

Online Appendix for
Teaching Election Inversions in the Electoral College: Two Simple Models

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Worksheet 1: Understanding Election Inversion in the Electoral College, Model 1

Your name: _____

Model 1 (page 1 of 2)

Consider the electoral map below. It consists of three states with equal populations. House seats are apportioned based on population, and the Electoral College votes are allocated according to the House seats + 2 rule.

State 1		State 2		State 3	
Population:	1,000	Population:	1,000	Population:	1,000
House seats:	10	House seats:	10	House seats:	10
Electoral College votes:	12	Electoral College votes:	12	Electoral College votes:	12

In a two-candidate election where every person votes, candidate A wins 55% of the popular votes in states 1 and 2, and 30% of the popular votes in state 3. Candidate B wins 45% of the popular votes in states 1 and 2, and 70% of the popular votes in state 3. The states' Electoral College votes are allocated on a winner-take-all basis.

Questions:

1. What are the candidates' popular vote totals and Electoral College vote totals, respectively? Use the following table to facilitate your calculations. One cell has been filled in as an example.

Hint: For the Electoral College votes, first determine if the candidate is the winner of the state(s). The winner receives all the Electoral College votes allocated to the state(s) per the winner-take-all rule, and the loser receives 0 Electoral College votes.

		States 1 and 2	State 3	Total
Candidate A	Popular votes	$55\% \times 1,000 \times 2 = 1,100$		
	Electoral College votes			
Candidate B	Popular votes			
	Electoral College votes			

2. Based on the results above, who wins the popular vote? Who wins the Electoral College vote? Why do you think this is the case?
3. If we remove the +2 seats and give 10 Electoral College votes to each state, the winner of the popular vote would not change. Who would then win the Electoral College vote?

Model 1 (page 2 of 2)

4. If we replace the voting blocs of 10 votes per state with single-member districts, giving one Electoral College vote to each state, who would be the winner of the Electoral College vote?
5. Consider the notion of “wasted votes,” defined as:
 - 1) All the votes a candidate receives in a losing state.
 - 2) The extra votes a candidate receives beyond “50% +1” in a winning state.

Use the table below to calculate the wasted vote totals for each candidate. Follow these steps:

- Copy the popular votes and Electoral College votes numbers from the previous page
- Examine each “wasted” votes cell individually.
- First, determine if the candidate wins or loses in the state(s) to identify which scenario applies.
- Then, apply the appropriate scenario to calculate the “wasted votes.”

One “wasted” votes cell has been filled in as an example. What do you find after you fill in all the cells?

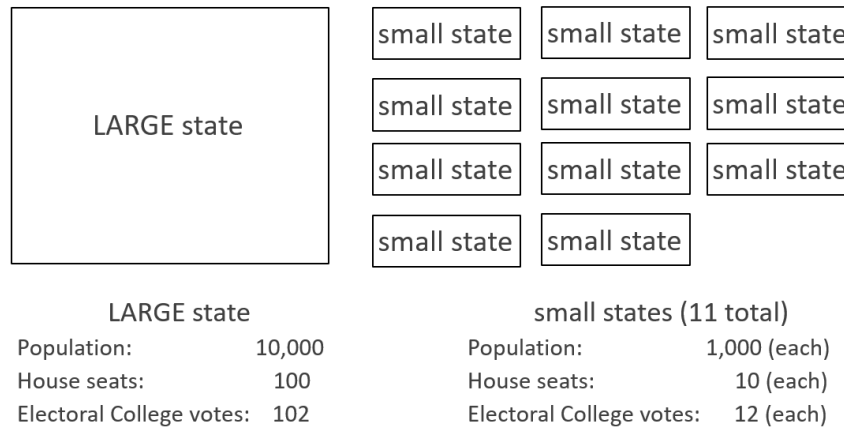
		States 1 and 2	State 3	Total
Candidate A	Popular votes	1,100		
	Electoral College votes			
	“wasted” votes	$1,100 - [(50\% \times 1,000 + 1) \times 2]$ $= 98$		
Candidate B	Popular votes			
	Electoral College votes			
	“wasted” votes			

Worksheet 2: Understanding Election Inversion in the Electoral College, Model 2

Your name: _____

Model 2 (page 1 of 2)

Consider the electoral map below that consists of 12 states. One state is a large state with 10 times the population of the small states. House seats are apportioned based on state population, and the Electoral College votes are allocated following the House seats + 2 rule.



In a two-candidate election where every person votes, candidate A wins 60% of the popular votes in the large state, 55% of the popular votes in one small state, and 40% of the popular votes in the other 10 small states. Candidate B wins 40% of the popular votes in the large state, 45% of the popular votes in one of the small states, and 60% of the popular votes in the other 10 small states.

Questions:

1. What are the candidates' popular vote totals and Electoral College vote totals, respectively? Use the following table to facilitate your calculations. One cell has been filled in as an example.

Hint: For the Electoral College votes, first determine if the candidate is the winner of the state(s). The winner receives all the Electoral College votes allocated to the state(s) per the winner-take-all rule, and the loser receives 0 Electoral College votes.

		Large state	One small state	The other 10 small states	Total
Candidate A	Popular votes	$60\% \times 10,000 = 6,000$			
	Electoral College votes				
Candidate B	Popular votes				
	Electoral College votes				

Model 2 (page 2 of 2)

2. Based on the results above, who wins the popular vote? Who wins the Electoral College vote? Why do you think this is the case?
3. If we remove the +2 seats and give 10 Electoral College votes to each small state and 100 Electoral College votes to the large state, the winner of the popular vote would not change. Who would then win the Electoral College vote?
4. Consider the notion of “wasted votes,” defined as:
 - 1) All the votes a candidate receives in a losing state.
 - 2) The extra votes a candidate receives beyond “50% +1” in a winning state.

Use the table below to calculate the wasted vote totals for each candidate. Follow these steps:

- Copy the popular votes and Electoral College votes numbers from the previous page
- Examine each “wasted” votes cell individually.
- First, determine if the candidate wins or loses in the state(s) to identify which scenario applies.
- Then, apply the appropriate scenario to calculate the “wasted votes.”

One “wasted” votes cell has been filled in as an example. What do you find after you fill in all the cells?

		Large state	One small state	Remaining 10 small states	Total
Candidate A	Popular votes	6,000			
	Electoral College votes				
	“wasted” votes	$6,000 - (50\% \times 10,000 + 1) = 999$			
Candidate B	Popular votes				
	Electoral College votes				
	“wasted” votes				

Worksheet 1/Model 1: Answers

Consider the electoral map below. It consists of three states with equal populations. House seats are apportioned based on population, and the Electoral College votes are allocated according to the House seats + 2 rule.

State 1		State 2		State 3	
Population:	1,000	Population:	1,000	Population:	1,000
House seats:	10	House seats:	10	House seats:	10
Electoral College votes:	12	Electoral College votes:	12	Electoral College votes:	12

In a two-candidate election where every person votes, candidate A wins 55% of the popular votes in states 1 and 2, and 30% of the popular votes in state 3. Candidate B wins 45% of the popular votes in states 1 and 2, and 70% of the popular votes in state 3. The states' Electoral College votes are allocated on a winner-take-all basis.

Questions:

- What are the candidates' popular vote totals and Electoral College vote totals, respectively? Use the following table to facilitate your calculations. One cell has been filled in as an example.

Hint: For the Electoral College votes, first determine if the candidate is the winner of the state(s). The winner receives all the Electoral College votes allocated to the state(s) per the winner-take-all rule, and the loser receives 0 Electoral College votes.

		States 1 and 2	State 3	Total
Candidate A	Popular votes	$55\% \times 1,000 \times 2 = 1,100$	$30\% \times 1,000 = 300$	1,400
	Electoral College votes	$12 \times 2 = 24$	0	24
	"wasted" votes	$(550 - 501) \times 2 = 98$	300	398
Candidate B	Popular votes	$45\% \times 1,000 \times 2 = 900$	$70\% \times 1,000 = 700$	1,600
	Electoral College votes	0	12	12
	"wasted" votes	900	$700 - 501 = 199$	1,099

- Based on the results above, who wins the popular vote? Who wins the Electoral College vote? Why do you think this is the case? **B wins the popular vote 1600 to 1400; A wins the EC vote 24 to 12. B "wasted" more popular votes than A.**
- If we remove the +2 seats and give 10 Electoral College votes to each state, the winner of the popular vote would not change. Who would then win the Electoral College vote? **A would still be the winner of the EC vote 20 to 10. This shows that even without the +2 seats, election inversions could still happen.**

4. If we replace the voting blocs of 10 votes per state with single-member districts, giving one Electoral College vote to each state, who would be the winner of the Electoral College vote? A would still be the winner of the EC vote 2 to 1. This shows that even without the +2 allocation rule or winner-take-all voting, election inversions could still occur. For example, if we were to (1) eliminate the +2 seat allocation rule by assigning each state the same number of Electoral College votes as its congressional districts, and (2) replace bloc voting with the Maine–Nebraska system, under which each congressional district determines one Electoral College vote based on the popular vote in that district, election inversions could still occur because they are possible in any two-stage voting system. See the following quote:

“The outcome of the electoral vote in a state, and hence the outcome of the Electoral College vote, depends on the margin of victory but not on voter participation rates in that state. The outcome of the nationwide popular vote depends on voter participation in every state but not on the margin of victory in each state” (Fon 2004 67-68)

5. Consider the notion of “wasted votes,” defined as:

- 1) All the votes a candidate receives in a losing state.
- 2) The extra votes a candidate receives beyond “50% +1” in a winning state.

Use the table below to calculate the wasted vote totals for each candidate. Follow these steps:

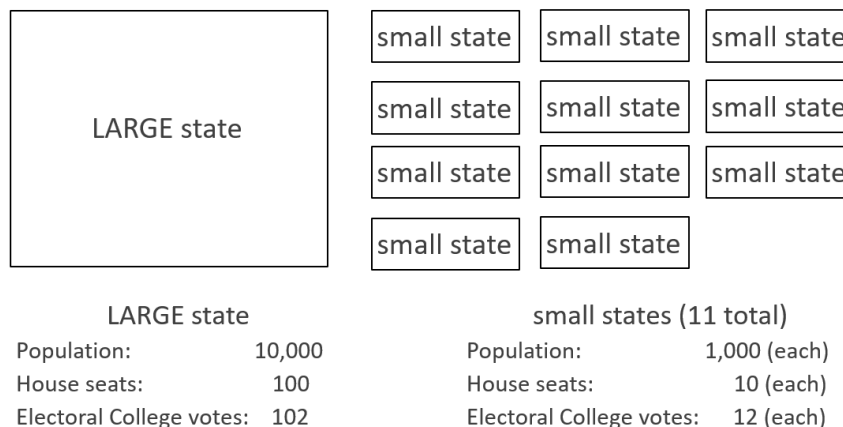
- Copy the popular votes and Electoral College votes numbers from the previous page
- Examine each “wasted” votes cell individually.
- First, determine if the candidate wins or loses in the state(s) to identify which scenario applies.
- Then, apply the appropriate scenario to calculate the “wasted votes.”

One “wasted” votes cell has been filled in as an example. What do you find after you fill in all the cells?

Answers are shown in the table. “Wasted votes” account for the inversion in Model 1 but not in Model 2. Clinton “wasted” more votes than Trump in 2016 and therefore lost the presidency, similar to the outcome in Worksheet 1/Model 1. Gore did not “waste” more votes than Bush in 2000; instead, he lost because of Bush’s small-state advantage, similar to the outcome in Worksheet 2/Model 2.

Worksheet 2/Model 2: Answers

Consider the electoral map below that consists of 12 states. One state is a large state with 10 times the population of the small states. House seats are apportioned based on state population, and the Electoral College votes are allocated following the House seats + 2 rule.



In a two-candidate election where every person votes, candidate A wins 60% of the popular votes in the large state, 55% of the popular votes in one small state, and 40% of the popular votes in the other 10 small states. Candidate B wins 40% of the popular votes in the large state, 45% of the popular votes in one of the small states, and 60% of the popular votes in the other 10 small states.

Questions:

- What are the candidates' popular vote totals and Electoral College vote totals, respectively? Use the following table to facilitate your calculations. One cell has been filled in as an example.

Hint: For the Electoral College votes, first determine if the candidate is the winner of the state(s). The winner receives all the Electoral College votes allocated to the state(s) per the winner-take-all rule, and the loser receives 0 Electoral College votes.

		Large state	One small state	Remaining 10 small states	Total
Candidate A	Popular votes	$60\% \times 10,000 = 6,000$	$55\% \times 1,000 = 550$	$40\% \times 1,000 \times 10 = 4,000$	10,550
	Electoral College votes	102	12	0	114
	"wasted" votes	$6,000 - 5,001 = 999$	$550 - 501 = 49$	4,000	5,048
Candidate B	Popular votes	$40\% \times 10,000 = 4,000$	$45\% \times 1,000 = 450$	$60\% \times 1,000 \times 10 = 6,000$	10,450
	Electoral College votes	0	0	120	120
	"wasted" votes	4,000	450	$(600 - 501) \times 10 = 990$	5,440

2. Based on the results above, who wins the popular vote? Who wins the Electoral College vote? Why do you think this is the case? **A wins the popular vote 10,550 to 10,450. B wins the EC vote 120 to 114. B wins more small states and the +2 rule gives small states extra advantages in the Electoral College.**
3. If we remove the +2 seats and give 10 Electoral College votes to each small state and 100 Electoral College votes to the large state, the winner of the popular vote would not change. Who would then win the Electoral College vote? **A would be the winner of the EC vote 110 to 100. This shows that in this example, the +2 rule contributes to the election inversion. Note however +2 is not the only reason why inversions take place, as shown in Model 1.**
4. Consider the notion of “wasted votes,” defined as:
 - 1) All the votes a candidate receives in a losing state.
 - 2) The extra votes a candidate receives beyond “50% +1” in a winning state.

Use the table below to calculate the wasted vote totals for each candidate. Follow these steps:

- Copy the popular votes and Electoral College votes numbers from the previous page
- Examine each “wasted” votes cell individually.
- First, determine if the candidate wins or loses in the state(s) to identify which scenario applies.
- Then, apply the appropriate scenario to calculate the “wasted votes.”

One “wasted” votes cell has been filled in as an example. What do you find after you fill in all the cells?

Answers are shown in the table. “Wasted votes” account for the inversion in Model 1 but not in Model 2. Clinton “wasted” more votes than Trump in 2016 and therefore lost the presidency, similar to the outcome in Worksheet 1/Model 1. Gore did not “waste” more votes than Bush in 2000; instead, he lost because of Bush’s small-state advantage, similar to the outcome in Worksheet 2/Model 2.

A List of Introductory Books on the Mathematics of Voting

El-Helaly, Sherif. 2019. *The Mathematics of Voting and Apportionment: An Introduction*. 2019 edition. New York, NY: Springer.

Heckelman, Jac C., and Nicholas R. Miller, eds. 2016. *Handbook of Social Choice and Voting*. Cheltenham, UK: Edward Elgar Publishing.

Hodge, Jonathan K., and Richard E. Klima. 2018. *The Mathematics of Voting and Elections: A Hands-On Approach*. American Mathematical Soc.

Robinson, E. Arthur, and Daniel H. Ullman. 2016. *The Mathematics of Politics*. 2nd edition. Boca Raton: CRC Press.

Saari, Donald G. 2001. *Chaotic Elections! A Mathematician Looks at Voting*. First Edition. Providence, R.I.: American Mathematical Society.

Sullivan, Brendan W. 2022. *An Introduction to the Math of Voting Methods*. 619 Wreath.

Student Learning Experience Survey Questionnaire (pre-survey)

Survey 1 (page 1 of 2)

This is an anonymous survey. There is no need to write down your name.

- 1. Do you major or minor in political science?**
 - a) Yes
 - b) No
- 2. Have you taken any political science courses (including AP courses) before?**
 - a) Yes
 - b) No
- 3. What is your current year in school?**
 - a) Freshman
 - b) Sophomore
 - c) Junior
 - d) Senior
 - e) Other
- 4. How closely have you followed the news about the 2024 presidential election?**
 - a) Very closely
 - b) Somewhat closely
 - c) Not too closely
 - d) Not at all
- 5. How would you rate your knowledge of how U.S. presidential elections work?**
 - a) Advanced knowledge
 - b) Moderate knowledge
 - c) Basic knowledge
 - d) No knowledge
- 6. Are you interested in learning more about how U.S. presidential elections work?**
 - a) Yes
 - b) No
 - c) Not sure
- 7. How much confidence do you have in the integrity of U.S. presidential elections?**
 - a) A great deal of confidence
 - b) A fair amount of confidence
 - c) Not too much confidence
 - d) No confidence at all

Survey 1 (page 2 of 2)

8. How much confidence do you have in democracy in America?

- a) A great deal of confidence
- b) A fair amount of confidence
- c) Not too much confidence
- d) No confidence at all

9. How familiar are you with the Electoral College system?

- a) Very familiar
- b) Somewhat familiar
- c) Not familiar at all

10. Based on what you know, is it possible for a candidate to lose the popular vote but still win the presidency?

- a) Yes
- b) No
- c) Not sure

11. Do you know how a candidate can lose the popular vote but still win the presidency?

- a) Yes
- b) No
- c) Not sure
- d) It is not possible for a candidate to lose the popular vote but still win the presidency.

Student Learning Experience Survey Questionnaire (post-survey)

Survey 2 (page 1 of 2)

This is an anonymous survey. There is no need to write down your name.

- 1. Overall, how accessible do you find the worksheet “Understanding Election Inversion in the Electoral College”?**
 - a) Very easy to understand
 - b) Somewhat easy to understand
 - c) Neither easy nor difficult to understand
 - d) Somewhat difficult to understand
 - e) Very difficult to understand

- 2. How would you describe your experience with the calculations in the worksheet?**
 - a) I can complete the calculations correctly on my own.
 - b) I can complete the calculations correctly with just a little help from my group.
 - c) I struggle with the calculations and need a lot of help from my group.
 - d) I am unable to complete the calculations, even with my group’s help.

- 3. Does the worksheet help you better understand the Electoral College?**
 - a) Yes, it helps a lot
 - b) Yes, it helps somewhat
 - c) It neither helps nor hinders my understanding
 - d) No, it doesn’t help much
 - e) No, it doesn’t help at all

- 4. How closely have you followed the news about the 2024 presidential election?**
 - a) Very closely
 - b) Somewhat closely
 - c) Not too closely
 - d) Not at all

- 5. How would you rate your knowledge of how U.S. presidential elections work?**
 - a) Advanced knowledge
 - b) Moderate knowledge
 - c) Basic knowledge
 - d) No knowledge

Survey 2 (page 2 of 2)

- 6. Are you interested in learning more about how U.S. presidential elections work?**
 - a) Yes
 - b) No
 - c) Not sure

- 7. How much confidence do you have in the integrity of U.S. presidential elections?**
 - a) A great deal of confidence
 - b) A fair amount of confidence
 - c) Not too much confidence
 - d) No confidence at all

- 8. How much confidence do you have in democracy in America?**
 - a) A great deal of confidence
 - b) A fair amount of confidence
 - c) Not too much confidence
 - d) No confidence at all

- 9. How familiar are you with the Electoral College system?**
 - a) Very familiar
 - b) Somewhat familiar
 - c) Not familiar at all

- 10. Based on what you know, is it possible for a candidate to lose the popular vote but still win the presidency?**
 - a) Yes
 - b) No
 - c) Not sure

- 11. Do you know how a candidate can lose the popular vote but still win the presidency?**
 - a) Yes
 - b) No
 - c) Not sure
 - d) It is not possible for a candidate to lose the popular vote but still win the presidency.

Please share anything else that you would like to say about your experience with the worksheet: