

The influence of online review expertise on performance of environmental enterprises

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Appendix A

This part provides an accessible reference point within our modeling framework, ensuring that the reader can follow the subsequent analysis without relying exclusively on external sources. We would like to clarify that our presentation of these two lemmas, while aligned in spirit with Papanastasiou and Savva (2017); Feldman *et al.* (2019), incorporates a simplified and additive utility specification, which allows for a more tractable and explicit derivation of the threshold condition for consumer purchasing behavior. In particular, the derivation of the threshold variable X and its implicit equation (below in Equation (A.2)) is presented in a more detailed and self-contained manner, which we believe enhances the clarity and pedagogical value of the model for readers.

Lemma A.1. *The mean value from “Updated Belief” q_u satisfies $\mathbb{E}q_u = q$, $\text{Var}(q_u) = \frac{m\beta}{\alpha + m\beta}\alpha^{-1}$.*

Proof: For Equation (2) when $t = 2$ we aim to calculate the expected value of u_{i2} for any visitor i , which is equivalent to calculating $\mathbb{E}_{\mathcal{Q}}\mathbb{E}(\hat{q}|\mathcal{Q}) = \mathbb{E}(q_u)$ and further the pdf of q_u . In the next, we will prove that $q_u \sim N(q, \frac{m\beta}{\alpha + m\beta}\alpha^{-1})$ viewed in the first period.

In fact, considering $R = \frac{\sum_{i=1}^m q_i}{m}$ and $q_i \sim N(\hat{q}, \beta^{-1})$, we have $R \sim N(\hat{q}, (m\beta)^{-1})$ ¹. Thus, combining with Equation (4) it easy to verify that:²

$$q_u|\hat{q} \sim N\left(\frac{\alpha}{\alpha + m\beta}q + \frac{m\beta}{\alpha + m\beta}\hat{q}, \left(\frac{m\beta}{\alpha + m\beta}\right)^2(m\beta)^{-1}\right). \quad (\text{A.1})$$

Noticing $\hat{q} = N(q, \alpha^{-1})$, we have:

$$\mathbb{E}q_u = \mathbb{E}(\mathbb{E}(q_u|\hat{q})) = \mathbb{E}\left(\frac{\alpha}{\alpha + m\beta}q + \frac{m\beta}{\alpha + m\beta}\hat{q}\right) = q,$$

and

$$\begin{aligned} \text{Var}(q_u) &= \mathbb{E}(\text{Var}(q_u|\hat{q})) + \text{Var}(\mathbb{E}(q_u|\hat{q})) \\ &= \left(\frac{m\beta}{\alpha + m\beta}\right)^2(m\beta)^{-1} + \left(\frac{m\beta}{\alpha + m\beta}\right)^2\alpha^{-1} = \frac{m\beta}{\alpha + m\beta}\alpha^{-1}. \end{aligned}$$

■

The results articulated in Lemma A.1 are consistent with the prevailing body of literature on social learning (refer to Cui *et al.* (2018); Belavina *et al.* (2020); Hu *et al.* (2016); and Feldman *et al.* (2019)). Specifically, the expectation of environmental quality within the posterior distribution does not differ from the prior expectation; however,

¹Noticing that from here we can only obtain $\mathbb{E}(R|\hat{q}) = \hat{q}$ and $\text{Var}(R|\hat{q}) = (m\beta)^{-1}$.

²We can also directly estimate $\mathbb{E}(q_u)$, $\text{Var}(q_u)$ using Equation (4) through $\mathbb{E}(R)$, $\text{Var}(R)$ via $\mathbb{E}(R|\hat{q}) = \hat{q}$ and $\text{Var}(R|\hat{q}) = (m\beta)^{-1}$.

there is a significant change in the variance associated with the environmental product. It is crucial to acknowledge that there are diverse evaluations of environmental products among various green consumers, yet the average expectation remains constant across individuals. Therefore, despite the product being promoted through a two-stage process, the overall evaluation of the environmental product remains stable among consumers across different temporal contexts. With respect to variance, an increase in the number of reviews is associated with a rise in the variance of both the pre- and post-distributions, although this variance consistently remains lower than that of the a priori assessment of environmental product quality. As the quantity of reviews increases, the posterior mean is likely to deviate further from the prior mean, which aligns with the observation that the preposterior distribution demonstrates greater variability. Consequently, concerning the influence of online media reviews, a limited number of reviews appears to enhance consumers' evaluations of the product. In contrast, an excessive number of reviews may lead to a decrease in the efficiency of the product evaluation process.

Lemma A.2. *For any given (p, q) a green consumer with type θ_i will purchase an environmental product in the first period provided $\theta_i \geq \bar{\theta}$, where $\bar{\theta}$ is the unique solution to the implicit equation:*

$$X + \delta \int_X^\infty \bar{\Phi}(s) ds = 0. \quad (\text{A.2})$$

Herein, $X = \frac{p - \bar{\theta} - q}{\sigma}$, $\sigma^2 = \frac{m\beta}{\alpha + m\beta} \alpha^{-1}$ and $m = 1 - G(\bar{\theta})$.

Proof: For consumer $\bar{\theta}$ who make no difference between the first and second stage, the threshold value satisfies:

$$\bar{\theta} + q - p = \delta \int_{p - \bar{\theta}}^\infty (\bar{\theta} + q_u - p) f(q_u; q, m) dq_u. \quad (\text{A.3})$$

We denote $f(q_u; q, m)$ as the corresponding pdf for q_u . In the second period, a consumer of type θ_i will visit only if his updated expected utility from visiting is positive. Thus, in the first period his expected utility from delaying purchase is:

$$\mathbb{E}[u_{i2}] = \mathbb{E}_{\mathcal{Q}} \mathbb{E}[u_{i2} | \mathcal{Q}] = \delta \int_{p - \theta_i}^\infty (\theta_i + q_u - p) f(q_u; q, m) dq_u. \quad (\text{A.4})$$

In the following discussion, we denote $f(s) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(s-q)^2}{2\sigma^2}}$. Thus, $f'(s) = -\frac{(s-q)e^{-\frac{(s-q)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma^3}$,

and $qf - xf = \sigma^2 f'$. Hence, for $\forall a$:

$$\begin{aligned} \int_a^\infty sf(s)ds &= \int_a^\infty (qf - \sigma^2 f')ds = q \int_a^\infty fds - \sigma^2 \int_a^\infty f'ds \\ &= q\bar{\Phi}\left(\frac{a-q}{\sigma}\right) - \sigma^2 f|_a^\infty = q\bar{\Phi}\left(\frac{a-q}{\sigma}\right) - \sigma^2 f(a) = q\bar{\Phi}\left(\frac{a-q}{\sigma}\right) + \sigma\varphi\left(\frac{a-q}{\sigma}\right). \end{aligned}$$

Thus, for $a = p - \bar{\theta}$ we have:

$$\delta \int_{p-\bar{\theta}}^\infty (\bar{\theta} + q_u - p)f(q_u; q, m)dq_u = \delta(\bar{\theta} - p) \int_{p-\bar{\theta}}^\infty f(q_u; q, m)dq_u + \delta \int_{p-\bar{\theta}}^\infty q_u f(q_u; q, m)dq_u. \quad (\text{A.5})$$

Inserting Equation (A.5) into Equation (A.3) we have:

$$\frac{\bar{\theta} + q - p}{\sigma} = \delta \left(\frac{\bar{\theta} - p + q}{\sigma} \bar{\Phi}\left(\frac{p - \bar{\theta} - q}{\sigma}\right) + \phi\left(\frac{p - \bar{\theta} - q}{\sigma}\right) \right).$$

Thus, denoting $X = \frac{p - \bar{\theta} - q}{\sigma}$, we have $-X = \delta (\varphi(X) - X\bar{\Phi}(X))$. The unique verification is similar to that in Feldman *et al.* (2019). $\blacksquare$

According to Lemma A.2, consumers categorized as higher-type green consumers tend to make purchases in the initial period to mitigate the effects of discounted utility, whereas lower-type green consumers exhibit a preference for deferring their purchases until they have acquired more information. While the direct correlation between the price and quality of environmental products and the number of purchasers in the first period is not explicitly established in this context, several intuitive insights can be drawn. For instance, if the intrinsic quality of environmental products is perceived to be high, a greater number of green consumers are likely to engage in purchases during the initial stage. This behavior indicates a lower level of patience among consumers, as the ex ante expected quality of the product is elevated, leading fewer consumers to deem it worthwhile to wait for additional information. Furthermore, it can be observed that an increase in the posterior variance regarding the assessment of product quality correlates with a higher propensity for purchases in the first stage. This phenomenon is intuitive, as the perceived quality of environmental products, particularly those evaluated through online reviews, may have diminished, prompting consumers to be more inclined to make purchases sooner rather than later.

Appendix B

Proof of Proposition 1.

The operator's expected profit at policy (p, q) ³ can be written as:

$$\pi(p, q) = p \left[\int_{\bar{\theta}}^1 g(\theta) d\theta + \int_{p-\bar{\theta}}^{\infty} \int_{p-q_u}^{\bar{\theta}} g(\theta) f(q_u; q, m) d\theta dq_u \right] - c(q). \quad (\text{A.6})$$

Here, $\bar{\theta}$ is determined from Lemma (A.2).

Note that the density function of a uniform distribution meets $g(\theta) = 1$, thus:

$$\begin{aligned} \pi(p, q) &= p \left[\int_{\bar{\theta}}^1 g(\theta) d\theta + \int_{p-\bar{\theta}}^{\infty} \int_{p-q_u}^{\bar{\theta}} g(\theta) f(q_u; q, m) d\theta dq_u \right] - c(q) \\ &= p \left[(1 - \bar{\theta}) + \int_{p-\bar{\theta}}^{\infty} (\bar{\theta} - p + q_u) f(q_u; q, m) dq_u \right] - c(q) \\ &= p [1 - \bar{\theta} + \sigma (\phi(X) - X\bar{\Phi}(X))] - c(q) = pS(p, q) - c(q). \end{aligned}$$

Herein, $S(p, q) = 1 - \bar{\theta} + \sigma (\phi(X) - X\bar{\Phi}(X))$ is the demand, $X = \frac{p-\bar{\theta}-q}{\sigma}$ and is determined from Lemma (A.2).

The first-order necessary conditions for optimality are:

$$\begin{cases} \pi_p = S + pS_p = 0, \\ \pi_q = pS_q - c'(q) = 0. \end{cases} \quad (\text{A.7})$$

We now use the equality of (A.7) to calculate $\frac{\partial p}{\partial \beta}, \frac{\partial q}{\partial \beta}$. Take the derivative of β on both sides of the equality (A.7):

$$\begin{cases} \frac{\partial S}{\partial \beta} + \frac{\partial p}{\partial \beta} S_p + p \frac{\partial S_p}{\partial \beta} = 0, \\ \frac{\partial p}{\partial \beta} S_q + p \frac{\partial S_q}{\partial \beta} - c''(q) \frac{\partial q}{\partial \beta} = 0. \end{cases} \quad (\text{A.8})$$

Considering the following relationship $S = S(\bar{\theta}, \beta)$ ⁴, $\bar{\theta} = \bar{\theta}(p, q, \beta)$, $p = p(\beta)$, $q = q(\beta)$, utilizing the chain rule we have:

$$\begin{cases} \frac{\partial S}{\partial \beta} = S_p \frac{\partial p}{\partial \beta} + S_q \frac{\partial q}{\partial \beta} + \frac{\partial S}{\partial \bar{\theta}} \bar{\theta}' + S', \\ \frac{\partial S_p}{\partial \beta} = S_{pp} \frac{\partial p}{\partial \beta} + S_{pq} \frac{\partial q}{\partial \beta} + \frac{\partial S_p}{\partial \bar{\theta}} \bar{\theta}' + S_{p'}, \\ \frac{\partial S_q}{\partial \beta} = S_{qp} \frac{\partial p}{\partial \beta} + S_{qq} \frac{\partial q}{\partial \beta} + \frac{\partial S_q}{\partial \bar{\theta}} \bar{\theta}' + S_{q'}, \end{cases} \quad (\text{A.9})$$

³We omit the star in subscript for convenience.

⁴ $S = S(\bar{\theta}, \beta)$ since S does not explicitly contain p and q .

and denoting $S_0 = S' = \frac{dS}{d\beta}$, $S_1 = S'_p = \frac{dS_p}{d\beta}$, $S_2 = S'_q = \frac{dS_q}{d\beta}$, $\theta_\beta = \bar{\theta}' = \frac{d\bar{\theta}}{d\beta}$ we have:

$$\frac{\partial q}{\partial \beta} = \frac{-S_{pp}p^2\theta_\beta S_{q\theta} + S_{qp}p^2\theta_\beta S_{p\theta} + S_1 S_{qp}p^2 - S_2 S_{pp}p^2 - 2S_p p \theta_\beta S_{q\theta} + \dots}{S_{pp}S_{qq}p^2 - S_{pq}S_{qp}p^2 + 2S_p S_{qq}p - S_{pp}cp - S_{pq}S_qp - S_q S_{qp}p - 2S_p c - S_q^2 + S_q p \theta_\beta S_{p\theta} + S_{qp}p \theta_\beta S_\theta + S_0 S_{qp}p + S_1 S_q p - 2S_2 S_p p + S_q \theta_\beta S_\theta + S_0 S_q}{S_{pp}S_{qq}p^2 - S_{pq}S_{qp}p^2 + 2S_p S_{qq}p - S_{pp}cp - S_{pq}S_qp - S_q S_{qp}p - 2S_p c - S_q^2}$$

In the following we need to calculate S_0, S_1, S_2 and $S_p, S_q, S_{pp}, S_{pq}, S_{qq}$. Noticing the relationship:

$$S_p + S_q = 0, S_1 + S_2 = 0, S_{p\theta} + S_{q\theta} = 0, S_{pp} = S_{qq} = -S_{pq} = -S_{qp},$$

we have (denoting $\mathcal{D}(\frac{\partial q}{\partial \beta})$, $\mathcal{N}(\frac{\partial q}{\partial \beta})$ are denominator and numerator of $\frac{\partial q}{\partial \beta}$, respectively)

$$\mathcal{D}(\frac{\partial q}{\partial \beta}) = -S_{pp}cp - 2S_p c - S_q^2,$$

and

$$\mathcal{N}(\frac{\partial q}{\partial \beta}) = -pS_p\theta_\beta S_{q\theta} + pS_{qp}\theta_\beta S_\theta + S_0 S_{qp}p - S_1 S_q p + S_q \theta_\beta S_\theta + S_0 S_q = S_0 S_{qp}p - S_1 S_q p + S_q \theta_\beta S_\theta + S_0 S_q.$$

The last equation holds since $S_q S_{p\theta} = S_\theta S_{pq}$.

(i) Case small β .

From $\lim_{\beta \rightarrow 0} S_p = -1$ and $\lim_{\beta \rightarrow 0} S_{qq} = 0$, we obtain $\lim_{\beta \rightarrow 0} \mathcal{D}(\frac{\partial q}{\partial \beta}) = 1$.

From $\lim_{\beta \rightarrow 0} S_\theta = -1$, we obtain:

$$\lim_{\beta \rightarrow 0} \mathcal{N}(\frac{\partial q}{\partial \beta}) = S_0 S_{qp}p - S_1 p - \theta_\beta + S_0.$$

Considering that $\lim_{\beta \rightarrow 0} S_0 S_{qp} = \frac{-F^2 - XF}{8\alpha^2\theta - 8\alpha^2} > 0$, $\lim_{\beta \rightarrow 0} S_1 p = -\infty$, $\lim_{\beta \rightarrow 0} (S_0 - \theta_\beta) = +\infty$, thus we have $\lim_{\beta \rightarrow 0} \frac{\partial q}{\partial \beta} \rightarrow +\infty > 0$.

(ii) Case large β .

We can also obtain $\lim_{\beta \rightarrow \infty} \mathcal{D}(\frac{\partial q}{\partial \beta}) = 1$ and $\lim_{\beta \rightarrow \infty} \mathcal{N}(\frac{\partial q}{\partial \beta}) = S_0 S_{qp}p - S_1 p - \theta_\beta + S_0$.

The limits of the above items when β goes to infinity are all zero, and we cannot determine their sign. However, we check that:

$$\theta_\beta S_{q\theta} - \theta_\beta S_{qp} = \frac{X^2(\bar{\theta} - 1)\beta^3(4\beta\bar{\theta} - \alpha - 4\beta)(F + X)}{\sqrt{\frac{(\bar{\theta}-1)\beta}{((\bar{\theta}-1)\beta-\alpha)\alpha}}(-2((\bar{\theta}-1)\beta-\alpha)^2\sqrt{\frac{(\bar{\theta}-1)\beta}{((\bar{\theta}-1)\beta-\alpha)\alpha}} + X\beta)^4\alpha} > 0,$$

$$S_0 - \theta_\beta = -\frac{(-2((\bar{\theta} - 1)\beta - \alpha)^2(F + X)\sqrt{\frac{(\bar{\theta}-1)\beta}{((\bar{\theta}-1)\beta-\alpha)\alpha}} + FX\beta)(\bar{\theta} - 1)}{2\sqrt{\frac{(\bar{\theta}-1)\beta}{((\bar{\theta}-1)\beta-\alpha)\alpha}}(-2((\bar{\theta} - 1)\beta - \alpha)^2\sqrt{\frac{(\bar{\theta}-1)\beta}{((\bar{\theta}-1)\beta-\alpha)\alpha}} + X\beta)((\bar{\theta} - 1)\beta - \alpha)^2} > 0,$$

$$S_0 S_{pq} = -\frac{4\beta^2 F((\bar{\theta} - 1)\beta - \frac{\alpha}{4})(\bar{\theta} - 1)(F + X)}{\sqrt{\frac{(\bar{\theta}-1)\beta}{((\bar{\theta}-1)\beta-\alpha)\alpha}}(-2((\bar{\theta} - 1)\beta - \alpha)^2\sqrt{\frac{(\bar{\theta}-1)\beta}{((\bar{\theta}-1)\beta-\alpha)\alpha}} + X\beta)^3\alpha} > 0,$$

$$S_1 = \frac{(\bar{\theta} - 1)(2\beta\bar{\theta} + \alpha - 2\beta)(F + X)\beta}{\sqrt{\frac{(\bar{\theta}-1)\beta}{((\bar{\theta}-1)\beta-\alpha)\alpha}}(-2((\bar{\theta} - 1)\beta - \alpha)^2\sqrt{\frac{(\bar{\theta}-1)\beta}{((\bar{\theta}-1)\beta-\alpha)\alpha}} + X\beta)^2\alpha}.$$

Here, we denote $F = \phi(X) - X\bar{\Phi}(X)$. Since $X < 0$, it can be checked that $F > 0$ and $F + X > 0$. Thus, to ensure the sign of $\frac{\partial q}{\partial \beta}$ we have to ensure the sign of S_1 is negative (or the sign of $\frac{\partial q}{\partial \beta}$ is ambiguous). This is equal to $\alpha > 2\beta(1 - \bar{\theta})$. Thus, if α is large enough $\alpha > 2\beta(1 - \bar{\theta})$ must hold for any given β .

Considering the two results (i) and (ii) we can draw the final conclusion. ■

Appendix C

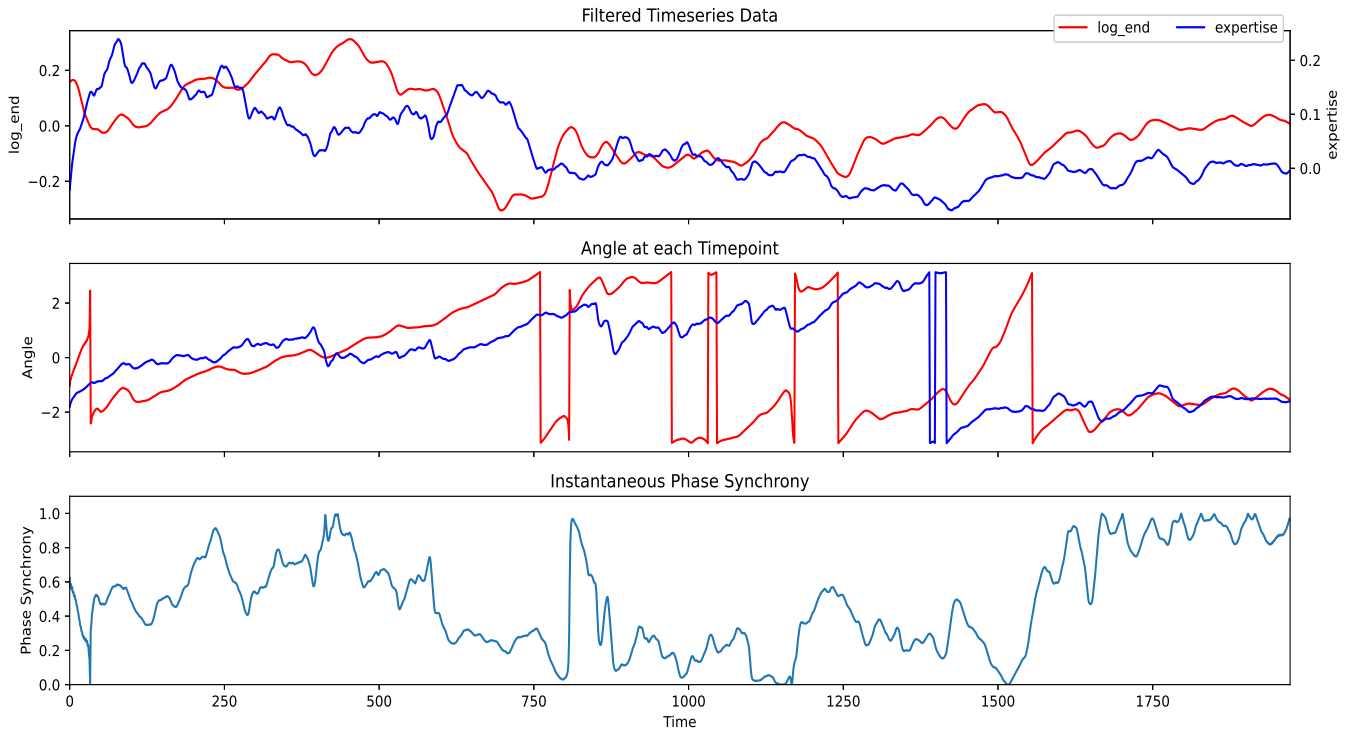


Figure A1: Results for instantaneous phase synchrony with experts' review.

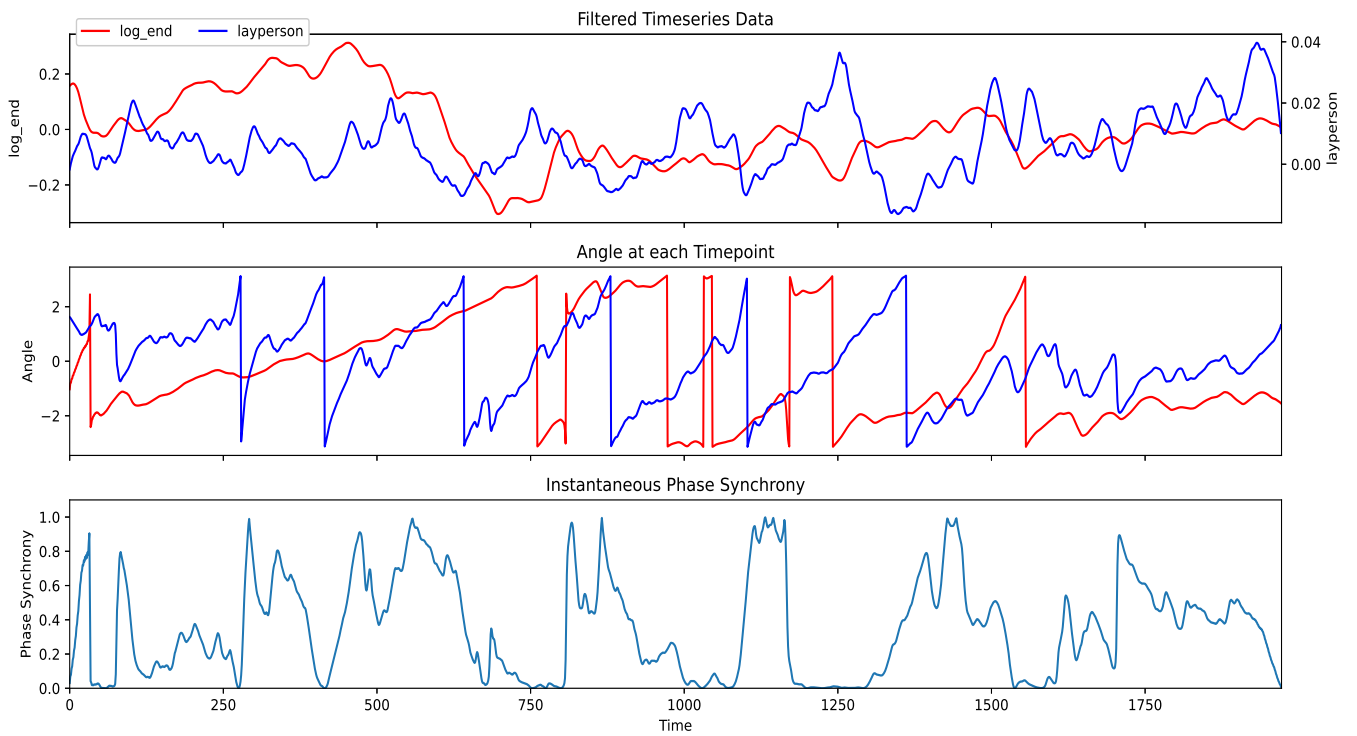


Figure A2: Results for instantaneous phase synchrony with laypersons' review.

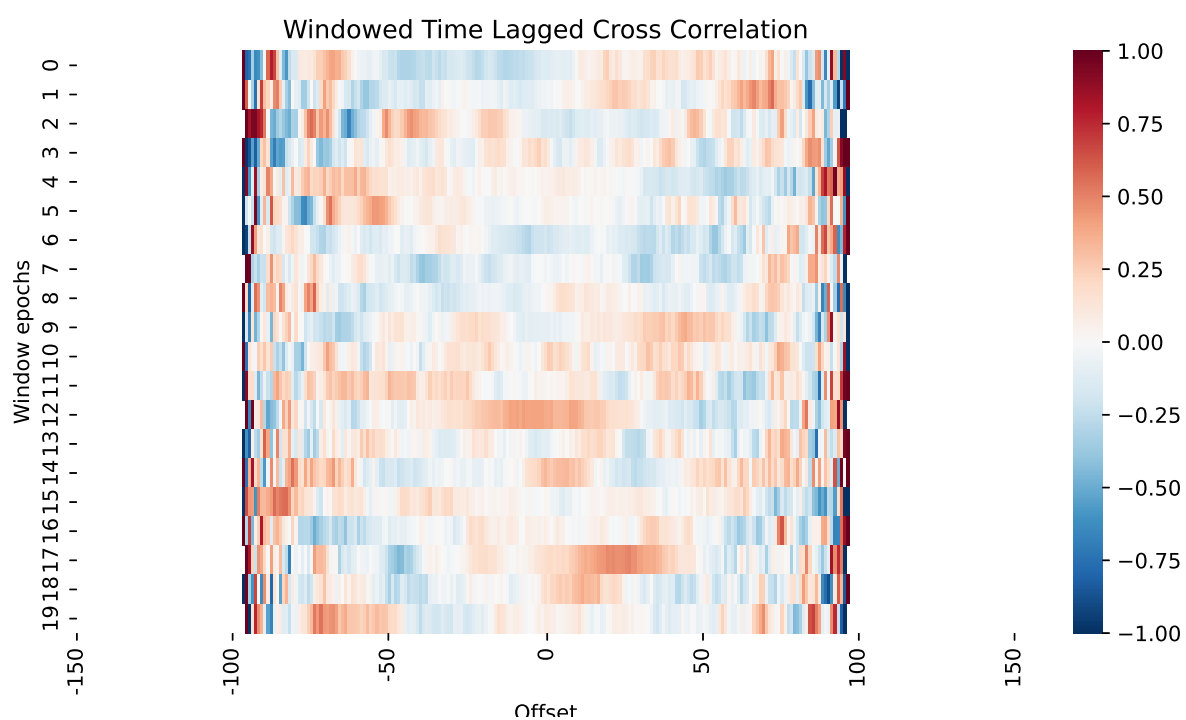


Figure A3: Results for instantaneous phase synchrony with experts' review.

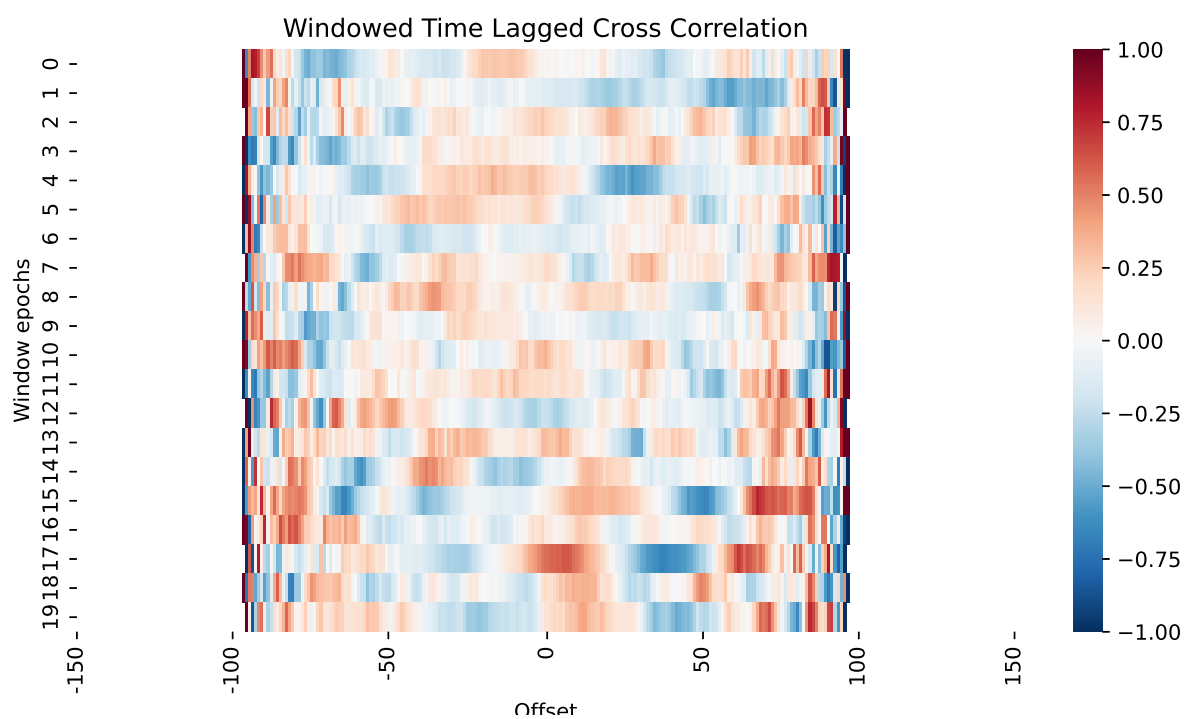


Figure A4: Results for instantaneous phase synchrony with laypersons' review.

Appendix D

Table A1: ADF, order and AIC in different case

Time period	Log_start ¹	AbLog_start ²	garchOrder	armaOrder	AIC
2015-12-09 2024-01-16	0.2952	0.005	(6,6)	(4,2)	-9.1059
2016-01-01 2019-12-31	0.4115	0.01	(3,2)	(3,2)	-9.0920
2020-01-01 2022-12-30	0.4791	0.01	(5,4)	(4,4)	-8.9426
2022-12-30 2024-01-16	0.0725	n.a.	(5,5)	(3,3)	-6.3005

¹ The p value obtained by using Log_start for the ADF test, where Log_start represents the logarithm of the stock's opening price on that day.

² The p value obtained by performing an ADF test using AbLog_end, where AbLog_end means the daily rate of change of Log_start.

Table A2: Weighted Ljung-Box Test on Standardized Squared Residuals

2017-01-01~2019-12-31			2020-01-01~2022-12-30		
	statistic	p-value		statistic	p-value
Lag[1]	5.195	0.2265	Lag[1]	0.01062	0.9179
Lag[2*(p+q)+(p+q)-1][11]	7.689	0.42016	Lag[2*(p+q)+(p+q)-1][8]	3.76495	0.9993
Lag[4*(p+q)+(p+q)-1][19]	11.258	0.57941	Lag[4*(p+q)+(p+q)-1][14]	9.56835	0.9985
d.o.f=5			d.o.f=9		
2022-12-30~2024-01-16			2015-12-09~2024-01-16		
number	group	statistic	number	group	statistic
Lag[1]	0.01578	0.9000	Lag[1]	0.008876	0.9249
Lag[2*(p+q)+(p+q)-1][8]	12.03534	0.7271	Lag[2*(p+q)+(p+q)-1][14]	5.704281	1.0000
Lag[4*(p+q)+(p+q)-1][14]	21.65366	0.7006	Lag[4*(p+q)+(p+q)-1][24]	9.857075	1.0000
d.o.f=10			d.o.f=12		

Table A3: Adjusted Pearson Goodness-of-Fit Test

2017-01-01 ~ 2019-12-31				2020-01-01 ~ 2022-12-30			
number	group	statistic	p-value(g-1)	number	group	statistic	p-value(g-1)
1	20	32.77	0.02552	1	20	27.27	0.009841
2	30	35.68	0.18317	2	30	40.71	0.07300
3	40	46.01	0.20479	3	40	47.49	0.16503
4	50	73.15	0.01425	4	50	54.14	0.28461
2022-12-30 ~ 2024-01-16				2015-12-09 ~ 2024-01-16			
number	group	statistic	p-value(g-1)	number	group	statistic	p-value(g-1)
1	20	26.79	0.197	1	20	34.58	0.015681
2	30	33.24	0.2680	2	30	53.58	0.003614
3	40	49.94	0.1127	3	40	48.84	0.134320
4	50	55.84	0.2333	4	50	60.24	0.130292

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