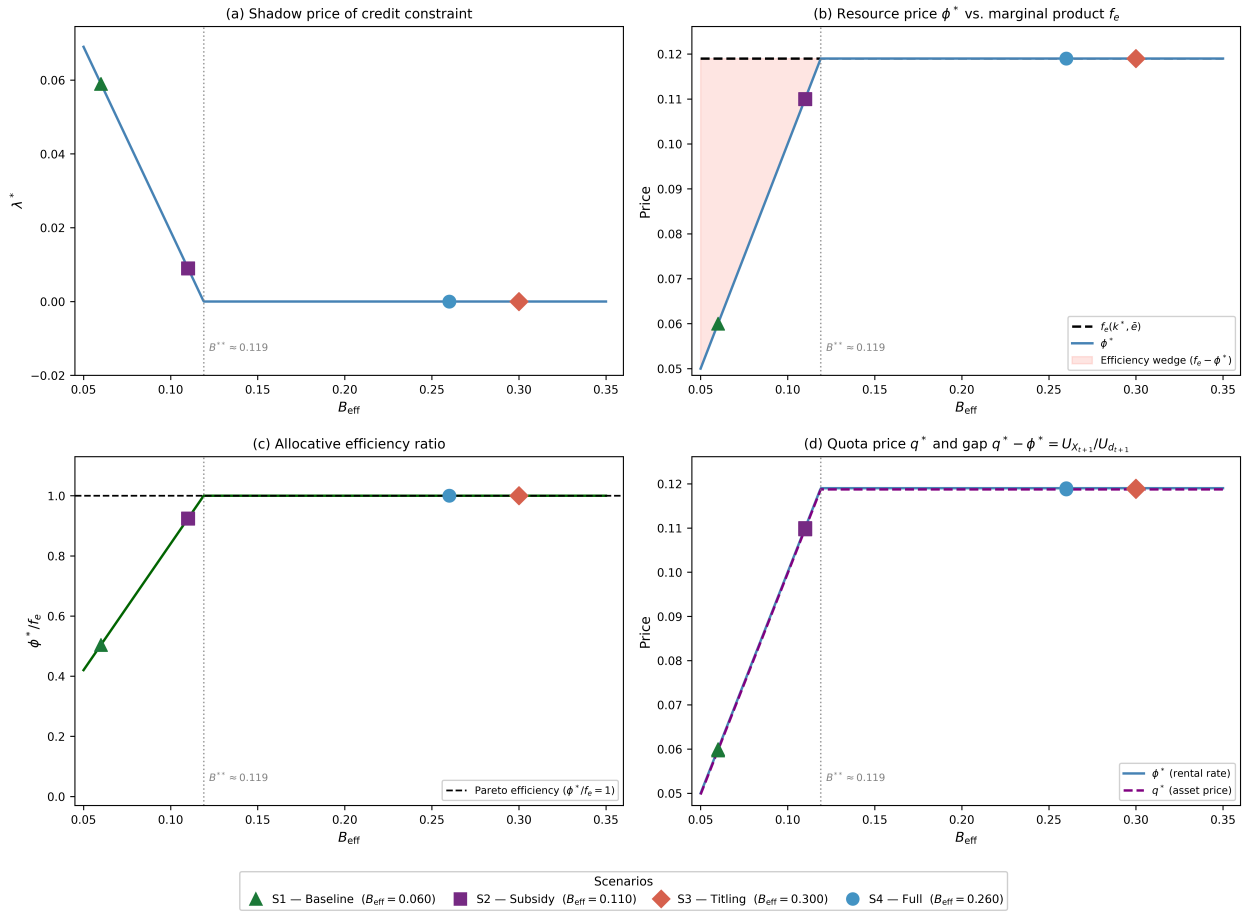


Figure A1. Steady-state equilibrium under varying credit access (B_{eff}).

Welfare interpretation: steady state versus transition. The invariance of c^*, d^*, and X^* means that the calibration cannot be read as showing steady-state welfare gains from credit relaxation. Instead, it illustrates the *price mechanism* through which the credit constraint prevents the ITQ system from achieving the social optimum. The welfare consequences emerge outside the steady-state comparison, along two channels.

First, the *transition path*. Relaxing the constraint raises e_t, temporarily increasing extraction: $\partial X_{t+1}/\partial z < 0$ from Equation (63) of the main text. This reduces the environmental component of retirement utility, $\mu \ln X_{t+1}$, even as consumption adjusts. The cost is transitional and dissipates as the economy converges to the new steady state.

Second, retirees' *portfolio allocation*. From Equation (44) of the main text, $d^* = R^* s^* + [\alpha^* \phi^* + (1 - \alpha^*) q^*] \tilde{e}$. A depressed ϕ^* alters the return differential between leasing

and selling quotas, modifying α^* as analyzed in Section 4.3 of the main text. While total d^* is pinned down by goods market clearing, the financing composition shifts between quota returns and capital returns.

Policy implications. Three conclusions follow. First, the constrained baseline confirms quantitatively that ITQs alone do not restore the social optimum when credit markets are imperfect: the resource price reflects only half of its marginal product. Second, the fiscal cost of closing the wedge is modest. The gap between the baseline ($B_{\text{eff}} = 0.06$) and the threshold ($B^{**} \approx 0.12$) corresponds to a collateral enhancement or guarantee of 0.06 per individual – comparable in magnitude to existing agricultural credit interventions (Guirkinger and Boucher, 2008; Banerjee et al., 2015). Third, the environmental cost of credit relaxation is mainly transitional: the short-run increase in extraction reduces X_{t+1} temporarily, but the long-run effect is to support a pricing regime that correctly internalizes both the productive and environmental value of the resource.

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