Choice of environmental policy instrument in developing countries: an application to fire regulation in the Brazilian Amazon

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ONLINE APPENDIX

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Appendix A. Marginal fire replacement cost as an opportunity cost

Following Morello (2023), the exogenous variables of the microeconomic models that based design of contract and CAC are the low and high levels of marginal abatement cost $\underline{\beta}$ and β, respectively, the intentional area burned without policy, f*, and the maximum accidentally burned area, E. The endogenous variables are (i) the (recommended) intentional burned area targets, $\underline{f}$ and f in the case of contracts and only "f" in the case of CAC, (ii) the (imposed) total burned area limit, $\underline{f}$ + E and f + E, for contracts, and f + E only for CAC, (iii) the compliance and non-compliance payments, t_c and t_{nc}, and (iv) the fine, g.

It is clarified that non-compliance is referred to as "shirking" in this appendix, the term adopted in contract theory (see Laffont and Martimort, 2002).

This subsection stablishes the microfoundations of $\underline{\beta}$, β and f*. If fire-based land preparation is the profit-maximizing technological option, there is an opportunity cost of replacing it. Following Moxey *et al.* (1999), such cost equals the difference between unrestricted profit level, i.e., the level achieved without the upper bound to intended burned area recommended by contracts, and the restricted level faced when a contract is signed, that is, $\pi^* - \pi_0$. The per hectare value of such cost is exactly what is here understood as β, that is, concentrating in the case of the high-cost agent, $\beta \equiv \frac{\pi^*-\pi_0}{f^*-f}$, with the denominator capturing, coherently, the extent of land removed from burnings (the analogous apply to the low-cost agent, which is thus abstracted from this subsection).

This general idea may be formalized in a simple way by focusing the farmer's problem of choosing how many hectares to allocate to fire-based (x) and fire-free (y) land preparation, as follows below[1]:

$$Max_{\{x,y\}} \{h(x,y) - c(x,y)\}\ s.t\ \ x + y = a$$

With output price normalized into one (p ≡ 1), h(.) is the production function, c(.) the total cost function and "a" the land area available to be prepared.[2] Let it be assumed, additionally, that the production function, h(x,y), has the perfect substitutes form.[3] That is:

[1] Since this is the only problem the contract is designed to intervene on, choices of production factors and output level are abstracted.

[2] It is worth clarifying that, if, on the one hand, f_d is restricted to the total area available for production ("a"), on the other hand, the area accidentally burned (f_a) is not restricted to such fraction of whole farm area. In fact, fire, by spreading uncontrollably through space, does not respect the inner boundaries subdividing farmland into parcels available and not available for preparation (Bowman *et al.*, 2008; Cammelli *et al.*, 2020). There is therefore no contradiction between assuming $f_d \leq a$, and the upper limit to f_a implicit in the empirical probability distribution adopted for f_a in the numerical simulation (see B.3 below).

[3] Land preparation, i.e., removal of (non-primary) spontaneous vegetation and incorporation of nutrients to soil, may be pursued with burnings or, among other options, with mechanized land preparation coupled with agrochemicals. These two routes are perfect substitutes since they are mutually exclusive ways to achieve, in a

$$Max_{\{x,y\}}\,\{u_0x + u_1y - c_0x - c_1y\} s.t.\ x + y = a \leftrightarrow$$

$$Max_{\{x,y\}}\,\{r_0x + r_1y\}\ s.t.\ x + y = a$$

There are three possible solutions to the problem, depending on whether r_0 is equal, larger or smaller than r_1, but only the possibility that r_0 is larger is consistent with the paper's goal of designing policy for reducing fire-based land preparation. Such technique is thus adopted for the whole land extension "a" (as indicated by point "A" in figure A1, left side).

Now considering the contract-restricted farmer's problem:

$$Max_{\{x,y\}}\,\{r_0x + r_1y\}\ s.t.\ x + y = a, x \leq f$$

The solution is indicated by point "B" in the right side of figure 2. It results from the following reasoning. First, it should be noted that for x < f and y = a - x, a land extension of f - x hectares is allocated to the less profitable fire-free land preparation, and thus a less-than-maximum profit level is achieved. Thus x* = f. Second, once the level of x is defined, it is optimal for the farmer to allocate all the remaining area to y in order to achieve maximum profit, i.e., y* = a – f.

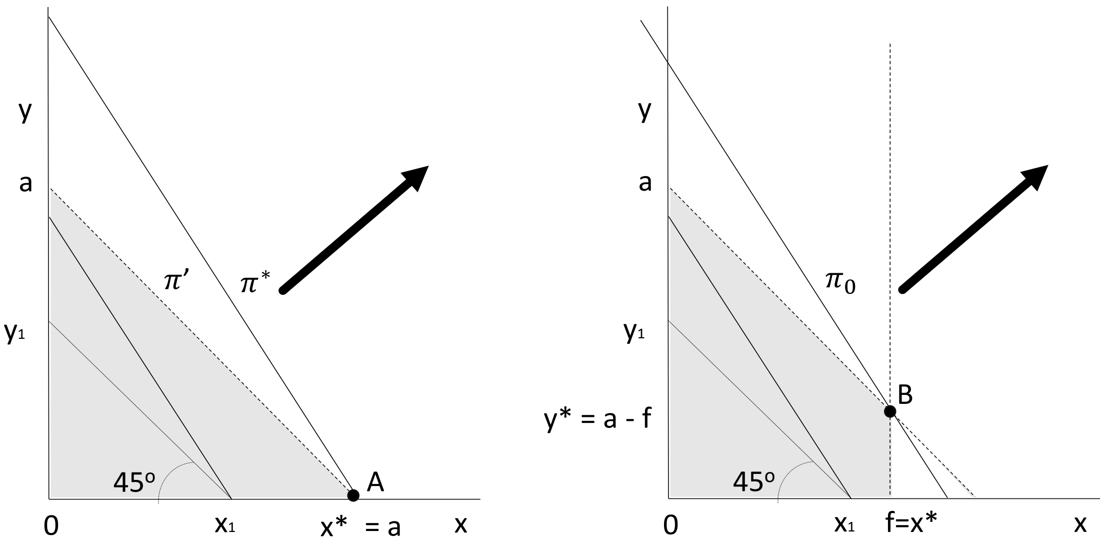

Figure A1. Land preparation technology decision without (left) and with a contract (right)*.
*Note: the isoprofit and land constraint lines are, respectively, indicated by the solid and dashed lines and profit increases in the direction indicated by the arrow (the grey region is the opportunity set).

The main point is that under the x ≤ f constraint, a lower profit level is achieved, since $\pi_0 = r_0.f + r_1(a\text{-}f) < \pi^* = r_0a$, mainly because part of available land is allocated to the less profitable fire-free option. Here lies the opportunity cost of signing a contract. A secondary

given land parcel, the same goal of preparing land. Indeed, two ways to prepare land are much less different than, for instance, production factors such as labour and capital, and there's no reason to believe that land allocated to one route constrains other route's productivity, such that complementarity between them is null (Nicholson and Snyder, 2008, chap.9).

consequence is that, assuming the contract is signed, and thus the constraint is in vigour, intended burned area is public information and profit loss from compliance is exactly compensated, then the farmer has incentive to resort to fire-free land preparation because so doing is more profitable than leaving part of land unprepared. Under such assumptions, therefore, the contract is able to induce replacement of fire-based for fire-free technology.

A final useful fact is that the marginal cost equals the per-hectare profit difference between the two land preparation routes. This is clear from the solution of the contract-constrained and unconstrained problems, since, considering that f* = a:

$$\beta \equiv \frac{\pi^* - \pi_0}{f^* - f} = \frac{r_0 f^* - \left(r_0 f + r_1(a - f)\right)}{f^* - f} = r_0 - r_1$$

This is another way to stablish that contracts should be supplied only when it is not in the best interest of the agent to opt for fire-free land preparation (i.e., $r_0 > r_1 \leftrightarrow \beta > 0$).

Appendix B. Detailed derivation of optimal contracts

B.1. Binding constraints, first order conditions and optimal target levels

A three-step approach was implemented to derive the solution (see details in sections B.1 and B.2). First, as standard in AS problems (Laffont, 1995, Moxey *et al.*, 1999; Laffont and Martimort, 2002, section 2.6; White and Hanley, 2016, section 3.7), it was assumed as binding (i) the participation constraint (PC) of the high-cost agent and (ii) the AS constraint of the low-cost agent (section B.1.2). In the second step, based in first step assumptions, optimal targets and compliance payments for the two contracts were obtained by maximizing the welfare function (sections B.1.3 and B.1.4). The satisfaction of remaining constraints was left for the third step (section B.2).

B.1.1. Expected marginal damage function

Expected damage as a function of f is denoted as $D^e(f)$. Considering both damage function's formula from Morello *et al.* (2023, section 3.3) and that, for a given f_d, F belongs to the $[f_d, f_d + E]$ interval, the functional form of $D^e(f)$ is:

$$D^e(f) = E[D(F)|f_d = f] = \int_f^{f+E} D(x)g(x)dx = \frac{1}{2}\left[s_0(E + 2f) + \frac{s_1}{3}[3f(f + E) + E^2]\right]$$

Thus marginal damage is:

$$\frac{d}{df}D^e(f) = s_0 + s_1\left[f + \frac{E}{2}\right]$$

Since welfare is the negative of expected damage, the derivative of the former, $W'(f)$, is:

$$W'(f) = -s_0 - s_1\left[f + \frac{E}{2}\right]$$

B.1.2. Binding constraints (first step)

With the standard solution to an adverse selection problem assumed, the adverse-selection constraint of low-cost agent ($\underline{A}$) and the participation constraint of the high-cost agent (D) are binding. What leads to the two equations below in which the compliance payments of the two contracts are functions strictly of targeted deliberately burned areas.

[$\underline{A}$] $\underline{t}_c - \underline{\beta}(f^*-\underline{f}) = t_c - \underline{\beta}(f^*-f)$

[PC] $t_c = \beta(f^*-f)$

Consequently:

[1] $t_c = \beta f^* - \beta f$

[2] $\underline{t}_c = \beta(f^*-f) - \underline{\beta}(f^*-f) + \underline{\beta}(f^*-\underline{f}) = \Delta\beta(f^*-f) + \underline{\beta}(f^*-\underline{f}) = (\Delta\beta+\underline{\beta})f^* - \Delta\beta f - \underline{\beta f} = \Delta\beta f^* + \underline{\beta}f^* - \Delta\beta f - \underline{\beta f}$

(where $\Delta\beta = \beta - \underline{\beta}$)

B.1.3. Welfare function and FOCs (second step): second-best problem

The expected welfare function and the principal's problem is:

$\text{Max}_{\{\underline{f},f\}}\ v\,[W(\underline{f}) + (\underline{t}_c - \underline{\beta}(f^*-\underline{f})) - (1+\lambda)\underline{t}_c] + (1-v)\,[W(f) + (t_c - \beta(f^*-f)) - (1+\lambda)t_c]$

Let the objective function be algebraically simplified. First, notice that it is equivalent to:

Applying equations [1] and [2]:

$\text{Max}_{\{\underline{f},f\}}\ v\,[W(\underline{f}) + \Delta\beta(f^*-f) - (1+\lambda)\,(\underline{\beta}(f^*-\underline{f}) + \Delta\beta(f^*-f))] + (1-v)\,[W(f) - (1+\lambda)(\beta(f^*-f))]$

Or, equivalently:

$\text{Max}_{\{\underline{f},f\}}\ v\,[W(\underline{f}) + \Delta\beta f^* - \Delta\beta f - (1+\lambda)(\Delta\beta+\underline{\beta})f^* + (1+\lambda)\Delta\beta f + (1+\lambda)\underline{\beta f}] + (1-v)\,[W(f) - (1+\lambda)\beta f^* + (1+\lambda)\beta f]$

$\text{Max}\{\underline{f},f\}\ v\,[W(\underline{f}) + f^*(\Delta\beta - (1+\lambda)(\Delta\beta+\underline{\beta})) + f(\lambda\Delta\beta) + (1+\lambda)\underline{\beta f}] + (1-v)\,[W(f) - (1+\lambda)\beta f^* + (1+\lambda)\beta f]$

FOCs are:

FOC $\underline{f}$: $v[W'(\underline{f}) + (1+\lambda)\underline{\beta}] = 0 \rightarrow -W'(\underline{f}) = (1+\lambda)\underline{\beta}$

FOC f: $v[\lambda\Delta\beta] + (1-v)[W'(f) + (1+\lambda)\beta] = 0 \rightarrow -W'(f) = v/(1-v)\lambda\Delta\beta + (1+\lambda)\beta$

Introducing the expected marginal welfare function formula from section B.1.1, $W'(f_d) = -s_0 - s_1\{f_d + E/2\}$, $f_d = f, \underline{f}$; one has:

FOC $\underline{f}$: $-W'(\underline{f}) = (1+\lambda)\underline{\beta} \rightarrow s_0 + s_1(\underline{f} + E/2) = (1+\lambda)\underline{\beta} \rightarrow \underline{f}^{SB} = [(1+\lambda)\underline{\beta} - s_0]/s_1 - E/2$

FOC f: $-W'(f) = v/(1-v)\lambda\Delta\beta + (1+\lambda)\beta \rightarrow s_0 + s_1(f + E/2) = v/(1-v)\lambda\Delta\beta + (1+\lambda)\beta$

$\rightarrow f^{SB} = [v/(1-v)\lambda\Delta\beta + (1+\lambda)\beta - s_0]/s_1 - E/2.$

Where SB stands for "second-best". Note thus that $f^{SB} = \underline{f}^{SB} + v/(1-v)(1+\lambda)\Delta\beta/s_1$. Therefore, f^{SB}

$> \underline{f}^{SB}$, since $v/(1-v)(1+\lambda)\Delta\beta/s_1 > 0$.

B.1.4. Welfare function and FOCs: first-best problem

In the case that both marginal cost and intended burned area are observed, the objective function is:

$$\text{Max}_{\{\underline{t},t,\underline{f},f\}}\ v[W(\underline{f}) + \underline{t} - \underline{\beta}(f^*-\underline{f}) - (1+\lambda)\,\underline{t}] + (1-v)[W(f) + t - \beta(f^*-f) - (1+\lambda)\,t]$$

Subjected to $\underline{t} - \underline{\beta}(f^*-\underline{f}) \geq 0$; $t - \beta(f^*-f) \geq 0$

Only the participation constraints apply, since adverse selection and moral hazard are impossible. In this case, it is sufficient to offer payments that make the two types of agents indifferent between participating or not, so the objective function is:

$$\text{Max}_{\{\underline{f},f\}}\ v[W(\underline{f}) - (1+\lambda)\underline{\beta}(f^*-\underline{f})] + (1-v)[W(f) - (1+\lambda)\beta(f^*-f)]$$

FOCs are:

FOC $\underline{f}$: $v[W'(\underline{f}) + (1+\lambda)\underline{\beta}] = 0 \rightarrow -W'(\underline{f}) = (1+\lambda)\underline{\beta}$

FOC f: $(1-v)[W'(f) + (1+\lambda)\beta] = 0 \rightarrow -W'(f) = (1+\lambda)\beta$

Applying the marginal expected welfare function of section B.1.1, one has:

FOC $\underline{f}$: $-W'(\underline{f}) = (1+\lambda)\underline{\beta} \rightarrow s_0+s_1\{\underline{f} + E/2\} = (1+\lambda)\underline{\beta} \rightarrow \underline{f}^{FB} = [(1+\lambda)\underline{\beta} - s_0]/s_1 - E/2$

FOC f: $-W'(f) = (1+\lambda)\beta \rightarrow s_0+s_1\{f + E[f_a]\} = (1+\lambda)\beta \rightarrow f^{FB} = [(1+\lambda)\beta - s_0]/s_1 - E/2$

It is notorious that $f^{FB} > \underline{f}^{FB}$ since $\beta > \underline{\beta}$.

B.2. Constraint satisfaction (third step)

The goal of the third step is to determine the upper bound for the non-compliance payment that verify all constraints that were not assumed to be binding. This is implemented in three steps. First, the constraints that are assumed to be binding and those that are, as a product, automatically verified, are omitted. Secondly, in the remaining constraints, the wedges are isolated. Third, a redundancy analysis is applied to identify dominating constraints understood as those restricting the range of payment wedges ($t_c - t_{nc}$) to the smallest interval for each contract.

Without loss of rigour, the explicit non-compliance constraints are ignored in subsections B.2.1 to B.2.4 and reintroduced in section B.2.5.

B.2.1. Full constraints' set

Low-cost agent

[A] (Compliance with wrong contract) $\underline{t_c} - \underline{\beta}(f^*-\underline{f}) \geq t_c - \underline{\beta}(f^*-f)$

[B] (Shirking with right contract) $\underline{t_c} - \underline{\beta}(f^*-\underline{f}) \geq \underline{t_c} - \underline{\varepsilon}[1/E.(\underline{t_c} - \underline{t_{nc}})] - \underline{\beta}(f^* - \underline{f} - \underline{\varepsilon})$

[C] (Shirking with wrong contract) $\underline{t_c} - \underline{\beta}(f^*-\underline{f}) \geq t_c - \underline{\varepsilon}'[1/E.(t_c - t_{nc})] - \underline{\beta}(f^* - f - \underline{\varepsilon}')$

[D] (Non-participation) $\underline{t}_c - \underline{\beta}(f^*-\underline{f}) \geq 0$

[E] (Explicit non-compliance with right contract) $\underline{t}_c - \underline{\beta}(f^*-\underline{f}) \geq \underline{t}_{nc}$

[F] (Explicit non-compliance with wrong contract) $\underline{t}_c - \underline{\beta}(f^*-\underline{f}) \geq t_{nc}$

High-cost agent

[A] (Compliance with wrong contract) $t_c - \beta(f^*-f) \geq \underline{t}_c - \beta(f^*-\underline{f})$

[B] (Shirking with right contract) $t_c - \beta(f^*-f) \geq t_c - \varepsilon[1/E.(t_c - t_{nc})] - \beta(f^* - f - \varepsilon)$

[C] (Shirking with wrong contract) $t_c - \beta(f^*-f) \geq \underline{t}_c - \varepsilon'[1/E.(\underline{t}_c - \underline{t}_{nc})] - \beta(f^* - \underline{f} - \varepsilon')$

[D] (Non-participation) $t_c - \beta(f^*-f) \geq 0$

[E] (Explicit non-compliance with right contract) $t_c - \beta(f^*-f) \geq t_{nc}$

[F] (Explicit non-compliance with wrong contract) $t_c - \beta(f^*-f) \geq \underline{t}_{nc}$

B.2.2. Optimal shirking conditions

The agent should make two choices regarding shirking, the first is about the optimal shirking level, the second is whether to shirk at the optimal level or not to shirk. As the latter decision is already encapsulated in the utility differentials of the previous sections, the former is tackled in this subsection.

Low-cost agent with right contract

In this case, shirking income is:

$\underline{t}_c - \varepsilon[1/E.(\underline{t}_c - \underline{t}_{nc})] - \underline{\beta}(f^* - \underline{f} - \underline{\varepsilon}) = \underline{t}_c - \underline{\varepsilon}[1/E.(\underline{t}_c - \underline{t}_{nc}) - \underline{\beta}] - \underline{\beta}(f^* - \underline{f}) = \underline{t}_c - 1/E.\underline{\varepsilon}[(\underline{t}_c - \underline{t}_{nc}) - \underline{\beta}E] - \underline{\beta}(f^* - \underline{f})$

Thus the optimal shirking level is:

$$\underline{\varepsilon}\begin{cases} = \min\{f^* - \underline{f}; E\}, if\ \underline{t_c} - \underline{t_{nc}} < \underline{\beta}E \\ \in \left[0\,; \min\left\{f^* - \underline{f}, E\right\}\right], if\ \underline{t_c} - \underline{t_{nc}} = \underline{\beta}E \\ = 0, if\ \underline{t_c} - \underline{t_{nc}} > \underline{\beta}E \end{cases}$$

where maximum shirking level is $\min\left\{f^* - \underline{f}, E\right\}$, as $f_d = f + \varepsilon \leq f^*$, so that $\varepsilon \leq f^* - f$, but it may be the case that $E < f^* - f$, so that $\underline{\varepsilon}$'s domain of variation is automatically reduced to the shorter sub-interval of non-explicit compliance (the $\underline{\varepsilon} \leq E$ interval).

Low-cost agent with wrong contract

The shirking income is:

$t_c - \underline{\varepsilon}'[1/E.(t_c - t_{nc})] - \underline{\beta}(f^* - f - \underline{\varepsilon}') = t_c - \underline{\varepsilon}'[1/E.(t_c - t_{nc}) - \underline{\beta}] - \underline{\beta}(f^* - f) = t_c - 1/E.\underline{\varepsilon}'[(t_c - t_{nc}) - \underline{\beta}E] - \underline{\beta}(f^* - f)$

The optimal shirking level is:

$$\underline{\varepsilon}' \begin{cases} = \min\{f^* - f; E\}, if\ t_c - t_{nc} < \underline{\beta}\ E \\ \in [\,0\,; \min\{f^* - f, E\}], if\ t_c - t_{nc} = \underline{\beta} E \\ = 0, if\ \ t_c - t_{nc} > \underline{\beta} E \end{cases}$$

<u>High-cost agent with right contract</u>

$t_c - \varepsilon[1/E.(t_c - t_{nc})] - \beta(f^*- f - \varepsilon) = t_c - \varepsilon[1/E.(t_c - t_{nc}) - \beta] - \beta(f^*- f) = t_c - 1/E.\varepsilon[(t_c - t_{nc}) - \beta E] - \beta(f^*- f)$

Optimal shirking level is:

$$\varepsilon \begin{cases} = \min\{f^* - f; E\}, if\ t_c - t_{nc} < \beta E \\ \in [\,0\,; \min\{f^* - f, E\}], if\ t_c - t_{nc} = \beta E \\ = 0, if\ \ t_c - t_{nc} > \beta E \end{cases}$$

<u>High-cost agent with wrong contract</u>

$\underline{t}_c - \varepsilon'[1/E.(\underline{t}_c - \underline{t}_{nc})] - \beta(f^*- \underline{f} - \varepsilon') = \underline{t}_c - \varepsilon'[1/E.(\underline{t}_c - \underline{t}_{nc}) - \beta] - \beta(f^*- \underline{f}) = \underline{t}_c - 1/E.\varepsilon'[(\underline{t}_c - \underline{t}_{nc}) - \beta E] - \beta(f^*- \underline{f})$

Best shirking level is:

$$\varepsilon' \begin{cases} = \min\{f^* - \underline{f}; E\}, if\ \underline{t_c} - \underline{t_{nc}} < \ \beta E \\ \in \left[\,0\,; \min\left\{f^* - \underline{f}, E\right\}\right], if\ \underline{t_c} - \underline{t_{nc}} = \beta E \\ = 0, if\ \underline{t_c} - \underline{t_{nc}} > \ \beta E \end{cases}$$

<u>Summary</u>

The principal should impose the minimum wedges below for making agent indifferent between zero and positive shirking:

[$\underline{S}$] $\underline{t}_c - \underline{t}_{nc} \geq \underline{\beta}E$

[$\underline{S}'$] $t_c - t_{nc} > \underline{\beta}E$

[S] $t_c - t_{nc} > \beta E$

[S'] $\underline{t}_c - \underline{t}_{nc} > \beta E$

B.2.3. Constraints' set reduced after the two binding constraints

Two constraints are automatically met after [$\underline{A}$] and [D] are assumed as binding, namely, [A] and [$\underline{D}$]. Let this be detailed. In the case of [$\underline{D}$], the cheating prevention premium makes participation always best, since $\underline{t}_c - \underline{\beta}(f^*-\underline{f}) = \Delta\beta(f^*-f) \geq 0$ (given that $\Delta\beta$ is positive and f*-f is non-negative). Now considering [A], it suffices to notice that [$\underline{A}$] leads to $\underline{t}_c = t_c - \underline{\beta}(f^*-f) + \underline{\beta}(f^*-\underline{f})$. Introducing this into [A], one has:

$t_c - \beta(f^*-f) \geq \{t_c - \underline{\beta}(f^*-f) + \underline{\beta}(f^*-\underline{f})\} - \beta(f^*-\underline{f}) \rightarrow \beta(f - \underline{f}) \geq \underline{\beta}(f - \underline{f})$, what is true since $\beta > \underline{\beta}$.

The remaining constraint set, which includes only MH constraints, is:

<u>Low-cost agent</u>

[B] (Shirking with right contract) $\underline{t_c} - \underline{\beta}(f^*-\underline{f}) \geq \underline{t_c} - \underline{\varepsilon}[1/E.(\underline{t_c} - \underline{t_{nc}})] - \underline{\beta}(f^* - \underline{f} - \underline{\varepsilon})$

[C] (Shirking with wrong contract) $\underline{t_c} - \underline{\beta}(f^*-\underline{f}) \geq t_c - \underline{\varepsilon}'[1/E.(t_c - t_{nc})] - \underline{\beta}(f^* - f - \underline{\varepsilon}')$

[E] (Explicit non-compliance with right contract) $\underline{t_c} - \underline{\beta}(f^*-\underline{f}) \geq \underline{t_{nc}}$

[F] (Explicit non-compliance with wrong contract) $\underline{t_c} - \underline{\beta}(f^*-\underline{f}) \geq t_{nc}$

High-cost agent

[B] (Shirking with right contract) $t_c - \beta(f^*-f) \geq t_c - \varepsilon[1/E.(t_c - t_{nc})] - \beta(f^* - f - \varepsilon)$

[C] (Shirking with wrong contract) $t_c - \beta(f^*-f) \geq \underline{t_c} - \varepsilon'[1/E.(\underline{t_c} - \underline{t_{nc}})] - \beta(f^* - \underline{f} - \varepsilon')$

[E] (Explicit non-compliance with right contract) $t_c - \beta(f^*-f) \geq t_{nc}$

[F] (Explicit non-compliance with wrong contract) $t_c - \beta(f^*-f) \geq \underline{t_{nc}}$

B.2.4. Isolation of payment wedges

B.2.4.1. Low-cost contract

[B. Shirking with right contract] $\underline{t_c} - \underline{\beta}(f^*-\underline{f}) \geq \underline{t_c} - \underline{\varepsilon}[1/E.(\underline{t_c} - \underline{t_{nc}})] - \underline{\beta}(f^* - \underline{f} - \underline{\varepsilon}) \leftrightarrow$

$\underline{t_c} - \underline{\beta}(f^*-\underline{f}) \geq \underline{t_c} - 1/E.\underline{\varepsilon}[(\underline{t_c} - \underline{t_{nc}}) - \underline{\beta}E] - \underline{\beta}(f^* - \underline{f}) \leftrightarrow$

$1/E.\underline{\varepsilon}[(\underline{t_c} - \underline{t_{nc}}) - \underline{\beta}E] \geq 0 \rightarrow \underline{t_c} - \underline{t_{nc}} \geq \underline{\beta}E$

[C. Shirking with wrong contract]

$t_c - \beta(f^*-f) \geq \underline{t_c} - \varepsilon'[1/E.(\underline{t_c} - \underline{t_{nc}})] - \beta(f^* - \underline{f} - \varepsilon') \leftrightarrow$

$t_c - \beta(f^*-f) \geq \underline{t_c} - 1/E.\varepsilon'[(\underline{t_c} - \underline{t_{nc}}) - \beta E] - \beta(f^* - \underline{f}) \leftrightarrow$ (incorporating [A])

$\{\underline{t_c} - \underline{\beta}(f^*-\underline{f}) + \underline{\beta}(f^*-f)\} - \beta(f^*-f) \geq \underline{t_c} - 1/E.\varepsilon'[(\underline{t_c} - \underline{t_{nc}}) - \beta E] - \beta(f^* - \underline{f}) \leftrightarrow$

$-\Delta\beta(f^*-f) \geq -1/E.\varepsilon'[(\underline{t_c} - \underline{t_{nc}}) - \beta E] - \Delta\beta(f^* - \underline{f}) \leftrightarrow$

$1/E.\varepsilon'[(\underline{t_c} - \underline{t_{nc}}) - \beta E] \geq -\Delta\beta(f - \underline{f}) \leftrightarrow$

$[(\underline{t_c} - \underline{t_{nc}}) - \beta E] \geq -\Delta\beta(f - \underline{f})\ E/\varepsilon' \leftrightarrow$

$\underline{t_c} - \underline{t_{nc}} \geq \beta E - \Delta\beta(f - \underline{f})\ E/\varepsilon'$

[E. Explicit non-compliance with right contract] $\underline{t_c} - \underline{\beta}(f^*-\underline{f}) \geq \underline{t_{nc}} \rightarrow \underline{t_c} - \underline{t_{nc}} \geq \underline{\beta}(f^*-\underline{f})$

[F. Explicit non-compliance with wrong contract] $t_c - \beta(f^*-f) \geq \underline{t_{nc}} \rightarrow$ (incorporating [A])

$\{\underline{t_c} - \underline{\beta}(f^*-\underline{f}) + \underline{\beta}(f^*-f)\} - \beta(f^*-f) \geq \underline{t_{nc}} \leftrightarrow$

$\underline{t_c} - \underline{\beta}(f^*-\underline{f}) - \Delta\beta(f^*-f) \geq \underline{t_{nc}} \leftrightarrow$

$\underline{t_c} - \underline{t_{nc}} \geq \underline{\beta}(f^*-\underline{f}) + \Delta\beta(f^*-f)$

(what is equivalent to $\underline{t_c} - \underline{t_{nc}} \geq \underline{t_c}$, i.e., $\underline{t_{nc}} \leq 0$).

B.2.4.2. High-cost contract

[B] (Shirking with right contract) $t_c - \beta(f^*-f) \geq t_c - \varepsilon[1/E.(t_c - t_{nc})] - \beta(f^* - f - \varepsilon) \leftrightarrow$

$0 \geq -\varepsilon[1/E.(t_c - t_{nc}) - \beta] \leftrightarrow \varepsilon[1/E.(t_c - t_{nc}) - \beta] \geq 0 \leftrightarrow 1/E.\varepsilon[(t_c - t_{nc}) - \beta E] \geq 0$

$\leftrightarrow t_c - t_{nc} \geq \beta E$

[C] (Shirking with wrong contract) $\underline{t_c} - \underline{\beta}(f^*-\underline{f}) \geq t_c - \underline{\varepsilon}'[1/E.(t_c - t_{nc})] - \underline{\beta}(f^* - f - \underline{\varepsilon}') \leftrightarrow$

$\underline{t}_c - \underline{\beta}(f^*-\underline{f}) \geq t_c - 1/E.\underline{\varepsilon}'[(t_c - t_{nc}) - \underline{\beta}E] - \underline{\beta}(f^*- f) \leftrightarrow$ (incorporating [$\underline{A}$])

$\{t_c - \underline{\beta}(f^*-f) + \underline{\beta}(f^*-\underline{f})\} - \underline{\beta}(f^*-\underline{f}) \geq t_c - 1/E.\underline{\varepsilon}'[(t_c - t_{nc}) - \underline{\beta}] - \underline{\beta}(f^*- f) \leftrightarrow$

$0 \geq - 1/E.\underline{\varepsilon}'[(t_c - t_{nc}) - \underline{\beta}E] \leftrightarrow$

$1/E.\underline{\varepsilon}'[(t_c - t_{nc}) - \underline{\beta}E] \geq 0 \leftrightarrow$

$t_c - t_{nc} \geq \underline{\beta}E$

[E] (Explicit non-compliance with right contract) $t_c - \beta(f^*-f) \geq t_{nc} \leftrightarrow t_c - t_{nc} \geq \beta(f^*-f)$

[$\underline{F}$] (Explicit non-compliance with wrong contract) $\underline{t}_c - \underline{\beta}(f^*-\underline{f}) \geq t_{nc} \leftrightarrow$ (incorporating [$\underline{A}$]) $\{t_c - \underline{\beta}(f^*-f) + \underline{\beta}(f^*-\underline{f})\} - \underline{\beta}(f^*-\underline{f}) \geq t_{nc} \leftrightarrow t_c - t_{nc} \geq \underline{\beta}(f^*-f)$.

B.2.4.3. Summary

<u>Low-cost contract</u>

[$\underline{B}$] $\underline{t}_c - \underline{t}_{nc} \geq \underline{\beta}E$

[C] $\underline{t}_c - \underline{t}_{nc} \leq \beta E - \Delta\beta(f - \underline{f})\, E/\varepsilon'$

[$\underline{E}$] $\underline{t}_c - \underline{t}_{nc} \geq \underline{\beta}(f^*-\underline{f})$

[F] $\underline{t}_c - \underline{t}_{nc} \geq \underline{\beta}(f^*-\underline{f}) + \Delta\beta(f^*-f)$

<u>High-cost contract</u>

[B] $t_c - t_{nc} \geq \beta E$

[$\underline{C}$] $t_c - t_{nc} \geq \underline{\beta}E$

[E] $t_c - t_{nc} \geq \beta(f^*-f)$

[$\underline{F}$] $t_c - t_{nc} \geq \underline{\beta}(f^*-f)$

B.2.5. Redundancy analysis

B.2.5.1. Low-cost contract

In the case that explicit non-compliance is possible ($f^* - \underline{f} > E$), the minimum wedge-constraints for the low-cost contract are:

[$\underline{B}$] $\underline{t}_c - \underline{t}_{nc} \geq \underline{\beta}E$

[C] $\underline{t}_c - \underline{t}_{nc} \geq \beta E - \Delta\beta(f - \underline{f})\, E/\varepsilon'$

[$\underline{E}$] $\underline{t}_c - \underline{t}_{nc} \geq \underline{\beta}(f^*-\underline{f})$

[F] $\underline{t}_c - \underline{t}_{nc} \geq \underline{\beta}(f^*-\underline{f}) + \Delta\beta(f^*-f)$

[$\underline{S}$] $\underline{t}_c - \underline{t}_{nc} \geq \underline{\beta}E$

[S'] $\underline{t}_c - \underline{t}_{nc} > \beta E$

Constraints, [$\underline{B}$], [C] and [$\underline{S}$] are redundant relative to [S'] (notice that $\Delta\beta(f - \underline{f})\, E/\varepsilon' > 0$, so that $\beta E - \Delta\beta(f - \underline{f})\, E/\varepsilon' < \beta E$). By the same coin, [$\underline{E}$] is redundant relative to [F]. The non-redundant constraints are:

[F] $\underline{t}_c - \underline{t}_{nc} \geq \underline{\beta}(f^*-\underline{f}) + \Delta\beta(f^*-f)$

[S’] $\underline{t}_c - \underline{t}_{nc} > \beta E$

If a large abatement is also the case in the high-cost contract, then redundancy of condition [S’] is clear based on the following manipulation of condition [F], after accounting for $f^* - f > E$:

$\underline{\beta}(f^*-\underline{f}) + \beta(f^*-f) - \underline{\beta}(f^*-f) = \underline{\beta}(f - \underline{f}) + \beta(f^*-f) > \underline{\beta}(f - \underline{f}) + \beta E > \beta E$

Anyway, in the case of large abatement in the low-cost contract, the wedge should be calculated as: $\underline{t}_c - \underline{t}_{nc} \geq \max\{\underline{\beta}(f^*-\underline{f}) + \Delta\beta(f^*-f); \beta E\}$

In the case of small abatement, in which explicit non-compliance is impossible, the constraint list is:

$[\underline{B}]$ $\underline{t}_c - \underline{t}_{nc} \geq \underline{\beta}E$

[C] $\underline{t}_c - \underline{t}_{nc} \geq \beta E - \Delta\beta(f - \underline{f})\, E/\varepsilon'$

$[\underline{S}]$ $\underline{t}_c - \underline{t}_{nc} \geq \underline{\beta}E$

[S’] $\underline{t}_c - \underline{t}_{nc} > \beta E$

The non-redundant constraint is [S’].

In summary, the wedge of low-cost contract is:

$\underline{t}_c - \underline{t}_{nc} \geq \max\{\underline{\beta}(f^*-\underline{f}) + \Delta\beta(f^*-f); \beta E\}$, if $f^* - \underline{f} > E$

$\underline{t}_c - \underline{t}_{nc} \geq \beta E$, otherwise.

B.2.5.2. High-cost contract

With explicit non-compliance being possible, i.e., $f^* - f > E$:

[B] $t_c - t_{nc} \geq \beta E$

$[\underline{C}]$ $t_c - t_{nc} \geq \underline{\beta}E$

[E] $t_c - t_{nc} \geq \beta(f^*-f)$

$[\underline{F}]$ $t_c - t_{nc} \geq \underline{\beta}(f^*-f)$

$[\underline{S'}]$ $t_c - t_{nc} > \underline{\beta}E$

[S] $t_c - t_{nc} > \beta E$

Constraints [B], $[\underline{C}]$, $[\underline{F}]$, $[\underline{S'}]$ and [S] are redundant relative to [E] which is the only constraint that remains.

With explicit non-compliance being impossible, the constraint list is:

[B] $t_c - t_{nc} \geq \beta E$

$[\underline{C}]$ $t_c - t_{nc} \geq \underline{\beta}E$

$[\underline{S'}]$ $t_c - t_{nc} > \underline{\beta}E$

[S] $t_c - t_{nc} > \beta E$

So that only [B]/[S] are not redundant.

In summary, wedge in the high cost constraint is:

$t_c - t_{nc} \geq \beta(f^*-f)$, if $f^* - f > E$

$t_c - t_{nc} \geq \beta E$, otherwise

B.2.5.3. Showing that non-positive non-compliance payments follow

Importantly, the payment wedges imposed result into non-positive non-compliance payments in the large abatement case.

$\underline{\text{High-cost contract}}$

[E] $t_c - t_{nc} \geq \beta(f^*-f)$.

From [D], $t_c = \beta(f^*-f) \rightarrow$ [E] $\beta(f^*-f) - t_{nc} \geq \beta(f^*-f) \rightarrow -t_{nc} \geq 0 \rightarrow t_{nc} \leq 0$

$\underline{\text{Low-cost contract}}$

Assuming large abatements in the two contracts:

$\underline{t}_c - \underline{t}_{nc} \geq \max\{\underline{\beta}(f^*-\underline{f}) + \Delta\beta(f^*-f);\ \beta E\} = \underline{\beta}(f^*-\underline{f}) + \Delta\beta(f^*-f)$

From [$\underline{A}$] $\underline{t}_c = \Delta\beta(f^*-f) + \underline{\beta}(f^*-\underline{f})$:

$\underline{t}_c - \underline{t}_{nc} \geq \underline{\beta}(f^*-\underline{f}) + \Delta\beta(f^*-f) = \underline{t}_c \rightarrow -\underline{t}_{nc} \geq 0 \rightarrow \underline{t}_{nc} \leq \underline{0}$

Appendix C. Command and control policy

C.1. Principal's problem

The problem of the command-and-control regulator is:

$$\text{Max}_{\{f,g\}}\ W(f) - v[\underline{\beta}(f^*-f)] - (1-v)[\beta(f^*-f)]$$

subject to:

1 Low-cost agent constraints

$\underline{\text{[CAC-A]}}$ (Shirking) $-\underline{\beta}(f^*-f) \geq -g\varepsilon/E - \underline{\beta}(f^*- f - \varepsilon)$

$\underline{\text{[CAC-B]}}$ (Explicit non-compliance) $-\underline{\beta}(f^*-f) \geq -g$

2 High-cost agent constraints

[CAC-A] (Shriking) $-\beta(f^*-f) \geq -g\varepsilon/E - \beta(f^*- f - \varepsilon)$

[CAC-B] (Explicit non-compliance) $-\beta(f^*-f) \geq -g$

(constraints are referred to by the type of non-compliance they make non-strictly preferred)

With utility symbols defined analogously as in section B.1. The number of constraints results from the two forms of non-compliance and the two agent types. If should be noted that, with $f > f^* - E$, explicit non-compliance is impossible, so only constraints CAC-A and CAC-$\underline{A}$ apply.

C.2. Utility differentials

$\underline{\text{High-cost agent, utility differentials}}$

[CAC-A] $-\beta(f^*-f) \geq -g\varepsilon/E - \beta(f^*- f - \varepsilon) \rightarrow 0 \geq -g\varepsilon/E + \beta\varepsilon \rightarrow g\varepsilon/E \geq \beta\varepsilon \rightarrow g \geq \beta E$

[CAC-B] $-\beta(f^*-f) \geq -g \rightarrow g \geq \beta(f^*-f)$

The optimal shirking problem of the agent is:

$$\varepsilon = \begin{cases} f^* - f, if\ g < \beta E \\ x \in [0; f^* - f], if\ g = \beta E \\ 0, \quad if\ g > \beta E \end{cases}$$

Low-cost agent, utility differentials

[CAC-A] $-\underline{\beta}(f^*-f) \geq -g\varepsilon/E - \underline{\beta}(f^*-f-\varepsilon) \rightarrow 0 \geq -g\varepsilon/E + \underline{\beta}\varepsilon \rightarrow g/E \geq \underline{\beta} \rightarrow g \geq \underline{\beta}E$

[CAC-B] $Uc - Ue \geq 0 \rightarrow -\underline{\beta}(f^*-f) - (-g) \geq 0 \rightarrow g \geq \underline{\beta}(f^*-f)$

The optimal shirking problem of the agent is:

$$\underline{\varepsilon} = \begin{cases} f^* - f, if\ g < \underline{\beta} E \\ x \in [0; f^* - f], if\ g = \underline{\beta} E \\ 0, \quad if\ g > \underline{\beta} E \end{cases}$$

C.3. Principal's problem with detailed utility differentials

$Max_{\{f,g\}}\ W(f) - v[\underline{\beta}(f^*-f)] - (1-v)[\beta(f^*-f)]$

s.t.:

[CAC-A] $g \geq \beta E$

[CAC-B] $g \geq \beta(f^*-f)$

[Null optimal shirking] $g > \beta E$; (this is hereafter omitted for being redundant with CAC-A)

[CAC-A] $g \geq \underline{\beta}E$

[CAC-B] $g \geq \underline{\beta}(f^*-f)$

[Null optimal shirking] $g > \underline{\beta}E$; (this is hereafter omitted for being redundant with CAC-A)

C.4. Lower bounds for fine

C.4.1. Situation 1, $f < f^* - E$

(A.1) Elimination of redundant constraints

The constraints are:

[CAC-A] $g \geq \beta E$

[CAC-B] $g \geq \beta(f^*-f)$

[CAC-A] $g \geq \underline{\beta}E$

[CAC-B] $g \geq \underline{\beta}(f^*-f)$

It is clear that, since $\beta > \underline{\beta}$, the constraints referring to the low-cost agent are dominated and thus automatically satisfied after the constraints for the high-cost agent are satisfied. Therefore, the non-redundant constraint set is:

[CAC-A] $g \geq \beta E$

[CAC-B] $g \geq \beta(f^*-f)$

(A.2) Determination of fine bounds satisfying non-redundant constraints

Since $f < f^* - E$, in situation 1, i.e., $E < f^* - f$, then $\beta(f^*-f) > \beta E$, so that CAC-B is dominant and thus defines the lower bound for the fine. In this situation, thus, a fine $w \geq \beta(f^*-f)$ should be charged.

C.4.2. Situation 2, $f \geq f^* - E$

(B.1) Elimination of redundant constraints

In this case, explicit non-compliance is impossible ($f + E > f^*$), so only shirking constraints apply:

[CAC-A] $g \geq \beta E$

[CAC-B] $g \geq \underline{\beta}(f^*-f)$

Again, the high-cost agent constraint dominates.

(B.2) Determination of fine bounds satisfying non-redundant constraints

With only one constraint, minimum fine level is promptly determined from $g \geq \beta E$.

C.4.3. Summary

The fine w is such that:

(a) If $f < f^* - E$, $g \geq \beta(f^*-f)$

(b) If $f \geq f^* - E$, $g \geq \beta E$

C.5. Optimal targets, second-best problem

From the principal's problem, one has that:

$Max_{\{f,g\}}\ W(f) - v[\underline{\beta}(f^*-f)] - (1-v)[\beta(f^*-f)]$

Or, equivalently:

$Max_{\{f,g\}}\ W(f) - v\underline{\beta}f^*+f[-v\Delta\beta + \beta] - (1-v)\beta f^*$

The FOC for f is:

$W'(f) - v\Delta\beta + \beta = 0 \rightarrow W'(f) = -(\beta + v\Delta\beta)$

Or, incorporating the functional form of W'(.) (see B.2 above for details):

$W'(f) = -D^{e\prime}(f) = -(s_0+s_1E/2 + s_1 f) = -(\beta - v\Delta\beta) \rightarrow f^{SB} = (\beta - v\Delta\beta - s_0)/s_1 - E/2$

C.6. Optimal targets, first-best problem

In this case both agent type and action are observable, so there are no AS or MH problems. Nevertheless, the FB case is examined here exclusively as a reference for target abatement levels, so constraints will be abstracted, with no loss of generality since they do not determine the targets – therefore, the FB direct regulation policy is ignored, as it was done for the contract policy. Since the principal observes agent type, it is possible to stablish type-specific targets and then, ex-post, verify whether each agent has complied with the targeted defined for him/her. The FB targets are obtained from the problem below.

$Max\{f,\underline{f},w,\underline{w}\}\ v[W(\underline{f}) - \underline{\beta}(f^*-\underline{f})] + (1-v)[W(f) - \beta(f^*-f)]$

Or

$Max\{f,\underline{f},w,\underline{w}\}\ v[-D^e(\underline{f}) - \underline{\beta}(f^*-\underline{f})] + (1-v)[-D^e(f) - \beta(f^*-f)]$

The FOCs are:

(High-cost agent) $(1-v)[-D^{e\prime}(f) + \beta] = 0 \rightarrow D^{e\prime}(f) = \beta$ (1)

(Low-cost agent) $v[-D^{e\prime}(\underline{f}) + \underline{\beta}] = 0 \rightarrow D^{e\prime}(\underline{f}) = \underline{\beta}$ (2)

Incorporating the functional form of $D^{e\prime}()$, one obtains:

$f^{FB} = (\beta - s_0)/s_1 - E/2$

$\underline{f}^{FB} = (\underline{\beta} - s_0)/s_1 - E/2$

C.7. Comparison of first-best and second-best target

The optimal target levels are:

$f^{FB} = (\beta - s_0)/s_1 - E/2$

$\underline{f}^{FB} = (\underline{\beta} - s_0)/s_1 - E/2$

$f^{SB} = (\beta - v\Delta\beta - s_0)/s_1 - E/2$

The second-best target is a weighted average of the first-best targets, as shown below.

$f^{SB} = (\beta - v\Delta\beta - s_0)/s_1 - E/2 = (\beta - v\beta + v\underline{\beta} - s_0)/s_1 - E/2 = ((1-v)\beta + v\underline{\beta} - s_0)/s_1 - E/2 =$
$((1-v)(\beta - s_0))/s_1 + (v(\underline{\beta} - s_0))/s_1 - (1-v)\ E/2 - vE/2 = (1-v)[(\beta - s_0)/s_1 - E/2] + v[(\underline{\beta} - s_0)\ /s_1 - E/2]$.

In summary, $f^{SB} = (1-v)[(\beta - s_0)/s_1 - E/2] + v[(\underline{\beta} - s_0)\ /s_1 - E/2]$.

Therefore, accounting for the fact that $v = 0.5$ is assumed in the main text, the pooling one-target regulation distorts target downward (smaller abatement, larger deliberately burned area) by the low-cost agent and upward by the high-cost agent, which is almost the opposite of the distortion observed under the optimal contract policy (which distorted the target of high-cost downwards, only).

C.8. Payment variance is directly proportional to the payment wedge

Such proportionality is clearly seen from the argument that follows, which is presented for the high-cost contract as the low-contract case is analogous.

$V[t] = E[(t - E[t])^2] = (t_c - E[t])^2(1-P(f_a > E - \varepsilon)) + (t_{nc} - E[t])^2 P(f_a > E - \varepsilon)$ (A).

It is known that $E[t] = t_c\ (1-P(f_a > E - \varepsilon)) + t_{nc}\ P(f_a > E - \varepsilon) = -m\ P(f_a > E - \varepsilon) + t_c$ (B), with "m" representing the wedge. Incorporating (B) into (A) yields:

$V[t] = m^2\ P(f_a > E - \varepsilon)^2(1-P(f_a > E - \varepsilon)) + m^2\ (P(f_a > E - \varepsilon) - 1)^2\ P(f_a > E - \varepsilon) = s(m^2),\ ds/dm > 0$.

Appendix D. Algebraic decomposition of the welfare surplus differential between contracts and CAC

The objective of this analysis is, from the algebraic decomposition of the welfare functions associated with contracts and CAC, to uncover the main sources of the trade-off between the instruments. In what follows, (second-best) welfare functions were first factored into components and then the component-specific differences were computed.

D.1. Welfare function of contracts

$$v\left[W\left(\underline{f}\right)+\left(\underline{t_c}-\underline{\beta}\left(f^*-\underline{f}\right)\right)-(1+\lambda)\underline{t_c}\right]$$
$$+(1-v)\left[W(f)+\left(t_c-\beta(f^*-f)\right)-(1+\lambda)t_c\right]=$$
$$v\left[W\left(\underline{f}\right)+\left(\underline{t_c}-\underline{\beta}\left(f^*-\underline{f}\right)\right)-(1+\lambda)\underline{t_c}\right]$$
$$+(1-v)\left[W(f)+\left(t_c-\beta(f^*-f)\right)-(1+\lambda)t_c\right]=$$
$$v\left[W\left(\underline{f}\right)-\underline{\beta}\left(f^*-\underline{f}\right)-\lambda\underline{t_c}\right]+(1-v)[W(f)-\beta(f^*-f)-\lambda t_c]=$$
$$v\left[W\left(\underline{f}\right)-\underline{\beta}\left(f^*-\underline{f}\right)\right]+(1-v)[W(f)-\beta(f^*-f)]-\lambda\left[\mathrm{v}\underline{t_c}+(1-\mathrm{v})t_c\right]=$$
$$vW\left(\underline{f}\right)+(1-v)W(f)-\lambda[\mathrm{v}\underline{t_c}+(1-\mathrm{v})t_c]-v\left[\underline{\beta}\left(f^*-\underline{f}\right)\right]-(1-v)[\beta(f^*-f)]$$

D.2. Welfare function of CAC

$$v\left[W(f^{CAC})-\underline{\beta}(f^*-f^{CAC})\right]+(1-v)[W(f^{CAC})-\beta(f^*-f^{CAC})]=$$
$$W(f^{CAC})-v\left[\underline{\beta}(f^*-f^{CAC})\right]-(1-v)[\beta(f^*-f^{CAC})]=$$

D.3. Welfare differential (Contracts – CAC)

$$vW\left(\underline{f}\right)+(1-v)W(f)-\lambda\left[\mathrm{v}\underline{t_c}+(1-\mathrm{v})t_c\right]-v\left[\underline{\beta}\left(f^*-\underline{f}\right)\right]-(1-v)[\beta(f^*-f)]-$$
$$W(f^{CAC})+v\left[\underline{\beta}(f^*-f^{CAC})\right]+(1-v)[\beta(f^*-f^{CAC})]=$$
$$vW\left(\underline{f}\right)+(1-v)W(f)-W(f^{CAC})-v\left[\underline{\beta}\left(f^*-\underline{f}\right)\right]-(1-v)[\beta(f^*-f)]$$
$$+v\left[\underline{\beta}(f^*-f^{CAC})\right]+(1-v)[\beta(f^*-f^{CAC})]-\lambda\left[\mathrm{v}\underline{t_c}+(1-\mathrm{v})t_c\right]=$$

$$vW\left(\underline{f}\right) + (1-v)W(f) - W(f^{CAC})$$
$$+\left\{v\left[\,\underline{\beta}(f^* - f^{CAC})\right] + (1-v)[\beta(f^* - f^{CAC})] - v\left[\underline{\beta}\left(f^* - \underline{f}\right)\right]\right.$$
$$\left. - (1-v)[\beta(f^* - f)]\right\} - \lambda\left[\mathrm{v}\underline{t_c} + (1-\mathrm{v})t_c\right]$$

D.4. Abatement cost differential (CAC - contracts)

$$v\left[\,\underline{\beta}(f^* - f^{CAC})\right] + (1-v)[\beta(f^* - f^{CAC})] - v\left[\underline{\beta}\left(f^* - \underline{f}\right)\right] - (1-v)[\beta(f^* - f)] =$$
$$v\underline{\beta}\left[\,f^* - f^{CAC} - f^* + \underline{f}\right] + (1-v)\beta[f^* - f^{CAC} - f^* + f] =$$
$$v\underline{\beta}\left[\,\underline{f} - f^{CAC}\right] + (1-v)\beta[f - f^{CAC}] = v\underline{\beta}[(A)] + (1-v)\beta[(B)]$$

Examining each abatement differential for its turn, one has:

$(A)\underline{f} - f^{CAC}$

From B.1.3: $f^{SB} = [(1+\lambda)\underline{\beta} - s_0]/s_1 - E/2$

From C.5:

$$f^{CAC} = \frac{1}{s_1}\left[v\underline{\beta} + (1-v)\beta - s_0\right] - \frac{E}{2}$$

Thus:

$$\underline{f} - f^{CAC} = \frac{1}{s_1}\left[(1+\lambda)\underline{\beta} - s_0\right] - \frac{E}{2} - \left(\frac{1}{s_1}\left[v\underline{\beta} + (1-v)\beta - s_0\right] - \frac{E}{2}\right) \rightarrow$$
$$\underline{f} - f^{CAC} = \frac{1}{s_1}\left[(1+\lambda)\underline{\beta}\right] - \frac{1}{s_1}\left[v\underline{\beta} + (1-v)\beta\right] \rightarrow$$
$$\underline{f} - f^{CAC} = \frac{1}{s_1}\left[v(1+\lambda)\underline{\beta} + (1-v)(1+\lambda)\underline{\beta}\right] - \frac{1}{s_1}\left[v\underline{\beta} + (1-v)\beta\right] \rightarrow$$
$$\underline{f} - f^{CAC} = \frac{1}{s_1}\left[v\lambda\underline{\beta} + (1-v)\left((1+\lambda)\underline{\beta} - \beta\right)\right] \rightarrow$$
$$\underline{f} - f^{CAC} = \frac{1}{s_1}\left[v\lambda\underline{\beta} + (1-v)\left(\lambda\underline{\beta} - \Delta\beta\right)\right] \quad (A)$$

Or, alternatively:

$$\underline{f} - f^{CAC} = \frac{1}{s_1}\left[v\lambda\underline{\beta} + (1-v)\underline{\beta}\left(\lambda - \frac{\Delta\beta}{\underline{\beta}}\right)\right]$$

Thus, only if $\Delta\beta/\beta > \lambda$ abatement is smaller in CEC than in the low-cost contract.

(B) $f - f^{CAC}$

From B.1.3: $f^{SM} = [v/(1-v)\lambda\Delta\beta + (1+\lambda)\beta - s_0]/s_1 - E/2$

From C.5:

$$f^{CAC} = \frac{1}{s_1}\left[v\underline{\beta} + (1-v)\beta - s_0\right] - \frac{E}{2}$$

Then:

$$f - f^{CAC} = \frac{1}{s_1}\left[\frac{v}{1-v}\lambda\Delta\beta + (1+\lambda)\beta - s_0\right] - \frac{E}{2} - \left(\frac{1}{s_1}\left[v\underline{\beta} + (1-v)\beta - s_0\right] - \frac{E}{2}\right) \rightarrow$$

$$f - f^{CAC} = \frac{1}{s_1}\left[\frac{v}{1-v}\lambda\Delta\beta + (1+\lambda)\beta\right] - \left(\frac{1}{s_1}\left[v\underline{\beta} + (1-v)\beta\right]\right) \rightarrow$$

$$f - f^{CAC} = \frac{1}{s_1}\left[\frac{v}{1-v}\lambda\Delta\beta + (1+\lambda)\beta\right] - \left(\frac{1}{s_1}\left[-v\Delta\beta + \beta\right]\right) \rightarrow$$

$$f - f^{CAC} = \frac{1}{s_1}\left[\Delta\beta\left(\frac{v}{1-v}\lambda + v\right) + \lambda\beta\right] \quad (B)$$

Since the final expression is positive, CAC is always more demanding upon the high-cost-agent (whom also generates smaller damage).

Incorporating (A) and (B) in the abatement cost differential between CAC and contract, one has:

$$v\underline{\beta}\left[\underline{f} - f^{CAC}\right] + (1-v)\beta[f - f^{CAC}] =$$

$$v\underline{\beta}\left[\frac{1}{s_1}\left[v\lambda\underline{\beta} + (1-v)\left((1+\lambda)\underline{\beta} - \beta\right)\right]\right] + (1-v)\beta\left[\frac{1}{s_1}\left[\Delta\beta\left(\frac{v}{1-v}\lambda + v\right) + \lambda\beta\right]\right] =$$

$$v\underline{\beta}\left[\frac{1}{s_1}\left[v\lambda\underline{\beta} + (1-v)\left(\lambda\underline{\beta} - \Delta\beta\right)\right]\right] + (1-v)\beta\left[\frac{1}{s_1}\left[\Delta\beta\left(\frac{v}{1-v}\lambda + v\right) + \lambda\beta\right]\right] =$$

$$v\underline{\beta}\left[\frac{1}{s_1}\left[v\lambda\underline{\beta} + (1-v)\lambda\underline{\beta} - (1-v)\Delta\beta\right]\right] + (1-v)\beta\left[\frac{1}{s_1}\left[\Delta\beta\left(\frac{v}{1-v}\lambda + v\right) + \lambda\beta\right]\right] =$$

$$v\underline{\beta}\left[\frac{1}{s_1}\left[\lambda\underline{\beta} - (1-v)\Delta\beta\right]\right] + (1-v)\beta\left[\frac{1}{s_1}\left[\Delta\beta\left(\frac{v}{1-v}\lambda + v\right) + \lambda\beta\right]\right] =$$

$$\frac{1}{s_1}\left\{v\underline{\beta}\left[\lambda\underline{\beta} - (1-v)\Delta\beta\right] + (1-v)\beta\left[\Delta\beta\left(\frac{v}{1-v}\lambda + v\right) + \lambda\beta\right]\right\} =$$

$$\frac{1}{s_1}\left\{\lambda\left[v\underline{\beta}^2 + (1-v)\beta^2\right] + \Delta\beta\left[(1-v)\beta\left(\frac{v}{1-v}\lambda + v\right) - v\underline{\beta}(1-v)\right]\right\} =$$

$$\frac{1}{s_1}\left\{\lambda\left[v\underline{\beta}^2 + (1-v)\beta^2\right] + \Delta\beta\left[(1-v)\left(\beta\left(\frac{v}{1-v}\lambda + v\right) - v\underline{\beta}\right)\right]\right\} =$$

$$\frac{1}{s_1}\left\{\lambda\left[v\underline{\beta}^2 + (1-v)\beta^2\right] + \Delta\beta\left[(1-v)\left(\beta v\left(\frac{1}{1-v}\lambda + \frac{1-v}{1-v}\right) - v\underline{\beta}\right)\right]\right\} =$$

$$\frac{1}{s_1}\left\{\lambda\left[v\underline{\beta}^2 + (1-v)\beta^2\right] + \Delta\beta\left[(1-v)\left(\beta v\left(\frac{\lambda + 1 - v}{1-v}\right) - v\underline{\beta}\right)\right]\right\} =$$

$$\frac{1}{s_1}\left\{\lambda\left[v\underline{\beta}^2+(1-v)\beta^2\right]+\Delta\beta\left[(1-v)\left(\beta v(\lambda+1-v)-v(1-v)\underline{\beta}\right)\right]\right\}=$$

$$\frac{1}{s_1}\left\{\lambda\left[v\underline{\beta}^2+(1-v)\beta^2\right]+\Delta\beta\left[(1-v)v\left(\beta(\lambda+1-v)-(1-v)\underline{\beta}\right)\right]\right\}=$$

$$\frac{1}{s_1}\left\{\lambda\left[v\underline{\beta}^2+(1-v)\beta^2\right]+\Delta\beta\left[(1-v)v\left(\beta(\lambda+1-v)-\underline{\beta}(1-v)\right)\right]\right\}=$$

$$\frac{1}{s_1}\left\{\lambda\left[v\underline{\beta}^2+(1-v)\beta^2\right]+\Delta\beta[(1-v)v(\beta\lambda+\Delta\beta(1-v))]\right\}=$$

Since all elements of the expression are certainly positive, there is always a larger abatement under CAC.

D.5. Abatement differential (CAC - contracts)

$$(f^*-f^{CAC})-v\left(f^*-\underline{f}\right)-(1-v)(f^*-f)=$$

$$v(f^*-f^{CAC})+(1-v)(f^*-f^{CAC})-v\left(f^*-\underline{f}\right)-(1-v)(f^*-f)$$

$$v\left(\underline{f}-f^{CAC}\right)+(1-v)(f-f^{CAC})=$$

$$v\left(\frac{1}{s_1}\left[v\lambda\underline{\beta}+(1-v)\left(\lambda\underline{\beta}-\Delta\beta\right)\right]\right)+(1-v)\left(\frac{1}{s_1}\left[\Delta\beta\left(\frac{v}{1-v}\lambda+v\right)+\lambda\beta\right]\right)=$$

$$\frac{1}{s_1}\left\{v\left[v\lambda\underline{\beta}+(1-v)\left(\lambda\underline{\beta}-\Delta\beta\right)\right]+(1-v)\left[\Delta\beta\left(\frac{v}{1-v}\lambda+v\right)+\lambda\beta\right]\right\}=$$

$$\frac{1}{s_1}\left\{v\left[\lambda\underline{\beta}-(1-v)\Delta\beta\right]+(1-v)\left[\Delta\beta\left(\frac{v}{1-v}\lambda+v\right)+\lambda\beta\right]\right\}=$$

$$\frac{1}{s_1}\left\{\lambda\left[v\underline{\beta}+(1-v)\beta\right]+\mathrm{v}(1-v)\Delta\beta\left[\left(\frac{\lambda}{1-v}+1\right)-1\right]\right\}=$$

$$\frac{1}{s_1}\left\{\lambda\left[v\underline{\beta}+(1-v)\beta\right]+\mathrm{v}(1-v)\Delta\beta\left[\frac{\lambda}{1-v}\right]\right\}$$

Since all elements are positive, abatement is larger under CAC.

An useful corollary is:

$$(f^*-f^{CAC})-v\left(f^*-\underline{f}\right)-(1-v)(f^*-f)>0$$

$$-f^{CAC}+v\underline{f}+(1-v)f>0\rightarrow v\underline{f}+(1-v)f>f^{CAC}$$

D.6. Expected damage differential (contracts – CAC)

$$vW\left(\underline{f}\right)+(1-v)W(f)-W(f^{CAC})=$$

$$-vD^e\left(\underline{f}\right)-(1-v)D^e(f)+D^e(f^{CAC})=$$

$$-\left[vD^e\left(\underline{f}\right)+(1-v)D^e(f)-D^e(f^{CAC})\right] \quad \text{(A1)}$$

It is noticed that:

$$vD^e\left(\underline{f}\right)+(1-v)D^e(f)>D^e\left(v\underline{f}+(1-v)f\right) \quad \text{(A2)}$$

This is because $D^e(.)$ is a strictly convex function, once its derivative is $s_1 > 0$ (see section B.1 and Mas-Collel *et al.*, 1995, appendix M.C). Hence, since $D^e(.)$ is an increasing function and vf +(1-v)f > f_{CEC} according with the corollary of the previous subsection, it is true that:

$$D^e\left(v\underline{f}+(1-v)f\right)>D^e(f^{CAC}) \quad \text{(A3)}$$

From (A2) and (A3):

$$vD^e\left(\underline{f}\right)+(1-v)D^e(f)>D^e\left(v\underline{f}+(1-v)f\right)>D^e(f^{CAC})\rightarrow$$

$$vD^e\left(\underline{f}\right)+(1-v)D^e(f)>D^e(f^{CAC})\rightarrow$$

$$vD^e\left(\underline{f}\right)+(1-v)D^e(f)-D^e(f^{CAC})>0 \quad \text{(A3}'\text{)}$$

Incorporating (A3′) in (A1):

$$-\left[vD^e\left(\underline{f}\right)+(1-v)D^e(f)-D^e(f^{CAC})\right]<0$$

Therefore, damage is always smaller under CAC.

D.7. Summary

In short, CAC systematically presented larger abatement, consequently smaller damage, and a larger abatement cost. Thus, greater welfare surplus under CAC is merely a matter of whether or not its lower damage and fiscal cost if avoids outweigh its greater abatement cost, which is an empirical question.

Appendix E. CAC under imperfect sanctioning

E.1. Non-judicial-based cancellation of non-compliance payment in contracts: justification

It is logical to wonder whether the farmer receiving a non-compliance payment would be successful if taking this to the judicial system, claiming the larger compliance payment, the same way s/he may achieve success in appealing to the judicial system under CAC. There are two convincing reasons for ruling out such possibility. First, the agent voluntarily signed the contract containing a clause that in case of exceedance of total burned area limit a lower payment would be made, what would be straightforward to use against him/her by a state prosecutor – or, to put alternatively, the argument that the violator ignored the rules and would have acted differently otherwise is less appealing in the voluntary case. Second, the evidences of judicial intervention on sanctioning that could be found refer to CAC exclusively, with the likelihood of judicial cancellation remaining unknown in the case of contracts.

E.2. Agent's problem under probable exogenous judicial intervention

With "j" indexing agent type, "k" indexing agent's action and x = 1 if the fine is actually paid (i.e., the judicial power does not intervene), agent's utility is found below.

$U_{jk} \equiv E[u_{jk}] = -E[g|f_{dk}] - \beta_j(f^* - f_{dk}) = -gP(f+\varepsilon + f_a > f + E, x = 1) - \beta_j(f^*-f-\varepsilon)$ = (accidental fire and judicial intervention are assumed to be two independent random events)

$-gP(f_a > E - \varepsilon).P(x = 1) - \beta_j(f^*-f-\varepsilon) = -g.\varepsilon/E.P(x = 1) - \beta_j(f^*-f-\varepsilon)$.

E.3. Optimal shirking under probable exogenous judicial intervention

Agent's shirking problem is:

$\mathrm{Max}\{\varepsilon\}\{-g.\varepsilon/E.P(x = 1) - \beta_j(f^*-f-\varepsilon)\} = \mathrm{Max}\{\varepsilon\}\{\varepsilon/E[\beta_j E - g.P(x = 1)] - \beta_j(f^*-f)\}$

The optimal non-compliance choice criteria are:

$$\varepsilon = \begin{cases} \min\{f^* - f, E\}, if\ \beta_j E - g.P(x = 1) > 0 \\ [0; \min\{f^* - f, E\}], if\ \beta_j E - g.P(x = 1) = 0 \\ 0, if \beta_j E - g.P(x = 1) < 0 \end{cases}$$

Or:

$$\varepsilon = \begin{cases} \min\{f^* - f, E\}, if\ g < \beta_j E/P(x = 1) \\ [0; \min\{f^* - f, E\}], if\ g = \beta_j E/P(x = 1) \\ 0, if\ g > \beta_j E/P(x = 1) \end{cases}$$

What makes clear that judicial intervention probability can be compensated by fine increasing. The constraint of null optimal shirking is thus:

$$g \geq \beta_j E/P(x = 1)$$

Particularizing for agent types:

$g \geq \underline{\beta}E/P(x=1)$

$g \geq \beta E/P(x=1)$

E.4. The possibility of shirking created by a legally bounded fine

The fine may be bounded above, by legislation, in a level that falls short of meeting one of the non-shirking constraints. Since there are two constraints, there are five possible scenarios, as described below along an increasing legal bound level, with the bound indicated as g_{legal}:

1. (Scenario 1) Most restrictive bound (lowest bound), i.e., $g_{legal} < \underline{\beta}E/P(x=1)$, in this case, the

two agent-types shirk;

2. Moderately restrictive bound, i.e., $\underline{\beta}E/P(x=1) \leq g_{legal} \leq \beta E/P(x=1)$. This unfolds into three scenarios:
    a. (Scenario 2) $g_{legal} = \underline{\beta}E/P(x=1)$: there is uncertain shirking by low-cost agent, which is indifferent between shirking or not, thus shirking in 50% of cases. The high-cost agent certainly shirks;
    b. (Scenario 3) $\underline{\beta}E/P(x=1) < g_{legal} < \beta E/P(x=1)$: no shirking by low-cost and (certain) shirking by high;
    c. (Scenario 4) $g_{legal} = \beta E/P(x=1)$: no shirking by low and uncertain shirking by high-cost;
3. (Scenario 5) Least restrictive bound (no bound), i.e., $g_{legal} > \beta E/P(x=1)$, in this case no agent-type shirks.

E.5. Principal's problem with probable exogenous judicial intervention for each legal fine scenario

It follows below the most general formula for principal's objective function. It accommodates shirking, either if certain or uncertain, and indicates such action with the "S" subscript, with "q" representing shirking probability. Compliance is indicated with "C". The non-distortionary nature of fine imposed on shirking is also recognized (as it is beneficial for replacing distortionary fiscal revenue sources). Importantly, non-judicial-intervention probability is treated as an exogenous relative frequency.

$$\underline{q}.v[W(\underline{f}_{d,S}) + E[u(\underline{g},\underline{f}_{d,S})] - (1+\lambda)[-g\varepsilon P(x=1)/E] + (1-\underline{q}).v[W(\underline{f}_{d,C}) + E[u(\underline{g},\underline{f}_{d,C})]] +$$
$$q.(1-v)[W(f_{d,S}) + E[u(g,f_{d,S})] - (1+\lambda)[-g\varepsilon P(x=1)/E] + (1-q).(1-v)[W(f_{d,C}) + E[u(g,f_{d,C})]] \quad (A4)$$

Utilities with shirking are:

$E[u(\underline{t},\underline{f}_d)] = \underline{\beta}\varepsilon - g\varepsilon P(x=1)/E - \underline{\beta}(f^* - f)$, and analogous for the high-cost agent (A5).

Shirking also affects intended burned area, which should be denoted as $\underline{f}_d = \underline{f} + \underline{\varepsilon}$ and $f_d = f + \underline{\varepsilon}$ (A6).

Introducing (A5) and (A6) in (A4):

$$\underline{q}.v[W(f+\underline{\varepsilon}) + \underline{\beta}\varepsilon - g\varepsilon P(x=1)/E - \underline{\beta}(f^* - f)] - (1+\lambda)(-g\varepsilon P(x=1)/E)] + (1-\underline{q}).v[W(f) - \underline{\beta}(f^* - f)] +$$
$$q.(1-v)[W(f+\varepsilon) + \beta\varepsilon - g\varepsilon P(x=1)/E - \beta(f^* - f)] - (1+\lambda)(-g\varepsilon P(x=1)/E)] + (1-q).(1-v)[W(f) - \beta(f^* - f)]$$

(A4′)

Since fine will be always fixed at the legal upper bound (g_{legal}), the objective function may be written as:

$$\underline{q}.v[W(f+\underline{\varepsilon}) + \underline{\beta}\varepsilon - g_{legal}\varepsilon P(x=1)/E - \underline{\beta}(f^* - f)] - (1+\lambda)(-g_{legal}\varepsilon P(x=1)/E)] + (1-\underline{q}).v[W(f) - \underline{\beta}(f^* - f)] +$$

$$q.(1-v)[W(f+\varepsilon) + \beta\varepsilon - g_{legal}\varepsilon P(x=1)/E - \beta(f^* - f)] - (1+\lambda)(-g_{legal}\varepsilon P(x=1)/E)] + (1-q).(1-v)[W(f) - \beta(f^* - f)] \quad (A4'')$$

Therefore, the objective function shifts, with fine increasing, between two extreme formulas. The first is the one for the second scenario of $g_{legal} < \underline{\beta}E/p(X=1)$, with shirking for both agent types:

$$v[W(f+\underline{\varepsilon}) + \underline{\beta}\underline{\varepsilon} - g_{legal}\underline{\varepsilon}P(x=1)/E - \underline{\beta}(f^* - f)] - (1+\lambda)(-g_{legal}\varepsilon P(x=1)/E)] +$$
$$(1-v)[W(f+\varepsilon) + \beta\varepsilon - g_{legal}\varepsilon P(x=1)/E - \beta(f^* - f)] - (1+\lambda)(-g_{legal}\varepsilon P(x=1)/E)]$$

The second corresponds to the fifth scenario with $g_{legal} > \beta E/p(X=1)$ and no shirking:

$$v[W(f) - \underline{\beta}(f^* - f)] + (1-v)[W(f) - \beta(f^* - f)] = W(f) - v\underline{\beta}(f^* - f) - (1-v)\beta(f^* - f)$$

It should be noticed that at this general level it is impossible to exclude the possibility that welfare may be higher in the double shirking case, since agents' utility may be positive and thus larger than in the no shirking case. What may be amplified by the gain of avoiding tax-distortion inherent to fine revenue.

Of the four scenarios with shirking by at least one agent, only two cases will be evaluated, the ones in which shirking level is certain/determinate (i.e., either null or full). The cases of indeterminate shirking, in which fine level is exact, of either $g = \underline{\beta}E/p(x=1)$ or $g = \beta E/p(x=1)$, do not lead to solutions. They are also negligibly likely to occur, since the legal bound on fine is a continuous variable that thus takes exact specific values with negligible likelihood.

E.5.1. Scenario 1: most restrictive legal bound

The objective function is:

$$v[W(f+\underline{\varepsilon}) + \underline{\beta}\varepsilon - g_{legal}\,\varepsilon P(x=1)/E - \underline{\beta}(f^* - f)] - (1+\lambda)(-g_{legal}\varepsilon P(x=1)/E)] +$$
$$(1-v)[W(f+\varepsilon) + \beta\varepsilon - g_{legal}\,\varepsilon P(x=1)/E - \beta(f^* - f)] - (1+\lambda)(-g_{legal}\varepsilon P(x=1)/E)]$$

Both agents shirk at maximum level, which will be denoted as $\min\{f^*-f,E\} = \varepsilon_{max}$, as whether $f^* - f > E$ or not is ultimately parametric.

$$v[W(f+\varepsilon_{max}) + \underline{\beta}\varepsilon_{max} - g_{legal}\,\varepsilon_{max}P(x=1)/E - \underline{\beta}(f^* - f)] - (1+\lambda)(-g_{legal}\varepsilon_{max}\,P(x=1)/E)] +$$
$$(1-v)[W(f+\varepsilon_{max}) + \beta\varepsilon_{max} - g_{legal}\,\varepsilon_{max}P(x=1)/E - \beta(f^* - f)] - (1+\lambda)(-g_{legal}\varepsilon_{max}\,P(x=1)/E)]$$

Or:

$$W(f+\varepsilon_{max}) + v[\underline{\beta}\varepsilon_{max} - \underline{\beta}(f^* - f)] + (1-v)[\beta\varepsilon_{max} - \beta(f^* - f)] + \lambda\,(g_{legal}\varepsilon_{max}\,P(x=1)/E) =$$

Isolating f and ε_{max}:

$W(f+\varepsilon_{max}) – v\underline{\beta}(f^* - f) – (1-v)\beta(f^* - f) + v\underline{\beta}\varepsilon_{max} + (1-v)\beta\varepsilon_{max} + \varepsilon_{max}\, \lambda\, (g_{legal}P(x=1)/E) =$

$W(f+\varepsilon_{max}) – v\underline{\beta}(f^* - f) – (1-v)\beta(f^* - f) + \varepsilon_{max}\, [v\underline{\beta}+ (1-v)\beta + \lambda\, g_{legal}P(x=1)/E]$

(it is clear that shirking both generates abatement cost economy and non-distortionary fiscal revenue)

Since ε_{max} is a function of f only in the <u>small abatement</u> case, the best target must be found for each level of abatement. For the small abatement case ($\varepsilon_{max} = f^*$-f), the objective function is:

$W(f^*) – v\underline{\beta}(f^* - f) – (1-v)\beta(f^* - f) + (f^*-f)[v\underline{\beta}+ (1-v)\beta + \lambda\, g_{legal}P(x=1)/E] =$

$W(f^*) -f(\lambda\, g_{legal}P(x=1)/E) – v\underline{\beta}f^* – (1-v)\beta f^* + f^*[v\underline{\beta}+ (1-v)\beta + \lambda\, g_{legal}P(x=1)/E]$

This linear function of f is maximized by setting $f^{CAC,sce.1,small} = 0$, which maximizes fine revenue. The solution's intuition is simple: since maximal shirking is unavoidable, the principal compensates maximal damage by increasing fine probability as much as possible.

Now for the <u>large abatement</u> case ($\varepsilon_{max} = E$):

$W(f+E) – v\underline{\beta}(f^* - f) – (1-v)\beta(f^* - f) + E[v\underline{\beta}+ (1-v)\beta + \lambda\, g_{legal}P(x=1)/E]$

The FOC is:

$W'(f+E) + v\underline{\beta} + (1-v)\beta = 0 \rightarrow - [s_0 + s_1(f+E/2)] = - [v\underline{\beta} + (1-v)\beta] \rightarrow$

$f^{CAC,sce.1,large} = f^{CAC} = [v\underline{\beta} + (1-v)\beta – s_0]/s_1 – E/2 = [\beta – v\Delta\beta – s_0]/s_1 – E/2$

This is the same optimal target as in the zero-shirking scenario.

Regarding whether abatement is small or large, and which target formula should be used, this is ultimately due to parameters and to which of the two cases is fully consistent with the two best targets. In this sense, it is relevant to notice that $f^*- f^{CAC,sce.1,large} = f^*- f^{CAC} \leq f^* - f^{CAC,sce.1,small} = f^*$. There are three possibilities to evaluate, (i) $f^{CAC} = 0$, (ii) $f^{CAC} > 0$ and $f^*- f^{CAC} < f^* \leq E$, (iii) $f^{CAC} > 0$ and $f^*- f^{CAC} \leq E < f^*$, (iv) $f^{CAC} > 0$ and $E < f^*- f^{CAC} < f^*$. Starting with the former, if $f^{CAC} = 0$, then there is only one best target, which is null, no matter the values of f* and E. In the second possibility abatement is always small, so that $f^{cac.sce1.small}$ prevails. The third possibility, is a contradiction, because small abatement is a large abatement and large abatement is a small abatement. In this possibility, there is no way to implement CAC as this would unavoidably entails a contradiction. In fourth sub-possibility, abatement is always large, then $f^{CAC.sce1.large} = f^{CAC}$ prevails.

E.5.2. Scenario 3: shirking only by the high-cost agent

In this case, the legal fine bound is g_{legal}: $\underline{\beta}E/P(x=1) < g_{legal} < \beta E/P(x=1)$, or, $g_{legal} = \gamma\beta E/P(x=1) + (1- \gamma)\underline{\beta}E/P(x=1)$, $0 < \gamma < 1$. Only the high-cost agent shirks, at the maximal level, and the objective function is:

$v[W(f) – \underline{\beta}(f^* - f)] + (1-v)[W(f+\varepsilon) + \beta\varepsilon – g_{legal}\varepsilon P(x=1)/E – \beta(f^* - f)] – (1+ \lambda)(- g_{legal}\varepsilon P(x=1)/E)]$

Isolating f and considering $\varepsilon = \varepsilon_{max}$:

$vW(f) + (1\text{-}v)W(f+\varepsilon_{max}) - v\underline{\beta}(f^* - f) - (1\text{-}v)\beta(f^* - f) + (1\text{-}v)[\beta\varepsilon_{max} - g_{legal}\varepsilon_{max}P(x=1)/E - (1+\lambda)(- g_{legal}\varepsilon_{max}P(x=1)/E)] =$

$vW(f) + (1\text{-}v)W(f+\varepsilon_{max}) - v\underline{\beta}(f^* - f) - (1\text{-}v)\beta(f^* - f) + (1\text{-}v)\varepsilon_{max}[\beta + \lambda g_{legal}P(x=1)/E]$

The best target with small abatement requires introducing $\varepsilon_{max} = f^*\text{-}f$:

$vW(f) + (1\text{-}v)W(f^*) - v\underline{\beta}(f^* - f) - (1\text{-}v)\beta(f^* - f) + (1\text{-}v)(f^*\text{-}f)[\beta + \lambda g_{legal}P(x=1)/E] =$

$vW(f) + (1\text{-}v)W(f^*) + v\underline{\beta}f + (1\text{-}v)\beta f - (1\text{-}v)f[\beta + \lambda g_{legal}P(x=1)/E] - v\underline{\beta}f^* - (1\text{-}v)\beta f^* + (1\text{-}v)f^*[\beta + \lambda g_{legal}P(x=1)/E] =$

The FOC is:

$vW'(f) + v\underline{\beta} + (1\text{-}v)\beta - (1\text{-}v)[\beta + \lambda g_{legal}P(x=1)/E] = 0 \rightarrow$

$vW'(f) + v\underline{\beta} - (1\text{-}v)\lambda g_{legal}P(x=1)/E = 0 \rightarrow$ (incorporating the formula of W'())

$-v[s_0 + s_1(f+E/2)] + v\underline{\beta} - (1\text{-}v)\lambda g_{legal}P(x=1)/E = 0 \rightarrow$

$f^{CAC,sce.3,small} = [\underline{\beta} - (1\text{-}v)\lambda g_{legal}P(x=1)/(vE) - s_0]/s_1 - E/2$

With large abatement, $\varepsilon_{max} = E$, the objective function is:

$vW(f) + (1\text{-}v)W(f+E) - v\underline{\beta}(f^* - f) - (1\text{-}v)\beta(f^* - f) + (1\text{-}v)E[\beta + \lambda g_{legal}P(x=1)/E]$

The FOC is:

$vW'(f) + (1\text{-}v)W'(f+E) + v\underline{\beta} + (1\text{-}v)\beta = 0 \rightarrow$

$-v[s_0 + s_1E/2 + s_1f] - (1\text{-}v)[s_0 + s_1E/2 + s_1f + s_1E] + v\underline{\beta} + (1\text{-}v)\beta = 0 \rightarrow$

$- [s_0 + s_1E/2 + s_1f + s_1E] + vs_1E + v\underline{\beta} + (1\text{-}v)\beta = 0 \rightarrow$

$- [s_0 + s_13E/2 + s_1f] = -vs_1E - v\underline{\beta} - (1\text{-}v)\beta \rightarrow$

$s_0 + s_13E/2 + s_1f = vs_1E + v\underline{\beta} + (1\text{-}v)\beta \rightarrow$

$s_1f = vs_1E + v\underline{\beta} + (1\text{-}v)\beta - s_0 - s_13E/2 \rightarrow$

$f = [v\underline{\beta} + (1\text{-}v)\beta - s_0]/s_1 - 3E/2 + vE \rightarrow$

$f^{CAC,sce.3,large} = [v\underline{\beta} + (1\text{-}v)\beta - s_0]/s_1 - E(3/2 - v) = [\beta - v\Delta\beta - s_0]/s_1 - E(3/2 - v)$

On which target between those for small and large abatement should apply is again dependent on parameters and consistency between targets and the case they should apply for. It is a matter of parameters whether $f^{CAC,sce.3,large} < f^{CAC,sce.3,small}$ or otherwise and each case convenes four possibilities:

<u>(Case a)</u> $f^* - f^{CAC,sce.3,small} < f^* - f^{CAC,sce.3,large}$

There are four possibilities to examine: (i) $f^{CAC,sce.3,small} = f^{CAC,sce.3,large} = 0$, (ii) $f^* - f^{CAC,sce.3,small} < f^* - f^{CAC,sce.3,large} \leq E$, (iii) $f^* - f^{CAC,sce.3,small} \leq E < f^* - f^{CAC,sce.3,large}$, (iv) $E < f^* - f^{CAC,sce.3,small} < f^* - f^{CAC,sce.3,large}$. In the first, there is only one abatement, which is full, and it should be imposed irrespective of parameters. In the second, abatement is always small, then the consistent target is $f^{CAC,sce.3,small}$. In the third possibility, the two targets are consistent (the smaller abatement is a small abatement and the larger is a large abatement), any of them could be imposed but the criterion of picking the welfare-maximizing one is adopted. In the fourth, abatement is always large, so the consistent target is $f^{CAC,sce.3,large}$.

(Case b) $f^* - f^{CAC,sce.3,small} \geq f^* - f^{CAC,sce.3,large}$

The four possibilities are: (i) $f^{CAC,sce.3,small} = f^{CAC,sce.3,large} = 0$, (ii) $f^* - f^{CAC,sce.3,large} \leq f^* - f^{CAC,sce.3,small} \leq E$, (iii) $f^* - f^{CAC,sce.3,large} \leq E < f^* - f^{CAC,sce.3,small}$, (iv) $E < f^* - f^{CAC,sce.3,large} \leq f^* - f^{CAC,sce.3,small}$. In the first, full abatement is imposed irrespective of parameters. In the second, abatement is always small, then the consistent target is $f^{CAC,sce.3,small}$. The third possibility is a contradiction in which case CAC cannot be implemented. In the fourth, abatement is always large, so the consistent target is $f^{CAC,sce.3,large}$.

E.6. The conceptual specificity of sanctioning imperfection

In this subsection it is argued that the CAC instrument derived in the previous sub-sections, whose imperfection is due to judicial intervention in sanctioning, is a specific policy instrument, with differential characteristics, which are not apparent in the mathematical derivation. More precisely, we show that two CAC instruments, one imperfect in terms of sanctioning, and the other imperfect in terms of monitoring, are, despite being mathematically equivalent, not conceptually equivalent.

The source of the difference is that, whereas monitoring-imperfect CAC is a two-stage Stackelberg game, sanctioning-imperfect CAC is a three-stage Stackelberg game as follows:

1. The regulator (executive power) plays first, defining the terms of the environmental law;
2. The farmer plays secondly, choosing whether to comply or not;
3. The judicial power plays thirdly, deciding whether, upon appeal from the farmer, to intervene in the sanctioning process started by the regulator. The intervention delays fine payment and may end with fine prescription or cancellation, and, thus, inevitably, in non-payment of the fine.

Since the regulator knows that the judicial power will play, s/he incorporates this in his/her decision by optimising contract terms accordingly. The agent also knows that the judicial power will play and accounts for that when making the compliance decision. It is implicitly assumed, for simplicity, that the agent always appeals to the judicial power when sanctioned (in accordance with the empirical evidence summarised in the previous part of this response).

This three-step game is conceptually different from the two-step imperfect monitoring game. The source of the difference lies in that the regulator must anticipate the action of two parties, the farmer and the court, whereas in the two-step game of monitoring-imperfect enforcement, only one party's action must be anticipated, that of the farmer. Likewise, the agent must account for the action of two parties, the principal and the judicial system, whereas in the standard principal-agent model, only one party, the principal, must be accounted for by the agent.

In the simplest specification considered in the submitted version of the paper, judicial intervention was modelled as an exogenous, random, process. In this case, the three-stage game takes the form of an optimisation problem which is, despite conceptually different, mathematically equal to that of the two-stage standard game of monitoring-imperfect enforcement. Such mathematical equivalence and the mathematical difference under endogenous judicial intervention, become clear when comparing the regulator's problem in three specifications, which will be now presented. We adopted the notational convention of subsuming the product of the probability of being caught violating, G(.), and probability of judicial intervention, p, to function B(.) = p.G(.).

1. The regulator's problem with imperfect monitoring (0 < G(.) < 1) and without exogenous judicial intervention.

$$Min_{\{g,y_p\}}\{-D^e(y_p) - v.c(y_p) - (1-v)\underline{c}(y_p)\}\ s.t. \begin{cases} c(y_p) \leq c\left(y_p + \varepsilon(g, y_p)\right) + G\left(\varepsilon(g, y_p)\right) g \\ \underline{c}(y_p) \leq \underline{c}(y_p + \underline{\varepsilon}(g, y_p)) + G(\underline{\varepsilon}(g, y_p))g \end{cases}$$

The notation is the same as in section 2.1 of the manuscript. That is, $D^e(.)$ is the expected damage from externality-generating activity (e.g., agricultural burnings), v is the proportion of low-cost agents in the population, c(.) is the abatement cost function, G(.) is the probability of non-compliance detection and sanctioning. Also, y_p is principal's optimal activity level ε is the extent of non-compliance and g the size of sanction (fine's value). The low-cost agent´s variables are underscored.

Notation is hereon simplified as follows:

$$Max_{\{g,y_p\}}\{L^e(y_p)\}\ s.t. \begin{cases} c(y_p) \leq c(g, y_p) + H(g, y_p) \\ \underline{c}(y_p) \leq \underline{c}(g, y_p) + H(g, y_p) \end{cases}$$

where:

$$L^e(y_p) \equiv -D^e(y_p) - v.c(y_p) - (1-v)\underline{c}(y_p)$$

$$c(g, y_p) + H(g, y_p) \equiv c\left(y_p + \varepsilon(g, y_p)\right) + G\left(\varepsilon(g, y_p)\right) g$$

The Lagrangian is:

$$Min_{\{g,y_p\}}\{L^e(y_p) + \lambda_1[c(g,y_p) + H(g,y_p) - c(y_p)] + \lambda_2[\underline{c}(g,y_p) + H(g,y_p) - \underline{c}(y_p)]\}$$

Where a further notational simplification, $\Delta c(g,y_p) = c(g,y_p) - c(y_p)$, is adopted.

$$Min_{\{g,y_p\}}\{L^e(y_p) + \lambda_1[\Delta c(g,y_p) + H(g,y_p)] + \lambda_2[\Delta\underline{c}(g,y_p) + H(g,y_p)]\}$$

2. The regulator's problem with monitoring-imperfect enforcement (0 < G(.) < 1) and with exogenous judicial intervention.

Denoting as "x" the exogenous probability of judicial intervention, one has:

$$Min_{\{g,y_p\}}\{L^e(y_p) + \lambda_1[\Delta c(g,y_p) + H(g,y_p)(1-x)] + \lambda_2[\Delta\underline{c}(g,y_p) + H(g,y_p)(1-x)]\}$$

Or, with the notational convention $H(g,y_p)(1+x) = B(g,y_p,x), with\ x\ being\ a\ constant$:

$$Min_{\{g,y_p\}}\{L^e(y_p) + \lambda_1[\Delta c(g,y_p) + B(g,y_p)] + \lambda_2[\Delta\underline{c}(g,y_p) + B(g,y_p,x)]\}$$

3. The regulator's problem with imperfect monitoring (0 < G(.) < 1) and endogenous judicial intervention.

Now, with fine cancelation being endogenous, and for simplicity, following a decision function z(ε), which is thus dependent on the size of violation, one has the problem below. The notation for judicial power's decision function containing CAC terms (g and y_p) was simplified to $z(\varepsilon(g, y_p)) \equiv z(g, y_p)$ - we are not, at this general stage of the argument, specifying the decision function formula, but it is reasonable to suppose that it entails a negative relationship with the size of violation (ε), so that larger violations are, the less likely is that the judicial power intervenes.

$$Min_{\{g,y_p\}}\left\{L^e(y_p) + \lambda_1\left[\Delta c(g,y_p) + H(g,y_p)\left(1 - z(g,y_p)\right)\right]\right.$$
$$\left. + \lambda_2\left[\Delta\underline{c}(g,y_p) + H(g,y_p)\left(1 - \underline{z}(g,y_p)\right)\right]\right\}$$

It is clear that problems 1 and 2, from one side, and 3, from the other side, lead to different solutions because their mathematical structures are different. This becomes clearer when the FOCs are considered.

Problem 1 FOCs:

$$\partial_{y_p}L^e + \lambda_1\left[\partial_{y_p}\Delta c + \partial_{y_p}H(.)\right] + \lambda_2\left[\partial_{y_p}\Delta\underline{c} + \partial_{y_p}H(.)\right] = 0$$

$$\lambda_1\left[\partial_g\Delta c + \partial_g\underline{H(.)}\right] + \lambda_2\left[\partial_g\Delta\underline{c} + \partial_g\underline{H(.)}\right] = 0$$

Problem 2 FOCs:

$$\partial_{y_p}L^e + \lambda_1\left[\partial_{y_p}\Delta c + \partial_{y_p}B(.)\right] + \lambda_2\left[\partial_{y_p}\Delta\underline{c} + \partial_{y_p}\underline{B(.)}\right] = 0$$

$$\lambda_1\left[\partial_g\Delta c + \partial_{y_p}\underline{B(.)}\right] + \lambda_2\left[\partial_{y_p}\Delta\underline{c} + \partial_{y_p}\underline{B(.)}\right] = 0$$

(notice that the difference with problem 1 FOCs is almost notational, with function B taking the place of function H)

Problem 3 FOCs:

$$\partial_{y_p} L^e + \lambda_1 \left[\partial_{y_p}\Delta c + \partial_{y_p} H(.).(1 - z(.)) - H(.).\partial_{y_p} z\right]$$
$$+ \lambda_2 \left[\partial_{y_p}\Delta \underline{c} + \partial_{y_p} H(.).\left(1 - \underline{z}(.)\right) - H(.).\partial_{y_p}\underline{z}(.)\right] = 0$$
$$\partial_g L^e + \lambda_1 \left[\partial_g \Delta c + \partial_g H(.).(1 - z(.)) - H(.).\partial_g z\right]$$
$$+ \lambda_2 \left[\partial_g \Delta \underline{c} + \partial_g H(.).\left(1 - \underline{z}(.)\right) - H(.)\partial_g \underline{z}(.)\right] = 0$$

(the dots outside parentheses, ".", were added to represent the multiplication operation and to better separate the functions being multiplied)

The difference between problem #3 and the remaining problems lies in the product rule of differentiation. That is, focusing on the case of the high-cost agent, whereas in problems #1 and #2, one has $\partial_{y_p} H(.)$ or $\partial_{y_p} B(.)$, in problem #3, the functionally analogous term is $\partial_{y_p} H(.).(1 - z(.)) - H(.).\partial_{y_p} z$. The source of this mathematical difference, i.e., of the product rule, is the conceptual difference of the anticipation, by the agent, of action of the judicial power which manifests mathematically as the derivative of the z function with respect to contract terms (p and g), which is the second component of the product rule ($-H(.).\partial_{y_p} z$ or $-H(.).\partial_g z$).

Another way to see the difference between the problems is by examining the functional form. Even if the monitoring imperfection and the non-judicial-intervention functions, H(.) and z(.) are both linear, their product, which corresponds to the probability of sanction function, is non-linear. However, this function is still linear in the case of exogenous judicial intervention. However, this is not the case for endogenous judicial intervention, as shown in the next subsection.

E.7. The solution to CAC under endogenous judicial intervention

The non-compliance ("shirking") problem of an agent facing a generic abatement cost β_j is:

$$\text{Max}\{\varepsilon\}\{-g.\varepsilon/E.z(\varepsilon) - \beta_j(f^* - f - \varepsilon)\} \quad (A7)$$

The possibility of endogenous judicial intervention will be introduced by assuming that the probability of no judicial intervention, z(ε), grows linearly with non-compliance according with the following functional form $z(\varepsilon) = \varepsilon/(\eta E)$, $\eta \geq 1$. This form is based on the notion that the judicial power has an implicit tolerance threshold to violation which can be expressed as multiples of the maximum size of an accidental burning, i.e., as ηE.[4] The larger the violation is, as a fraction of this reference threshold, the more likely it is that the judicial power does not accept the agent's request for intervention. The $\varepsilon/(\eta E)$ ratio is thus a metric of violation's saliency from judicial power's perspective. The agent´s problem is:

$$\text{Max}\{\varepsilon_j\}\{-g.\varepsilon_j/E.[\varepsilon_j/(\eta E)] - \beta_j(f^*-f-\varepsilon_j)\} = \text{Min}\{\varepsilon_j\}\{g.\varepsilon_j^2/(\eta E^2) - \beta_j\varepsilon_j + \beta_j(f^*-f)\}$$

FOC:

$2g.\varepsilon_j/(\eta E^2) = \beta_j \rightarrow \varepsilon_j = \beta_j\, \eta E^2/(2g)$ (A8), and analogously for the low-cost agent.

Four consequences follow:

Consequence 1: Non-compliance extent is a decaying function of fine level.

This is evident from the inspection of the optimal non-compliance formula, not requiring a proof.

Consequence 2: The optimal level of non-compliance is positive.

Proof: Since all the parameters in the numerator are not null but positive, and none of the denominator parameters tends to infinity, it results in non-compliance being optimal for both agents.

Consequence 3: The benefit of non-compliance is always larger than its variable cost.

Proof: Referring to the negative of the non-compliance utility function (optimised in this sub-section) as the "expected cost of shirking function" (CS), such function is:

$$CS_j = \frac{g}{\eta E^2}\varepsilon_j^2 - \beta_j\varepsilon_j + \beta_j(f^* - f) = \varepsilon_j\left(\frac{g}{\eta E^2}\varepsilon_j - \beta_j\right) + \beta_j(f^* - f)$$

Replacing optimal non-compliance:

$$CS_j = \left[\frac{\beta_j \eta E^2}{2g}\right]\left(\frac{g}{\eta E^2}\left[\frac{\beta_j \eta E^2}{2g}\right] - \beta_j\right) + \beta_j(f^* - f)$$

Or, equivalently:

$$CS_j = -\left[\frac{\beta_j \eta E^2}{2g}\right]\frac{\beta_j}{2} + \beta_j(f^* - f) \text{ (A9)}$$

Therefore, the portion of the net cost of non-compliance ("shirking") under agent's control, which is captured by the first expression at the RHS, is always negative, meaning that the benefit from shirking (abatement cost saved) is always larger than the "variable" cost of shirking, the latter being equal to expected fine – the $\beta_j(f^*-f)$ component is a "fixed" cost, uncontrollable by the agent.

Consequence 4: non-compliance is always optimal. Or, alternatively, compliance is never optimal at a finite-fine rate.

---

[4] Since $\varepsilon = \min(f^*-f, E)$, then $0 < z(\varepsilon) < 1$.

Proof: for logical consistency, it should be noted that consequence 1 is not a sufficient condition for the impossibility of inducing compliance. This is because the net benefit from optimal non-compliance may be smaller than the net benefit from compliance. An additional step is needed to show that the principal is powerless, in this case of endogenous judicial intervention, to induce compliance. This follows directly from (A9), after noticing that $\beta_j(f^* - f)$ is the cost of compliance (CC):

$$\mathrm{CS_j} - CC = -\left[\frac{\beta_j \eta E^2}{2g}\right]\frac{\beta_j}{2} < 0$$

Consequence 5: the fine level and target cannot be chosen separately by the principal but only jointly by balancing the trade-off between smaller damage and larger abatement cost.

Proof: Let the principal's problem be examined, and the costs of non-compliance ("shirking") facing low-cost and high-cost agents to be denoted as $\underline{CS}$ and CS, respectively. These are agents' disutilities but the principal's goal is maximising welfare augmented by utilities and fine revenue, that is, schematically:

$$\text{Max}\{g,f\}\ \{v.[\text{Welfare} - \underline{CS} + (1+\lambda)\ \underline{\text{fine\_revenue}}] + (1-v).[\text{ Welfare} - \underline{CS} + (1+\lambda)\ \text{fine\_revenue}]\}$$

Formally, by incorporating agents' optimal choice (which is an implicit incorporation of the participation constraint):

$$\text{Max}\{g,f\}\ \{v.[W(f+\underline{\varepsilon(g)}) - \underline{CS}(f,g) + (1+\lambda)g.\underline{\varepsilon}^2/(\eta E^2)] + (1-v).[W(f+\varepsilon(g)) - CS(f,g) + (1+\lambda)\varepsilon^2/(\eta E^2)]\}$$

s.t.: $g \leq g_{legal}$

It should be noted that since two of the terms are positively related to fine value, welfare, and fine revenue, whereas one is negatively related, the negative of the non-compliance cost, then fine value choice is not trivial but results from the balancing of a trade-off – i.e., $g = g_{legal}$ is not necessarily the best choice.

The four consequences are of great importance. Consequence 2 is due to the control upon the probability of judiciary intervention that the endogenous nature of such probability gives the agent. In fact, the agent is assumed to know that such probability is a function of non-compliance extent and to make use of this fact. Also, consequences 3 and 4 establish that the principal is powerless to induce compliance at a finite-fine rate. Conversely, in the exogenous judiciary case, a finite fine, if high enough, could drive compliance optimal. Regarding consequence 5, its main implication is that the solution to the principal's problem in the endogenous case is more complex. In fact, it does not have a simple analytical solution. We thus solved it numerically by resorting to Matlab®'s built-in optimiser.

E.8. Conventions adopted in the numerical simulation of imperfect CAC

E.8.1. Exogenously imperfect CAC

In the simulation, the legal bound on fine was calculated in consistency with the minimum shirking-preventing fine(s) defining the scenario, as follows.

(A) Scenario 1: $g_{legal} < \underline{\beta}E/P(x=1) \rightarrow g_{legal} = \underline{\beta}E/P(x=1) - c_1,\ c_1 > 0 \rightarrow g_{legal} = [\underline{\beta}E - c_1P(x=1)]/P(x=1)$

(B) Scenario 3:

> (B.1) If $E/P(x=1)(v\underline{\beta}+(1-v)\beta) > \beta E/P(x=1)$, $g_{legal} = E/P(x=1)(v\underline{\beta} + (1-v)\beta)$ (in this case, fine has no effect);
>
> (B.2) Else, $g_{legal} = E/P(x=1)(v\underline{\beta} + (1-v)\beta) + c_3,\ c_3 > 0$.

It must be informed that in scenario 3 and for some municipalities, the probability of non-judicial intervention had no effect on CAC's efficiency. This was due to the parametric values of the municipality greatly constraining the legal bound levels consistent with scenario 3's definition. In these cases, it was thus easier to assume a legal bound equal to the medium point between the minimum (shirking-avoiding) fines for low and high-cost agents (i.e., condition (B.2) was applied, what drove the probability mentioned to be cancelled-out in the calculations). Three levels of the probability of fine not being cancelled (P(x=1)) were adopted, 1%, 10% and 50%. The former was based on evidence. Sousa (2016) calculated a share of fines paid below 1% of total fines issued from 2004 to 2012 due to flora-related infractions in Mato Grosso, a top state in deforestation and fires in the Amazon. The two other levels were to be larger than 1% and saliently different between themselves. In most of the results presented in the main text (figures and one table), the moderate 10% level was adopted.

E.8.2. Endogenously imperfect CAC

The legal fine rate was calculated the same way as in scenario 1 of the exogenously imperfect CAC with P(x=1) = 10%.

Appendix F. Further details about the numerical simulation

F.1. Marginal abatement cost: computation

The adoption index had approximately the same weights for the three variables (which ranged from 0.5751 and 0.5801), what was also the case for the correlations with the index (which ranged from 0.8076 to 0.8145). An attestation of the high complementarity of the three inputs (figure A2).

The adoption cost index's sign was switched, thus generating a non-adoption index, and then the first and third quartiles (p25 and p75) were taken as estimates for β and β̲. For simplicity, β̲ was normalized to one, so that β/β̲ could be estimated as the p75/p25 ratio. In order to avoid a theoretically inconsistent negative ratio, which occurred for municipalities with quartiles of opposite signs, the β/β̲ ratio was in fact estimated as (p75 + |min|)/(p25 + |min|), with the absolute value of the minimum indicated as |min|. This formula was applied to all municipalities, to ensure comparability of the result - and also avoiding cases with p75 < 0 and, thus, p75/p25 = |p75|/|p25| < 1. Outliers with a ratio greater than the 95th percentile were excluded. In the end, also considering the exclusion due to missing values, it was possible to estimate β/β̲ for 702 municipalities (70 were thus excluded, i.e., 9%).

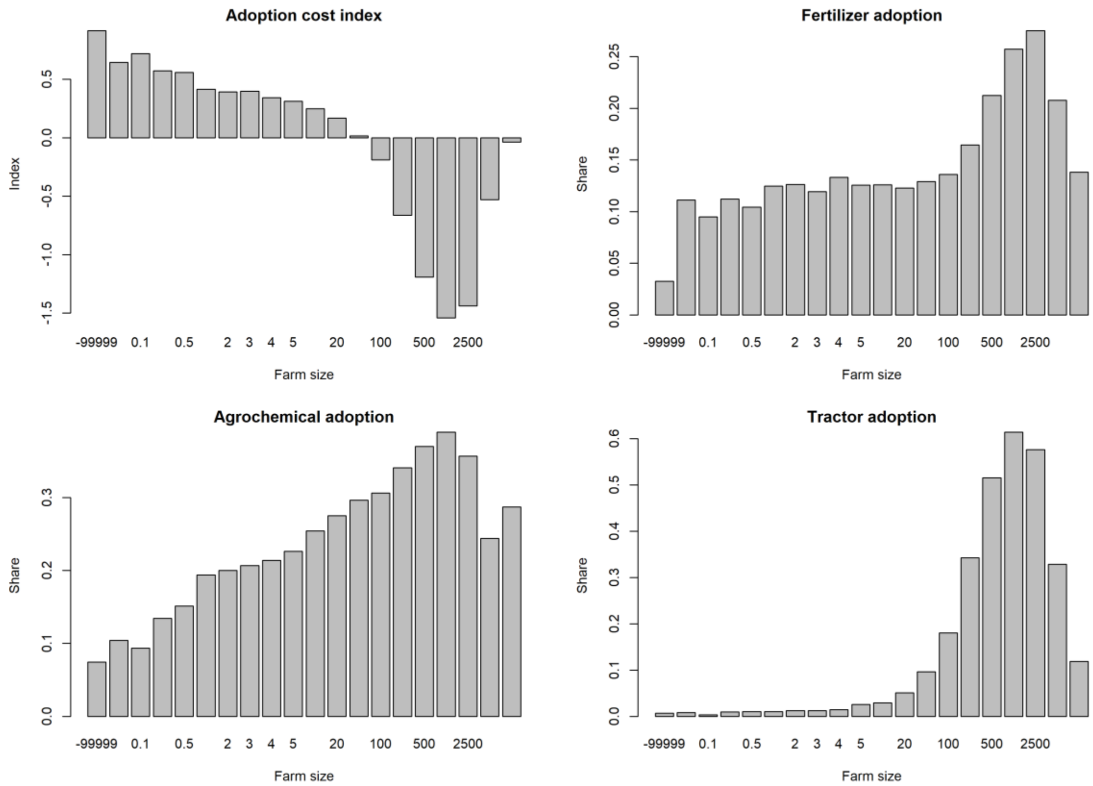

Figure A2. Index for the cost of adoption of alternatives to fires and its components.
Note: the horizontal axis shows the lower bound of area size bins (measured in hectares), with -9999 indicating the case of landless farmers. The vertical axis shows the share of adoption, or, in the "cost of adoption" graph, the principal component index.

F.2. Account of excluded municipalities

Table A1. Reason for and count of municipalities excluded from the numerical simulation

Reason	Count	%
Abatement cost: Missing or outlier	70	9%
Non-eligibility or non-additionality: deforestation fires and low frequency of non-deforestation agricultural fires	510	66%
Non-regular land tenure	13	2%
Non-available burned area without fires (f*)	0	0%
Remaining in the simulation	179	23%
Total	**772**	**100%**

F.3. References for damage function parameters' ranges

In Perman et al. (2011, box 5.2, p.148) the damage function $D(M) = M^2$ is assumed, where M is the pollution flow, that is, $s_0 = 0$, $s_1/2 = 1$. The authors used a different function for stock pollutants, $D(A) = 0.2A^2$, where A is the pollutant stock (Box 5.3, p.150), i.e., $s_0 = 0$, $s_1/2 = 0.2$. In Conrad (2010, chap.6, section 6.3, p.209), the function $D(Z) = c.Z^2$, $c > 0$, is applied to stock pollutants subjected to natural degradation, with $c = 0.1$, assumed in a numerical simulation; thus $s_0 = 0$ and $s_1 = 0.1$. The same function was also applied by the author to non-self-degradable waste, with $c = 0.5$ (Conrad, 2010, section 6.5) and to stock subject to degradation, with $c = 0.02$ (Conrad, 2010, section 6.6, p. 233). Phaneuf and Requate (2016, pp. 56, 105, 214) also adopted, in their numerical exercises and examples, damage functions with $s_0 = 0$. In summary, in the references reviewed, the range of levels of s_1 was of [0.04; 2], and a null s_0 was common.

F.4. Calibration of damage function parameters

F.4.1. Conceptual basis

In each municipality two types of tangible assets particularly relevant from a fire damage perspective are invested on, public and private assets, which are owned by government and farmers. Nevertheless, to simplify calibration we will understand these two assets as owned by a higher level entity, the municipal society, and take the averages of their benefits and costs. The key assumption is that the sensitiveness of the marginal damage function to intentional burnings, measured by the slope coefficient s_1, is directly proportional to the municipal value of assets that are commonly reported to be damaged by fires in the Brazilian Amazon. This economically sound property was introduced in the model with a linear mapping between the observed range of an empirical measure of asset value, herein "asset value ratio", and the range of s_1 as informed by the references cited in section F.3. The asset value ratio was measured as the net-benefit/cost quotient, averaged across two types of assets, public and private, as follows:

$$asset\ value\ ratio\ measure = \frac{1}{2}\left(\frac{\mathrm{b}_{pub} - \mathrm{c}_{pub}}{\mathrm{c}_{pub}} + \frac{\mathrm{b}_{priv} - \mathrm{c}_{priv}}{\mathrm{c}_{priv}}\right)$$

The mapping applied is illustrated in the figure below. Despite the parameters it was based on were fixed across all municipalities, s_1 varied with the asset value ratio across municipalities.

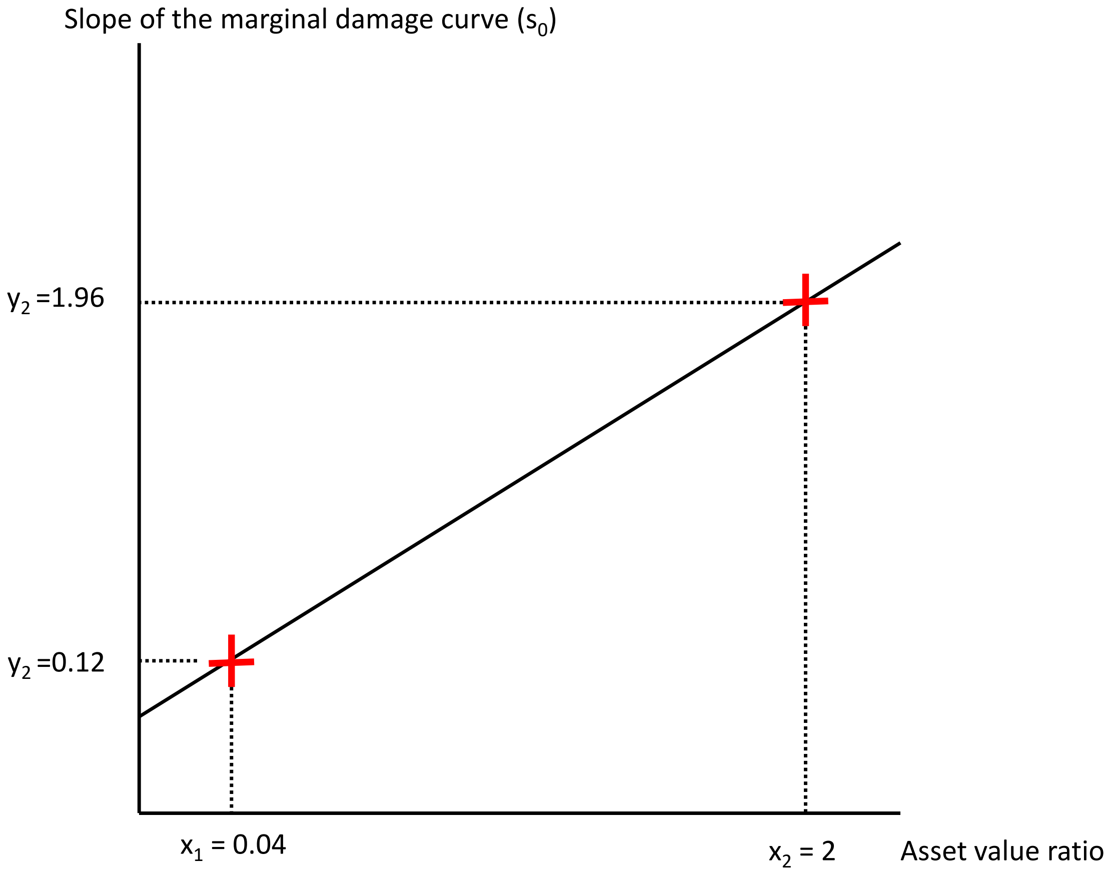

Figure A3. Mappping between asset value ratio and s_0.

The intercept of the marginal damage function, s_0, was assigned a value ensuring consistency of the policy incentive problems, i.e., that policy was minimally effective in abating externalities. More precisely, s_0 was calculated to ensure non-null first-best abatements in first-best CAC and contracts. To minimise the restriction imposed on modelling results, the requirement was imposed only on the low-cost agent. The mathematical steps to obtain s_0 are detailed below.

Starting with first-best contracts, the relevant condition was $f^{CON,fb} < f^*$. Incorporating the optimal level of the threshold, it became:

$[(1+\lambda)\underline{\beta} - s_0]/s_1 - E/2 < f^* \rightarrow [(1+\lambda)\underline{\beta} - s_0]/s_1 < f^* + E/2$

$\rightarrow (1+\lambda)\underline{\beta} - s_0 < s_1(f^* + E/2) \rightarrow (1+\lambda)\underline{\beta} - s_1(f^* + E/2) < s_0$

Or, synthetically: $s_0 > (1+\lambda)\underline{\beta} - s_1(f^* + E/2)$ (A10)

Now turning to first-best CAC, the condition is $f^{CAC,fb} < f^*$. Manipulating one has:

$(\underline{\beta} - s_0)/s_1 - E/2 < f^* \rightarrow (\underline{\beta} - s_0)/s_1 < f^* + E/2 \rightarrow \underline{\beta} - s_0 < s_1(f^* + E/2) \rightarrow$

$\underline{\beta} - s_1(f^* + E/2) < s_0 \rightarrow s_0 > \underline{\beta} - s_1(f^* + E/2)$ (A11)

Combining the two previous inequalities, one arrives at:

$s_0 > \max\,[(1+\lambda)\underline{\beta} - s_1(f^* + E/2); \underline{\beta} - s_1(f^* + E/2)] \rightarrow s_0 > (1+\lambda)\underline{\beta} - s_1(f^* + E/2)$

An equation, which allows for exact calculation of s_0, is found by selecting a small real number, κ, so that $s_0 = (1+\lambda)\underline{\beta} - s_1(f^* + E/2) + \kappa$ (A12).

A third condition was imposed, that $s_0 \geq 0$, or, more rigorously, that:

$$s_0 = (1+\lambda)\underline{\beta} - s_1(f^* + E/2) + \kappa, \text{ if } (1+\lambda)\underline{\beta} - s_1(f^* + E/2) + \kappa > 0$$

$$s_0 = \begin{cases} (1+\lambda)\underline{\beta} - s_1\left(f^* + \frac{1}{2}\right) + \kappa, if (1+\lambda)\underline{\beta} - s_1\left(f^* + \frac{1}{2}\right) + \kappa \geq 0 \\ 0,\ if (1+\lambda)\underline{\beta} - s_1\left(f^* + \frac{1}{2}\right) + \kappa \geq 0 \end{cases} \quad (A12')$$

F.4.2 Data 1: public assets

We measure the value of both public and private assets as their return as perceived by investors. In the Amazon, most electricity utilities which build and maintain powerlines are state-owned and thus public, but, at the same time, transmission lines are operated by private companies (Eletronorte, 2025). These are the two reasons ensuring plausibility of the valuation of the public asset based on private return. The net return of a power line is assumed to equal the profit it yields to the company winning the public auction granting building and operation rights. In fact this profit is the key element taken into account when private companies decide to enter or not such type of auction, with the minimal investment (i.e., the cost) being pre-specified before the auction as an obligation to be fulfilled, together with the maximum payment (revenue) requested by the operator (Tolmasquin *et al.*, 2021; Brandão and Ehrl, 2022). For a given municipality we calculated power line profit as revenue minus cost based on auctions. The data used captured all powerline concessions in Brazil since 1999 (ANEEL, 2025).

Cost corresponded to the annualised value of investment, which was calculated assuming a 30 years concession period (ANEEL, 2024, 2025) and the interest rate charged by the Brazilian development bank on loans (6.66% (fixed component, BNDES, 2025) + 0.52% (IPCA inflation index) on December 2024, the latest year in the dataset). Then we took the median of asset value ratio within each state (profit/cost), to estimate the state-level ratio - this statistic was chosen instead of the trimmed mean excluding ratios which were negative or above 10, because it exhibited greater variability across states. The range across states was of [0.23;0.53].

F.4.3. Data 2: private assets

Perennial crops, frequently mentioned in the literature as one of the targets of fire damages, were taken as the representative private asset. Its net benefit was calculated as revenue minus expenditure, the latter also including plantation (establishment investment). For this, we relied on data from the latest Brazilian Agricultural Census of 2017 (IBGE, 2019). Municipalities for which revenue or expenditure on perennial crops was not available were assigned with the asset value ratio of their contiguous neighbours, an imputation procedure which was repeated to second-order contiguous neighbourhood when needed. The range of the net-benefit vs. cost ratio across municipalities was of [0.0082;3.48].

Appendix G. Supplementary tables

Table A2. CAC under different institutional states and the non-intervention scenario, subsample with CAC SCE.1 better than perfect CAC, representative municipality

Statistic / Instrument & information [b]	Without policy	Second best perfect CAC	Second best CAC imperfect exog. (sce.1)	Second best CAC imperfect exog. (sce.3)	Second best CAC imperfect endo.
Required abatement	0.00	0.06	0.45	0.44	0.45
Effective abatement	0.00	0.06	0.00	0.22	0.05
Abatement cost (A)	0.00	0.06	0.00	0.22	0.05
Subsidy cost (B)[c]	0.00	0.00	0.00	0.00	0.00
Total cost (C=A+B)	0.00	0.06	0.00	0.22	0.05
Expected damage (D)	-0.73	-0.67	-0.73	-0.51	-0.67
Net payment (E) [d]	0.00	0.00	0.00	0.00	0.00
Fine revenue (F)	0.00	0.00	0.09	0.05	0.05
Welfare (D-C+E+F)	-0.73	-0.73	-0.65	-0.68	-0.68

Note: The same notes from table 2 in the main text apply, with the correction that the representative municipality was selected within the particular subsample in which exogenous imperfect CAC, in scenario 1, was better than perfect CAC.

Table A3. CAC under different institutional states and the non-intervention scenario, full sample, representative municipality

Statistic / Instrument & information [b]	Without policy	Second best perfect CAC	Second best CAC imperfect exog. (sce.1)	Second best CAC imperfect exog. (sce.3)	Second best CAC imperfect endo.
Required abatement	0.00	2.39	2.39	2.89	3.26
Effective abatement	0.00	2.39	1.39	2.39	2.39
Abatement cost (A)	0.00	2.82	1.64	2.73	2.80
Subsidy cost (B)[c]	0.00	0.00	0.00	0.00	0.00
Total cost (C=A+B)	0.00	2.82	1.64	2.73	2.80
Expected damage (D)	-5.67	-1.49	-2.91	-1.55	-1.50
Net payment (E) [d]	0.00	0.00	0.00	0.00	0.00
Fine revenue (F)	0.00	0.00	0.19	0.12	0.10
Welfare (D-C+E+F)	-5.67	-4.32	-4.36	-4.16	-4.19

Note: The notes from table 2 in the main text are also valid, except that the representative municipality was selected from the full sample.

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