Parameterizing Spatial Weight Matrices in Spatial Econometric Models

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A Online Appendix: Supplemental Material

A.1 Row and scalar normalization of distance decay matrices

For $i \neq j$, the ijth element of the negative exponential matrix in non-normalized or raw (r) form is given by $w_{ij}(\alpha_k)^r = e^{-d_{ij}\alpha_k}$ and the ijth element of the inverse distance matrix by $w_{ij}(\alpha_k)^r = 1/d_{ij}^{\alpha_k}$, where d_{ij} denotes the generalized distance between units i and j and α_k the distance decay parameter of the kth spatial lag. $\boldsymbol{W}^r$ is the corresponding $N \times N$ spatial weight matrix in raw form. Its diagonal elements are assumed to be zero to prevent units from predicting themselves.

For the purpose of identifying the parameters of the spatial lags $(\rho, \gamma_1, \ldots, \gamma_K)$, the spatial weight matrices need to be normalized. We consider two frequently used normalizations: normalization by rows and scalar normalization by the largest eigenvalue. Row normalization is generally applied for the two following reasons. It facilitates the interpretation of operations with the weight matrix as an averaging of neighboring values (Anselin and Bera 1998), and the spatial autoregressive parameter ρ takes values in the parameter space $(1/\lambda_{min}, 1)$, where λ_{min} is the smallest negative eigenvalue of $\boldsymbol{W}$ (Ord 1981). However, row normalization has also been criticized. Kelejian and Prucha (2010) and Neymayer and Plümper (2016) demonstrate that normalization of the elements of the spatial weight matrix by a different factor for each row as opposed to a single factor is not neutral as it imposes the restriction that if one unit has fewer ties to other units than another unit, then each tie is assumed to be more important. This especially concerns distance decay matrices (Anselin 1988, pp. 23-24), because it changes the relative relevance of senders across recipients. Therefore, Kelejian and Prucha (2010) propose a normalization procedure where each element of $\boldsymbol{W}^r$ is divided by its largest eigenvalue λ^r_{max}, to get $\boldsymbol{W} = (1/\lambda^r_{max})\boldsymbol{W}^r$. When applying this normalization, ρ again takes values in the parameter space $(1/\lambda_{min}, 1)$, although the value of the smallest negative eigenvalue of this matrix is different. Since some rows will sum up to values greater than one and other to values smaller than one, a scalar normalized weight matrix can no longer be interpreted as an averaging of neighboring values. In empirical work both types of normalizations are applied. Since there is no unifying consensus in the literature in favor of one of these normalizations, we also consider

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scalar normalized (by its largest eigenvalue) forms of the negative exponential and inverse distance matrices.

To generalize the mathematical expressions in the subsequent sections of this appendix, i.e., to avoid that they all need to be repeated for both types of spatial weight matrices and both types of normalizations, we first introduce the symbol $\boldsymbol{Z}_{\alpha_k}$ representing the first order derivative of the row or scalar normalized spatial weight matrix with respect to α_k, $\boldsymbol{Z}_{\alpha_k} = \frac{\partial \boldsymbol{W}(\alpha_k)}{\partial \alpha_k}$. We have $w_{ij}^r(\alpha_k) = e^{-d_{ij}\alpha_k}$ and $w_{ij}(\alpha_k) = \frac{e^{-d_{ij}\alpha_k}}{\sum_j e^{-d_{ij}\alpha_k}}$ for the negative exponential decay matrix before and after row normalization, respectively. Similarly, we have $w_{ij}^r(\alpha_k) = d_{ij}{}^{-\alpha_k}$ and $w_{ij}(\alpha_k) = \frac{d_{ij}{}^{-\alpha_k}}{\sum_j d_{ij}{}^{-\alpha_k}}$ for the inverse distance decay matrix before and after row normalization, respectively.

Under row normalization, the elements of the matrix of first order derivatives for the negative exponential and inverse distance matrix are

$$z_{ij}(\alpha_k) = \frac{(\sum_j d_{ij} e^{-d_{ij}\alpha_k}) e^{-d_{ij}\alpha_k} - (\sum_j e^{-d_{ij}\alpha_k}) d_{ij} e^{-d_{ij}\alpha_k}}{(\sum_j e^{-d_{ij}\alpha_k})^2}, \tag{A.1}$$

$$z_{ij}(\alpha_k) = \frac{(\sum_j ln(d_{ij}) d_{ij}^{-\alpha_k}) d_{ij}{}^{-\alpha_k} - (\sum_j d_{ij}{}^{-\alpha_k}) ln(d_{ij}) d_{ij}{}^{-\alpha_k}}{(\sum_j d_{ij}{}^{-\alpha_k})^2}, \tag{A.2}$$

respectively.

If the matrices are scalar normalized by the largest eigenvalue, we respectively have $w_{ij} = e^{-d_{ij}\alpha_k}/\lambda_{max}^r(\alpha_k)$ and $w_{ij} = d_{ij}{}^{-\alpha_k}/\lambda_{max}^r(\alpha_k)$. In this case, the matrix $\boldsymbol{Z}_{\alpha_k}$ of first order derivatives is easier expressed in full form.

Since $\boldsymbol{W}^r(\alpha_k)$ is real and symmetric, it has distinct eigenvalues λ_i^r and eigenvectors $\boldsymbol{v}_i^r$ $(i = 1, \ldots, N)$ with properties $\boldsymbol{v}_i^{r\prime} \boldsymbol{v}_i^r = 1$ and $\partial \lambda_i = \boldsymbol{v}_i^{r\prime} \partial \boldsymbol{W}^r(\alpha_k) \boldsymbol{v}_i$ (Magnus 1985, equation 4). Using these properties for the largest eigenvalue and corresponding eigenvector of $\boldsymbol{W}^r(\alpha_k)$, the matrix of first order derivatives for the negative exponential and inverse distance decay matrices are

$$\boldsymbol{Z}_{\alpha_k} = \frac{\frac{\partial \boldsymbol{W}^r(\alpha_k)}{\partial \alpha_k} \lambda_{max}^r(\alpha_k) - \boldsymbol{v}^{r\prime}(\alpha_k) \frac{\partial \boldsymbol{W}^r(\alpha_k)}{\partial \alpha_k} \boldsymbol{v}(\alpha_k) \boldsymbol{W}^r(\alpha_k)}{[\lambda_{max}^r(\alpha_k)]^2}, \tag{A.3}$$

$$\boldsymbol{Z}_{\alpha_k} = \frac{\frac{\partial \boldsymbol{W}^r(\alpha_k)}{\partial \alpha_k} \lambda_{max}^r(\alpha_k) - \boldsymbol{v}^{r\prime}(\alpha_k) \frac{\partial \boldsymbol{W}^r(\alpha_k)}{\partial \alpha_k} \boldsymbol{v}^r(\alpha_k) \boldsymbol{W}^r(\alpha_k)}{[\lambda_{max}^r(\alpha_k)]^2}, \tag{A.4}$$

respectively, where the typical element of $\frac{\partial \boldsymbol{W}^r(\alpha_k)}{\partial \alpha_k} = -d_{ij} e^{-\alpha_k d_{ij}}$ for the negative exponential and $\frac{\partial \boldsymbol{W}^r(\alpha_k)}{\partial \alpha_k} = -ln(d_{ij}) d_{ij}^{-\alpha_k}$ for the inverse distance decay matrix.

A.2 Estimation

Generally, individual fixed effects are concentrated out by demeaning the variables by their individual-specific means.[2] The dependent variable of this demeaned model reads as $y_{it}^* = y_{it} - \frac{1}{T}\sum_{t=1}^{T} y_{it}$. Similar transformations are applied to right-hand side elements of the regression equation. The demeaned model for time period t is

$$y_{it}^* = \rho \sum_{j=1}^{N} w_{ij}(\alpha_0) y_{jt}^* + \sum_{k=1}^{K} x_{kit}^* \beta_k + \sum_{k=1}^{K}\sum_{j=1}^{N} w_{ij}(\alpha_k) x_{kjt}^* \gamma_k + \xi_t^* + \varepsilon_{it}^*. \quad (A.5)$$

Although this transformation does eliminate c_i, it induces linear dependence of the transformed errors ε_{it}^* over time. Consequently $\hat{\sigma}^2$ will be biased when T is small or fixed. To get an unbiased estimate, Arellano (2003, p.24) and Lee and Yu (2010) propose the following bias-correction (bc) $\hat{\sigma}_{bc}^2 = \frac{T}{T-1}\hat{\sigma}^2$. This correction can easily be carried out after the parameters of the model have been estimated.

When stacking the individual observations in each time period t, the demeaned SD model of equation (A.5) can be rewritten as

$$\boldsymbol{y}_t^* = \rho \boldsymbol{W}(\alpha_0)\boldsymbol{y}_t^* + \sum_{k=1}^{K} \boldsymbol{x}_{kt}^* \beta_k + \sum_{k=1}^{K} \boldsymbol{W}(\alpha_k)\boldsymbol{x}_{kt}^* \gamma_k + \boldsymbol{\xi}_t^* \boldsymbol{\iota}_N + \boldsymbol{\varepsilon}_t^*, \quad t = 1, \ldots, T, \quad (A.6)$$

where $\boldsymbol{y}_t^*$, $\boldsymbol{x}_{kt}^*$, and $\boldsymbol{\varepsilon}_t^*$ are $N \times 1$ vectors, and $\boldsymbol{\iota}_N$ is an $N \times 1$ vector of ones. When also stacking the data over time, the model reads as

$$\boldsymbol{y}^* = \rho(\boldsymbol{I}_T \otimes \boldsymbol{W}(\alpha_0))\, \boldsymbol{y}^* + \boldsymbol{X}^* \boldsymbol{\zeta} + \boldsymbol{\varepsilon}^*, \quad (A.7)$$

where $\boldsymbol{y}^*$ is a $NT \times 1$ vector containing observations of the dependent variable first sorted by time and then by spatial unit. $\boldsymbol{\varepsilon}^*$ is defined analogously. The $NT \times (2K + T - 1)$ matrix $\boldsymbol{X}^*$ is the corresponding sorted matrix of all explanatory variables, their spatial lags and dummy variables for time periods, and is given by

$$\boldsymbol{X}^* = [\boldsymbol{x}_1^*, \ldots, \boldsymbol{x}_K^*, (\boldsymbol{I}_T \otimes \boldsymbol{W}(\alpha_1))\boldsymbol{x}_1^*, \ldots, (\boldsymbol{I}_T \otimes \boldsymbol{W}(\alpha_K))\boldsymbol{x}_K^*, (\boldsymbol{\ell}_1, \boldsymbol{\ell}_2, ..., \boldsymbol{\ell}_{T-1}) \otimes \boldsymbol{\iota}_N], \quad (A.8)$$

where $\boldsymbol{\ell}_j$ are $T \times 1$ unit vectors, which contain zeros except for the jth element which contains a one. The corresponding parameters are summarized in the $(2K + T - 1) \times 1$ vector $\boldsymbol{\zeta} = (\boldsymbol{\beta}', \boldsymbol{\gamma}', \boldsymbol{\xi}^{*'})'$. The remaining parameters to be estimated are the spatial autoregressive parameter ρ, the distance decay parameters for all spatial lags $\boldsymbol{\alpha} = (\alpha_0, \alpha_1, ..., \alpha_K)'$ and σ^2.

Lee and Yu (2010) set out the assumptions under which the ML estimator of the model parameters in equation (1) are identified, consistent, and asymptotically normal, both when

[2] This is because these fixed effects are not of interest (or not reported), cannot be estimated consistently when T is small, and might affect the accuracy of parameter estimates when taking them up as part of the regressors if N grows large.

assuming that the error terms are normally distributed or not, and provided that the $\boldsymbol{W}$ matrices are nonstochastic, i.e., not parameterized. The regular rate of convergence is $\sqrt{N}$, provided that T is small or fixed. In Appendix A.6 we discuss all assumptions, as well as those that need to be adapted, such that the proof by Lee and Yu (2010) carries over to the model in this study.

Defining the full set of parameters as $\boldsymbol{\theta} = (\boldsymbol{\zeta}', \rho, \sigma^2, \boldsymbol{\alpha}')'$, the log-likelihood function of the model is given by

$$\text{log}L(\boldsymbol{\theta}) = -\frac{NT}{2}\text{log}(2\pi\sigma^2) + T\text{log}|\boldsymbol{I}_N - \rho\boldsymbol{W}(\alpha_0)| - \frac{1}{2\sigma^2}\,\boldsymbol{\varepsilon}^{*'}\,\boldsymbol{\varepsilon}^*, \tag{A.9}$$

where

$$\boldsymbol{\varepsilon}^* = \boldsymbol{y}^* - \rho(\boldsymbol{I}_T \otimes \boldsymbol{W}(\alpha_0))\,\boldsymbol{y}^* - \boldsymbol{X}^*\boldsymbol{\zeta}. \tag{A.10}$$

The ML estimator can be obtained by maximizing the log-likelihood function with respect to $\boldsymbol{\theta}$.[3] The parameter vector $\boldsymbol{\zeta}$ and the scalar σ^2 can be solved analytically from their first order conditions, given ρ and $\boldsymbol{\alpha} = (\alpha_0, \ldots, \alpha_K)'$, which yields

$$\hat{\boldsymbol{\zeta}}(\rho, \boldsymbol{\alpha}) = (\boldsymbol{X}^{*'}\boldsymbol{X}^*)^{-1}\boldsymbol{X}^{*'}\tilde{\boldsymbol{S}}\boldsymbol{y}^*, \tag{A.11}$$

$$\hat{\sigma}^2(\rho, \boldsymbol{\alpha}) = \frac{1}{NT}(\tilde{\boldsymbol{S}}\boldsymbol{y}^* - \boldsymbol{X}^*\zeta)'(\tilde{\boldsymbol{S}}\boldsymbol{y}^* - \boldsymbol{X}^*\boldsymbol{\zeta}), \tag{A.12}$$

where $\tilde{\boldsymbol{S}} = \boldsymbol{I}_T \otimes \boldsymbol{S}$ with $\boldsymbol{S} = \boldsymbol{I}_N - \rho\boldsymbol{W}(\alpha_0)$. By substituting these solutions for $\hat{\boldsymbol{\zeta}}$ and $\hat{\sigma}^2$ into equation (A.9), the concentrated log-likelihood function with respect to the $K+2$ remaining parameters ρ and $\hat{\alpha}_k$ ($k = 0, \ldots, K$) is obtained. Given the solution of this maximization problem for $\hat{\rho}$ and $\hat{\boldsymbol{\alpha}}$, we can subsequently determine the unconditional ML estimates of $\hat{\boldsymbol{\zeta}}$ and $\hat{\sigma}^2$, as well as the bias-corrected outcome $\hat{\sigma}^2_{bc} = \frac{T}{T-1}\hat{\sigma}^2$.

To find $\hat{\rho}$ and $\hat{\alpha}_k$ ($k = 0, \ldots, K$), we maximize the concentrated log-likelihood function of ρ and α_k ($k = 0, \ldots, K$), which is

$$\text{log}L(\rho, \boldsymbol{\alpha}|\hat{\boldsymbol{\zeta}}, \hat{\sigma}^2) = -\frac{NT}{2}\text{log}(2\pi\hat{\sigma}^2) + T\text{log}|\boldsymbol{I}_N - \rho\boldsymbol{W}(\alpha_0)| - \frac{NT}{2}, \tag{A.13}$$

where $\hat{\sigma}^2$ is programmed as in (A.12), and $\hat{\boldsymbol{\zeta}}$ as part of this expression is programmed as in (A.11). This has the effect that if one or more values of ρ and $\boldsymbol{\alpha}$ change, the estimates for $\hat{\boldsymbol{\zeta}}(\rho, \boldsymbol{\alpha})$ and $\hat{\sigma}^2(\rho, \boldsymbol{\alpha})$ change accordingly.

One final issue is the estimation of ρ and α_0 if $\boldsymbol{W}(\alpha_0)$ is not row but scalar normalized. The problem is that $\rho\boldsymbol{W}(\alpha_0) = \rho\boldsymbol{W}^r(\alpha_0)/\lambda^r_{max}$ is equivalent with $\rho^*\boldsymbol{W}^*(\alpha_0)$, where $\rho^* = c\rho$ and $\boldsymbol{W}^*(\alpha_0) = \boldsymbol{W}(\alpha_0)/c$ and c denotes any scalar factor (Kelejian and Prucha 2010, p.55).

[3] Mathematical expressions for the first and second order conditions and the information matrix, which will be used to determine the variance-covariance matrix of the parameters, are reported in Appendix A.3, A.4 and A.5, respectively.

We apply the following approach. First, ρ is estimated setting $\alpha_0 = 1$ (the default value) and $\boldsymbol{W}(\alpha_0)$ is scalar normalized, which yields $\rho = \rho_{initial}$. This $\rho_{initial}$ is then kept fixed when maximizing the concentrated log-likelihood function for α_0 and the other α_k $(k = 1, \ldots, K)$. During this iterative process $\boldsymbol{W}(\alpha_0)$ is scalar normalized every time α_0 changes. Only after the optimal values for α_k $(k = 0, \ldots, K)$ are found, among which $\widehat{\alpha}_{0ML}$, ρ is estimated again to get $\rho = \widehat{\rho}_{ML}$.

We developed an estimation routine both in Matlab and R, which enables estimating the proposed model for different normalizations and distance decay functions. A description of the routine can be found in the Appendix A.7.

The best option to find the global optimum of the decay parameters is to scale distance such that the exponential distance decay parameters of each spatial lag take values around 1 within the interval (0,10]. This interval is used to set the lower and the upper bounds in the maximization process and based on the properties that the spatial matrix converges to a sparse or dense matrix if α_k goes to infinity or zero, respectively. Although it does not matter which metric is used to measure distance, since the exponential distance decay parameters change accordingly, this scaling speeds up the maximization process and avoids non-convergence. This property does not apply to the inverse distance matrix because distance is taken to a power (Neumayer and Plümper 2016, p.184). However, the metric used to scale the exponential distance parameters also tends to work well for the inverse distance parameters.

A.3 First-order derivatives of the parameters

Without loss of generality, the presentation with respect to the distance decay parameters is limited to α_0, α_1 and α_2. This also implies that only two explanatory variables are considered $(K = 2)$. For the same reason, time dummies are left aside. The first-order derivatives are

$$\frac{\partial \underline{log} L(\boldsymbol{\theta})}{\partial \boldsymbol{\zeta}} = \frac{1}{\sigma^2}\boldsymbol{X}^{*'}\boldsymbol{\varepsilon}^* = \frac{1}{\sigma^2}\boldsymbol{X}^{*'}[\boldsymbol{y}^* - \rho(\boldsymbol{I}_T \otimes \boldsymbol{W}(\alpha_0)\boldsymbol{y}^* - \boldsymbol{X}^*\boldsymbol{\zeta}], \tag{A.14}$$

$$\frac{\partial \underline{log} L(\boldsymbol{\theta})}{\partial \rho} = -\text{tr}(\tilde{\boldsymbol{\Pi}}) + \frac{1}{\sigma^2}[(\tilde{\boldsymbol{\Pi}}\boldsymbol{X}^*\boldsymbol{\zeta})'\boldsymbol{\varepsilon}^* + \boldsymbol{\varepsilon}^{*'}\tilde{\boldsymbol{\Pi}}\boldsymbol{\varepsilon}^*], \tag{A.15}$$

$$\frac{\partial \underline{log} L(\boldsymbol{\theta})}{\partial \sigma^2} = -\frac{NT}{2\sigma^2} + \frac{\boldsymbol{\varepsilon}^{*'}\boldsymbol{\varepsilon}^*}{2\sigma^4}, \tag{A.16}$$

$$\frac{\partial \underline{log} L(\boldsymbol{\theta})}{\partial \alpha_0} = -\rho\text{tr}(\tilde{\boldsymbol{\Delta}}) + \frac{\rho}{\sigma^2}[(\tilde{\boldsymbol{\Delta}}\boldsymbol{X}^*\boldsymbol{\zeta})'\boldsymbol{\varepsilon}^* + \boldsymbol{\varepsilon}^{*'}\tilde{\boldsymbol{\Delta}}\boldsymbol{\varepsilon}^*], \tag{A.17}$$

$$\frac{\partial \underline{log} L(\boldsymbol{\theta})}{\partial \alpha_1} = \frac{\boldsymbol{\varepsilon}^{*'}\boldsymbol{Z}_{\alpha_1}\,\boldsymbol{x^*}_1\,\gamma_1}{\sigma^2} = \frac{(\boldsymbol{y}^* - \rho\tilde{\boldsymbol{W}}(\alpha_0)\boldsymbol{y}^* - \boldsymbol{X}^*\boldsymbol{\zeta})'\boldsymbol{Z}_{\alpha_1}\,\boldsymbol{x^*}_1\,\gamma_1}{\sigma^2}, \tag{A.18}$$

$$\frac{\partial \underline{log} L(\boldsymbol{\theta})}{\partial \alpha_2} = \frac{\boldsymbol{\varepsilon}^{*'}\boldsymbol{Z}_{\alpha_2}\,\boldsymbol{x^*}_2\,\gamma_2}{\sigma^2} = \frac{(\boldsymbol{y}^* - \rho\tilde{\boldsymbol{W}}(\alpha_0)\boldsymbol{y}^* - \boldsymbol{X}^*\boldsymbol{\zeta})'\boldsymbol{Z}_{\alpha_2}\,\boldsymbol{x^*}_2\,\gamma_2}{\sigma^2}, \tag{A.19}$$

where $\boldsymbol{\varepsilon}^* = \boldsymbol{y}^* - \rho\tilde{\boldsymbol{W}}(\alpha_0)\boldsymbol{y}^* - \boldsymbol{X}^*\boldsymbol{\zeta}$. To shorten notation, we further use $\boldsymbol{\Pi} = \boldsymbol{W}(\alpha_0)\boldsymbol{S}^{-1}$ and

$\boldsymbol{\Delta} = \boldsymbol{Z}_{\alpha_0}\boldsymbol{S}^{-1}$. A matrix $\boldsymbol{M}$ with a tilde is obtained by $\tilde{\boldsymbol{M}} = \boldsymbol{I}_T \otimes \boldsymbol{M}$ for $\boldsymbol{M} = \boldsymbol{\Pi}, \boldsymbol{W}, \boldsymbol{Z}_{\boldsymbol{\alpha_k}}, \boldsymbol{\Delta}$.

A.4 Second-order derivatives of the parameters

In line with $\boldsymbol{\theta} = (\boldsymbol{\zeta}', \rho, \sigma^2, \boldsymbol{\alpha}')'$, the Hessian matrix can be partitioned into a block-matrix consisting of six rows and column groups with respect to $\boldsymbol{\zeta}'$, ρ, σ^2, α_0, α_1, and α_2. Since the Hessian matrix is symmetric, we provide the expressions for the diagonal and upper-diagonal blocks of this matrix only.

The submatrices in the first row group of the Hessian matrix are

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial \boldsymbol{\zeta} \partial \boldsymbol{\zeta}'} = -\frac{\boldsymbol{X}^{*'}\boldsymbol{X}^*}{\sigma^2}, \tag{A.20}$$

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial \boldsymbol{\zeta} \partial \rho} = -\frac{\boldsymbol{X}^{*'}\tilde{\boldsymbol{W}}(\alpha_0)\boldsymbol{y}^*}{\sigma^2} = -\frac{\boldsymbol{X}^{*'}\tilde{\boldsymbol{\Pi}}\boldsymbol{X}^*\boldsymbol{\zeta} + \boldsymbol{X}^{*\prime}\tilde{\boldsymbol{\Pi}}\boldsymbol{\varepsilon}^*}{\sigma^2}, \tag{A.21}$$

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial \boldsymbol{\zeta} \partial \sigma^2} = -\frac{\boldsymbol{X}^{*'}\boldsymbol{\varepsilon}^*}{\sigma^4}, \tag{A.22}$$

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial \boldsymbol{\zeta} \partial \alpha_0} = -\frac{\rho \boldsymbol{X}^{*'}\tilde{\boldsymbol{Z}}_{\alpha_0}\boldsymbol{Y}^*}{\sigma^2} = -\frac{\rho \boldsymbol{X}^{*'}\tilde{\boldsymbol{\Delta}}\boldsymbol{X}^*\boldsymbol{\zeta} + \rho \boldsymbol{X}^{*'}\tilde{\boldsymbol{\Delta}}\boldsymbol{\varepsilon}^*}{\sigma^2}, \tag{A.23}$$

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial \boldsymbol{\zeta} \partial \alpha_1} = \frac{(\boldsymbol{Z}_{\alpha_1}\boldsymbol{x^*}_1)'\boldsymbol{\varepsilon}^* - \boldsymbol{X}^{*'}\boldsymbol{Z}_{\alpha_1}\,\boldsymbol{x^*}_1\,\gamma_1}{\sigma^2}, \tag{A.24}$$

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial \boldsymbol{\zeta} \partial \alpha_2} = \frac{(\boldsymbol{Z}_{\alpha_2}\boldsymbol{x^*}_2)'\boldsymbol{\varepsilon}^* - \boldsymbol{X}^{*'}\boldsymbol{Z}_{\alpha_2}\,\boldsymbol{x^*}_2\,\gamma_2}{\sigma^2}. \tag{A.25}$$

The submatrices in the second row group are

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial \rho^2} = -\text{tr}(\tilde{\boldsymbol{\Pi}}\tilde{\boldsymbol{\Pi}}) - \frac{(\boldsymbol{X}^*\boldsymbol{\zeta} + \boldsymbol{\varepsilon}^*)'\tilde{\boldsymbol{\Pi}}'\tilde{\boldsymbol{\Pi}}(\boldsymbol{X}^*\boldsymbol{\zeta} + \boldsymbol{\varepsilon}^*)}{\sigma^2}, \tag{A.26}$$

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial \rho \partial \sigma^2} = -\frac{(\tilde{\boldsymbol{W}}(\alpha_0)\boldsymbol{y}^*)'\boldsymbol{\varepsilon}^*}{\sigma^4} = -\frac{(\tilde{\boldsymbol{\Pi}}\boldsymbol{X}^*\boldsymbol{\zeta})'\boldsymbol{\varepsilon}^* + \boldsymbol{\varepsilon}^{*'}\tilde{\boldsymbol{\Pi}}\boldsymbol{\varepsilon}^*}{\sigma^4}, \tag{A.27}$$

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial \rho \partial \alpha_0} = -\text{tr}(\tilde{\boldsymbol{\Delta}}) - \rho\text{tr}(\tilde{\boldsymbol{\Pi}}\tilde{\boldsymbol{\Delta}}) + \frac{(\tilde{\boldsymbol{\Delta}}\boldsymbol{X}^*\boldsymbol{\zeta} + \tilde{\boldsymbol{\Delta}}\boldsymbol{\varepsilon}^*)'\boldsymbol{\varepsilon}^* - \rho(\tilde{\boldsymbol{\Pi}}\boldsymbol{X}^*\boldsymbol{\zeta} + \tilde{\boldsymbol{\Pi}}\boldsymbol{\varepsilon}^*)'(\tilde{\boldsymbol{\Delta}}\boldsymbol{X}^*\boldsymbol{\zeta} + \tilde{\boldsymbol{\Delta}}\boldsymbol{\varepsilon}^*)}{\sigma^2}, \tag{A.28}$$

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial \rho \partial \alpha_1} = -\frac{(\tilde{\boldsymbol{W}}(\alpha_0)\boldsymbol{Y}^*)'\boldsymbol{Z}_{\alpha_1}\,\boldsymbol{x^*}_1\,\gamma_1}{\sigma^2} = -\frac{(\tilde{\boldsymbol{\Pi}}\boldsymbol{X}^*\boldsymbol{\beta} + \tilde{\boldsymbol{\Pi}}\boldsymbol{\varepsilon}^*)'\boldsymbol{Z}_{\alpha_1}\,\boldsymbol{x^*}_1\,\gamma_1}{\sigma^2}, \tag{A.29}$$

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial \rho \partial \alpha_2} = -\frac{(\tilde{\boldsymbol{W}}(\alpha_0)\boldsymbol{y}^*)'\boldsymbol{Z}_{\alpha_2}\,\boldsymbol{x^*}_2\,\gamma_2}{\sigma^2} = -\frac{(\tilde{\boldsymbol{\Pi}}\boldsymbol{X}^*\boldsymbol{\zeta} + \tilde{\boldsymbol{\Pi}}\boldsymbol{\varepsilon}^*)'\boldsymbol{Z}_{\alpha_2}\,\boldsymbol{x^*}_2\,\gamma_2}{\sigma^2}. \tag{A.30}$$

The submatrices in the third row group are

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial(\sigma^2)^2} = \frac{NT}{2\sigma^4} - \frac{\boldsymbol{\varepsilon}^{*'}\boldsymbol{\varepsilon}^*}{\sigma^6}, \tag{A.31}$$

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial\sigma^2\partial\alpha_0} = \frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial\alpha_0\partial\sigma^2} = -\frac{\rho(\tilde{\boldsymbol{Z}}_{\alpha_0}\boldsymbol{y}^*)'\boldsymbol{\varepsilon}^*}{\sigma^4} = -\frac{\rho(\tilde{\boldsymbol{\Delta}}\boldsymbol{X}^*\boldsymbol{\zeta} + \tilde{\boldsymbol{\Delta}}\boldsymbol{\varepsilon}^*)'\boldsymbol{\varepsilon}^*}{\sigma^4}, \tag{A.32}$$

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial\sigma^2\partial\alpha_1} = \frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial\alpha_1\partial\sigma^2} = -\frac{\boldsymbol{\varepsilon}^{*'}\boldsymbol{Z}_{\alpha_1}\,\boldsymbol{x^*}_1\,\gamma_1}{\sigma^4}, \tag{A.33}$$

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial\sigma^2\partial\alpha_2} = \frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial\alpha_2\partial\sigma^2} = -\frac{\boldsymbol{\varepsilon}^{*'}\boldsymbol{Z}_{\alpha_2}\,\boldsymbol{x^*}_2\,\gamma_2}{\sigma^4}. \tag{A.34}$$

The submatrices in the fourth row group are

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial\alpha_0^2} = -\rho\mathrm{tr}(\tilde{\boldsymbol{\Lambda}}) - \rho^2\mathrm{tr}(\tilde{\boldsymbol{\Delta}}\tilde{\boldsymbol{\Delta}}) + \frac{\rho(\tilde{\boldsymbol{\Lambda}}\boldsymbol{X}^*\boldsymbol{\zeta}+\tilde{\boldsymbol{\Lambda}}\boldsymbol{\varepsilon}^*)'\,\boldsymbol{\varepsilon}^* - \rho^2(\tilde{\boldsymbol{\Delta}}\boldsymbol{X}^*\boldsymbol{\zeta}+\tilde{\boldsymbol{\Delta}}\boldsymbol{\varepsilon}^*)'(\tilde{\boldsymbol{\Delta}}\boldsymbol{X}^*\boldsymbol{\zeta}+\tilde{\boldsymbol{\Delta}}\boldsymbol{\varepsilon}^*)}{\sigma^2}, \tag{A.35}$$

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial\alpha_0\partial\alpha_1} = -\frac{\rho(\tilde{\boldsymbol{Z}}_{\alpha_0}\boldsymbol{Y}^*)'\boldsymbol{V_1}\boldsymbol{\zeta}}{\sigma^2} = -\frac{\rho(\tilde{\boldsymbol{\Delta}}\boldsymbol{X}^*\boldsymbol{\zeta}+\tilde{\boldsymbol{\Delta}}\boldsymbol{\varepsilon}^*)'\boldsymbol{Z}_{\alpha_1}\,\boldsymbol{x^*}_1\,\gamma_1}{\sigma^2}, \tag{A.36}$$

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial\alpha_0\partial\alpha_2} = -\frac{\rho(\tilde{\boldsymbol{Z}}_{\alpha_0}\boldsymbol{Y}^*)'\boldsymbol{V_2}\boldsymbol{\zeta}}{\sigma^2} = -\frac{\rho(\tilde{\boldsymbol{\Delta}}\boldsymbol{X}^*\boldsymbol{\zeta}+\tilde{\boldsymbol{\Delta}}\boldsymbol{\varepsilon}^*)'\boldsymbol{Z}_{\alpha_2}\,\boldsymbol{x^*}_2\,\gamma_2}{\sigma^2}. \tag{A.37}$$

The submatrices in the fifth row group are

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial\alpha_1^2} = \frac{\boldsymbol{\varepsilon}^{*'}\frac{\partial^2 \boldsymbol{W}_{(\alpha_1)}}{\partial\alpha_1^2}x_1^*\gamma_1 - (\boldsymbol{Z}_{\alpha_1}\,\boldsymbol{x^*}_1\,\gamma_1)'\boldsymbol{Z}_{\alpha_1}\,\boldsymbol{x^*}_1\,\gamma_1}{\sigma^2}, \tag{A.38}$$

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial\alpha_1\partial\alpha_2} = -\frac{(\boldsymbol{Z}_{\alpha_2}\,\boldsymbol{x^*}_2\,\gamma_2)'\boldsymbol{Z}_{\alpha_2}\,\boldsymbol{x^*}_2\,\gamma_2}{\sigma^2}. \tag{A.39}$$

Finally, the submatrix in the sixth row group is

$$\frac{\partial^2 \underline{log} L(\boldsymbol{\theta})}{\partial\alpha_2^2} = \frac{\boldsymbol{\varepsilon}^{*'}\frac{\partial^2 \boldsymbol{W}_{(\alpha_2)}}{\partial\alpha_2^2}x_2^*\gamma_2 - (\boldsymbol{Z}_{\alpha_2}\,\boldsymbol{x^*}_2\,\gamma_2)'\boldsymbol{Z}_{\alpha_2}\,\boldsymbol{x^*}_2\,\gamma_2}{\sigma^2}. \tag{A.40}$$

A.5 Information matrix

The submatrices in the first row group of the information matrix $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$ are

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial\boldsymbol{\zeta}\partial\boldsymbol{\zeta}'}\right] = \frac{1}{\sigma^2}\boldsymbol{X}^{*'}\boldsymbol{X}^{*}, \tag{A.41}$$

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial\boldsymbol{\zeta}\partial\rho}\right] = \frac{1}{\sigma^2}\boldsymbol{X}^{*'}\tilde{\boldsymbol{\Pi}}\boldsymbol{X}^{*}\boldsymbol{\zeta}, \tag{A.42}$$

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial\boldsymbol{\zeta}\partial\sigma^2}\right] = \mathbf{0}, \tag{A.43}$$

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial\boldsymbol{\zeta}\partial\alpha_0}\right] = \frac{\rho}{\sigma^2}\boldsymbol{X}^{*'}\tilde{\boldsymbol{\Delta}}\boldsymbol{X}^{*}\boldsymbol{\zeta}, \tag{A.44}$$

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial\boldsymbol{\zeta}\partial\alpha_1}\right] = \frac{1}{\sigma^2}\boldsymbol{X}^{*'}\boldsymbol{Z}_{\alpha_1}\,\boldsymbol{x^*}_1\,\gamma_1, \tag{A.45}$$

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial\boldsymbol{\zeta}\partial\alpha_2}\right] = \frac{1}{\sigma^2}\boldsymbol{X}^{*'}\boldsymbol{Z}_{\alpha_2}\,\boldsymbol{x^*}_2\,\gamma_2. \tag{A.46}$$

The submatrices in the second row group of $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$ are

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial\rho^2}\right] = \mathrm{tr}(\tilde{\boldsymbol{\Pi}}\tilde{\boldsymbol{\Pi}}) + \frac{1}{\sigma^2}(\boldsymbol{X}^{*}\boldsymbol{\zeta})'\tilde{\boldsymbol{\Pi}}'\tilde{\boldsymbol{\Pi}}\boldsymbol{X}^{*}\boldsymbol{\zeta} + \mathrm{tr}(\tilde{\boldsymbol{\Pi}}'\tilde{\boldsymbol{\Pi}}), \tag{A.47}$$

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial\rho\partial\sigma^2}\right] = \frac{1}{\sigma^2}\mathrm{tr}(\tilde{\boldsymbol{\Pi}}), \tag{A.48}$$

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial\rho\partial\alpha_0}\right] = \rho\mathrm{tr}(\tilde{\boldsymbol{\Pi}}\tilde{\boldsymbol{\Delta}}) + \frac{\rho}{\sigma^2}(\boldsymbol{X}^{*}\boldsymbol{\zeta})'\tilde{\boldsymbol{\Pi}}'\tilde{\boldsymbol{\Delta}}\boldsymbol{X}^{*}\boldsymbol{\zeta} + \rho\mathrm{tr}(\tilde{\boldsymbol{\Pi}}'\tilde{\boldsymbol{\Delta}}), \tag{A.49}$$

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial\rho\partial\alpha_1}\right] = \frac{1}{\sigma^2}(\boldsymbol{X}^{*}\boldsymbol{\zeta})'\tilde{\boldsymbol{\Pi}}'\boldsymbol{Z}_{\alpha_1}\,\boldsymbol{x^*}_1\,\gamma_1, \tag{A.50}$$

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial\rho\partial\alpha_2}\right] = \frac{1}{\sigma^2}(\boldsymbol{X}^{*}\boldsymbol{\beta})'\tilde{\boldsymbol{\Pi}}'\boldsymbol{Z}_{\alpha_2}\,\boldsymbol{x^*}_2\,\gamma_2. \tag{A.51}$$

The submatrices in the third row group of $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$ are

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial(\sigma^2)^2}\right] = \frac{NT}{2\sigma^4}, \tag{A.52}$$

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial\sigma^2\partial\alpha_0}\right] = \frac{\rho}{\sigma^2}\mathrm{tr}(\tilde{\boldsymbol{\Delta}}), \tag{A.53}$$

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial\sigma^2\partial\alpha_1}\right] = \mathbf{0}, \tag{A.54}$$

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial\sigma^2\partial\alpha_2}\right] = \mathbf{0}. \tag{A.55}$$

The submatrices in the fourth row group of $\boldsymbol{\Sigma_\theta}$ are

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial \alpha_0^2}\right] = \rho^2 \mathrm{tr}(\tilde{\boldsymbol{\Delta}}\tilde{\boldsymbol{\Delta}}) + \frac{\rho^2}{\sigma^2}(\boldsymbol{X}^*\boldsymbol{\zeta})'\tilde{\boldsymbol{\Delta}}'\tilde{\boldsymbol{\Delta}}\boldsymbol{X}^*\boldsymbol{\zeta} + \rho^2 \mathrm{tr}(\tilde{\boldsymbol{\Delta}}'\tilde{\boldsymbol{\Delta}}), \tag{A.56}$$

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial \alpha_0 \partial \alpha_1}\right] = \frac{\rho}{\sigma^2}(\tilde{\boldsymbol{\Delta}}\boldsymbol{X}^*\boldsymbol{\zeta})'\boldsymbol{Z}_{\alpha_1}\,\boldsymbol{x^*}_1\,\gamma_1, \tag{A.57}$$

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial \alpha_0 \partial \alpha_2}\right] = \frac{\rho}{\sigma^2}(\tilde{\boldsymbol{\Delta}}\boldsymbol{X}^*\boldsymbol{\zeta})'\boldsymbol{Z}_{\alpha_2}\,\boldsymbol{x^*}_2\,\gamma_2. \tag{A.58}$$

The submatrices in the fifth row group of $\boldsymbol{\Sigma_\theta}$ are

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial \alpha_1^2}\right] = \frac{1}{\sigma^2}(\boldsymbol{Z}_{\alpha_1}\,\boldsymbol{x^*}_1\,\gamma_1)'\boldsymbol{Z}_{\alpha_1}\,\boldsymbol{x^*}_1\,\gamma_1, \tag{A.59}$$

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial \alpha_1 \alpha_2}\right] = \frac{1}{\sigma^2}(\boldsymbol{Z}_{\alpha_2}\,\boldsymbol{x^*}_2\,\gamma_2)'\boldsymbol{Z}_{\alpha_1}\,\boldsymbol{x^*}_1\,\gamma_1. \tag{A.60}$$

Finally, the submatrix in the sixth row group is

$$-\underline{E}\left[\frac{\partial^2 \underline{logL}(\boldsymbol{\theta})}{\partial \alpha_2^2}\right] = \frac{1}{\sigma^2}(\boldsymbol{Z}_{\alpha_2}\,\boldsymbol{x^*}_2\,\gamma_2)'\boldsymbol{Z}_{\alpha_2}\,\boldsymbol{x^*}_2\,\gamma_2. \tag{A.61}$$

A.6 Asymptotic normality

Assumptions similar to Lee and Yu (2010) under which the ML estimator of the model parameters in equation (1) are identified, consistent, and asymptotically normal, both when assuming that the error terms are normally distributed or not, are labeled by **(LY)** and those that are different to considering spatial weight matrices that are parameterized by **(PW)**.

Assumption 1 ***(LY)*** *The idiosyncratic errors ε_{it}, $i = 1, 2, ..., N$ and $t = 1, 2, ..., T$ are normally distributed or are iid across i and t with mean zero, variance σ^2 and finite fourth moment.*

Assumption 2 ***(PW)*** *The matrices $\boldsymbol{W}(\alpha_k)(k = 0, 1, ...K)$ are stochastic but exogenous with diagonal elements equal to zero. Before normalization, the row and column sums of the spatial weight matrix $\boldsymbol{W}(\alpha_k)(k = 0, 1, ...K)$ are uniformly bounded or, alternatively, diverge to infinity at a rate slower than N.*

Assumption 3 ***(LY)*** *$\boldsymbol{S} = \boldsymbol{I}_N - \rho\boldsymbol{W}(\alpha_0)$ is invertible and uniformly bounded in row and column sums in absolute value. ρ is in the interior of $\Re$, where $\Re$ is an open but bounded interval.* ***(PW)*** *ρ is bounded away from zero.*

Assumption 4 ***(PW)*** *γ_k for $k = 1, \ldots, K$ is bounded away from zero.*

Assumption 5 ***(PW)*** *For all k $\rho\beta_k\boldsymbol{W}(\alpha_0) + \gamma_k\boldsymbol{W}(\alpha_k) \neq \boldsymbol{0}_N$.*

Assumption 6 ***(LY)*** *The regressors x_k are nonstochastic and bounded uniformly. There is no multicollinearity among the regressors and their spatially lagged counterparts.*

If the error terms are assumed to be normally distributed, the parameters can be estimated by ML. If they are not truly normally distributed, they can be estimated by quasi (Q)ML based on the properties specified in Assumption 1. Assumption 2 allows for weak divergence of the spatial weight matrix. In the case of a non-stochastic $\boldsymbol{W}$, $\boldsymbol{W}$ is assumed to be uniformly bounded in both row and column sums in absolute value (Lee and Yu 2010; Kelejian and Prucha 2010). We are dealing with spatial weight matrices that depend on distance decay parameters α_k ($k = 0, 1, ..., K$), which makes the spatial weight matrices stochastic in the sense that they are subject to uncertainty and consequently with a margin of error. Gupta (2019) shows that many established estimation methods also work with an exogenous stochastic spatial weight matrix.[4] However, the fact that the spatial weight matrix is stochastic, needs to be formalized in the weak divergence assumption, which requires that the row and column sums of the stochastic spatial weight matrix may diverge to infinity as long as the sample size N diverges to infinity faster. This is in line with Assumption 2 in Gupta (2019). Assumption 2 can be fulfilled when the spatial weight matrix has a distance decay functional form. For a negative exponential distance decay matrix the condition of uniformly boundedness corresponds to a distance decay parameter of $\alpha_k > 0$, and for an inverse distance matrix to $\alpha_k > 1$ (Elhorst et al. 2021).[5] Lee (2004) shows that the row and column sums of the spatial weight matrix in a cross-sectional setting may also diverge to infinity as long as the sample size N diverges to infinity faster.[6] For this reason, we also consider and experiment with an inverse distance matrix using a lower bound in the interval $(0, 1]$ in Appendix A.9.

Assumption 3 is a modification of Assumption 3 in Lee and Yu (2010). The invertibility of $\boldsymbol{S}$ guarantees that equation (A.63) below is valid. For ρ we use the interval $(-1, +1)$, with the exception of $\rho = 0$. If $\boldsymbol{W}(\alpha_0)$ is scalar or row normalized the upper bound is 1 by construction. The lower bound might be smaller than -1 when scalar normalization is applied, but negative

[4]There are also studies that allow for endogenous spatial weight matrices (Qu and Lee 2015; Qu et al. 2016). Endogeneity can arise due to feedback effects, e.g. if the spatial weight matrix is based on economic variables, which also depend on the dependent variable. However, if the spatial weight matrix depends on geographic distance between countries, counties or cities, such feedback effects do not occur, since distance is exogenous. This property does not change when distance is parameterized.

[5]When $\boldsymbol{W}$ is a parameterized negative exponential distance matrix, the corresponding row or column sum of this series in a continuous space (to ease calculations) can be calculated as the integral $\int_1^N e^{-\alpha x} dx = (1/\alpha)e^{-\alpha}(1 - e^{-N})$. The row or column sums represented by this integral are upper bounded for $\alpha > 0$ if N goes to infinity. Similarly, the integral $\int_1^N \frac{1}{x^\alpha} dx = 1/(1-\alpha)(N^{1-\alpha} - 1)$ is upper bounded for $\alpha > 1$ if N goes to infinity.

[6]Although not part of the formal proof in Lee and Yu (2010), they state that this result also carries over to a panel data setting. This point occurs at $1/(1-\alpha)(N^{1-\alpha} - 1)/N = 1/(1-\alpha)(1/N^\alpha - 1/N)$, which converges to zero and thus is upper bounded for $\alpha > 0$ if N goes to infinity.

values for ρ in models with only one spatial lag in the dependent variable are considerably less common than positive ones, let alone negative values smaller than -1 (Elhorst 2014, Section 2.5). Therefore, imposing this lower bound is hardly restrictive. ρ should be bounded away from zero. If ρ equals zero, then the distance decay parameter α_0 is not identified. This follows from the information matrix of the parameters derived in Appendix A.5.[7] The same non-identification problem occurs for α_k if $\gamma_k = 0$ $(k = 1, \ldots, K)$, which is excluded in Assumption 4.

Assumption 5 states further identification assumptions. The first part is different and less restrictive than the condition $\rho\beta_k + \gamma_k \neq 0$ for all k in Bramoullé et al. (2009, Proposition 1) and Lee and Yu (2016, Lemmas 2 and 3). This is due to the fact that we have not one common but different weight matrices for each spatial lag.

Model (1) can be rewritten in matrix notation. If we stack the individual observations for each time period t, the model reads as

$$\boldsymbol{y}_t = \rho \boldsymbol{W}(\alpha_0)\boldsymbol{y}_t + \sum_{k=1}^{K} \boldsymbol{x}_{kt}\beta_k + \sum_{k=1}^{K} \boldsymbol{W}(\alpha_k)\boldsymbol{x}_{kt}\gamma_k + \boldsymbol{c} + \xi_t \boldsymbol{\iota}_N + \boldsymbol{\varepsilon}_t, \quad t = 1, \ldots, T, \tag{A.62}$$

where $\boldsymbol{y}_t$, $\boldsymbol{x}_{kt}$, $\boldsymbol{c}$, $\boldsymbol{\varepsilon}_t$ are $N \times 1$ vectors, and $\boldsymbol{\iota}_N$ is an $N \times 1$ vector of ones. $\boldsymbol{W}(\alpha_0)$ and $\boldsymbol{W}(\alpha_k)$ are $N \times N$ matrices describing the connectivity of all N cross-sectional units in the sample. The corresponding reduced form of equation (A.62) reads as

$$\boldsymbol{y}_t = (\boldsymbol{I}_N - \rho \boldsymbol{W}(\alpha_0))^{-1} \left(\sum_{k=1}^{K} \boldsymbol{x}_{kt}\beta_k + \sum_{k=1}^{K} \boldsymbol{W}(\alpha_k)\boldsymbol{x}_{kt}\gamma_k + \boldsymbol{c} + \xi_t \boldsymbol{\iota}_N + \boldsymbol{\varepsilon}_t \right), \quad t = 1, \ldots, T. \tag{A.63}$$

Rewriting the reduced form in equation (A.63) using the fact that $(\boldsymbol{I}_N - \rho \boldsymbol{W}(\alpha_0))^{-1} = \boldsymbol{I}_N + \rho \boldsymbol{W}(\alpha_0) + \rho^2 \boldsymbol{W}^2(\alpha_0) + \ldots$ yields

$$\begin{aligned} \boldsymbol{y}_t &= (\boldsymbol{I}_N - \rho \boldsymbol{W}(\alpha_0))^{-1} \left[\sum_{k=1}^{K} (\boldsymbol{x}_{kt}\beta_k + \boldsymbol{W}(\alpha_k)\boldsymbol{x}_{kt}\gamma_k) + \boldsymbol{c} + \xi_t \boldsymbol{\iota}_N + \boldsymbol{\varepsilon}_t \right] \\ &= \sum_{k=1}^{K} \left[\boldsymbol{x}_{kt}\beta_k + [\rho\beta_k \boldsymbol{W}(\alpha_0) + \gamma_k \boldsymbol{W}(\alpha_k)][\boldsymbol{I}_N + \sum_{g=2}^{\infty} \rho^{g-1} \boldsymbol{W}^{g-1}(\alpha_0)\boldsymbol{x}_{kt}] \right] \\ &\quad + (\boldsymbol{I}_N - \rho \boldsymbol{W}(\alpha_0))^{-1} [\boldsymbol{c} + \xi_t \boldsymbol{\iota}_N + \boldsymbol{\varepsilon}_t]. \end{aligned} \tag{A.64}$$

This expression shows that the spatial lags $\boldsymbol{W}(\alpha_0)\boldsymbol{y}_t$ and $\boldsymbol{W}(\alpha_k)\boldsymbol{x}_{kt}$ $(k = 1, \ldots, K)$ cancel each other out if $\rho\beta_k + \gamma_k = 0$ for all k and $\alpha_0 = \alpha_1 = \ldots = \alpha_K$. Under these restrictions, the SD model reduces to the spatial error model (SEM) (Burridge 1981; Juhl 2021), as a result of which the coefficients of the spatial lags $\boldsymbol{W}(\alpha_0)\boldsymbol{y}_t$ and $\boldsymbol{W}(\alpha_k)\boldsymbol{x}_{kt}$ $(k = 1, \ldots, K)$ are not identified.[8]

[7] If ρ equals zero, then the elements in the information matrix that are based on second order derivatives involving α_0 equal zero (see equations A.44,A.49,A.53,A.56,A.57,A.58). Consequently one row and column contains zeros only. Thus the information matrix is not invertible and the variance covariance matrix is not defined.

[8] We also validated these findings as part of our Monte Carlo simulation experiment.

However, in empirical work we do not expect the estimates for the decay parameters α_k to be identical. Thus this assumption is not very restrictive.

As in Lee and Yu (2010), we assume exogenous and uniformly bounded regressors. Given the exogeneity of the explanatory variables and the weak divergence of the corresponding spatial weight matrices, the spatially lagged regressors are also bounded uniformly. Assumption 6 also rules out multicollinearity among the regressors.

Lee and Yu (2010) show in Theorem 2(1) and Lemma A.4 that the asymptotic distribution of the ML estimator is given by

$$\sqrt{NT}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \xrightarrow{d} N(0, \boldsymbol{\Upsilon}_{\boldsymbol{\theta}}). \tag{A.65}$$

where $\boldsymbol{\Upsilon}_{\boldsymbol{\theta}} = \lim \frac{T}{T-1}(\frac{1}{NT}\boldsymbol{\Sigma}_{\boldsymbol{\theta}})^{-1}(\frac{1}{NT}\boldsymbol{\Sigma}_{\boldsymbol{\theta}} + \boldsymbol{\Omega}_{\boldsymbol{\theta}})(\frac{1}{NT}\boldsymbol{\Sigma}_{\boldsymbol{\theta}})^{-1}$, and $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$ is specified in Appendix A.5. The matrix $\boldsymbol{\Omega}_{\boldsymbol{\theta}}$ reads as

$$\boldsymbol{\Omega}_{\boldsymbol{\theta}} = \frac{(T-1)}{T}\frac{\mu_4 - 3\sigma^4}{\sigma^4}\begin{pmatrix} \mathbf{0}_{2K\times 2K} & * & * & * & * \\ \mathbf{0}_{1\times 2K} & \frac{1}{N}\sum_{i=1}^{N}(\boldsymbol{\Pi})_{ii}^2 & * & * & * \\ \mathbf{0}_{1\times 2K} & \frac{1}{2\sigma^2 N}\mathrm{tr}(\boldsymbol{\Pi}) & \frac{1}{4\sigma^4} & * & * \\ \mathbf{0}_{1\times 2K} & \frac{\rho}{N}\sum_{i=1}^{N}(\boldsymbol{\Pi})_{ii}(\boldsymbol{\Delta})_{ii} & \frac{\rho}{2\sigma^2 N}\mathrm{tr}(\boldsymbol{\Delta}) & \frac{\rho^2}{N}\sum_{i=1}^{N}(\boldsymbol{\Delta})_{ii}^2 & * \\ \mathbf{0}_{2\times 2K} & \mathbf{0}_{2\times 1} & \mathbf{0}_{2\times 1} & \mathbf{0}_{2\times 1} & \mathbf{0}_{2\times 2} \end{pmatrix} \tag{A.66}$$

where μ_4 is the fourth moment of the error term. If the error terms are assumed to be normally distributed, the matrix $\boldsymbol{\Omega}_{\boldsymbol{\theta}}$ cancels out since $\mu_4 - 3\sigma^4 = 0$ under this circumstance. This yields $\boldsymbol{\Upsilon}_{\boldsymbol{\theta}} = \lim \frac{T}{T-1}\frac{1}{NT}{\boldsymbol{\Sigma}_{\boldsymbol{\theta}}}^{-1}$.

A.7 Direct and indirect effects: the delta method

To draw statistical inferences on the direct and indirect effects, expressions for standard errors are needed. Two methods can be used: bootrapping or the delta method. To save computation time, we use the delta method which is an extension of the method described in Arbia et al. (2020). We depart from $\sqrt{NT}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \xrightarrow{d} N(0, \boldsymbol{\Upsilon}_{\boldsymbol{\theta}})$, derived in Appendix A.5, but instead of $\boldsymbol{\theta}$ and $\boldsymbol{\Upsilon}_{\boldsymbol{\theta}}$ or $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$, we consider $\boldsymbol{\varphi} = (\boldsymbol{\beta}', \boldsymbol{\gamma}', \rho, \boldsymbol{\alpha}')'$ and $\boldsymbol{\Sigma}_{\boldsymbol{\varphi}}$, i.e., after rows and columns for the time dummies and σ^2 have been removed since they are not needed to determine the direct and indirect effects. Applying propositions 1 and 2 and remark 2 in Arbia et al. (2020), we get

$$\sqrt{N}[DE_k(\hat{\boldsymbol{\varphi}}) - DE_k(\boldsymbol{\varphi})] \xrightarrow{d} N[0, \boldsymbol{A}_k^{DE}(\boldsymbol{\varphi}){\boldsymbol{\Sigma}_{\boldsymbol{\varphi}}}^{-1}\boldsymbol{A}_k^{DE}(\boldsymbol{\varphi})'], \tag{A.67}$$

$$\sqrt{N}[IE_k(\hat{\boldsymbol{\varphi}}) - IE_k(\boldsymbol{\varphi})] \xrightarrow{d} N[0, \boldsymbol{A}_k^{IE}(\boldsymbol{\varphi}){\boldsymbol{\Sigma}_{\boldsymbol{\varphi}}}^{-1}\boldsymbol{A}_k^{IE}(\boldsymbol{\varphi})']. \tag{A.68}$$

where $\boldsymbol{A}_k^{DE}(\boldsymbol{\varphi}) = \frac{\partial DE_k(\boldsymbol{\varphi})}{\partial \boldsymbol{\varphi}'}$ and $\boldsymbol{A}_k^{IE}(\boldsymbol{\varphi}) = \frac{\partial IE_k(\boldsymbol{\varphi})}{\partial \boldsymbol{\varphi}'}$. These first order derivatives for the direct effects take the form

$$\frac{\partial DE_k(\boldsymbol{\varphi})}{\partial \beta_k} = \frac{1}{N} tr(\boldsymbol{S}^{-1}), \tag{A.69}$$

$$\frac{\partial DE_k(\boldsymbol{\varphi})}{\partial \gamma_k} = \frac{1}{N} tr\{\boldsymbol{S}^{-1}\boldsymbol{W}(\alpha_k)\}, \tag{A.70}$$

$$\frac{\partial DE_k(\boldsymbol{\varphi})}{\partial \rho} = \frac{1}{N} tr\{\boldsymbol{S}^{-1}\boldsymbol{W}(\alpha_0)\boldsymbol{S}^{-1}\boldsymbol{C}\}, \tag{A.71}$$

$$\frac{\partial DE_k(\boldsymbol{\varphi})}{\partial \alpha_0} = \frac{1}{N} tr\{\rho\boldsymbol{S}^{-1}\boldsymbol{Z}_{\alpha_0}\boldsymbol{S}^{-1}\boldsymbol{C}\}, \tag{A.72}$$

$$\frac{\partial DE_k(\boldsymbol{\varphi})}{\partial \alpha_k} = \frac{1}{N} tr\{\boldsymbol{S}^{-1}\boldsymbol{Z}_{\alpha_k}\gamma_k\}. \tag{A.73}$$

and for the indirect effects take the form

$$\frac{\partial IE_k(\boldsymbol{\varphi})}{\partial \beta_k} = \frac{1}{N}\boldsymbol{\tau_N}'\boldsymbol{S}^{-1}\boldsymbol{\tau_N} - \frac{1}{N} tr(\boldsymbol{S}^{-1}), \tag{A.74}$$

$$\frac{\partial IE_k(\boldsymbol{\varphi})}{\partial \gamma_k} = \frac{1}{N}\boldsymbol{\tau_N}'\{\boldsymbol{S}^{-1}\boldsymbol{W}(\alpha_k)\}\boldsymbol{\tau_N} - \frac{1}{N} tr\{\boldsymbol{S}^{-1}\boldsymbol{W}(\alpha_k)\}, \tag{A.75}$$

$$\frac{\partial IE_k(\boldsymbol{\varphi})}{\partial \rho} = \frac{1}{N}\boldsymbol{\tau_N}'\{\boldsymbol{S}^{-1}\boldsymbol{W}(\alpha_0)\boldsymbol{S}^{-1}\boldsymbol{C}\}\boldsymbol{\tau_N} - \frac{1}{N} tr\{\boldsymbol{S}^{-1}\boldsymbol{W}(\alpha_0)\boldsymbol{S}^{-1}\boldsymbol{C}\}, \tag{A.76}$$

$$\frac{\partial IE_k(\boldsymbol{\varphi})}{\partial \alpha_0} = \frac{1}{N}\boldsymbol{\tau_N}'\{\rho\boldsymbol{S}^{-1}\boldsymbol{Z}_{\alpha_0}\boldsymbol{S}^{-1}\boldsymbol{C}\}\boldsymbol{\tau_N} - \frac{1}{N} tr\{\rho\boldsymbol{S}^{-1}\boldsymbol{Z}_{\alpha_0}\boldsymbol{S}^{-1}\boldsymbol{C}\}, \tag{A.77}$$

$$\frac{\partial IE_k(\boldsymbol{\varphi})}{\partial \alpha_k} = \frac{1}{N}\boldsymbol{\tau_N}'\{\boldsymbol{S}^{-1}\boldsymbol{Z}_{\alpha_k}\gamma_k\}\boldsymbol{\tau_N} - \frac{1}{N} tr\{\boldsymbol{S}^{-1}\boldsymbol{Z}_{\alpha_k}\gamma_k\}. \tag{A.78}$$

where $\boldsymbol{S} = \boldsymbol{I}_N - \rho\boldsymbol{W}(\alpha_0)$, and $\boldsymbol{C} = \boldsymbol{I}_N \cdot \beta_k + \boldsymbol{W}(\alpha_k)\cdot\gamma_k$. Contrary to Arbia et al. (2020), our effect estimates contain the terms $\boldsymbol{Z}_{\alpha_k}$, which represent the first order derivative of the spatial weight matrices with respect to α_k. The mathematical expressions of this derivative for the inverse and negative exponential distance decay matrices and for both normalizations can be found in Appendix A.1.

A.8 PWFE: A routine for practitioners

To be able to estimate the parameters of the model set out in this paper, we developed a routine in both Matlab and R. The routine entitled PWFE has the following options:

1. Type of spatial weight matrix: negative exponential distance decay matrix (Edist) or inverse distance matrix (Idist).

2. Type of normalization of the spatial weight matrix: row normalization (rsn) or scalar normalization by the largest eigenvalue (men).

3. Type of model: different distance decay parameters for each spatial lag (multi), one common distance decay parameter for all spatial lags (same), and one common distance

decay parameter for all spatial lags in the explanatory variables, but not the dependent variable (one).

4. Method to determine significance levels of the direct and indirect effects: the recommended delta method (del) or the bootstrap method based on LeSage and Pace (2009) (bt).

5. gamma0, lb and ub are respectively the starting values, the lower bounds and the upper bounds on ρ and the distance decay parameters when using row normalization, and on the distance decay parameters when using scalar normalization by the largest eigenvalue.

A.9 Additional simulation designs and results

The main text only reports simulation results regarding the Bias, RMSE, Mbias and Mabias of each parameter estimate, direct and indirect effect for the row normalized negative exponential distance decay matrix in the base run (Case I): $\beta_1 = -1, \beta_2 = 0.2, \gamma_1 = 1.5, \gamma_2 = -0.3, \rho = 0.5, \sigma^2 = 1, \alpha_0 = 2, \alpha_1 = 1.5, \alpha_2 = 3$. This appendix contains additional simulation results.

It first reports the mean and standard deviations of the p-values of these estimates in Table A.1 for the row normalized negative exponential distance decay matrix in the base run (Case I). If the underlying asymptotic distribution is true, then under the null the p-values should follow a $U(0,1)$ distribution, and thus should have a mean p-value of 0.5 and a standard deviation of approximately 0.29.

Table 1 in the main text and Table A1 in this appendix are followed with similar Tables A.2 and A.3 for the scalar normalized negative exponential distance decay matrix. Since we used rectangles of 10×20 for $N = 200$ and 20×40 for $N = 800$, the row and scalar normalized matrices are different by construction. Further note that similar values of ρ and the γs in a row normalized and scalar normalized matrix do not have the same interpretation, as explained in Section 3.3. Next, Tables A.4 to A.6 report similar simulation results for both the row and scalar normalized inverse distance matrix.

Tables A.1 and A.3 show that when using different parameterized distance decay matrices for all spatial lags, denoted by PWFE, average p-values and their standard deviation closely fluctuate around 0.5 and 0.29, respectively. Only for the coefficient ρ differences are slightly higher when scalar rather than row normalization is used. For the negative exponential matrix mean and standard deviation are 0.65 and 0.23, and for the inverse distance matrix (see Table A.6) they are 0.69 and of 0.21, respectively. However, this has no adverse effect on the p-values of the direct and indirect effects derived from ρ and other coefficients. It was found in the main text that especially the indirect effects are sensitive to an incorrect choice of the spatial weight matrix. The biases reported in Table 1 when using one common spatial weight matrix for all spatial lags, denoted by WFE, are also reflected in the p-values in Tables A.1, A.3 and

A.6. Except for the direct effect estimates, the p-values of indirect effects and the parameter estimates are far off the desired values.

In addition to the base run in Case I, we also ran several simulations to investigate the parameter spaces of the spatial autoregressive parameter and the distance decay parameters in greater detail. Specifically, we modified ρ, α_0, α_1, and α_2. This yields seven additional parameter configurations, summarized in Table A.7. The corresponding simulation results are reported in Tables B.1-B.18.

Changing the ρ parameter from rather strong (0.5 in Case I) to mild spatial dependence (Case II, $\rho = 0.25$), or even to a negative value (Case III, $\rho = -0.25$), does not affect the pattern of results. By contrast, it does change when setting $\rho = 0.01$ (Case IV), which simulates the case where a spatial lag hardly matters and its decay parameter is difficult to identify. This is confirmed in the Monte Carlo simulation results: bias and RMSE of α_0 increase substantially and to unacceptable levels. In case of the negative exponential matrix, the bias in ρ remains more or less the same, while in case of the inverse distance matrix, both the bias and the RMSE of this parameter increase more. Importantly, this appears to have no effect on the bias and RMSE of the direct and indirect effects. This is reassuring news for practitioners mainly interested in direct and indirect effects estimates of the explanatory variables.

In the next two experiments, we investigate the consequences when the distance decay parameter is close to its lower bound, which is 0 for the negative exponential and 1 for the inverse distance matrix. We consider $\alpha_2 = 1$ (Case V) and $\alpha_2 = 0.5$ (Case VI). At $\alpha_2 = 0.5$ the bias reduction becomes smaller. On the other hand, the median bias always remains close to zero. For the inverse distance matrix we see no improvement, but a deterioration, although the median bias remains close to zero, especially for the larger value of $N = 800$. This implies that the distance decay parameter of a negative exponential distance decay matrix can be estimated with greater accuracy than that of an inverse distance matrix. This is because of the property that $\boldsymbol{W}(\alpha_k)$ of the negative exponential matrix is uniformly bounded in both row and column sums in absolute value for $\alpha_k > 0$, whereas this is not the case for the inverse distance matrix. Then this property only holds if $\alpha_k > 1$. This implies that values of $\alpha_k < 1$ need to be handled with care when employing inverse distance matrices.

In Table C.1 we report additional estimation results of our empirical analysis on military expenditures. Instead of one common binary contiguity matrix for all spatial lags in the SD model (as in YE), we use one common row and scalar normalized parameterized negative exponential or inverse distance matrix. This is the first step a practitioner can undertake to determine the best performing spatial weight matrix empirically. The estimated distance decay parameters amount to 2.022 and 2.305 for the row and scalar normalized negative exponential and to 2.113 and 0.766 for the row and scalar normalized inverse distance matrices, and are all significant. However, when comparing the performance of the SD model for these matrices with that of the binary contiguity matrix, measured in terms of the log-likelihood function value (LogL), only one common parameterized spatial weight matrix for all spatial lags means no improvement. Just as for the binary contiguity matrix, no empirical evidence in favor of the SD model is found: at most one spatial lag of the explanatory variables appears to have a coefficient and one indirect effect that is (weakly) significant; and also the Wald tests do not reject.

In Table D.1 and D.2 we report the results of the empirical illustration based on military expenditures obtained by using Matlab (version 2022a) instead of R. The results are almost similar. Only the t-value of the spatially lagged dependent variable is slightly higher.

Additional References

Anselin, L. 1988. *Spatial Econometrics: Methods and Models*. Dordrecht, Netherlands: Kluwer.

Anselin, L., and A. Bera. 1998. "Spatial Dependence in Linear Regression Models with an Introduction to Spatial Econometrics. In *Handbook of Applied Economics Statistics*, edited by A. Ullah, and D. Giles, 237-289. New York: Marcel Dekker.

Arbia, G., A.K. Bera, O. Dogan, and S. Taspınar. 2020. "Testing Impact Measures in Spatial Autoregressive Models." *International Regional Science Review* 43(1-2):40–75.

Arellano, M. 2003. *Panel Data Econometrics*. Oxford: Oxford University Press.

Bramoullé, Y., H. Djebbari, and B. Fortin. 2009. "Identification of Peer Effects Through Social Networks." *Journal of Econometrics* 150(1):41-55.

Burridge, P. 1981. "Testing for a Common Factor in a Spatial Autoregression Model." *Environment and Planning A* 13(7):795-400.

Elhorst, J.P. 2014. *Spatial Econometrics: From Cross-Sectional Data to Spatial Panels*. Heidelberg: Springer.

Elhorst, J.P., M. Gross, and E. Tereanu. 2021. "Spillovers in Space and Time: Where Spatial Econometrics and Global VAR Models Meet. *Journal of Economic Surveys* 35(1):192-226.

Kelejian, H.H., and I.R. Prucha. 2010. "Specification and Estimation of Spatial Autoregressive Models with Autoregressive and Heteroskedastic Disturbances." *Journal of Econometrics* 157(1):53–67.

Lee, L.F. 2004. "Asymptotic Distributions of Quasi-Maximum Likelihood Estimators for Spatial Autoregressive Models." *Econometrica* 72(6):1899–1925.

Lee, L.F., and J. Yu. 2016. “Identification of Spatial Durbin Panel Models.” *Journal of Applied Econometrics* 31:133–162.

Magnus, J.R. 1985. “On Differentiating Eigenvalues and Eigenvectors.” *Econometric Theory* 1(2):179-191.

Ord, J.K. 1981. “Towards a Theory of Spatial Statistics: A Comment.” *Geographical Analysis* 13:91-93.

Qu, X., and L.F. Lee. 2015. “Estimating a Spatial Autoregressive Model with an Endogenous Spatial Weight Matrix.” *Journal of Econometrics* 184:209-232.

Qu, X., X. Wang, and L.F. Lee. 2016. “Instrumental Variables Estimation of a Spatial Dynamic Panel Model with Endogenous Spatial Weights when T is Small.” *The Econometrics Journal* 19(3):261-290.

Settings				Negative exponential: row normalized											
	T	N		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(0.5)**	α_0**(2)**	α_1**(1.5)**	α_2**(3)**	DE_x_1	DE_x_2	IE_x_1	IE_x_2
WFE	**5**	**200**	**Mean**	0.255	0.387	0.006	0.000	0.341				0.501	0.513	0.059	0.002
			Std	0.286	0.300	0.034	0.004	0.311				0.287	0.282	0.134	0.009
TWFE	**5**	**200**	**Mean**	0.491	0.497	0.509	0.508	0.491				0.511	0.510	0.486	0.502
			Std	0.290	0.280	0.288	0.281	0.288				0.283	0.282	0.297	0.296
PWFE	**5**	**200**	**Mean**	0.481	0.494	0.486	0.499	0.461	0.482	0.477	0.504	0.491	0.497	0.477	0.491
			Std	0.292	0.282	0.293	0.285	0.299	0.288	0.297	0.282	0.290	0.287	0.288	0.290
WFE	**5**	**800**	**Mean**	0.180	0.226	0.000	0.000	0.449				0.523	0.504	0.000	0.000
			Std	0.246	0.262	0.000	0.000	0.297				0.289	0.280	0.000	0.000
TWFE	**5**	**800**	**Mean**	0.501	0.483	0.499	0.502	0.511				0.517	0.500	0.480	0.505
			Std	0.288	0.284	0.286	0.293	0.288				0.294	0.283	0.293	0.294
PWFE	**5**	**800**	**Mean**	0.497	0.484	0.498	0.498	0.481	0.463	0.495	0.493	0.500	0.487	0.500	0.495
			Std	0.291	0.287	0.293	0.295	0.291	0.294	0.297	0.294	0.298	0.285	0.294	0.292

Table A.1: Mean and standard deviation of the p-values of the parameters and direct (DE) and indirect (IE) effects of variables x_1 and x_2 in Table 1

Settings				Exponential: scalar normalized									Direct/indirect effects			
	T	N		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(0.5)**	σ^2**(1)**	α_0**(2)**	α_1**(1.5)**	α_2**(3)**	DE_x_1	DE_x_2	IE_x_1	IE_x_2
WFE	**5**	**200**	**Bias**	0.024	-0.006	0.343	-0.188	-0.070	0.072				0.006	0.001	0.502	-0.291
			RMSE	0.030	0.010	0.360	0.193	0.107	0.091				0.019	0.008	0.559	0.303
			Mbias	0.024	-0.007	0.340	-0.187	-0.067	0.068				0.005	0.001	0.491	-0.286
			Mabias	0.024	0.008	0.340	0.187	0.072	0.069				0.013	0.006	0.491	0.286
TWFE	**5**	**200**	**Bias**	0.001	-0.001	0.000	0.002	-0.008	-0.005				0.000	0.000	-0.001	0.003
			RMSE	0.018	0.008	0.072	0.020	0.050	0.053				0.019	0.008	0.159	0.039
			Mbias	0.001	-0.001	0.001	0.003	-0.006	-0.009				0.000	0.000	-0.003	0.005
			Mabias	0.013	0.005	0.048	0.013	0.034	0.038				0.012	0.005	0.108	0.025
PWFE	**5**	**200**	**Bias**	0.001	-0.001	0.008	-0.001	-0.012	-0.008	0.083	0.020	0.136	0.000	0.000	0.006	0.000
			RMSE	0.018	0.008	0.130	0.030	0.055	0.053	0.413	0.203	0.762	0.019	0.008	0.249	0.055
			Mbias	0.001	-0.001	0.005	0.001	-0.011	-0.011	0.033	0.004	0.007	0.001	0.000	-0.009	0.003
			Mabias	0.013	0.005	0.090	0.020	0.037	0.038	0.234	0.130	0.402	0.012	0.005	0.174	0.037
WFE	**5**	**800**	**Bias**	0.015	-0.006	0.376	-0.211	0.004	0.072				0.001	-0.001	0.706	-0.391
			RMSE	0.018	0.007	0.380	0.212	0.041	0.077				0.010	0.004	0.721	0.394
			Mbias	0.016	-0.006	0.375	-0.211	0.006	0.070				0.002	-0.001	0.709	-0.391
			Mabias	0.016	0.006	0.375	0.211	0.029	0.070				0.007	0.003	0.709	0.391
TWFE	**5**	**800**	**Bias**	0.000	0.000	-0.002	0.001	-0.001	-0.001				0.000	0.000	-0.003	0.001
			RMSE	0.009	0.004	0.038	0.011	0.025	0.026				0.010	0.004	0.085	0.022
			Mbias	0.001	0.000	-0.001	0.001	-0.001	-0.003				0.000	0.000	0.001	0.002
			Mabias	0.006	0.003	0.026	0.007	0.017	0.017				0.007	0.003	0.060	0.014
PWFE	**5**	**800**	**Bias**	0.000	0.000	0.000	-0.001	-0.002	-0.002	0.010	0.004	0.019	0.000	0.000	0.000	-0.001
			RMSE	0.009	0.004	0.067	0.017	0.027	0.026	0.168	0.096	0.325	0.010	0.004	0.131	0.032
			Mbias	0.000	0.000	0.001	0.000	0.000	-0.004	-0.010	-0.001	-0.001	0.000	0.000	0.001	0.000
			Mabias	0.006	0.003	0.042	0.011	0.018	0.017	0.112	0.060	0.216	0.007	0.003	0.086	0.021

Table A.2: Simulation results scalar normalized negative exponential matrix (Case I): $\rho = 0.5$, $\alpha_0 = 2$, $\alpha_1 = 1.5$, $\alpha_2 = 3$

	T	N		Negative exponential: scalar normalized											
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(0.5)**	α_0**(2)**	α_1**(1.5)**	α_2**(3)**	DE_x_1	DE_x_2	IE_x_1	IE_x_2
WFE	**5**	**200**	**Mean**	0.305	0.413	0.019	0.001	0.359				0.503	0.511	0.095	0.005
			Std	0.302	0.301	0.072	0.006	0.311				0.290	0.283	0.176	0.018
TWFE	**5**	**200**	**Mean**	0.489	0.496	0.505	0.511	0.494				0.509	0.507	0.472	0.501
			Std	0.287	0.282	0.293	0.287	0.293				0.285	0.283	0.296	0.294
PWFE	**5**	**200**	**Mean**	0.483	0.492	0.495	0.503	0.546	0.585	0.487	0.509	0.493	0.496	0.474	0.497
			Std	0.288	0.282	0.286	0.284	0.269	0.261	0.294	0.283	0.291	0.286	0.287	0.289
WFE	**5**	**800**	**Mean**	0.224	0.255	0.000	0.000	0.449				0.516	0.501	0.000	0.000
			Std	0.265	0.276	0.000	0.000	0.297				0.290	0.282	0.001	0.000
TWFE	**5**	**800**	**Mean**	0.501	0.483	0.501	0.497	0.503				0.514	0.499	0.474	0.499
			Std	0.289	0.284	0.288	0.292	0.282				0.295	0.283	0.292	0.296
PWFE	**5**	**800**	**Mean**	0.499	0.484	0.508	0.491	0.578	0.600	0.498	0.493	0.500	0.487	0.504	0.490
			Std	0.291	0.287	0.288	0.287	0.266	0.252	0.293	0.293	0.298	0.285	0.292	0.289

Table A.3: Mean and standard deviation of the p-values of the parameters and direct (DE) and indirect (IE) effects of variables x_1 and x_2 in Table A.2

	T	N		Inverse distance: row normalized								
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(0.5)**	σ^2**(1)**	α_0**(2)**	α_1**(1.5)**	α_2**(3)**
WFE	**5**	**200**	**Bias**	0.0141	-0.0078	0.1346	-0.4401	-0.4512	0.0898			
			RMSE	0.0232	0.0113	0.4182	0.4700	0.5409	0.1073			
			Mbias	0.0144	-0.0080	0.1396	-0.4117	-0.4276	0.0864			
			Mabias	0.0163	0.0085	0.2875	0.4117	0.4276	0.0864			
TWFE	**5**	**200**	**Bias**	0.0005	-0.0006	-0.0092	0.0023	-0.0201	-0.0047			
			RMSE	0.0175	0.0077	0.1677	0.0232	0.0816	0.0526			
			Mbias	0.0004	-0.0009	-0.0025	0.0026	-0.0166	-0.0083			
			Mabias	0.0119	0.0053	0.1093	0.0153	0.0517	0.0372			
PWFE	**5**	**200**	**Bias**	0.0018	-0.0009	0.0848	-0.0113	-0.0779	-0.0081	0.4430	0.0059	0.0891
			RMSE	0.0179	0.0079	0.4686	0.0653	0.1520	0.0530	1.5931	0.3404	0.8459
			Mbias	0.0016	-0.0013	0.0347	-0.0012	-0.0753	-0.0115	0.2026	-0.0183	-0.0194
			Mabias	0.0125	0.0055	0.2763	0.0374	0.1036	0.0375	0.3500	0.1850	0.3655
WFE	**5**	**800**	**Bias**	0.0018	-0.0056	0.4593	-0.9452	-0.1267	0.0705			
			RMSE	0.0095	0.0069	0.5481	0.9539	0.2888	0.0757			
			Mbias	0.0021	-0.0057	0.4574	-0.9293	-0.0988	0.0690			
			Mabias	0.0066	0.0057	0.4576	0.9293	0.1825	0.0690			
TWFE	**5**	**800**	**Bias**	0.0000	-0.0001	-0.0122	0.0008	-0.0058	-0.0011			
			RMSE	0.0092	0.0041	0.1316	0.0144	0.0561	0.0254			
			Mbias	0.0002	-0.0002	-0.0037	0.0011	-0.0026	-0.0030			
			Mabias	0.0063	0.0029	0.0839	0.0095	0.0372	0.0171			
PWFE	**5**	**800**	**Bias**	0.0002	-0.0002	0.0099	-0.0055	-0.0364	-0.0019	0.1230	0.0183	0.0078
			RMSE	0.0092	0.0041	0.3613	0.0388	0.1125	0.0255	0.3714	0.2003	0.2799
			Mbias	0.0006	-0.0002	-0.0154	-0.0013	-0.0336	-0.0036	0.0635	-0.0017	-0.0086
			Mabias	0.0063	0.0029	0.2460	0.0240	0.0756	0.0171	0.1766	0.1267	0.1865

	T	N		Inverse distance: scalar normalized								
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(0.5)**	σ^2**(1)**	α_0**(2)**	α_1**(1.5)**	α_2**(3)**
WFE	**5**	**200**	**Bias**	0.0127	-0.0063	0.1199	-0.3613	-0.4039	0.0727			
			RMSE	0.0223	0.0102	0.4120	0.3931	0.4976	0.0926			
			Mbias	0.0128	-0.0064	0.1238	-0.3362	-0.3817	0.0696			
			Mabias	0.0158	0.0074	0.2731	0.3362	0.3817	0.0702			
TWFE	**5**	**200**	**Bias**	0.0006	-0.0006	-0.0105	0.0025	-0.0238	-0.0047			
			RMSE	0.0175	0.0077	0.1754	0.0264	0.0892	0.0526			
			Mbias	0.0006	-0.0008	-0.0022	0.0032	-0.0198	-0.0081			
			Mabias	0.0119	0.0053	0.1147	0.0179	0.0566	0.0373			
PWFE	**5**	**200**	**Bias**	0.0011	-0.0007	0.0181	-0.0039	-0.0388	-0.0079	0.0392	0.0299	0.1415
			RMSE	0.0177	0.0078	0.3684	0.0612	0.0999	0.0530	0.3652	0.3144	0.7820
			Mbias	0.0011	-0.0010	0.0020	0.0024	-0.0350	-0.0111	0.0486	0.0068	0.0001
			Mabias	0.0124	0.0055	0.2452	0.0400	0.0645	0.0372	0.2117	0.1900	0.3978
WFE	**5**	**800**	**Bias**	0.0004	-0.0046	0.3785	-0.8645	-0.1042	0.0615			
			RMSE	0.0093	0.0062	0.4802	0.8733	0.2709	0.0673			
			Mbias	0.0007	-0.0047	0.3681	-0.8495	-0.0700	0.0601			
			Mabias	0.0064	0.0048	0.3732	0.8495	0.1679	0.0601			
TWFE	**5**	**800**	**Bias**	0.0000	-0.0001	-0.0130	0.0010	-0.0067	-0.0011			
			RMSE	0.0092	0.0041	0.1406	0.0157	0.0607	0.0254			
			Mbias	0.0002	-0.0001	-0.0038	0.0015	-0.0039	-0.0032			
			Mabias	0.0063	0.0029	0.0893	0.0107	0.0401	0.0171			
PWFE	**5**	**800**	**Bias**	0.0001	-0.0002	0.0073	-0.0046	-0.0131	-0.0018	0.0023	0.0109	0.0139
			RMSE	0.0092	0.0041	0.3237	0.0424	0.0670	0.0255	0.1258	0.1895	0.3068
			Mbias	0.0004	-0.0002	-0.0008	-0.0012	-0.0110	-0.0038	0.0081	-0.0032	-0.0108
			Mabias	0.0062	0.0029	0.2299	0.0266	0.0428	0.0171	0.0782	0.1236	0.2063

Table A.4: Simulation results row and scalar normalized inverse distance matrix (Case I): $\rho = 0.5$, $\alpha_0 = 2$, $\alpha_1 = 1.5$, $\alpha_2 = 3$

Settings				Inverse distance: row normalized				Inverse distance: scalar normalized			
	T	**N**		DE_x_1	DE_x_2	IE_x_1	IE_x_2	DE_x_1	DE_x_2	IE_x_1	IE_x_2
WFE	**5**	**200**	**Bias**	0.0009	0.0001	-0.2218	-0.3957	0.0014	0.0001	-0.1509	-0.3534
			RMSE	0.0183	0.0079	0.5706	0.4202	0.0182	0.0078	0.5454	0.3795
			Mbias	0.0017	0.0001	-0.2607	-0.3844	0.0019	0.0000	-0.1911	-0.3462
			Mabias	0.0123	0.0054	0.4179	0.3844	0.0123	0.0054	0.3945	0.3462
TWFE	**5**	**200**	**Bias**	0.0002	-0.0002	-0.0131	0.0066	0.0002	-0.0002	-0.0166	0.0065
			RMSE	0.0181	0.0078	0.4030	0.0545	0.0180	0.0078	0.3953	0.0556
			Mbias	0.0003	-0.0003	-0.0232	0.0090	0.0005	-0.0003	-0.0311	0.0113
			Mabias	0.0122	0.0054	0.2866	0.0357	0.0122	0.0055	0.2715	0.0361
PWFE	**5**	**200**	**Bias**	0.0003	-0.0002	0.0286	0.0022	0.0003	-0.0002	0.0063	-0.0011
			RMSE	0.0183	0.0078	0.7548	0.1079	0.0181	0.0078	0.7039	0.1125
			Mbias	0.0007	-0.0002	-0.0416	0.0146	0.0007	-0.0003	-0.0457	0.0148
			Mabias	0.0125	0.0055	0.5050	0.0684	0.0124	0.0055	0.4778	0.0756
WFE	**5**	**800**	**Bias**	-0.0015	-0.0018	0.9264	-1.7962	-0.0024	-0.0016	0.8289	-1.6770
			RMSE	0.0095	0.0046	1.5574	2.0526	0.0097	0.0044	1.3745	1.8995
			Mbias	-0.0015	-0.0017	0.6457	-1.5246	-0.0023	-0.0015	0.6096	-1.4567
			Mabias	0.0064	0.0031	0.6825	1.5246	0.0065	0.0030	0.6493	1.4567
TWFE	**5**	**800**	**Bias**	-0.0001	-0.0001	-0.0093	0.0011	-0.0001	-0.0001	-0.0084	0.0012
			RMSE	0.0093	0.0040	0.3278	0.0372	0.0093	0.0040	0.3299	0.0370
			Mbias	0.0001	-0.0001	-0.0175	0.0029	0.0001	-0.0001	-0.0218	0.0031
			Mabias	0.0063	0.0028	0.2153	0.0231	0.0064	0.0028	0.2184	0.0228
PWFE	**5**	**800**	**Bias**	-0.0001	-0.0001	-0.0078	-0.0040	0.0000	-0.0001	0.0245	-0.0079
			RMSE	0.0093	0.0040	0.6964	0.0853	0.0093	0.0040	0.6513	0.0839
			Mbias	0.0001	-0.0001	-0.0376	0.0112	0.0001	-0.0001	-0.0215	0.0007
			Mabias	0.0064	0.0028	0.5097	0.0496	0.0064	0.0028	0.4648	0.0514

Table A.5: Direct (DE) and indirect (IE) effects of variables x_1 and x_2 for Case I using the parameter estimates in Table A.4

Settings				Inverse distance: row normalized											
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(0.5)**	α_0**(2)**	α_1**(1.5)**	α_2**(3)**	DE_x_1	DE_x_2	IE_x_1	IE_x_2
WFE	**5**	**200**	**Mean**	0.4099	0.3638	0.3871	0.0057	0.1014				0.4994	0.5118	0.3406	0.0336
			Std	0.3050	0.2943	0.3159	0.0238	0.2046				0.2876	0.2807	0.3185	0.0667
TWFE	**5**	**200**	**Mean**	0.4909	0.4979	0.5013	0.5074	0.4970				0.4935	0.5057	0.4548	0.5033
			Std	0.2898	0.2803	0.2908	0.2830	0.2880				0.2892	0.2829	0.2977	0.2993
PWFE	**5**	**200**	**Mean**	0.4850	0.4888	0.4794	0.4893	0.4138	0.5033	0.4815	0.4922	0.4879	0.4953	0.4780	0.4715
			Std	0.2917	0.2785	0.2972	0.2979	0.3079	0.2675	0.2989	0.2961	0.2880	0.2852	0.3123	0.3043
WFE	**5**	**800**	**Mean**	0.4929	0.2903	0.1766	0.0000	0.1790				0.4957	0.4706	0.2400	0.0000
			Std	0.2896	0.2974	0.2531	0.0000	0.2673				0.2919	0.2910	0.2829	0.0000
TWFE	**5**	**800**	**Mean**	0.4977	0.4849	0.5082	0.4996	0.5093				0.4987	0.4955	0.4594	0.5002
			Std	0.2940	0.2874	0.2839	0.2924	0.2908				0.2964	0.2857	0.2936	0.2986
PWFE	**5**	**800**	**Mean**	0.4946	0.4825	0.4598	0.4863	0.4565	0.4834	0.4768	0.4903	0.4973	0.4859	0.4537	0.4847
			Std	0.2916	0.2861	0.2929	0.2924	0.3112	0.2849	0.2908	0.2918	0.2935	0.2882	0.3002	0.3008
				Inverse distance: scalar normalized											
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(0.5)**	α_0**(2)**	α_1**(1.5)**	α_2**(3)**	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.4230	0.4043	0.3955	0.0236	0.1364				0.4980	0.5095	0.3567	0.0617
			Std	0.3043	0.2981	0.3124	0.0683	0.2371				0.2876	0.2810	0.3073	0.1085
TWFE	**5**	**200**	**Mean**	0.4915	0.4976	0.4960	0.5099	0.4961				0.4949	0.5030	0.4532	0.4985
			Std	0.2895	0.2817	0.2917	0.2867	0.2890				0.2888	0.2832	0.2959	0.2948
PWFE	**5**	**200**	**Mean**	0.4877	0.4921	0.5245	0.5050	0.5905	0.6342	0.5085	0.5109	0.4896	0.4950	0.5013	0.4899
			Std	0.2882	0.2815	0.2847	0.2928	0.2516	0.2407	0.2829	0.2846	0.2874	0.2846	0.3021	0.2994
WFE	**5**	**800**	**Mean**	0.5025	0.3417	0.2348	0.0000	0.2024				0.4884	0.4766	0.2619	0.0000
			Std	0.2932	0.3083	0.2762	0.0000	0.2829				0.2908	0.2892	0.2834	0.0000
TWFE	**5**	**800**	**Mean**	0.4977	0.4838	0.5083	0.4934	0.5125				0.4982	0.4946	0.4629	0.4956
			Std	0.2939	0.2868	0.2865	0.2902	0.2930				0.2963	0.2861	0.2978	0.2959
PWFE	**5**	**800**	**Mean**	0.4966	0.4830	0.4898	0.4930	0.6501	0.7221	0.4935	0.4877	0.4972	0.4863	0.4764	0.4971
			Std	0.2919	0.2858	0.2852	0.2954	0.2371	0.2035	0.2825	0.2823	0.2943	0.2881	0.2931	0.2931

Table A.6: Case I: Mean and standard deviation of the p-values of the parameters (Table A.4) and the direct (DE) and indirect (IE) effects of variables x_1 and x_2 (Table A.5)

	ρ	α_0	α_1	α_2
Case I	0.5	2	1.5	3
Case II	0.25	2	1.5	3
Case III	-0.25	2	1.5	3
Case IV	0.01	2	1.5	3
Case V	0.5	2	1.5	1
Case VI	0.5	2	1.5	0.5
Case VII	0.5	2	10	3
Case VIII	0.01	2	10	3

Table A.7: Summary of cases

	T	N		Negative exponential: row normalized									Negative exponential: scalar normalized								
				β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.25)	σ^2(1)	α_0(2)	α_1(1.5)	α_2(3)	β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.25)	σ^2(1)	α_0(2)	α_1(1.5)	α_2(3)
WFE	5	200	Bias	0.0352	-0.0078	0.4014	-0.1804	-0.2435	0.0715				0.0301	-0.0065	0.3927	-0.1966	-0.2462	0.0590			
			RMSE	0.0402	0.0112	0.4134	0.1843	0.2628	0.0912				0.0356	0.0103	0.4085	0.2015	0.2709	0.0812			
			Mbias	0.0351	-0.0080	0.4025	-0.1798	-0.2417	0.0683				0.0301	-0.0069	0.3911	-0.1960	-0.2454	0.0560			
			Mabias	0.0351	0.0086	0.4025	0.1798	0.2417	0.0686				0.0303	0.0077	0.3911	0.1960	0.2454	0.0581			
TWFE	5	200	Bias	0.0009	-0.0005	0.0009	0.0016	-0.0059	-0.0053				0.0010	-0.0005	0.0002	0.0017	-0.0079	-0.0053			
			RMSE	0.0185	0.0079	0.0640	0.0177	0.0512	0.0526				0.0183	0.0079	0.0734	0.0198	0.0584	0.0527			
			Mbias	0.0011	-0.0006	0.0020	0.0021	-0.0057	-0.0090				0.0010	-0.0007	0.0015	0.0026	-0.0055	-0.0090			
			Mabias	0.0130	0.0053	0.0420	0.0119	0.0344	0.0368				0.0129	0.0055	0.0491	0.0133	0.0394	0.0368			
PWFE	5	200	Bias	0.0020	-0.0007	0.0105	0.0004	-0.0052	-0.0091	0.9247	0.0065	0.1548	0.0012	-0.0004	0.0048	0.0000	-0.0081	-0.0089	0.3730	0.0242	0.1618
			RMSE	0.0195	0.0081	0.1169	0.0263	0.0788	0.0531	3.1547	0.1803	0.9911	0.0188	0.0080	0.1294	0.0303	0.0634	0.0531	1.3587	0.2004	0.7791
			Mbias	0.0019	-0.0011	0.0043	0.0016	-0.0088	-0.0128	0.1548	-0.0038	0.0476	0.0009	-0.0007	0.0020	0.0013	-0.0070	-0.0117	0.0790	0.0126	0.0449
			Mabias	0.0141	0.0054	0.0806	0.0177	0.0548	0.0375	0.6732	0.1204	0.3806	0.0133	0.0055	0.0882	0.0199	0.0440	0.0380	0.4923	0.1323	0.4070
WFE	5	800	Bias	0.0259	-0.0075	0.4349	-0.2215	-0.1907	0.0695				0.0202	-0.0062	0.4323	-0.2221	-0.1738	0.0619			
			RMSE	0.0279	0.0086	0.4378	0.2228	0.1979	0.0746				0.0225	0.0075	0.4363	0.2235	0.1832	0.0677			
			Mbias	0.0260	-0.0076	0.4340	-0.2221	-0.1891	0.0675				0.0207	-0.0061	0.4324	-0.2223	-0.1710	0.0606			
			Mabias	0.0260	0.0077	0.4340	0.2221	0.1891	0.0675				0.0207	0.0062	0.4324	0.2223	0.1710	0.0606			
TWFE	5	800	Bias	0.0002	-0.0002	0.0008	0.0000	-0.0008	-0.0016				0.0001	-0.0001	-0.0018	0.0007	-0.0013	-0.0012			
			RMSE	0.0097	0.0040	0.0338	0.0100	0.0272	0.0252				0.0095	0.0041	0.0387	0.0109	0.0293	0.0254			
			Mbias	0.0004	-0.0002	-0.0005	0.0002	-0.0005	-0.0028				0.0004	-0.0001	-0.0006	0.0009	-0.0001	-0.0031			
			Mabias	0.0067	0.0026	0.0222	0.0065	0.0174	0.0174				0.0063	0.0029	0.0267	0.0072	0.0201	0.0170			
PWFE	5	800	Bias	0.0004	-0.0002	0.0008	-0.0009	0.0014	-0.0026	0.1030	0.0042	0.0236	0.0001	-0.0001	-0.0009	-0.0004	-0.0004	-0.0021	0.0451	0.0060	0.0253
			RMSE	0.0101	0.0041	0.0576	0.0161	0.0439	0.0253	0.5707	0.0868	0.3192	0.0096	0.0041	0.0668	0.0169	0.0322	0.0255	0.3771	0.0968	0.3309
			Mbias	0.0004	-0.0004	-0.0004	-0.0008	-0.0002	-0.0041	0.0272	0.0029	-0.0054	0.0003	-0.0001	-0.0008	0.0002	0.0000	-0.0039	-0.0208	-0.0007	0.0007
			Mabias	0.0069	0.0027	0.0402	0.0112	0.0294	0.0173	0.3298	0.0628	0.1954	0.0065	0.0029	0.0430	0.0111	0.0208	0.0171	0.2258	0.0610	0.2172

	T	N		Inverse distance: row normalized									Inverse distance: scalar normalized								
				β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.25)	σ^2(1)	α_0(2)	α_1(1.5)	α_2(3)	β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.25)	σ^2(1)	α_0(2)	α_1(1.5)	α_2(3)
WFE	5	200	Bias	0.0199	-0.0081	0.2984	-0.4610	-0.8595	0.0761				0.0175	-0.0065	0.2767	-0.3783	-0.7679	0.0619			
			RMSE	0.0272	0.0115	0.5110	0.4904	0.9481	0.0966				0.0254	0.0104	0.4957	0.4107	0.8621	0.0847			
			Mbias	0.0199	-0.0083	0.3102	-0.4373	-0.8337	0.0725				0.0176	-0.0067	0.2710	-0.3541	-0.7478	0.0587			
			Mabias	0.0207	0.0088	0.3647	0.4373	0.8337	0.0735				0.0186	0.0076	0.3511	0.3541	0.7478	0.0610			
TWFE	5	200	Bias	0.0007	-0.0006	-0.0078	0.0024	-0.0195	-0.0053				0.0007	-0.0006	-0.0093	0.0026	-0.0232	-0.0053			
			RMSE	0.0176	0.0078	0.1711	0.0232	0.0959	0.0526				0.0176	0.0077	0.1787	0.0264	0.1043	0.0526			
			Mbias	0.0007	-0.0009	-0.0015	0.0027	-0.0190	-0.0091				0.0008	-0.0008	0.0018	0.0034	-0.0200	-0.0087			
			Mabias	0.0121	0.0053	0.1135	0.0156	0.0611	0.0370				0.0121	0.0052	0.1209	0.0178	0.0670	0.0371			
PWFE	5	200	Bias	0.0018	-0.0008	0.0635	-0.0066	-0.0665	-0.0091	1.8672	0.0130	0.1290	0.0010	-0.0006	0.0089	-0.0032	-0.0373	-0.0087	0.2969	0.0435	0.1590
			RMSE	0.0181	0.0080	0.4653	0.0605	0.1951	0.0530	4.9330	0.3141	0.9857	0.0178	0.0079	0.3685	0.0611	0.1294	0.0530	1.6143	0.3162	0.8008
			Mbias	0.0016	-0.0012	0.0115	0.0024	-0.0694	-0.0125	0.3975	-0.0068	0.0086	0.0011	-0.0008	-0.0144	0.0034	-0.0311	-0.0119	0.0789	0.0161	0.0172
			Mabias	0.0126	0.0055	0.2599	0.0369	0.1174	0.0376	0.8757	0.1817	0.3698	0.0124	0.0055	0.2374	0.0401	0.0775	0.0375	0.4849	0.1892	0.4039
WFE	5	800	Bias	0.0053	-0.0035	1.0618	-1.0953	-0.4958	0.0588				0.0020	-0.0037	0.6806	-0.8860	-0.4912	0.0564			
			RMSE	0.0108	0.0054	1.0955	1.1036	0.6175	0.0647				0.0096	0.0056	0.7568	0.8959	0.6158	0.0628			
			Mbias	0.0054	-0.0036	1.0614	-1.0815	-0.4649	0.0579				0.0023	-0.0039	0.7026	-0.8728	-0.4602	0.0548			
			Mabias	0.0075	0.0039	1.0614	1.0815	0.4653	0.0579				0.0066	0.0041	0.7026	0.8728	0.4608	0.0548			
TWFE	5	800	Bias	0.0001	-0.0002	-0.0040	0.0001	-0.0057	-0.0016				0.0000	-0.0001	-0.0115	0.0010	-0.0061	-0.0012			
			RMSE	0.0091	0.0039	0.1202	0.0144	0.0628	0.0251				0.0092	0.0041	0.1436	0.0157	0.0694	0.0254			
			Mbias	0.0002	-0.0002	-0.0064	0.0001	-0.0026	-0.0031				0.0002	-0.0002	-0.0032	0.0016	-0.0043	-0.0033			
			Mabias	0.0063	0.0026	0.0829	0.0094	0.0424	0.0173				0.0063	0.0029	0.0929	0.0107	0.0457	0.0171			
PWFE	5	800	Bias	0.0004	-0.0002	0.0004	-0.0049	-0.0177	-0.0026	0.7358	0.0139	0.0233	0.0001	-0.0002	0.0078	-0.0044	-0.0089	-0.0020	0.0017	0.0143	0.0169
			RMSE	0.0092	0.0040	0.2718	0.0401	0.1269	0.0252	2.6918	0.1614	0.3002	0.0092	0.0041	0.3284	0.0424	0.0794	0.0254	0.3482	0.1949	0.3089
			Mbias	0.0005	-0.0003	-0.0182	-0.0021	-0.0290	-0.0042	0.1366	0.0053	0.0019	0.0004	-0.0002	-0.0037	-0.0012	-0.0054	-0.0038	0.0111	-0.0036	-0.0093
			Mabias	0.0063	0.0026	0.1738	0.0258	0.0881	0.0174	0.4089	0.1042	0.1828	0.0063	0.0029	0.2347	0.0264	0.0494	0.0171	0.1645	0.1224	0.2073

Table B.1: Simulation results for Case II: $\rho = 0.25$, $\alpha_0 = 2$, $\alpha_1 = 1.5$, $\alpha_2 = 3$

	T	N		Negative exponential: row normalized									Negative exponential: scalar normalized								
				β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(-0.25)	σ^2(1)	α_0(2)	α_1(1.5)	α_2(3)	β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(-0.25)	σ^2(1)	α_0(2)	α_1(1.5)	α_2(3)
WFE	**5**	**200**	**Bias**	0.0566	-0.0113	0.4367	-0.1901	-0.7372	0.0534				0.0496	-0.0093	0.4347	-0.2074	-0.7642	0.0445			
			RMSE	0.0602	0.0139	0.4503	0.1945	0.7488	0.0795				0.0537	0.0124	0.4529	0.2128	0.7807	0.0732			
			Mbias	0.0565	-0.0115	0.4393	-0.1894	-0.7381	0.0517				0.0499	-0.0097	0.4344	-0.2081	-0.7641	0.0431			
			Mabias	0.0565	0.0115	0.4393	0.1894	0.7381	0.0573				0.0499	0.0098	0.4344	0.2081	0.7641	0.0497			
TWFE	**5**	**200**	**Bias**	0.0007	-0.0004	0.0016	0.0015	-0.0037	-0.0057				0.0010	-0.0005	0.0008	0.0017	-0.0063	-0.0059			
			RMSE	0.0194	0.0081	0.0663	0.0181	0.0574	0.0527				0.0191	0.0080	0.0758	0.0202	0.0666	0.0527			
			Mbias	0.0009	-0.0004	0.0015	0.0023	-0.0033	-0.0083				0.0011	-0.0006	0.0020	0.0025	-0.0059	-0.0094			
			Mabias	0.0136	0.0055	0.0441	0.0120	0.0378	0.0361				0.0134	0.0056	0.0497	0.0134	0.0453	0.0368			
PWFE	**5**	**200**	**Bias**	0.0027	-0.0009	0.0187	-0.0023	-0.0712	-0.0108	0.3684	0.0034	0.0654	0.0018	-0.0007	0.0091	-0.0011	-0.0227	-0.0094	0.0534	0.0107	0.0874
			RMSE	0.0213	0.0083	0.1153	0.0266	0.1682	0.0535	2.5218	0.1722	0.7426	0.0200	0.0082	0.1258	0.0305	0.0786	0.0531	0.7854	0.1888	0.7150
			Mbias	0.0017	-0.0012	0.0142	-0.0012	-0.0457	-0.0133	-0.1817	-0.0030	-0.0145	0.0012	-0.0011	0.0087	-0.0003	-0.0222	-0.0125	-0.0614	-0.0099	-0.0202
			Mabias	0.0151	0.0057	0.0797	0.0179	0.0865	0.0375	0.6703	0.1180	0.3781	0.0145	0.0057	0.0864	0.0201	0.0532	0.0376	0.3978	0.1225	0.3979
WFE	**5**	**800**	**Bias**	0.0415	-0.0093	0.4611	-0.2163	-0.6868	0.0598				0.0371	-0.0088	0.4850	-0.2360	-0.7194	0.0539			
			RMSE	0.0429	0.0103	0.4651	0.2179	0.6907	0.0663				0.0386	0.0097	0.4896	0.2377	0.7244	0.0608			
			Mbias	0.0420	-0.0092	0.4612	-0.2166	-0.6884	0.0595				0.0374	-0.0087	0.4877	-0.2361	-0.7189	0.0534			
			Mabias	0.0420	0.0092	0.4612	0.2166	0.6884	0.0595				0.0374	0.0087	0.4877	0.2361	0.7189	0.0534			
TWFE	**5**	**800**	**Bias**	0.0002	-0.0001	-0.0020	0.0006	-0.0014	-0.0013				0.0001	-0.0001	-0.0019	0.0007	-0.0012	-0.0013			
			RMSE	0.0099	0.0042	0.0370	0.0104	0.0315	0.0253				0.0098	0.0042	0.0402	0.0111	0.0342	0.0253			
			Mbias	0.0007	-0.0001	-0.0017	0.0008	0.0001	-0.0031				0.0005	-0.0001	-0.0008	0.0009	-0.0003	-0.0030			
			Mabias	0.0065	0.0029	0.0260	0.0070	0.0209	0.0169				0.0065	0.0029	0.0278	0.0074	0.0235	0.0170			
PWFE	**5**	**800**	**Bias**	0.0005	-0.0002	0.0022	-0.0011	-0.0161	-0.0025	0.0685	0.0019	0.0013	0.0003	-0.0002	0.0016	-0.0009	-0.0039	-0.0022	0.0371	0.0003	0.0033
			RMSE	0.0105	0.0042	0.0603	0.0156	0.0685	0.0254	0.6735	0.0896	0.3091	0.0101	0.0042	0.0663	0.0171	0.0372	0.0254	0.3373	0.0953	0.3285
			Mbias	0.0008	-0.0002	0.0011	-0.0006	-0.0077	-0.0041	-0.0123	-0.0019	-0.0144	0.0005	-0.0002	0.0025	-0.0005	-0.0030	-0.0038	0.0112	-0.0088	-0.0258
			Mabias	0.0071	0.0030	0.0404	0.0099	0.0397	0.0170	0.3716	0.0578	0.2038	0.0068	0.0029	0.0428	0.0113	0.0252	0.0171	0.2104	0.0603	0.2129

	T	N		Inverse distance: row normalized									Inverse distance: scalar normalized								
				β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(-0.25)	σ^2(1)	α_0(2)	α_1(1.5)	α_2(3)	β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(-0.25)	σ^2(1)	α_0(2)	α_1(1.5)	α_2(3)
WFE	**5**	**200**	**Bias**	0.0323	-0.0099	0.4593	-0.5126	-1.9527	0.0571				0.0281	-0.0079	0.4584	-0.4177	-1.7420	0.0470			
			RMSE	0.0377	0.0130	0.6570	0.5419	2.0311	0.0856				0.0340	0.0114	0.6522	0.4517	1.8271	0.0774			
			Mbias	0.0322	-0.0100	0.4815	-0.4942	-1.9292	0.0558				0.0284	-0.0080	0.4827	-0.3987	-1.7251	0.0444			
			Mabias	0.0324	0.0102	0.5080	0.4942	1.9292	0.0600				0.0287	0.0086	0.5184	0.3987	1.7251	0.0519			
TWFE	**5**	**200**	**Bias**	0.0008	-0.0006	-0.0041	0.0025	-0.0166	-0.0059				0.0009	-0.0006	-0.0059	0.0026	-0.0206	-0.0060			
			RMSE	0.0179	0.0079	0.1739	0.0235	0.1124	0.0527				0.0178	0.0078	0.1819	0.0266	0.1223	0.0527			
			Mbias	0.0010	-0.0008	0.0051	0.0022	-0.0170	-0.0091				0.0011	-0.0007	0.0020	0.0032	-0.0168	-0.0094			
			Mabias	0.0124	0.0054	0.1210	0.0150	0.0728	0.0368				0.0123	0.0054	0.1255	0.0179	0.0801	0.0368			
PWFE	**5**	**200**	**Bias**	0.0039	-0.0013	0.1350	-0.0163	-0.2663	-0.0115	0.4975	-0.0309	0.0251	0.0019	-0.0009	0.0290	-0.0038	-0.0734	-0.0094	0.0337	0.0138	0.1187
			RMSE	0.0190	0.0081	0.4688	0.0669	0.4194	0.0535	3.7403	0.2866	0.8177	0.0182	0.0079	0.3662	0.0608	0.1691	0.0531	1.0957	0.2900	0.7473
			Mbias	0.0034	-0.0016	0.0886	-0.0067	-0.2100	-0.0147	-0.4670	-0.0433	-0.0740	0.0021	-0.0013	0.0162	0.0028	-0.0674	-0.0130	-0.0689	-0.0039	-0.0107
			Mabias	0.0133	0.0055	0.2890	0.0383	0.2212	0.0382	0.7982	0.1832	0.3752	0.0131	0.0055	0.2494	0.0394	0.1064	0.0378	0.3397	0.1806	0.3893
WFE	**5**	**800**	**Bias**	0.0081	-0.0042	1.0386	-1.0267	-2.0253	0.0593				0.0055	-0.0032	0.9690	-0.9387	-1.8309	0.0547			
			RMSE	0.0127	0.0061	1.1313	1.0398	2.1097	0.0665				0.0111	0.0054	1.0676	0.9523	1.9203	0.0621			
			Mbias	0.0088	-0.0042	1.0700	-1.0217	-2.0194	0.0582				0.0062	-0.0033	1.0019	-0.9297	-1.8188	0.0536			
			Mabias	0.0098	0.0045	1.0700	1.0217	2.0194	0.0582				0.0080	0.0038	1.0019	0.9297	1.8188	0.0536			
TWFE	**5**	**800**	**Bias**	0.0001	-0.0001	-0.0097	0.0008	-0.0048	-0.0013				0.0001	-0.0001	-0.0096	0.0010	-0.0052	-0.0013			
			RMSE	0.0092	0.0041	0.1364	0.0146	0.0746	0.0253				0.0092	0.0041	0.1460	0.0158	0.0804	0.0253			
			Mbias	0.0005	-0.0002	-0.0005	0.0013	-0.0020	-0.0033				0.0004	-0.0002	-0.0018	0.0013	-0.0035	-0.0033			
			Mabias	0.0061	0.0029	0.0883	0.0097	0.0496	0.0169				0.0062	0.0029	0.0940	0.0106	0.0548	0.0171			
PWFE	**5**	**800**	**Bias**	0.0005	-0.0003	0.0427	-0.0064	-0.1638	-0.0026	0.3932	0.0034	-0.0078	0.0003	-0.0003	0.0225	-0.0048	-0.0186	-0.0022	0.0204	0.0037	0.0087
			RMSE	0.0093	0.0041	0.3656	0.0387	0.3337	0.0255	2.6566	0.1918	0.2758	0.0092	0.0041	0.3342	0.0424	0.0876	0.0254	0.4632	0.1899	0.3055
			Mbias	0.0008	-0.0002	0.0170	-0.0022	-0.0846	-0.0045	-0.1847	-0.0083	-0.0262	0.0006	-0.0002	0.0067	-0.0013	-0.0164	-0.0039	-0.0104	-0.0119	-0.0180
			Mabias	0.0061	0.0029	0.2477	0.0231	0.1496	0.0177	0.5736	0.1261	0.1834	0.0063	0.0029	0.2367	0.0266	0.0576	0.0174	0.1628	0.1245	0.2047

Table B.2: Simulation results for Case III: $\rho = -0.25$, $\alpha_0 = 2$, $\alpha_1 = 1.5$, $\alpha_2 = 3$

	T	N		β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.01)	σ^2(1)	α_0(2)	α_1(1.5)	α_2(3)	β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.01)	σ^2(1)	α_0(2)	α_1(1.5)	α_2(3)
				Negative exponential: row normalized									**Negative exponential: scalar normalized**								
WFE	5	200	Bias	0.0444	-0.0091	0.4256	-0.1863	-0.4625	0.0623				0.0382	-0.0075	0.4210	-0.2029	-0.4722	0.0519			
			RMSE	0.0487	0.0122	0.4381	0.1904	0.4774	0.0847				0.0431	0.0110	0.4375	0.2080	0.4920	0.0767			
			Mbias	0.0443	-0.0094	0.4274	-0.1853	-0.4625	0.0597				0.0385	-0.0079	0.4210	-0.2025	-0.4719	0.0495			
			Mabias	0.0443	0.0097	0.4274	0.1853	0.4625	0.0609				0.0385	0.0084	0.4210	0.2025	0.4719	0.0552			
TWFE	5	200	Bias	0.0008	-0.0005	0.0011	0.0016	-0.0049	-0.0056				0.0010	-0.0005	0.0004	0.0017	-0.0072	-0.0057			
			RMSE	0.0189	0.0080	0.0651	0.0178	0.0555	0.0526				0.0187	0.0079	0.0747	0.0200	0.0635	0.0526			
			Mbias	0.0009	-0.0006	0.0027	0.0022	-0.0043	-0.0089				0.0012	-0.0006	0.0016	0.0025	-0.0071	-0.0089			
			Mabias	0.0132	0.0054	0.0435	0.0120	0.0376	0.0367				0.0133	0.0055	0.0496	0.0134	0.0431	0.0366			
PWFE	5	200	Bias	0.0020	-0.0007	0.0157	-0.0010	-0.0283	-0.0096	3.0267	0.0012	0.1023	0.0017	-0.0007	0.0075	-0.0005	-0.0191	-0.0088	1.6990	0.0039	0.0964
			RMSE	0.0210	0.0083	0.1165	0.0266	0.1495	0.0531	7.0660	0.1712	0.8550	0.0195	0.0081	0.1125	0.0300	0.1015	0.0530	3.9540	0.1403	0.6696
			Mbias	0.0013	-0.0010	0.0098	0.0004	0.0068	-0.0130	-0.0523	-0.0014	0.0119	0.0021	-0.0009	0.0059	0.0009	-0.0143	-0.0123	0.0209	-0.0045	0.0104
			Mabias	0.0146	0.0054	0.0796	0.0178	0.0785	0.0375	1.5000	0.1193	0.3772	0.0141	0.0056	0.0797	0.0200	0.0661	0.0374	1.4300	0.0953	0.3816
WFE	5	800	Bias	0.0338	-0.0083	0.4695	-0.2282	-0.4129	0.0643				0.0298	-0.0077	0.4780	-0.2499	-0.4310	0.0575			
			RMSE	0.0354	0.0093	0.4725	0.2296	0.4179	0.0699				0.0316	0.0088	0.4817	0.2515	0.4371	0.0636			
			Mbias	0.0337	-0.0084	0.4693	-0.2288	-0.4114	0.0628				0.0301	-0.0079	0.4788	-0.2507	-0.4298	0.0561			
			Mabias	0.0337	0.0084	0.4693	0.2288	0.4114	0.0628				0.0301	0.0079	0.4788	0.2507	0.4298	0.0561			
TWFE	5	800	Bias	0.0002	-0.0002	0.0008	0.0000	-0.0008	-0.0017				0.0001	-0.0002	0.0010	0.0001	-0.0007	-0.0017			
			RMSE	0.0099	0.0040	0.0344	0.0101	0.0301	0.0251				0.0098	0.0040	0.0379	0.0107	0.0328	0.0251			
			Mbias	0.0002	-0.0002	0.0004	0.0002	-0.0001	-0.0030				0.0000	-0.0003	0.0012	0.0000	0.0003	-0.0030			
			Mabias	0.0070	0.0027	0.0225	0.0067	0.0196	0.0176				0.0070	0.0027	0.0252	0.0071	0.0217	0.0176			
PWFE	5	800	Bias	0.0000	-0.0001	0.0009	-0.0011	0.0011	-0.0027	2.9763	0.0070	0.0226	0.0002	-0.0002	0.0020	-0.0012	-0.0002	-0.0025	1.8593	0.0030	0.0143
			RMSE	0.0107	0.0042	0.0588	0.0162	0.0739	0.0252	6.7608	0.0876	0.3174	0.0102	0.0041	0.0559	0.0170	0.0480	0.0252	4.0778	0.0706	0.3085
			Mbias	-0.0002	-0.0003	-0.0019	-0.0010	0.0104	-0.0041	0.0229	0.0071	-0.0052	0.0001	-0.0003	0.0013	-0.0012	0.0058	-0.0040	0.1341	0.0032	-0.0080
			Mabias	0.0074	0.0028	0.0408	0.0111	0.0386	0.0176	1.4509	0.0616	0.1966	0.0069	0.0027	0.0379	0.0119	0.0320	0.0175	1.3867	0.0490	0.1934

	T	N		β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.01)	σ^2(1)	α_0(2)	α_1(1.5)	α_2(3)	β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.01)	σ^2(1)	α_0(2)	α_1(1.5)	α_2(3)
				Inverse distance: row normalized									**Inverse distance: scalar normalized**								
WFE	5	200	Bias	0.0257	-0.0088	0.3960	-0.4856	-1.3482	0.0665				0.0224	-0.0071	0.3811	-0.3972	-1.2034	0.0543			
			RMSE	0.0319	0.0121	0.5926	0.5149	1.4322	0.0904				0.0292	0.0108	0.5791	0.4304	1.2936	0.0804			
			Mbias	0.0256	-0.0090	0.4138	-0.4670	-1.3184	0.0636				0.0227	-0.0073	0.3953	-0.3737	-1.1809	0.0511			
			Mabias	0.0256	0.0092	0.4482	0.4670	1.3184	0.0659				0.0229	0.0080	0.4360	0.3737	1.1809	0.0565			
TWFE	5	200	Bias	0.0007	-0.0006	-0.0060	0.0025	-0.0183	-0.0056				0.0008	-0.0006	-0.0077	0.0027	-0.0221	-0.0057			
			RMSE	0.0177	0.0078	0.1729	0.0233	0.1053	0.0526				0.0177	0.0078	0.1806	0.0265	0.1144	0.0527			
			Mbias	0.0008	-0.0008	0.0033	0.0027	-0.0186	-0.0089				0.0010	-0.0008	0.0011	0.0032	-0.0192	-0.0094			
			Mabias	0.0123	0.0054	0.1171	0.0153	0.0672	0.0371				0.0123	0.0053	0.1235	0.0176	0.0746	0.0367			
PWFE	5	200	Bias	0.0029	-0.0010	0.1245	-0.0136	-0.1695	-0.0102	2.2950	-0.0174	0.0740	0.0016	-0.0008	0.0227	-0.0040	-0.0737	-0.0089	0.9509	0.0184	0.1283
			RMSE	0.0190	0.0081	0.5072	0.0697	0.3869	0.0532	6.3857	0.3090	0.9102	0.0181	0.0079	0.3484	0.0611	0.2016	0.0530	3.0259	0.2656	0.7444
			Mbias	0.0028	-0.0012	0.0527	-0.0017	-0.0593	-0.0138	-0.2146	-0.0249	-0.0289	0.0016	-0.0010	0.0171	0.0017	-0.0531	-0.0126	0.0302	-0.0085	0.0002
			Mabias	0.0130	0.0054	0.2689	0.0378	0.1440	0.0377	1.3633	0.1885	0.3775	0.0129	0.0055	0.2382	0.0395	0.1107	0.0374	1.1632	0.1688	0.3872
WFE	5	800	Bias	0.0075	-0.0032	1.2104	-1.1313	-1.1243	0.0571				0.0069	-0.0031	1.0376	-1.0739	-1.1134	0.0504			
			RMSE	0.0122	0.0053	1.2493	1.1409	1.2189	0.0636				0.0118	0.0052	1.0885	1.0847	1.2160	0.0575			
			Mbias	0.0077	-0.0033	1.2174	-1.1192	-1.1025	0.0556				0.0071	-0.0032	1.0464	-1.0573	-1.0841	0.0482			
			Mabias	0.0087	0.0038	1.2174	1.1192	1.1025	0.0556				0.0083	0.0037	1.0464	1.0573	1.0841	0.0482			
TWFE	5	800	Bias	0.0002	-0.0002	-0.0032	0.0001	-0.0054	-0.0017				0.0002	-0.0002	-0.0037	0.0002	-0.0059	-0.0017			
			RMSE	0.0092	0.0040	0.1210	0.0145	0.0681	0.0251				0.0092	0.0039	0.1323	0.0156	0.0751	0.0251			
			Mbias	0.0001	-0.0003	-0.0062	0.0001	-0.0020	-0.0032				0.0000	-0.0002	-0.0053	0.0001	-0.0033	-0.0032			
			Mabias	0.0064	0.0026	0.0840	0.0095	0.0459	0.0177				0.0063	0.0026	0.0909	0.0103	0.0502	0.0177			
PWFE	5	800	Bias	0.0004	-0.0002	0.0193	-0.0058	-0.0775	-0.0027	2.0525	0.0071	0.0122	0.0003	-0.0003	0.0016	-0.0062	-0.0336	-0.0025	0.7461	0.0111	0.0143
			RMSE	0.0094	0.0040	0.2770	0.0405	0.2966	0.0252	5.8118	0.1576	0.2976	0.0092	0.0040	0.2582	0.0444	0.1617	0.0252	2.6828	0.1538	0.3224
			Mbias	0.0005	-0.0004	-0.0084	-0.0021	0.0054	-0.0043	-0.1380	0.0051	-0.0062	0.0004	-0.0004	-0.0128	-0.0028	-0.0168	-0.0043	0.0251	0.0027	-0.0252
			Mabias	0.0065	0.0027	0.1725	0.0253	0.0766	0.0178	1.2664	0.1011	0.1856	0.0064	0.0026	0.1713	0.0286	0.0746	0.0176	0.7691	0.1010	0.1994

Table B.3: Simulation results for Case IV: $\rho = 0.01$, $\alpha_0 = 2$, $\alpha_1 = 1.5$, $\alpha_2 = 3$

				Negative exponential: row normalized									Negative exponential: scalar normalized								
	T	N		β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.5)	σ^2(1)	α_0(2)	α_1(1.5)	α_2(1)	β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.5)	σ^2(1)	α_0(2)	α_1(1.5)	α_2(1)
WFE	5	200	**Bias**	0.0161	-0.0024	0.3432	0.0284	0.0486	0.0327				0.0143	-0.0019	0.3516	0.0387	0.0422	0.0269			
			RMSE	0.0239	0.0084	0.3549	0.0453	0.0768	0.0615				0.0233	0.0080	0.3655	0.0551	0.0794	0.0608			
			Mbias	0.0159	-0.0025	0.3424	0.0282	0.0499	0.0315				0.0144	-0.0019	0.3504	0.0392	0.0458	0.0234			
			Mabias	0.0173	0.0056	0.3424	0.0321	0.0578	0.0424				0.0162	0.0056	0.3504	0.0415	0.0578	0.0396			
TWFE	5	200	**Bias**	0.0007	-0.0004	-0.0047	-0.0005	-0.0055	-0.0046				0.0009	-0.0004	0.0000	0.0032	-0.0089	-0.0046			
			RMSE	0.0172	0.0080	0.0656	0.0340	0.0434	0.0507				0.0179	0.0077	0.0720	0.0373	0.0493	0.0528			
			Mbias	0.0009	-0.0003	-0.0055	-0.0003	-0.0034	-0.0063				0.0013	-0.0005	0.0006	0.0027	-0.0070	-0.0089			
			Mabias	0.0117	0.0055	0.0445	0.0217	0.0290	0.0335				0.0126	0.0054	0.0480	0.0246	0.0332	0.0375			
PWFE	5	200	**Bias**	0.0021	-0.0006	0.0142	-0.0112	-0.0198	-0.0078	0.2024	0.0012	0.0173	0.0013	-0.0005	0.0048	-0.0036	-0.0154	-0.0080	0.0964	0.0235	0.0736
			RMSE	0.0180	0.0080	0.1253	0.0639	0.0682	0.0514	0.8152	0.1877	0.2620	0.0183	0.0077	0.1287	0.0669	0.0552	0.0534	0.4106	0.1995	0.5300
			Mbias	0.0020	-0.0006	0.0103	-0.0055	-0.0166	-0.0088	0.0984	-0.0192	-0.0192	0.0012	-0.0006	0.0012	0.0006	-0.0132	-0.0109	0.0447	0.0052	0.0019
			Mabias	0.0122	0.0056	0.0867	0.0404	0.0449	0.0339	0.3020	0.1248	0.1578	0.0133	0.0053	0.0876	0.0447	0.0375	0.0383	0.2362	0.1247	0.1752
WFE	5	800	**Bias**	0.0142	-0.0010	0.3273	0.0216	0.0599	0.0377				0.0104	-0.0002	0.3597	0.0304	0.0907	0.0310			
			RMSE	0.0171	0.0040	0.3307	0.0304	0.0684	0.0461				0.0141	0.0041	0.3635	0.0379	0.0974	0.0407			
			Mbias	0.0140	-0.0010	0.3285	0.0208	0.0610	0.0373				0.0108	-0.0002	0.3590	0.0311	0.0929	0.0295			
			Mabias	0.0141	0.0026	0.3285	0.0222	0.0610	0.0374				0.0112	0.0028	0.3590	0.0312	0.0929	0.0298			
TWFE	5	800	**Bias**	0.0000	0.0002	-0.0033	-0.0004	-0.0022	-0.0017				0.0001	-0.0001	-0.0019	0.0009	-0.0016	-0.0011			
			RMSE	0.0093	0.0038	0.0346	0.0204	0.0230	0.0256				0.0094	0.0040	0.0379	0.0223	0.0244	0.0256			
			Mbias	-0.0001	0.0002	-0.0031	-0.0007	-0.0030	-0.0020				0.0005	-0.0002	-0.0005	0.0012	-0.0009	-0.0031			
			Mabias	0.0063	0.0025	0.0226	0.0128	0.0157	0.0179				0.0063	0.0028	0.0260	0.0149	0.0166	0.0171			
PWFE	5	800	**Bias**	0.0004	0.0001	0.0011	-0.0039	-0.0058	-0.0025	0.0400	0.0001	0.0038	0.0002	-0.0001	-0.0011	-0.0028	-0.0026	-0.0019	0.0139	0.0056	0.0044
			RMSE	0.0093	0.0038	0.0610	0.0355	0.0344	0.0259	0.2283	0.0835	0.1267	0.0094	0.0040	0.0661	0.0381	0.0272	0.0257	0.1638	0.0939	0.1322
			Mbias	0.0001	0.0001	-0.0006	-0.0021	-0.0047	-0.0024	0.0224	-0.0041	-0.0014	0.0004	-0.0001	0.0001	-0.0011	-0.0017	-0.0036	0.0015	-0.0011	-0.0035
			Mabias	0.0063	0.0025	0.0397	0.0230	0.0235	0.0178	0.1420	0.0566	0.0782	0.0064	0.0028	0.0427	0.0256	0.0180	0.0171	0.1117	0.0613	0.0878

				Inverse distance: row normalized									Inverse distance: scalar normalized								
	T	N		β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.5)	σ^2(1)	α_0(2)	α_1(1.5)	α_2(1)	β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.5)	σ^2(1)	α_0(2)	α_1(1.5)	α_2(1)
WFE	5	200	**Bias**	0.0068	0.0005	0.5730	0.1215	0.0885	0.0117				0.0068	0.0005	0.5672	0.1186	0.0614	0.0092			
			RMSE	0.0183	0.0081	0.6477	0.1464	0.1407	0.0525				0.0191	0.0077	0.6399	0.1427	0.1301	0.0541			
			Mbias	0.0066	0.0004	0.5738	0.1240	0.1033	0.0098				0.0066	0.0004	0.5712	0.1207	0.0716	0.0071			
			Mabias	0.0125	0.0055	0.5738	0.1244	0.1149	0.0364				0.0132	0.0053	0.5712	0.1210	0.0949	0.0361			
TWFE	5	200	**Bias**	0.0004	-0.0004	-0.0245	-0.0052	-0.0202	-0.0045				0.0006	-0.0004	-0.0087	0.0020	-0.0238	-0.0047			
			RMSE	0.0166	0.0079	0.1742	0.0782	0.0789	0.0505				0.0175	0.0077	0.1742	0.0777	0.0828	0.0526			
			Mbias	0.0005	-0.0004	-0.0196	-0.0016	-0.0147	-0.0070				0.0006	-0.0005	-0.0064	0.0009	-0.0182	-0.0084			
			Mabias	0.0118	0.0054	0.1162	0.0511	0.0534	0.0337				0.0120	0.0054	0.1152	0.0509	0.0515	0.0376			
PWFE	5	200	**Bias**	0.0021	-0.0005	0.1018	-0.0263	-0.1041	-0.0079	0.7775	0.0335	0.1044	0.0012	-0.0003	0.0520	0.0005	-0.0440	-0.0077	0.0459	0.0242	0.0702
			RMSE	0.0175	0.0080	0.5891	0.1315	0.1689	0.0510	2.4137	0.4828	1.4794	0.0177	0.0077	0.4383	0.1042	0.0961	0.0530	0.3547	0.3374	0.7799
			Mbias	0.0023	-0.0006	0.0229	-0.0227	-0.1002	-0.0101	0.2757	-0.0184	-0.0827	0.0014	-0.0003	-0.0072	0.0009	-0.0420	-0.0103	0.0496	-0.0012	-0.0437
			Mabias	0.0119	0.0055	0.3501	0.0843	0.1172	0.0343	0.3801	0.2333	0.2716	0.0122	0.0054	0.2907	0.0714	0.0628	0.0375	0.1969	0.2168	0.2619
WFE	5	800	**Bias**	0.0024	0.0006	0.5738	0.1659	0.1884	0.0116				0.0029	0.0004	0.5665	0.1344	0.1673	0.0107			
			RMSE	0.0093	0.0039	0.6141	0.1924	0.2067	0.0283				0.0097	0.0040	0.6173	0.1610	0.1901	0.0278			
			Mbias	0.0024	0.0006	0.5739	0.1646	0.1959	0.0111				0.0031	0.0005	0.5653	0.1336	0.1814	0.0089			
			Mabias	0.0061	0.0025	0.5739	0.1647	0.1959	0.0187				0.0066	0.0029	0.5653	0.1336	0.1815	0.0175			
TWFE	5	800	**Bias**	-0.0001	0.0002	-0.0207	0.0055	-0.0103	-0.0018				0.0001	-0.0001	-0.0150	0.0039	-0.0092	-0.0011			
			RMSE	0.0090	0.0038	0.1246	0.0971	0.0561	0.0255				0.0092	0.0040	0.1387	0.0901	0.0599	0.0254			
			Mbias	-0.0001	0.0002	-0.0144	0.0059	-0.0075	-0.0024				0.0003	-0.0001	-0.0057	0.0027	-0.0060	-0.0031			
			Mabias	0.0059	0.0024	0.0832	0.0662	0.0385	0.0176				0.0062	0.0028	0.0918	0.0604	0.0390	0.0171			
PWFE	5	800	**Bias**	0.0002	0.0002	0.0006	-0.0116	-0.0556	-0.0026	0.1675	0.0088	0.2784	0.0002	-0.0001	0.0044	-0.0042	-0.0205	-0.0017	0.0177	0.0148	0.1169
			RMSE	0.0089	0.0038	0.2859	0.1702	0.1202	0.0256	0.3829	0.1659	1.7678	0.0092	0.0040	0.3269	0.1228	0.0659	0.0254	0.1214	0.1901	0.9220
			Mbias	0.0001	0.0002	-0.0171	0.0014	-0.0577	-0.0033	0.1106	0.0136	-0.0164	0.0005	-0.0001	-0.0086	-0.0020	-0.0184	-0.0037	0.0225	-0.0002	-0.0250
			Mabias	0.0059	0.0024	0.1784	0.1116	0.0856	0.0178	0.1865	0.1078	0.2627	0.0062	0.0027	0.2316	0.0833	0.0413	0.0171	0.0827	0.1254	0.2154

Table B.4: Simulation results for Case V: $\rho = 0.5$, $\alpha_0 = 2$, $\alpha_1 = 1.5$, $\alpha_2 = 1$

	T	N		$\beta_1(-1)$	$\beta_2(0.2)$	$\gamma_1(1.5)$	$\gamma_2(-0.3)$	$\rho(0.5)$	$\sigma^2(1)$	$\alpha_0(2)$	$\alpha_1(1.5)$	$\alpha_2(0.5)$	$\beta_1(-1)$	$\beta_2(0.2)$	$\gamma_1(1.5)$	$\gamma_2(-0.3)$	$\rho(0.5)$	$\sigma^2(1)$	$\alpha_0(2)$	$\alpha_1(1.5)$	$\alpha_2(0.5)$
				Negative exponential: row normalized									**Negative exponential: scalar normalized**								
WFE	**5**	**200**	**Bias**	0.0127	-0.0002	0.3491	0.1690	0.0782	0.0380				0.0114	0.0002	0.3631	0.1783	0.0787	0.0297			
			RMSE	0.0224	0.0077	0.3594	0.1722	0.0972	0.0673				0.0215	0.0077	0.3758	0.1825	0.1017	0.0624			
			Mbias	0.0130	-0.0004	0.3487	0.1688	0.0826	0.0351				0.0114	0.0001	0.3611	0.1782	0.0835	0.0265			
			Mabias	0.0154	0.0054	0.3487	0.1688	0.0833	0.0441				0.0152	0.0054	0.3611	0.1782	0.0851	0.0417			
TWFE	**5**	**200**	**Bias**	0.0009	-0.0003	0.0005	0.0045	-0.0069	-0.0046				0.0009	-0.0003	0.0001	0.0048	-0.0087	-0.0046			
			RMSE	0.0180	0.0077	0.0629	0.0525	0.0419	0.0528				0.0179	0.0077	0.0717	0.0617	0.0474	0.0528			
			Mbias	0.0010	-0.0004	0.0016	0.0024	-0.0051	-0.0084				0.0008	-0.0003	0.0010	0.0018	-0.0071	-0.0086			
			Mabias	0.0126	0.0054	0.0414	0.0339	0.0277	0.0376				0.0127	0.0054	0.0472	0.0396	0.0312	0.0377			
PWFE	**5**	**200**	**Bias**	0.0020	-0.0004	0.0117	-0.0101	-0.0237	-0.0077	0.2021	0.0119	0.0276	0.0011	-0.0003	0.0030	0.0000	-0.0158	-0.0079	0.0945	0.0252	0.0694
			RMSE	0.0187	0.0077	0.1188	0.0894	0.0679	0.0534	0.5384	0.1774	0.3286	0.0182	0.0077	0.1287	0.0867	0.0536	0.0533	0.3798	0.1967	0.5238
			Mbias	0.0022	-0.0005	0.0046	-0.0067	-0.0216	-0.0104	0.1219	-0.0015	-0.0222	0.0016	-0.0004	-0.0007	-0.0007	-0.0139	-0.0109	0.0584	0.0079	-0.0122
			Mabias	0.0135	0.0054	0.0802	0.0575	0.0438	0.0380	0.2852	0.1168	0.1114	0.0132	0.0054	0.0870	0.0578	0.0357	0.0381	0.2162	0.1216	0.1023
WFE	**5**	**800**	**Bias**	0.0130	0.0007	0.3110	0.1741	0.0737	0.0478				0.0109	0.0008	0.3419	0.1837	0.0928	0.0395			
			RMSE	0.0162	0.0040	0.3143	0.1755	0.0809	0.0546				0.0145	0.0040	0.3454	0.1853	0.0996	0.0474			
			Mbias	0.0129	0.0006	0.3091	0.1733	0.0752	0.0465				0.0110	0.0008	0.3419	0.1824	0.0950	0.0383			
			Mabias	0.0129	0.0027	0.3091	0.1733	0.0752	0.0465				0.0112	0.0026	0.3419	0.1824	0.0950	0.0383			
TWFE	**5**	**800**	**Bias**	0.0002	-0.0002	0.0008	-0.0011	-0.0012	-0.0015				0.0002	-0.0002	0.0009	-0.0014	-0.0013	-0.0016			
			RMSE	0.0095	0.0038	0.0332	0.0366	0.0220	0.0253				0.0094	0.0038	0.0364	0.0419	0.0239	0.0253			
			Mbias	0.0004	-0.0002	0.0001	-0.0010	-0.0004	-0.0027				0.0004	-0.0002	0.0010	-0.0027	0.0001	-0.0027			
			Mabias	0.0065	0.0026	0.0225	0.0246	0.0142	0.0173				0.0064	0.0026	0.0243	0.0293	0.0154	0.0172			
PWFE	**5**	**800**	**Bias**	0.0004	-0.0002	0.0026	-0.0133	-0.0042	-0.0024	0.0387	0.0041	0.0007	0.0002	-0.0002	0.0003	-0.0120	-0.0027	-0.0024	0.0191	0.0076	0.0066
			RMSE	0.0097	0.0038	0.0589	0.0676	0.0338	0.0256	0.2208	0.0848	0.1069	0.0096	0.0038	0.0611	0.0720	0.0264	0.0256	0.1500	0.0889	0.1182
			Mbias	0.0005	-0.0001	0.0029	-0.0104	-0.0037	-0.0044	0.0187	0.0026	-0.0072	0.0003	-0.0002	-0.0007	-0.0072	-0.0021	-0.0044	0.0145	0.0050	-0.0074
			Mabias	0.0066	0.0026	0.0415	0.0428	0.0227	0.0174	0.1376	0.0608	0.0695	0.0065	0.0026	0.0413	0.0444	0.0175	0.0174	0.0999	0.0633	0.0743

	T	N		$\beta_1(-1)$	$\beta_2(0.2)$	$\gamma_1(1.5)$	$\gamma_2(-0.3)$	$\rho(0.5)$	$\sigma^2(1)$	$\alpha_0(2)$	$\alpha_1(1.5)$	$\alpha_2(0.5)$	$\beta_1(-1)$	$\beta_2(0.2)$	$\gamma_1(1.5)$	$\gamma_2(-0.3)$	$\rho(0.5)$	$\sigma^2(1)$	$\alpha_0(2)$	$\alpha_1(1.5)$	$\alpha_2(0.5)$
				Inverse distance: row normalized									**Inverse distance: scalar normalized**								
WFE	**5**	**200**	**Bias**	0.0065	0.0013	0.6937	0.2154	0.1616	0.0139				0.0061	0.0013	0.6491	0.1972	0.1394	0.0112			
			RMSE	0.0190	0.0078	0.7480	0.2260	0.1867	0.0553				0.0189	0.0078	0.7083	0.2101	0.1705	0.0545			
			Mbias	0.0064	0.0011	0.6843	0.2187	0.1718	0.0111				0.0061	0.0012	0.6496	0.1995	0.1495	0.0086			
			Mabias	0.0131	0.0053	0.6843	0.2187	0.1726	0.0360				0.0130	0.0053	0.6496	0.1995	0.1511	0.0361			
WFE	**5**	**200**	**Bias**	0.0005	-0.0003	-0.0119	-0.0042	-0.0219	-0.0046				0.0006	-0.0003	-0.0118	-0.0058	-0.0251	-0.0046			
			RMSE	0.0175	0.0077	0.1694	0.0888	0.0782	0.0526				0.0175	0.0077	0.1773	0.0920	0.0837	0.0526			
			Mbias	0.0004	-0.0003	-0.0076	0.0008	-0.0186	-0.0078				0.0007	-0.0004	-0.0078	0.0005	-0.0191	-0.0078			
			Mabias	0.0118	0.0054	0.1103	0.0585	0.0489	0.0376				0.0120	0.0054	0.1164	0.0607	0.0530	0.0373			
WFE	**5**	**200**	**Bias**	0.0018	-0.0004	0.0944	-0.0211	-0.0985	-0.0080	0.5470	0.0344	0.0432	0.0010	-0.0002	0.0496	-0.0075	-0.0442	-0.0077	0.0784	0.0347	0.0370
			RMSE	0.0178	0.0077	0.5563	0.1259	0.1607	0.0530	1.4688	0.3866	1.3426	0.0176	0.0077	0.4418	0.1085	0.0965	0.0530	0.3275	0.3390	0.6663
			Mbias	0.0017	-0.0004	-0.0050	-0.0189	-0.1009	-0.0114	0.3080	0.0023	-0.1881	0.0014	-0.0003	-0.0082	-0.0054	-0.0418	-0.0110	0.0650	0.0113	-0.0502
			Mabias	0.0122	0.0054	0.3179	0.0775	0.1157	0.0377	0.3808	0.2097	0.4000	0.0122	0.0054	0.2904	0.0734	0.0627	0.0374	0.1879	0.2142	0.3730
WFE	**5**	**800**	**Bias**	0.0028	0.0007	0.5987	0.3281	0.2093	0.0134				0.0025	0.0006	0.5600	0.3022	0.1951	0.0112			
			RMSE	0.0095	0.0039	0.6331	0.3425	0.2254	0.0289				0.0094	0.0039	0.5990	0.3181	0.2134	0.0278			
			Mbias	0.0029	0.0007	0.6019	0.3271	0.2173	0.0121				0.0025	0.0005	0.5576	0.3008	0.2057	0.0097			
			Mabias	0.0066	0.0026	0.6019	0.3271	0.2173	0.0191				0.0066	0.0026	0.5576	0.3008	0.2057	0.0179			
TWFE	**5**	**800**	**Bias**	0.0002	-0.0002	-0.0077	-0.0036	-0.0098	-0.0014				0.0002	-0.0002	-0.0094	-0.0044	-0.0111	-0.0014			
			RMSE	0.0091	0.0038	0.1167	0.1551	0.0516	0.0251				0.0091	0.0038	0.1267	0.1545	0.0563	0.0252			
			Mbias	0.0003	-0.0002	-0.0093	-0.0036	-0.0064	-0.0031				0.0002	-0.0002	-0.0094	-0.0032	-0.0081	-0.0030			
			Mabias	0.0062	0.0026	0.0794	0.1038	0.0336	0.0174				0.0063	0.0026	0.0893	0.1068	0.0385	0.0174			
PWFE	**5**	**800**	**Bias**	0.0005	-0.0002	0.0018	-0.0229	-0.0571	-0.0024	0.1819	0.0214	0.2154	0.0002	-0.0002	-0.0130	-0.0092	-0.0209	-0.0023	0.0357	0.0256	0.1679
			RMSE	0.0092	0.0039	0.2814	0.1973	0.1150	0.0252	0.3681	0.1635	1.6751	0.0091	0.0038	0.2699	0.1759	0.0621	0.0253	0.1217	0.1685	0.9381
			Mbias	0.0005	-0.0002	-0.0224	-0.0143	-0.0575	-0.0042	0.1365	0.0137	-0.0547	0.0002	-0.0002	-0.0262	-0.0087	-0.0179	-0.0039	0.0353	0.0163	0.0095
			Mabias	0.0063	0.0026	0.1872	0.1328	0.0777	0.0172	0.1975	0.1068	0.3457	0.0063	0.0026	0.1828	0.1205	0.0404	0.0173	0.0792	0.1077	0.3043

Table B.5: Simulation results for Case VI: $\rho = 0.5$, $\alpha_0 = 2$, $\alpha_1 = 1.5$, $\alpha_2 = 0.5$

	T	N		Negative exponential: row normalized									Negative exponential: scalar normalized								
				β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.5)	σ^2(1)	α_0(2)	α_1(10)	α_2(3)	β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.5)	σ^2(1)	α_0(2)	α_1(10)	α_2(3)
WFE	5	200	Bias	0.1457	-0.0232	1.5329	-0.2129	-0.7391	0.7008				0.1342	-0.0200	1.6631	-0.2219	-0.8034	0.6237			
			RMSE	0.1480	0.0250	1.5413	0.2228	0.7516	0.7075				0.1367	0.0220	1.6731	0.2340	0.8196	0.6307			
			Mbias	0.1460	-0.0232	1.5313	-0.2126	-0.7343	0.7006				0.1353	-0.0198	1.6594	-0.2239	-0.7952	0.6201			
			Mabias	0.1460	0.0232	1.5313	0.2126	0.7343	0.7006				0.1353	0.0198	1.6594	0.2239	0.7952	0.6201			
TWFE	5	200	Bias	0.0008	-0.0004	-0.0036	0.0001	-0.0038	-0.0048				0.0011	-0.0005	-0.0047	0.0003	-0.0059	-0.0046			
			RMSE	0.0193	0.0082	0.0358	0.0182	0.0444	0.0507				0.0191	0.0081	0.0384	0.0201	0.0505	0.0507			
			Mbias	0.0008	-0.0004	-0.0031	-0.0001	-0.0017	-0.0060				0.0016	-0.0004	-0.0038	-0.0005	-0.0038	-0.0057			
			Mabias	0.0131	0.0055	0.0246	0.0121	0.0302	0.0339				0.0130	0.0055	0.0261	0.0135	0.0343	0.0337			
PWFE	5	200	Bias	0.0025	-0.0006	0.0028	-0.0033	-0.0058	-0.0077	0.0814	2.4753	0.0384	0.0027	-0.0006	0.0027	-0.0043	-0.0069	-0.0072	-0.0070	0.8482	0.0412
			RMSE	0.0209	0.0082	0.0485	0.0282	0.0637	0.0510	0.5652	5.7870	0.6058	0.0201	0.0082	0.0470	0.0324	0.0558	0.0509	0.2957	3.3829	0.6853
			Mbias	0.0024	-0.0005	0.0037	-0.0021	-0.0029	-0.0089	-0.0055	0.1893	-0.0379	0.0026	-0.0005	0.0021	-0.0038	-0.0051	-0.0087	-0.0267	0.0575	-0.0438
			Mabias	0.0137	0.0056	0.0325	0.0180	0.0446	0.0337	0.3072	2.7060	0.3691	0.0135	0.0057	0.0323	0.0208	0.0376	0.0342	0.1867	2.7792	0.4100
WFE	5	800	Bias	0.1435	-0.0197	1.5524	-0.2676	-0.7046	0.6851				0.1357	-0.0189	1.6726	-0.2913	-0.7654	0.6426			
			RMSE	0.1441	0.0202	1.5549	0.2702	0.7089	0.6866				0.1363	0.0194	1.6755	0.2943	0.7706	0.6442			
			Mbias	0.1432	-0.0197	1.5510	-0.2687	-0.7036	0.6833				0.1358	-0.0188	1.6723	-0.2926	-0.7660	0.6413			
			Mabias	0.1432	0.0197	1.5510	0.2687	0.7036	0.6833				0.1358	0.0188	1.6723	0.2926	0.7660	0.6413			
TWFE	5	800	Bias	0.0001	0.0001	-0.0019	-0.0001	-0.0017	-0.0018				0.0002	0.0001	-0.0021	-0.0001	-0.0021	-0.0018			
			RMSE	0.0102	0.0039	0.0183	0.0104	0.0234	0.0256				0.0101	0.0039	0.0192	0.0108	0.0256	0.0256			
			Mbias	0.0002	0.0002	-0.0012	-0.0001	-0.0022	-0.0021				0.0003	0.0002	-0.0014	0.0001	-0.0025	-0.0022			
			Mabias	0.0067	0.0026	0.0124	0.0069	0.0167	0.0180				0.0068	0.0026	0.0129	0.0072	0.0178	0.0178			
PWFE	5	800	Bias	0.0003	0.0001	-0.0007	-0.0011	-0.0022	-0.0026	0.0256	1.6319	0.0170	0.0004	0.0001	-0.0009	-0.0012	-0.0022	-0.0025	0.0017	0.8025	0.0141
			RMSE	0.0110	0.0039	0.0246	0.0155	0.0342	0.0258	0.2433	4.2299	0.2991	0.0105	0.0039	0.0234	0.0167	0.0288	0.0257	0.1297	2.5923	0.3105
			Mbias	0.0000	0.0002	-0.0005	-0.0002	-0.0027	-0.0027	0.0103	0.1010	-0.0077	0.0003	0.0001	-0.0005	-0.0005	-0.0013	-0.0028	-0.0008	0.0099	-0.0143
			Mabias	0.0079	0.0026	0.0168	0.0104	0.0238	0.0181	0.1585	1.4390	0.1901	0.0075	0.0026	0.0159	0.0113	0.0190	0.0180	0.0858	1.4201	0.1993

	T	N		Inverse distance: row normalized									Inverse distance scalar normalization								
				β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.5)	σ^2(1)	α_0(2)	α_1(10)	α_2(3)	β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.5)	σ^2(1)	α_0(2)	α_1(10)	α_2(3)
WFE	5	200	Bias	0.1752	-0.0242	7.2181	-0.4224	-2.6522	1.0537				0.1613	-0.0206	6.8883	-0.3427	-2.6303	0.9028			
			RMSE	0.1809	0.0266	7.2921	0.5844	2.8198	1.0748				0.1668	0.0230	6.9727	0.5242	2.7932	0.9226			
			Mbias	0.1746	-0.0239	7.1481	-0.3895	-2.6158	1.0478				0.1605	-0.0201	6.8338	-0.3215	-2.5753	0.9018			
			Mabias	0.1746	0.0239	7.1481	0.4032	2.6158	1.0478				0.1605	0.0201	6.8338	0.3520	2.5753	0.9018			
TWFE	5	200	Bias	0.0013	-0.0005	-0.0042	0.0002	-0.0113	-0.0049				0.0016	-0.0005	-0.0053	0.0003	-0.0153	-0.0048			
			RMSE	0.0190	0.0081	0.0360	0.0242	0.0810	0.0504				0.0187	0.0081	0.0383	0.0271	0.0875	0.0504			
			Mbias	0.0016	-0.0004	-0.0040	0.0001	-0.0061	-0.0066				0.0020	-0.0004	-0.0052	-0.0008	-0.0102	-0.0067			
			Mabias	0.0128	0.0054	0.0249	0.0161	0.0544	0.0339				0.0129	0.0055	0.0260	0.0178	0.0578	0.0341			
PWFE	5	200	Bias	0.0018	-0.0006	0.0061	-0.0142	-0.0463	-0.0084	0.3317	1.5625	0.0370	0.0034	-0.0007	0.0022	-0.0096	-0.0235	-0.0076	-0.0139	0.5795	0.0645
			RMSE	0.0213	0.0081	0.0502	0.0638	0.1336	0.0507	1.4353	4.4605	0.5998	0.0205	0.0081	0.0519	0.0607	0.0968	0.0507	0.2899	2.7348	0.6513
			Mbias	0.0016	-0.0005	0.0051	-0.0081	-0.0362	-0.0096	0.0851	-0.0080	-0.0549	0.0032	-0.0006	0.0004	-0.0073	-0.0153	-0.0093	0.0081	-0.1221	-0.0453
			Mabias	0.0145	0.0055	0.0344	0.0384	0.0922	0.0342	0.2988	1.7265	0.3481	0.0140	0.0056	0.0341	0.0400	0.0659	0.0338	0.1711	1.7958	0.3639
WFE	5	800	Bias	0.1446	-0.0144	10.4701	-1.7657	-4.1500	1.1636				0.1299	-0.0137	10.2618	-1.6937	-4.1029	1.0752			
			RMSE	0.1475	0.0156	10.5594	1.8323	4.3777	1.1710				0.1328	0.0149	10.3424	1.7669	4.3362	1.0819			
			Mbias	0.1442	-0.0143	10.3971	-1.7425	-4.1109	1.1683				0.1283	-0.0137	10.2267	-1.6735	-4.0196	1.0768			
			Mabias	0.1442	0.0143	10.3971	1.7425	4.1109	1.1683				0.1283	0.0137	10.2267	1.6735	4.0196	1.0768			
TWFE	5	800	Bias	0.0003	0.0001	-0.0020	-0.0001	-0.0054	-0.0019				0.0003	0.0001	-0.0023	0.0000	-0.0065	-0.0019			
			RMSE	0.0102	0.0039	0.0186	0.0147	0.0552	0.0255				0.0102	0.0038	0.0196	0.0157	0.0605	0.0255			
			Mbias	0.0003	0.0001	-0.0012	-0.0002	-0.0050	-0.0026				0.0005	0.0002	-0.0015	0.0003	-0.0048	-0.0027			
			Mabias	0.0069	0.0025	0.0124	0.0096	0.0363	0.0174				0.0068	0.0025	0.0129	0.0104	0.0410	0.0175			
PWFE	5	800	Bias	0.0000	0.0001	0.0002	-0.0042	-0.0180	-0.0028	0.0819	0.5029	0.0222	0.0006	0.0001	-0.0013	-0.0054	-0.0070	-0.0026	-0.0028	0.3211	0.0179
			RMSE	0.0113	0.0039	0.0253	0.0374	0.1048	0.0256	0.3645	2.0831	0.2883	0.0106	0.0039	0.0261	0.0426	0.0654	0.0256	0.1011	1.6400	0.3163
			Mbias	-0.0004	0.0002	0.0000	-0.0014	-0.0183	-0.0036	0.0305	0.0286	0.0064	0.0005	0.0000	-0.0008	-0.0017	-0.0051	-0.0034	-0.0038	-0.0467	-0.0064
			Mabias	0.0079	0.0025	0.0171	0.0236	0.0739	0.0178	0.1727	0.8695	0.1819	0.0074	0.0026	0.0173	0.0276	0.0439	0.0179	0.0662	0.8307	0.2046

Table B.6: Simulation results for Case VII: $\rho = 0.5$, $\alpha_0 = 2$, $\alpha_1 = 10$, $\alpha_2 = 3$

	T	N		Negative exponential: row normalized									Negative exponential: scalar normalized								
				β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.01)	σ^2(1)	α_0(2)	α_1(10)	α_2(3)	β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.01)	σ^2(1)	α_0(2)	α_1(10)	α_2(3)
WFE	5	200	Bias	0.2004	-0.0348	1.4153	-0.1869	-1.5530	0.5643				0.1914	-0.0310	1.5643	-0.1983	-1.7261	0.4985			
			RMSE	0.2023	0.0359	1.4236	0.1966	1.5615	0.5740				0.1936	0.0323	1.5747	0.2109	1.7391	0.5094			
			Mbias	0.1999	-0.0349	1.4163	-0.1864	-1.5553	0.5605				0.1907	-0.0313	1.5650	-0.1997	-1.7281	0.4933			
			Mabias	0.1999	0.0349	1.4163	0.1864	1.5553	0.5605				0.1907	0.0313	1.5650	0.1997	1.7281	0.4933			
TWFE	5	200	Bias	0.0007	-0.0004	0.0003	0.0014	-0.0029	-0.0056				0.0012	-0.0005	-0.0003	0.0016	-0.0057	-0.0056			
			RMSE	0.0216	0.0080	0.0390	0.0177	0.0555	0.0526				0.0213	0.0080	0.0422	0.0199	0.0644	0.0527			
			Mbias	0.0003	-0.0008	-0.0006	0.0020	-0.0034	-0.0092				0.0009	-0.0008	-0.0004	0.0023	-0.0051	-0.0088			
			Mabias	0.0152	0.0054	0.0246	0.0117	0.0375	0.0367				0.0154	0.0056	0.0263	0.0130	0.0430	0.0365			
PWFE	5	200	Bias	0.0026	-0.0007	0.0054	-0.0010	-0.0253	-0.0090	2.8172	2.3581	0.0831	0.0024	-0.0006	0.0051	-0.0005	-0.0130	-0.0086	1.6421	0.9760	0.0885
			RMSE	0.0239	0.0081	0.0449	0.0267	0.1364	0.0529	6.6412	5.6701	0.8206	0.0231	0.0081	0.0496	0.0297	0.0996	0.0529	3.8817	3.3059	0.6762
			Mbias	0.0028	-0.0011	0.0041	0.0003	-0.0093	-0.0124	-0.0929	0.0957	-0.0045	0.0017	-0.0010	0.0044	0.0005	-0.0050	-0.0117	0.0438	0.2781	-0.0185
			Mabias	0.0169	0.0052	0.0290	0.0177	0.0676	0.0371	1.5000	2.6506	0.3741	0.0165	0.0055	0.0335	0.0196	0.0645	0.0371	1.1884	2.5545	0.3815
WFE	5	800	Bias	0.1809	-0.0302	1.4790	-0.2369	-1.5570	0.5706				0.1728	-0.0301	1.6181	-0.2501	-1.6929	0.5281			
			RMSE	0.1814	0.0306	1.4818	0.2401	1.5600	0.5728				0.1734	0.0304	1.6211	0.2539	1.6968	0.5305			
			Mbias	0.1810	-0.0301	1.4767	-0.2366	-1.5580	0.5698				0.1727	-0.0301	1.6159	-0.2500	-1.6922	0.5277			
			Mabias	0.1810	0.0301	1.4767	0.2366	1.5580	0.5698				0.1727	0.0301	1.6159	0.2500	1.6922	0.5277			
TWFE	5	800	Bias	0.0003	-0.0001	-0.0008	0.0006	-0.0016	-0.0012				0.0002	-0.0001	-0.0006	0.0007	-0.0013	-0.0012			
			RMSE	0.0109	0.0041	0.0205	0.0103	0.0299	0.0253				0.0109	0.0041	0.0215	0.0110	0.0326	0.0253			
			Mbias	0.0004	-0.0002	-0.0005	0.0010	-0.0016	-0.0032				0.0004	-0.0002	0.0002	0.0011	-0.0018	-0.0032			
			Mabias	0.0074	0.0029	0.0141	0.0069	0.0196	0.0171				0.0073	0.0029	0.0143	0.0074	0.0218	0.0171			
PWFE	5	800	Bias	0.0005	-0.0002	-0.0006	-0.0005	0.0001	-0.0021	2.1963	1.7196	0.0108	0.0003	-0.0001	0.0000	-0.0006	0.0002	-0.0020	1.5392	0.8897	0.0115
			RMSE	0.0120	0.0042	0.0248	0.0156	0.0723	0.0254	5.7637	4.3262	0.2981	0.0118	0.0042	0.0261	0.0167	0.0482	0.0254	3.7583	2.6025	0.3032
			Mbias	0.0003	-0.0001	-0.0012	0.0003	0.0064	-0.0042	-0.2557	0.1839	0.0045	0.0001	0.0000	-0.0002	-0.0001	0.0048	-0.0042	-0.0616	0.1383	-0.0020
			Mabias	0.0083	0.0031	0.0175	0.0102	0.0358	0.0174	1.4999	1.4744	0.1989	0.0079	0.0030	0.0178	0.0110	0.0333	0.0174	1.1559	1.4028	0.1999

	T	N		Inverse distance: row normalized									Inverse distance scalar normalization								
				β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.01)	σ^2(1)	α_0(2)	α_1(10)	α_2(3)	β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.01)	σ^2(1)	α_0(2)	α_1(10)	α_2(3)
WFE	5	200	Bias	0.2257	-0.0333	6.9018	-0.3780	-5.0820	0.8925				0.2064	-0.0286	6.5419	-0.2866	-4.9425	0.7766			
			RMSE	0.2316	0.0351	6.9604	0.5300	5.2213	0.9250				0.2122	0.0305	6.6099	0.4712	5.0851	0.8068			
			Mbias	0.2268	-0.0334	6.8631	-0.3581	-5.1170	0.8737				0.2074	-0.0288	6.5006	-0.2695	-4.9821	0.7625			
			Mabias	0.2268	0.0334	6.8631	0.3730	5.1170	0.8737				0.2074	0.0288	6.5006	0.3184	4.9821	0.7625			
TWFE	5	200	Bias	0.0013	-0.0005	-0.0004	0.0021	-0.0104	-0.0056				0.0017	-0.0005	-0.0010	0.0023	-0.0160	-0.0057			
			RMSE	0.0210	0.0078	0.0378	0.0232	0.1014	0.0526				0.0208	0.0078	0.0409	0.0264	0.1141	0.0527			
			Mbias	0.0011	-0.0008	-0.0013	0.0023	-0.0097	-0.0089				0.0015	-0.0007	-0.0002	0.0030	-0.0144	-0.0088			
			Mabias	0.0149	0.0052	0.0236	0.0151	0.0685	0.0367				0.0148	0.0054	0.0257	0.0173	0.0763	0.0368			
PWFE	5	200	Bias	0.0030	-0.0008	0.0056	-0.0126	-0.1158	-0.0097	2.1636	1.4176	0.0675	0.0030	-0.0007	0.0041	-0.0040	-0.0513	-0.0089	0.9349	0.6777	0.1157
			RMSE	0.0244	0.0080	0.0480	0.0667	0.3215	0.0530	5.9421	4.3208	0.8604	0.0231	0.0079	0.0518	0.0576	0.1903	0.0529	2.9728	2.6813	0.7305
			Mbias	0.0031	-0.0011	0.0038	-0.0027	-0.0407	-0.0131	-0.1092	0.0087	-0.0367	0.0025	-0.0009	0.0028	-0.0005	-0.0361	-0.0121	-0.0291	0.0077	-0.0311
			Mabias	0.0172	0.0053	0.0316	0.0377	0.1150	0.0376	1.4632	1.7671	0.3802	0.0164	0.0055	0.0347	0.0377	0.1073	0.0371	0.9140	1.6937	0.3788
WFE	5	800	Bias	0.1565	-0.0195	11.2877	-1.4102	-7.6537	0.9802				0.1392	-0.0168	10.7103	-1.2560	-7.4590	0.9426			
			RMSE	0.1609	0.0207	11.3646	1.4817	7.8633	0.9923				0.1437	0.0180	10.7934	1.3368	7.6825	0.9540			
			Mbias	0.1589	-0.0199	11.2669	-1.4098	-7.7835	0.9758				0.1409	-0.0171	10.6479	-1.2566	-7.5171	0.9383			
			Mabias	0.1589	0.0199	11.2669	1.4098	7.7835	0.9758				0.1409	0.0171	10.6479	1.2566	7.5171	0.9383			
TWFE	5	800	Bias	0.0004	-0.0001	-0.0009	0.0008	-0.0044	-0.0012				0.0003	-0.0001	-0.0007	0.0009	-0.0041	-0.0012			
			RMSE	0.0107	0.0041	0.0196	0.0145	0.0650	0.0253				0.0106	0.0041	0.0206	0.0158	0.0722	0.0253			
			Mbias	0.0007	-0.0002	-0.0006	0.0013	-0.0032	-0.0032				0.0005	-0.0002	0.0000	0.0013	-0.0041	-0.0033			
			Mabias	0.0073	0.0029	0.0133	0.0096	0.0434	0.0171				0.0071	0.0029	0.0138	0.0109	0.0481	0.0171			
PWFE	5	800	Bias	0.0004	-0.0002	-0.0005	-0.0046	-0.0699	-0.0023	2.2109	0.6187	0.0092	0.0005	-0.0002	-0.0011	-0.0045	-0.0201	-0.0021	0.7462	0.4318	0.0136
			RMSE	0.0120	0.0041	0.0267	0.0384	0.2941	0.0254	6.0041	2.2832	0.2794	0.0116	0.0041	0.0276	0.0418	0.1551	0.0254	2.6925	1.7107	0.3022
			Mbias	0.0004	-0.0001	-0.0009	0.0009	0.0031	-0.0043	-0.1911	0.0635	-0.0053	0.0008	-0.0001	-0.0020	-0.0016	-0.0034	-0.0041	-0.0114	0.0516	-0.0088
			Mabias	0.0081	0.0030	0.0184	0.0231	0.0734	0.0175	1.3603	0.9589	0.1834	0.0079	0.0030	0.0187	0.0270	0.0713	0.0174	0.6273	0.8948	0.2062

Table B.7: Simulation results for Case VIII: $\rho = 0.01$, $\alpha_0 = 2$, $\alpha_1 = 10$, $\alpha_2 = 3$

$\rho = 0.25$				Exponential: row normalized				Exponential: scalar normalized				Inverse-dis: row normalized				Inverse-dis: scalar normalized			
	T	N		DE_x_1	DE_x_2	IE_x_1	IE_x_2	DE_x_1	DE_x_2	IE_x_2	IE_x_2	DE_x_1	DE_x_2	IE_x_1	IE_x_2	DE_x_1**1**	DE_x_2	IE_x_1	IE_x_2
WFE	**5**	**200**	**Bias**	0.0032	0.0017	0.2822	-0.1605	0.0037	0.0015	0.2615	-0.1608	0.0023	-0.0004	-0.1057	-0.2252	0.0026	-0.0003	-0.0498	-0.2046
			RMSE	0.0181	0.0079	0.3135	0.1664	0.0181	0.0079	0.2968	0.1674	0.0178	0.0077	0.3456	0.2397	0.0178	0.0077	0.3371	0.2208
			Mbias	0.0036	0.0016	0.2784	-0.1597	0.0038	0.0014	0.2577	-0.1607	0.0028	-0.0004	-0.1296	-0.2240	0.0032	-0.0003	-0.0639	-0.2023
			Mabias	0.0124	0.0054	0.2784	0.1597	0.0124	0.0053	0.2577	0.1607	0.0120	0.0054	0.2510	0.2240	0.0121	0.0054	0.2379	0.2023
TWFE	**5**	**200**	**Bias**	0.0002	-0.0002	0.0011	0.0020	0.0002	-0.0002	0.0005	0.0020	0.0001	-0.0003	-0.0041	0.0038	0.0002	-0.0003	-0.0063	0.0038
			RMSE	0.0177	0.0078	0.1006	0.0251	0.0177	0.0078	0.1041	0.0257	0.0177	0.0077	0.2658	0.0350	0.0176	0.0077	0.2628	0.0361
			Mbias	0.0002	-0.0003	-0.0007	0.0022	0.0004	-0.0003	0.0007	0.0031	0.0006	-0.0003	-0.0085	0.0052	0.0005	-0.0003	-0.0144	0.0061
			Mabias	0.0118	0.0055	0.0676	0.0160	0.0118	0.0055	0.0701	0.0166	0.0118	0.0054	0.1885	0.0226	0.0119	0.0054	0.1792	0.0231
PWFE	**5**	**200**	**Bias**	0.0002	-0.0002	0.0169	-0.0003	0.0003	-0.0002	0.0065	-0.0002	0.0001	-0.0002	0.0216	-0.0004	0.0002	-0.0003	0.0017	-0.0021
			RMSE	0.0178	0.0078	0.1654	0.0361	0.0177	0.0078	0.1670	0.0375	0.0177	0.0077	0.5060	0.0722	0.0176	0.0077	0.4767	0.0765
			Mbias	0.0001	-0.0003	0.0026	0.0015	0.0003	-0.0003	-0.0047	0.0015	0.0004	-0.0003	-0.0114	0.0080	0.0008	-0.0003	-0.0330	0.0085
			Mabias	0.0117	0.0055	0.1115	0.0245	0.0119	0.0055	0.1169	0.0250	0.0121	0.0054	0.3347	0.0457	0.0122	0.0054	0.3238	0.0510
WFE	**5**	**800**	**Bias**	0.0015	0.0000	0.3563	-0.2173	-0.0003	0.0003	0.3396	-0.2031	-0.0001	0.0002	0.7475	-0.9149	-0.0027	-0.0006	0.4470	-0.7281
			RMSE	0.0092	0.0039	0.3644	0.2192	0.0093	0.0040	0.3487	0.2052	0.0090	0.0039	0.9747	0.9814	0.0096	0.0041	0.6961	0.7777
			Mbias	0.0015	0.0001	0.3555	-0.2179	-0.0001	0.0003	0.3433	-0.2030	0.0000	0.0001	0.6348	-0.8454	-0.0025	-0.0006	0.3618	-0.6734
			Mabias	0.0064	0.0026	0.3555	0.2179	0.0064	0.0028	0.3433	0.2030	0.0062	0.0026	0.6348	0.8454	0.0064	0.0028	0.3752	0.6734
TWFE	**5**	**800**	**Bias**	0.0001	-0.0001	0.0016	-0.0001	-0.0001	-0.0001	-0.0018	0.0007	0.0001	-0.0001	0.0012	-0.0001	0.0000	-0.0001	-0.0057	0.0009
			RMSE	0.0091	0.0039	0.0534	0.0142	0.0093	0.0040	0.0559	0.0145	0.0090	0.0039	0.1929	0.0233	0.0092	0.0040	0.2150	0.0232
			Mbias	0.0002	-0.0001	-0.0002	-0.0001	0.0001	0.0000	0.0002	0.0014	0.0002	-0.0002	-0.0087	0.0001	0.0002	-0.0001	-0.0042	0.0021
			Mabias	0.0064	0.0026	0.0361	0.0093	0.0064	0.0028	0.0396	0.0094	0.0062	0.0026	0.1314	0.0157	0.0063	0.0028	0.1439	0.0149
PWFE	**5**	**800**	**Bias**	0.0001	-0.0002	0.0040	-0.0019	0.0000	-0.0001	0.0001	-0.0007	0.0001	-0.0001	0.0023	-0.0069	0.0000	-0.0001	0.0214	-0.0061
			RMSE	0.0091	0.0039	0.0811	0.0224	0.0093	0.0040	0.0878	0.0218	0.0090	0.0039	0.3724	0.0582	0.0092	0.0040	0.4440	0.0561
			Mbias	0.0002	-0.0002	0.0007	-0.0022	0.0002	0.0000	0.0005	0.0003	0.0002	-0.0002	-0.0289	0.0007	0.0003	-0.0001	-0.0090	-0.0006
			Mabias	0.0064	0.0026	0.0558	0.0157	0.0064	0.0028	0.0574	0.0143	0.0062	0.0026	0.2388	0.0359	0.0062	0.0028	0.3179	0.0344

$\rho = -0.25$				Exponential: row normalized				Exponential: scalar normalized				Inverse-dis: row normalized				Inverse-dis: scalar normalized			
	T	N		DE_x_1	DE_x_2	IE_x_1	IE_x_2	DE_x_1	DE_x_2	IE_x_2	IE_x_2	DE_x_1	DE_x_2	IE_x_1	IE_x_2	DE_x_1**1**	DE_x_2	IE_x_1	IE_x_2
WFE	**5**	**200**	**Bias**	0.0013	0.0003	0.1014	-0.0725	0.0032	0.0007	0.1024	-0.0751	0.0035	-0.0022	-0.0716	-0.1131	0.0038	-0.0020	-0.0216	-0.1047
			RMSE	0.0176	0.0078	0.1230	0.0762	0.0179	0.0078	0.1266	0.0792	0.0179	0.0080	0.1943	0.1229	0.0179	0.0079	0.1891	0.1158
			Mbias	0.0017	0.0001	0.0997	-0.0728	0.0037	0.0007	0.1019	-0.0759	0.0039	-0.0023	-0.0779	-0.1125	0.0040	-0.0020	-0.0193	-0.1038
			Mabias	0.0123	0.0054	0.1001	0.0728	0.0122	0.0054	0.1025	0.0759	0.0121	0.0057	0.1390	0.1125	0.0122	0.0057	0.1312	0.1038
TWFE	**5**	**200**	**Bias**	0.0000	-0.0003	0.0021	0.0012	0.0000	-0.0003	0.0015	0.0012	0.0000	-0.0003	0.0012	0.0020	0.0001	-0.0003	-0.0004	0.0020
			RMSE	0.0176	0.0078	0.0594	0.0156	0.0176	0.0077	0.0624	0.0163	0.0175	0.0077	0.1560	0.0207	0.0175	0.0077	0.1564	0.0216
			Mbias	0.0006	-0.0004	0.0001	0.0017	0.0005	-0.0003	0.0015	0.0022	0.0003	-0.0003	0.0018	0.0030	0.0005	-0.0003	-0.0030	0.0029
			Mabias	0.0121	0.0055	0.0401	0.0100	0.0120	0.0055	0.0421	0.0107	0.0118	0.0054	0.1094	0.0138	0.0118	0.0054	0.1087	0.0139
PWFE	**5**	**200**	**Bias**	0.0000	-0.0004	-0.0014	0.0015	0.0002	-0.0004	0.0044	-0.0001	0.0003	-0.0004	0.0285	0.0014	0.0002	-0.0004	0.0185	-0.0011
			RMSE	0.0176	0.0078	0.0943	0.0218	0.0176	0.0078	0.0980	0.0233	0.0175	0.0077	0.2949	0.0427	0.0175	0.0077	0.2959	0.0459
			Mbias	0.0005	-0.0005	-0.0050	0.0023	0.0007	-0.0004	0.0040	0.0007	0.0006	-0.0005	0.0005	0.0062	0.0004	-0.0004	-0.0046	0.0044
			Mabias	0.0119	0.0055	0.0659	0.0150	0.0120	0.0055	0.0682	0.0154	0.0121	0.0055	0.1930	0.0274	0.0120	0.0054	0.2033	0.0309
WFE	**5**	**800**	**Bias**	0.0005	-0.0007	0.1181	-0.0877	0.0003	-0.0008	0.1225	-0.0889	-0.0018	-0.0015	0.1045	-0.2711	-0.0027	-0.0011	0.1384	-0.2693
			RMSE	0.0092	0.0041	0.1244	0.0889	0.0092	0.0041	0.1290	0.0902	0.0093	0.0043	0.2321	0.2780	0.0096	0.0042	0.2544	0.2768
			Mbias	0.0008	-0.0007	0.1186	-0.0871	0.0006	-0.0008	0.1249	-0.0886	-0.0016	-0.0015	0.0927	-0.2683	-0.0026	-0.0011	0.1184	-0.2645
			Mabias	0.0063	0.0028	0.1186	0.0871	0.0062	0.0029	0.1249	0.0886	0.0063	0.0029	0.1447	0.2683	0.0065	0.0029	0.1637	0.2645
TWFE	**5**	**800**	**Bias**	0.0000	-0.0001	-0.0015	0.0005	0.0000	-0.0001	-0.0011	0.0005	0.0000	-0.0001	-0.0043	0.0006	0.0000	-0.0001	-0.0035	0.0006
			RMSE	0.0092	0.0041	0.0331	0.0089	0.0092	0.0040	0.0338	0.0092	0.0092	0.0040	0.1223	0.0131	0.0092	0.0040	0.1260	0.0135
			Mbias	0.0003	-0.0001	-0.0016	0.0009	0.0002	-0.0001	-0.0005	0.0010	0.0003	-0.0001	0.0008	0.0014	0.0002	-0.0001	-0.0007	0.0011
			Mabias	0.0063	0.0028	0.0225	0.0059	0.0062	0.0028	0.0236	0.0061	0.0061	0.0028	0.0802	0.0085	0.0061	0.0027	0.0815	0.0090
PWFE	**5**	**800**	**Bias**	0.0000	-0.0001	-0.0021	-0.0001	0.0000	-0.0001	0.0009	-0.0006	0.0000	-0.0001	0.0040	0.0020	0.0000	-0.0001	0.0211	-0.0032
			RMSE	0.0092	0.0041	0.0512	0.0132	0.0092	0.0040	0.0526	0.0136	0.0092	0.0040	0.2736	0.0315	0.0092	0.0040	0.2717	0.0328
			Mbias	0.0002	-0.0001	-0.0013	0.0007	0.0002	-0.0001	0.0017	0.0000	0.0002	-0.0001	-0.0191	0.0062	0.0003	-0.0001	0.0041	-0.0007
			Mabias	0.0063	0.0029	0.0336	0.0088	0.0062	0.0028	0.0346	0.0090	0.0061	0.0027	0.1953	0.0202	0.0061	0.0027	0.1924	0.0205

Table B.8: Direct (DE) and indirect (IE) effects of variables x_1 and x_2 for Case II and III using the parameter estimates in Table B.1 and B.2

$\alpha_2 = 1$				Exponential: row normalized				Exponential: scalar normalized				Inverse-dis: row normalized				Inverse-dis: scalar normalized			
	T	N		DE_x_1	DE_x_2	IE_x_1	IE_x_2	DE_x_1	DE_x_2	IE_x_2	IE_x_2	DE_x_1	DE_x_2	IE_x_1	IE_x_2	DE_x_1**1**	DE_x_2	IE_x_1	IE_x_2
WFE	**5**	**200**	**Bias**	0.0069	-0.0013	0.9266	0.0346	0.0057	-0.0007	0.8169	0.0532	0.0080	0.0011	1.7971	0.2749	0.0077	0.0012	1.5994	0.2526
			RMSE	0.0193	0.0086	0.9744	0.0898	0.0196	0.0081	0.8660	0.0955	0.0198	0.0083	2.0942	0.3537	0.0203	0.0079	1.8386	0.3144
			Mbias	0.0063	-0.0016	0.9225	0.0351	0.0056	-0.0009	0.8024	0.0549	0.0082	0.0011	1.6703	0.2566	0.0081	0.0011	1.5392	0.2420
			Mabias	0.0132	0.0058	0.9225	0.0593	0.0135	0.0056	0.8024	0.0654	0.0139	0.0057	1.6703	0.2574	0.0139	0.0054	1.5392	0.2427
TWFE	**5**	**200**	**Bias**	-0.0001	-0.0004	-0.0092	-0.0009	0.0003	-0.0002	-0.0029	0.0057	-0.0002	-0.0004	-0.0432	-0.0043	0.0002	-0.0003	-0.0195	0.0103
			RMSE	0.0178	0.0085	0.1639	0.0698	0.0185	0.0081	0.1587	0.0681	0.0172	0.0081	0.4174	0.1514	0.0180	0.0078	0.3839	0.1409
			Mbias	-0.0007	-0.0007	-0.0094	-0.0012	0.0003	-0.0004	-0.0080	0.0054	-0.0003	-0.0005	-0.0637	-0.0020	0.0004	-0.0003	-0.0283	0.0044
			Mabias	0.0122	0.0058	0.1165	0.0430	0.0125	0.0056	0.1049	0.0431	0.0121	0.0055	0.2914	0.0987	0.0121	0.0054	0.2510	0.0914
PWFE	**5**	**200**	**Bias**	-0.0001	-0.0003	0.0033	-0.0150	0.0002	-0.0003	-0.0037	-0.0048	0.0001	-0.0002	0.0285	-0.0073	0.0005	-0.0002	0.0541	0.0153
			RMSE	0.0179	0.0085	0.2506	0.1173	0.0186	0.0081	0.2437	0.1186	0.0180	0.0081	0.9371	0.2048	0.0182	0.0078	0.8141	0.1858
			Mbias	0.0000	-0.0007	0.0083	-0.0078	0.0004	-0.0005	-0.0177	0.0028	0.0000	-0.0004	-0.0711	-0.0138	0.0009	-0.0002	-0.0640	0.0115
			Mabias	0.0127	0.0057	0.1753	0.0789	0.0124	0.0056	0.1766	0.0774	0.0124	0.0055	0.6028	0.1278	0.0124	0.0054	0.5604	0.1234
WFE	**5**	**800**	**Bias**	0.0035	-0.0004	0.9177	0.0188	0.0028	0.0003	0.9781	0.0288	0.0024	0.0006	2.7079	0.4255	0.0029	0.0004	2.3958	0.3045
			RMSE	0.0101	0.0041	0.9319	0.0550	0.0101	0.0042	0.9927	0.0594	0.0095	0.0039	2.9968	0.5466	0.0098	0.0041	2.6316	0.4155
			Mbias	0.0034	-0.0003	0.9107	0.0193	0.0029	0.0004	0.9863	0.0312	0.0023	0.0006	2.5046	0.4126	0.0029	0.0003	2.2509	0.2944
			Mabias	0.0067	0.0026	0.9107	0.0359	0.0067	0.0029	0.9863	0.0412	0.0063	0.0026	2.5046	0.4192	0.0066	0.0028	2.2509	0.3019
TWFE	**5**	**800**	**Bias**	-0.0004	0.0002	-0.0080	-0.0005	-0.0001	-0.0001	-0.0033	0.0013	-0.0003	0.0002	-0.0362	0.0060	0.0000	-0.0001	-0.0183	0.0037
			RMSE	0.0094	0.0040	0.0870	0.0422	0.0097	0.0042	0.0843	0.0418	0.0090	0.0038	0.3114	0.1980	0.0093	0.0040	0.3222	0.1756
			Mbias	-0.0003	0.0003	-0.0107	-0.0004	0.0001	0.0000	0.0018	0.0026	-0.0003	0.0002	-0.0463	0.0125	0.0001	-0.0001	-0.0324	0.0044
			Mabias	0.0065	0.0025	0.0600	0.0264	0.0066	0.0029	0.0592	0.0276	0.0061	0.0024	0.2123	0.1358	0.0064	0.0028	0.2156	0.1208
PWFE	**5**	**800**	**Bias**	-0.0003	0.0002	-0.0054	-0.0059	-0.0001	-0.0001	-0.0033	-0.0050	-0.0003	0.0002	-0.0631	-0.0048	0.0000	-0.0001	0.0027	-0.0090
			RMSE	0.0094	0.0040	0.1268	0.0697	0.0097	0.0042	0.1294	0.0692	0.0090	0.0038	0.5476	0.2997	0.0093	0.0040	0.6448	0.2374
			Mbias	-0.0003	0.0003	-0.0099	-0.0029	0.0001	0.0000	-0.0024	-0.0018	-0.0002	0.0002	-0.1037	0.0058	0.0002	-0.0001	-0.0470	0.0010
			Mabias	0.0066	0.0025	0.0865	0.0443	0.0065	0.0029	0.0831	0.0464	0.0061	0.0024	0.3538	0.2111	0.0064	0.0028	0.4471	0.1622

$\alpha_2 = 0.5$				Exponential: row normalized				Exponential: scalar normalized				Inverse-dis: row normalized				Inverse-dis: scalar normalized			
	T	N		DE_x_1	DE_x_2	IE_x_1	IE_x_2	DE_x_1	DE_x_2	IE_x_2	IE_x_2	DE_x_1	DE_x_2	IE_x_1	IE_x_2	DE_x_1**1**	DE_x_2	IE_x_1	IE_x_2
WFE	**5**	**200**	0.0073	0.0000	1.0689	0.3634	0.0067	0.0006	0.9768	0.3519	0.0132	0.0021	2.7834	0.5863	0.0117	0.0018	2.3873	0.4997	
			RMSE	0.0204	0.0081	1.1145	0.3713	0.0199	0.0081	1.0266	0.3611	0.0236	0.0082	3.0805	0.6459	0.0224	0.0081	2.6441	0.5565
			Mbias	0.0073	-0.0002	1.0474	0.3655	0.0065	0.0005	0.9603	0.3548	0.0133	0.0019	2.5985	0.5618	0.0122	0.0016	2.2590	0.4848
			Mabias	0.0140	0.0056	1.0474	0.3655	0.0137	0.0055	0.9603	0.3548	0.0162	0.0055	2.5985	0.5618	0.0154	0.0054	2.2590	0.4848
TWFE	**5**	**200**	**Bias**	0.0002	-0.0002	-0.0028	0.0087	0.0003	-0.0003	-0.0032	0.0083	0.0001	-0.0003	-0.0241	0.0027	0.0001	-0.0003	-0.0257	0.0015
			RMSE	0.0188	0.0081	0.1549	0.1055	0.0185	0.0081	0.1560	0.1114	0.0181	0.0078	0.4014	0.1622	0.0180	0.0078	0.3916	0.1613
			Mbias	0.0003	-0.0002	-0.0069	0.0029	0.0002	-0.0003	-0.0082	0.0060	0.0004	-0.0004	-0.0334	0.0033	0.0005	-0.0004	-0.0417	0.0037
			Mabias	0.0125	0.0057	0.1026	0.0686	0.0125	0.0057	0.1019	0.0721	0.0122	0.0054	0.2586	0.1089	0.0121	0.0054	0.2602	0.1079
PWFE	**5**	**200**	**Bias**	-0.0001	-0.0001	-0.0101	-0.0089	0.0001	-0.0003	-0.0082	0.0015	0.0002	-0.0002	0.0192	0.0020	0.0005	-0.0003	0.0494	0.0069
			RMSE	0.0188	0.0081	0.2360	0.1637	0.0185	0.0081	0.2430	0.1571	0.0184	0.0078	0.8791	0.1945	0.0183	0.0078	0.8204	0.1872
			Mbias	0.0005	-0.0002	-0.0249	-0.0079	0.0002	-0.0005	-0.0284	0.0032	0.0003	-0.0002	-0.1059	-0.0112	0.0009	-0.0002	-0.0749	0.0013
			Mabias	0.0125	0.0057	0.1721	0.1117	0.0125	0.0056	0.1733	0.1050	0.0124	0.0054	0.5873	0.1266	0.0124	0.0054	0.5474	0.1218
WFE	**5**	**800**	**Bias**	0.0030	0.0001	0.9395	0.3753	0.0029	0.0004	0.9494	0.3641	0.0034	0.0008	3.0886	1.0502	0.0030	0.0007	2.7388	0.8996
			RMSE	0.0099	0.0041	0.9532	0.3786	0.0098	0.0041	0.9645	0.3677	0.0099	0.0041	3.3894	1.1438	0.0097	0.0040	3.0303	0.9876
			Mbias	0.0031	0.0002	0.9350	0.3742	0.0029	0.0004	0.9419	0.3632	0.0031	0.0009	2.8755	0.9919	0.0027	0.0007	2.5187	0.8547
			Mabias	0.0069	0.0027	0.9350	0.3742	0.0069	0.0027	0.9419	0.3632	0.0067	0.0027	2.8755	0.9919	0.0067	0.0026	2.5187	0.8547
TWFE	**5**	**800**	**Bias**	0.0001	-0.0002	0.0017	-0.0027	0.0001	-0.0002	0.0022	-0.0032	0.0001	-0.0002	-0.0140	-0.0097	0.0001	-0.0002	-0.0146	-0.0099
			RMSE	0.0094	0.0041	0.0820	0.0744	0.0094	0.0041	0.0820	0.0770	0.0091	0.0039	0.2861	0.3063	0.0091	0.0039	0.2952	0.2954
			Mbias	0.0002	-0.0002	-0.0011	-0.0030	0.0003	-0.0003	0.0010	-0.0057	0.0000	-0.0002	-0.0225	-0.0031	0.0000	-0.0002	-0.0303	-0.0094
			Mabias	0.0064	0.0028	0.0571	0.0514	0.0064	0.0028	0.0557	0.0550	0.0063	0.0027	0.2026	0.2074	0.0063	0.0027	0.2071	0.2012
PWFE	**5**	**800**	**Bias**	0.0001	-0.0002	0.0000	-0.0245	0.0001	-0.0002	-0.0011	-0.0219	0.0001	-0.0002	-0.0730	-0.0208	0.0001	-0.0003	-0.0377	-0.0136
			RMSE	0.0095	0.0041	0.1208	0.1316	0.0094	0.0041	0.1207	0.1302	0.0091	0.0039	0.5361	0.3393	0.0091	0.0039	0.5414	0.3283
			Mbias	0.0001	-0.0002	-0.0051	-0.0203	0.0002	-0.0002	-0.0023	-0.0133	0.0002	-0.0003	-0.1206	-0.0157	0.0001	-0.0004	-0.0810	-0.0107
			Mabias	0.0065	0.0028	0.0816	0.0815	0.0065	0.0028	0.0827	0.0819	0.0062	0.0027	0.3693	0.2376	0.0063	0.0027	0.3648	0.2289

Table B.9: Direct (DE) and indirect (IE) effects of variables x_1 and x_2 for Case V and VI using the parameter estimates in Table B.4 and B.5

$\rho = 0.01$				Exponential: row normalized				Exponential: scalar normalized				Inverse-dis: row normalized				Inverse-dis: scalar normalized			
	T	N		DE_x_1	DE_x_2	IE_x_1	IE_x_2	DE_x_1	DE_x_2	IE_x_2	IE_x_2	DE_x_1	DE_x_2	IE_x_1	IE_x_2	DE_x_1**1**	DE_x_2	IE_x_1	IE_x_2
WFE	**5**	**200**	**Bias**	0.0019	0.0013	0.1659	-0.1047	0.0031	0.0014	0.1612	-0.1067	0.003	-0.001	-0.080	-0.154	0.0033	-0.0011	-0.0274	-0.1414
			RMSE	0.0176	0.0078	0.1905	0.1091	0.0177	0.0078	0.1889	0.1117	0.018	0.008	0.250	0.165	0.0178	0.0077	0.2454	0.1544
			Mbias	0.0021	0.0011	0.1620	-0.1050	0.0032	0.0013	0.1597	-0.1071	0.003	-0.001	-0.090	-0.154	0.0037	-0.0012	-0.0310	-0.1415
			Mabias	0.0119	0.0053	0.1620	0.1050	0.0124	0.0053	0.1597	0.1071	0.012	0.005	0.178	0.154	0.0121	0.0054	0.1723	0.1415
TWFE	**5**	**200**	**Bias**	0.0001	-0.0003	0.0018	0.0015	0.0001	-0.0003	0.0012	0.0015	0.000	0.000	-0.001	0.003	0.0001	-0.0003	-0.0024	0.0027
			RMSE	0.0175	0.0077	0.0752	0.0191	0.0175	0.0077	0.0785	0.0198	0.018	0.008	0.199	0.026	0.0175	0.0077	0.1983	0.0272
			Mbias	0.0004	-0.0004	0.0004	0.0021	0.0004	-0.0003	0.0014	0.0023	0.000	0.000	-0.001	0.004	0.0003	-0.0004	-0.0042	0.0043
			Mabias	0.0118	0.0053	0.0516	0.0124	0.0118	0.0054	0.0532	0.0130	0.012	0.005	0.140	0.017	0.0117	0.0054	0.1377	0.0172
PWFE	**5**	**200**	**Bias**	0.0002	-0.0003	0.0077	0.0002	0.0002	-0.0003	0.0052	0.0000	0.000	0.000	0.034	0.000	0.0002	-0.0003	0.0097	-0.0010
			RMSE	0.0175	0.0077	0.1157	0.0271	0.0175	0.0077	0.1117	0.0283	0.018	0.008	0.377	0.054	0.0175	0.0077	0.3423	0.0571
			Mbias	0.0001	-0.0004	0.0017	0.0007	0.0002	-0.0004	0.0018	0.0006	0.001	0.000	0.006	0.006	0.0006	-0.0004	-0.0116	0.0043
			Mabias	0.0116	0.0054	0.0822	0.0181	0.0117	0.0054	0.0783	0.0189	0.012	0.005	0.241	0.034	0.0119	0.0054	0.2342	0.0371
WFE	**5**	**800**	**Bias**	0.0011	0.0001	0.2106	-0.1394	0.0007	0.0000	0.1994	-0.1406	0.000	0.000	0.367	-0.508	-0.0001	0.0000	0.3170	-0.4859
			RMSE	0.0091	0.0038	0.2169	0.1408	0.0091	0.0038	0.2064	0.1421	0.009	0.004	0.480	0.526	0.0090	0.0039	0.4445	0.5045
			Mbias	0.0011	0.0001	0.2105	-0.1397	0.0007	-0.0001	0.1990	-0.1409	0.000	0.000	0.332	-0.487	-0.0001	0.0000	0.2875	-0.4646
			Mabias	0.0063	0.0026	0.2105	0.1397	0.0062	0.0026	0.1990	0.1409	0.006	0.003	0.332	0.487	0.0061	0.0026	0.2912	0.4646
TWFE	**5**	**800**	**Bias**	0.0001	-0.0001	0.0013	-0.0001	0.0001	-0.0001	0.0015	0.0000	0.000	0.000	0.001	0.000	0.0001	-0.0001	0.0016	0.0000
			RMSE	0.0090	0.0038	0.0401	0.0108	0.0090	0.0038	0.0414	0.0110	0.009	0.004	0.142	0.017	0.0090	0.0038	0.1494	0.0173
			Mbias	0.0001	-0.0002	-0.0002	0.0001	0.0001	-0.0002	0.0004	0.0000	0.000	0.000	-0.007	0.000	0.0001	-0.0002	-0.0026	-0.0001
			Mabias	0.0062	0.0026	0.0270	0.0073	0.0062	0.0026	0.0272	0.0074	0.006	0.003	0.097	0.011	0.0062	0.0026	0.1039	0.0116
PWFE	**5**	**800**	**Bias**	0.0001	-0.0001	0.0024	-0.0015	0.0001	-0.0001	0.0029	-0.0014	0.000	0.000	-0.001	-0.003	0.0001	-0.0002	0.0036	-0.0050
			RMSE	0.0090	0.0038	0.0592	0.0171	0.0090	0.0038	0.0569	0.0169	0.009	0.004	0.268	0.044	0.0090	0.0038	0.2704	0.0437
			Mbias	0.0001	-0.0002	-0.0002	-0.0017	0.0001	-0.0002	0.0020	-0.0011	0.000	0.000	-0.018	0.002	0.0001	-0.0002	-0.0183	-0.0017
			Mabias	0.0062	0.0026	0.0396	0.0118	0.0062	0.0026	0.0383	0.0115	0.006	0.003	0.180	0.029	0.0062	0.0026	0.1795	0.0279
$\alpha_1 = 10$				Exponential: row normalized				Exponential: scalar normalized				Inverse-dis: row normalized				Inverse-dis: scalar normalized			
	T	N		DE_b1	DE_b2	IE_b1	IE_b2	DE_b1	DE_b2	IE_b1	IE_b2	DE_b1	DE_b2	IE_b1	IE_b2	DE_b1	DE_b2	IE_b1	IE_b2
WFE	**5**	**200**	**Bias**	-0.0018	0.0014	0.7764	-0.0751	0.0020	0.0017	0.7546	-0.0726	0.0143	-0.0052	1.6929	0.0339	0.0163	-0.0040	1.6749	0.0308
			RMSE	0.0179	0.0086	0.8007	0.0948	0.0178	0.0085	0.7803	0.0932	0.0223	0.0098	1.8650	0.1277	0.0236	0.0092	1.8176	0.1271
			Mbias	-0.0018	0.0012	0.7773	-0.0742	0.0019	0.0014	0.7504	-0.0719	0.0145	-0.0053	1.5474	0.0368	0.0164	-0.0041	1.5915	0.0320
			Mabias	0.0122	0.0058	0.7773	0.0752	0.0120	0.0058	0.7504	0.0725	0.0162	0.0068	1.5474	0.0849	0.0176	0.0063	1.5915	0.0824
TWFE	**5**	**200**	**Bias**	-0.0002	-0.0003	-0.0047	-0.0002	-0.0002	-0.0003	-0.0074	0.0002	-0.0001	-0.0003	-0.0027	-0.0004	-0.0001	-0.0003	-0.0078	0.0002
			RMSE	0.0177	0.0084	0.1121	0.0394	0.0175	0.0084	0.1085	0.0394	0.0170	0.0081	0.1747	0.0576	0.0169	0.0081	0.1617	0.0582
			Mbias	-0.0005	-0.0005	-0.0071	0.0008	-0.0004	-0.0006	-0.0109	0.0007	-0.0004	-0.0005	-0.0121	0.0016	-0.0005	-0.0005	-0.0205	0.0014
			Mabias	0.0121	0.0058	0.0775	0.0257	0.0122	0.0056	0.0732	0.0265	0.0117	0.0054	0.1212	0.0380	0.0117	0.0054	0.1127	0.0389
PWFE	**5**	**200**	**Bias**	0.0002	-0.0002	0.0118	-0.0073	0.0002	-0.0002	0.0078	-0.0079	-0.0002	-0.0002	-0.0209	-0.0132	0.0000	-0.0003	0.0000	-0.0155
			RMSE	0.0177	0.0084	0.1345	0.0578	0.0176	0.0083	0.1228	0.0598	0.0170	0.0081	0.2439	0.1114	0.0170	0.0081	0.1905	0.1142
			Mbias	0.0000	-0.0005	0.0046	-0.0043	-0.0004	-0.0006	0.0082	-0.0047	-0.0001	-0.0004	-0.0608	-0.0032	-0.0003	-0.0005	-0.0195	-0.0050
			Mabias	0.0122	0.0057	0.0897	0.0361	0.0120	0.0056	0.0823	0.0383	0.0117	0.0056	0.1759	0.0710	0.0117	0.0055	0.1251	0.0756
WFE	**5**	**800**	**Bias**	0.0132	0.0022	0.8150	-0.1248	0.0126	0.0017	0.7742	-0.1240	0.0336	0.0030	1.5438	-0.2439	0.0284	0.0020	1.5974	-0.2519
			RMSE	0.0162	0.0046	0.8221	0.1298	0.0157	0.0043	0.7819	0.1291	0.0351	0.0049	1.7180	0.3009	0.0301	0.0044	1.7714	0.3065
			Mbias	0.0131	0.0023	0.8102	-0.1241	0.0125	0.0018	0.7687	-0.1227	0.0336	0.0031	1.3954	-0.2127	0.0286	0.0021	1.4534	-0.2227
			Mabias	0.0132	0.0032	0.8102	0.1241	0.0126	0.0030	0.7687	0.1227	0.0336	0.0036	1.3954	0.2127	0.0286	0.0031	1.4534	0.2227
TWFE	**5**	**800**	**Bias**	-0.0004	0.0002	-0.0043	0.0001	-0.0004	0.0002	-0.0045	0.0002	-0.0003	0.0002	-0.0020	-0.0008	-0.0003	0.0002	-0.0019	-0.0007
			RMSE	0.0094	0.0040	0.0581	0.0225	0.0093	0.0040	0.0574	0.0219	0.0090	0.0038	0.1173	0.0388	0.0090	0.0038	0.1151	0.0382
			Mbias	-0.0004	0.0002	-0.0051	0.0008	-0.0005	0.0002	-0.0057	0.0011	-0.0002	0.0002	-0.0114	0.0022	-0.0002	0.0002	-0.0104	0.0028
			Mabias	0.0064	0.0025	0.0413	0.0145	0.0063	0.0025	0.0401	0.0141	0.0061	0.0024	0.0810	0.0259	0.0062	0.0024	0.0811	0.0247
PWFE	**5**	**800**	**Bias**	-0.0003	0.0002	-0.0011	-0.0021	-0.0003	0.0002	-0.0016	-0.0020	-0.0003	0.0002	0.0066	-0.0096	-0.0003	0.0002	0.0023	-0.0113
			RMSE	0.0094	0.0040	0.0717	0.0328	0.0094	0.0040	0.0658	0.0319	0.0090	0.0038	0.2180	0.0873	0.0090	0.0038	0.1332	0.0847
			Mbias	-0.0004	0.0002	-0.0068	-0.0003	-0.0004	0.0002	-0.0039	-0.0004	-0.0004	0.0003	-0.0282	-0.0034	-0.0003	0.0002	-0.0094	-0.0012
			Mabias	0.0066	0.0025	0.0492	0.0215	0.0066	0.0025	0.0445	0.0216	0.0062	0.0024	0.1458	0.0558	0.0062	0.0024	0.0888	0.0534

Table B.10: Direct (DE) and indirect (IE) effects of variables x_1 and x_2 for Case IV and VII using the parameter estimates in Table B.3 and B.6

	$\rho = 0.01$			Exponential: row normalized				Exponential: scalar normalized				Inverse-dis: row normalized				Inverse-dis: scalar normalized			
WFE	**5**	**200**	**Bias**	0.0074	-0.0003	0.3225	-0.0257	0.0122	0.0007	0.3182	-0.0271	0.0297	-0.0088	0.7609	0.0261	0.0292	-0.0072	0.7832	0.0225
			RMSE	0.0190	0.0077	0.3319	0.0364	0.0213	0.0077	0.3294	0.0385	0.0350	0.0121	0.8026	0.0671	0.0346	0.0109	0.8248	0.0677
			Mbias	0.0074	-0.0007	0.3215	-0.0264	0.0120	0.0006	0.3161	-0.0272	0.0303	-0.0091	0.7269	0.0265	0.0300	-0.0074	0.7531	0.0219
			Mabias	0.0133	0.0054	0.3215	0.0276	0.0153	0.0052	0.3161	0.0286	0.0303	0.0094	0.7269	0.0455	0.0300	0.0081	0.7531	0.0468
TWFE	**5**	**200**	**Bias**	0.0001	-0.0003	0.0015	0.0012	0.0001	-0.0003	0.0007	0.0013	0.0001	-0.0003	0.0012	0.0017	0.0001	-0.0003	0.0000	0.0019
			RMSE	0.0175	0.0077	0.0490	0.0189	0.0175	0.0077	0.0497	0.0198	0.0175	0.0077	0.0657	0.0261	0.0175	0.0077	0.0650	0.0274
			Mbias	0.0004	-0.0003	-0.0001	0.0018	0.0004	-0.0003	0.0004	0.0019	0.0005	-0.0004	-0.0030	0.0027	0.0004	-0.0004	-0.0037	0.0028
			Mabias	0.0119	0.0053	0.0317	0.0122	0.0118	0.0054	0.0333	0.0127	0.0119	0.0054	0.0433	0.0173	0.0118	0.0054	0.0433	0.0174
PWFE	**5**	**200**	**Bias**	0.0001	-0.0002	0.0032	0.0001	0.0002	-0.0002	0.0059	-0.0005	0.0000	-0.0003	-0.0160	-0.0022	0.0001	-0.0003	-0.0019	-0.0026
			RMSE	0.0175	0.0077	0.0733	0.0270	0.0175	0.0077	0.0633	0.0284	0.0175	0.0077	0.1230	0.0540	0.0175	0.0077	0.0888	0.0551
			Mbias	0.0004	-0.0003	0.0031	0.0001	0.0006	-0.0002	0.0049	0.0002	0.0002	-0.0003	-0.0105	0.0028	0.0005	-0.0003	-0.0053	0.0009
			Mabias	0.0118	0.0054	0.0473	0.0182	0.0118	0.0054	0.0432	0.0183	0.0117	0.0054	0.0731	0.0351	0.0118	0.0054	0.0554	0.0357
WFE	**5**	**800**	**Bias**	0.0156	0.0008	0.3282	-0.0442	0.0151	-0.0009	0.3249	-0.0414	0.0301	0.0008	0.8939	-0.0836	0.0252	0.0011	0.8923	-0.0819
			RMSE	0.0181	0.0041	0.3310	0.0472	0.0177	0.0041	0.3280	0.0448	0.0321	0.0045	0.9357	0.1059	0.0276	0.0045	0.9376	0.1062
			Mbias	0.0159	0.0009	0.3291	-0.0437	0.0154	-0.0009	0.3274	-0.0412	0.0307	0.0010	0.8366	-0.0768	0.0260	0.0014	0.8421	-0.0745
			Mabias	0.0159	0.0029	0.3291	0.0437	0.0154	0.0029	0.3274	0.0412	0.0307	0.0031	0.8366	0.0773	0.0260	0.0032	0.8421	0.0752
TWFE	**5**	**800**	0.0000	-0.0001	-0.0007	0.0006	0.0000	-0.0001	-0.0003	0.0006	0.0000	-0.0001	-0.0004	0.0007	0.0000	-0.0001	0.0003	0.0006	
			RMSE	0.0092	0.0040	0.0261	0.0109	0.0092	0.0040	0.0263	0.0112	0.0092	0.0040	0.0396	0.0166	0.0092	0.0040	0.0400	0.0173
			Mbias	0.0002	-0.0001	-0.0007	0.0011	0.0002	-0.0001	0.0006	0.0010	0.0002	-0.0001	-0.0020	0.0022	0.0003	-0.0001	0.0005	0.0016
			Mabias	0.0061	0.0027	0.0183	0.0072	0.0061	0.0028	0.0187	0.0073	0.0061	0.0027	0.0275	0.0109	0.0061	0.0028	0.0282	0.0113
PWFE	**5**	**800**	0.0000	-0.0001	0.0023	-0.0009	0.0000	-0.0001	0.0014	-0.0008	0.0000	-0.0001	-0.0049	-0.0025	-0.0001	-0.0001	0.0005	-0.0050	
			RMSE	0.0092	0.0040	0.0401	0.0164	0.0092	0.0040	0.0332	0.0166	0.0092	0.0040	0.1174	0.0425	0.0092	0.0040	0.0757	0.0436
			Mbias	0.0003	-0.0001	0.0017	-0.0001	0.0002	-0.0001	0.0021	0.0001	0.0002	-0.0001	-0.0045	0.0030	0.0002	-0.0001	-0.0031	-0.0003
			Mabias	0.0061	0.0027	0.0263	0.0103	0.0061	0.0028	0.0230	0.0108	0.0062	0.0028	0.0457	0.0265	0.0062	0.0028	0.0375	0.0266

Table B.11: Direct (DE) and indirect (IE) effects of variables x_1 and x_2 for Case VIII using the parameter estimates in Table B.7

Settings				Negative exponential: row normalized											
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(0.25)**	α_0**(2)**	α_1**(1.5)**	α_2**(3)**	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.1784	0.3733	0.0028	0.0002	0.0440				0.5371	0.5171	0.0866	0.0033
			Std	0.2341	0.2987	0.0216	0.0014	0.1102				0.2767	0.2799	0.1758	0.0142
TWFE	**5**	**200**	**Mean**	0.4903	0.4972	0.5105	0.5088	0.4921				0.5261	0.5143	0.4814	0.4986
			Std	0.2890	0.2805	0.2879	0.2834	0.2883				0.2796	0.2801	0.2979	0.2947
PWFE	**5**	**200**	**Mean**	0.4761	0.4949	0.4870	0.4988	0.4661	0.5313	0.4741	0.5036	0.4917	0.4951	0.4840	0.4966
			Std	0.2913	0.2838	0.2890	0.2852	0.3032	0.2884	0.2951	0.2798	0.2904	0.2861	0.2878	0.2886
WFE	**5**	**800**	**Mean**	0.0634	0.1706	0.0000	0.0000	0.0044				0.5360	0.5271	0.0001	0.0000
			Std	0.1342	0.2298	0.0000	0.0000	0.0371				0.2804	0.2740	0.0011	0.0000
TWFE	**5**	**800**	**Mean**	0.4783	0.5037	0.5088	0.5076	0.5075				0.5248	0.5197	0.4773	0.4931
			Std	0.2869	0.2861	0.2819	0.2856	0.2918				0.2844	0.2770	0.2911	0.2862
PWFE	**5**	**800**	**Mean**	0.4766	0.4998	0.4914	0.4835	0.4714	0.4834	0.4707	0.4974	0.4946	0.5032	0.4945	0.4826
			Std	0.2912	0.2846	0.2863	0.2898	0.2962	0.2991	0.2873	0.2980	0.2932	0.2817	0.2888	0.2863
				Negative exponential: scalar normalized											
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(0.25)**	α_0**(2)**	α_1**(1.5)**	α_2**(3)**	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.2358	0.4058	0.0093	0.0005	0.0718				0.5297	0.5139	0.1156	0.0062
			Std	0.2745	0.3002	0.0499	0.0033	0.1512				0.2800	0.2792	0.2083	0.0289
TWFE	**5**	**200**	**Mean**	0.4892	0.4962	0.5075	0.5125	0.4949				0.5231	0.5117	0.4728	0.5015
			Std	0.2872	0.2816	0.2936	0.2884	0.2920				0.2812	0.2806	0.2969	0.2943
PWFE	**5**	**200**	**Mean**	0.4832	0.4937	0.4961	0.5017	0.5758	0.6133	0.4864	0.5081	0.4928	0.4948	0.4779	0.4991
			Std	0.2878	0.2839	0.2865	0.2854	0.2682	0.2497	0.2927	0.2818	0.2908	0.2855	0.2889	0.2892
WFE	**5**	**800**	**Mean**	0.1395	0.2566	0.0000	0.0000	0.0168				0.5392	0.5090	0.0003	0.0000
			Std	0.2186	0.2770	0.0000	0.0000	0.0600				0.2838	0.2823	0.0022	0.0000
TWFE	**5**	**800**	**Mean**	0.5024	0.4827	0.5028	0.4972	0.5050				0.5248	0.5032	0.4725	0.4923
			Std	0.2880	0.2840	0.2898	0.2914	0.2871				0.2883	0.2837	0.2954	0.2954
PWFE	**5**	**800**	**Mean**	0.5001	0.4835	0.5098	0.4901	0.6023	0.6214	0.4982	0.4921	0.4980	0.4874	0.5038	0.4905
			Std	0.2894	0.2868	0.2893	0.2871	0.2548	0.2449	0.2926	0.2931	0.2960	0.2882	0.2907	0.2890
				Inverse distance: row normalized											
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(0.25)**	α_0**(2)**	α_1**(1.5)**	α_2**(3)**	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.3435	0.3543	0.2951	0.0044	0.0289				0.5078	0.5141	0.3294	0.0341
			Std	0.3038	0.2933	0.2930	0.0262	0.1124				0.2835	0.2786	0.3144	0.0873
TWFE	**5**	**200**	**Mean**	0.4903	0.4988	0.5016	0.5081	0.4957				0.4979	0.5087	0.4562	0.4994
			Std	0.2893	0.2802	0.2895	0.2833	0.2865				0.2885	0.2817	0.2942	0.2960
PWFE	**5**	**200**	**Mean**	0.4837	0.4892	0.4925	0.4916	0.4163	0.6217	0.4951	0.5003	0.4899	0.4945	0.4912	0.4833
			Std	0.2912	0.2803	0.2987	0.2940	0.3195	0.2741	0.2991	0.2905	0.2897	0.2835	0.3100	0.2995
WFE	**5**	**800**	**Mean**	0.4378	0.3887	0.0056	0.0000	0.0855				0.5082	0.5160	0.0855	0.0000
			Std	0.2964	0.3016	0.0521	0.0000	0.2016				0.2904	0.2780	0.1896	0.0000
TWFE	**5**	**800**	**Mean**	0.4890	0.5022	0.4989	0.5066	0.5076				0.4976	0.5121	0.4554	0.4839
			Std	0.2928	0.2845	0.2794	0.2882	0.2873				0.2934	0.2766	0.2958	0.2933
PWFE	**5**	**800**	**Mean**	0.4874	0.4997	0.4967	0.4855	0.4527	0.5485	0.4975	0.5050	0.4953	0.5017	0.4991	0.4914
			Std	0.2919	0.2828	0.2916	0.2846	0.3240	0.2783	0.2931	0.2920	0.2936	0.2789	0.2987	0.2911
				Inverse distance: scalar normalized											
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(0.25)**	α_0**(2)**	α_1**(1.5)**	α_2**(3)**	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.3706	0.3952	0.3154	0.0203	0.0476				0.5052	0.5113	0.3412	0.0617
			Std	0.3053	0.2969	0.3030	0.0710	0.1425				0.2844	0.2796	0.3049	0.1275
TWFE	**5**	**200**	**Mean**	0.4907	0.4983	0.4976	0.5111	0.4983				0.4991	0.5056	0.4557	0.4989
			Std	0.2887	0.2815	0.2918	0.2874	0.2919				0.2887	0.2821	0.2952	0.2927
PWFE	**5**	**200**	**Mean**	0.4869	0.4922	0.5261	0.5034	0.6069	0.6963	0.5108	0.5109	0.4903	0.4945	0.5045	0.4912
			Std	0.2873	0.2830	0.2860	0.2939	0.2626	0.2229	0.2835	0.2830	0.2886	0.2840	0.2997	0.2986
WFE	**5**	**800**	**Mean**	0.4881	0.3836	0.0862	0.0000	0.0993				0.4939	0.4946	0.2205	0.0000
			Std	0.2884	0.3050	0.1911	0.0000	0.2150				0.2927	0.2845	0.2782	0.0000
TWFE	**5**	**800**	**Mean**	0.4986	0.4836	0.5087	0.4946	0.5104				0.4984	0.4968	0.4708	0.4919
			Std	0.2938	0.2862	0.2868	0.2902	0.2896				0.2937	0.2859	0.2993	0.2976
PWFE	**5**	**800**	**Mean**	0.4969	0.4826	0.4875	0.4932	0.6886	0.7536	0.4913	0.4875	0.4959	0.4866	0.4782	0.5006
			Std	0.2915	0.2859	0.2857	0.2966	0.2271	0.1913	0.2832	0.2824	0.2926	0.2890	0.2924	0.2945

Table B.12: Case II: Mean and standard deviation of the p-values of the parameters (Table B.1) and the direct (DE) and indirect (IE) effects of variables x_1 and x_2 (Table B.8, upper panel)

Settings				Negative exponential: row normalized											
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(-0.25)**	α_0**(2)**	α_1**(1.5)**	α_2**(3)**	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.0450	0.2758	0.0026	0.0001	0.0000				0.5937	0.5423	0.1816	0.0142
			Std	0.1052	0.2786	0.0244	0.0010	0.0000				0.2570	0.2687	0.2542	0.0583
TWFE	**5**	**200**	**Mean**	0.4883	0.4984	0.5104	0.5100	0.4962				0.5665	0.5275	0.4904	0.5059
			Std	0.2854	0.2831	0.2875	0.2865	0.2928				0.2688	0.2750	0.2932	0.2905
PWFE	**5**	**200**	**Mean**	0.4728	0.4966	0.4858	0.5012	0.4509	0.4764	0.4778	0.5009	0.4941	0.4922	0.4774	0.4926
			Std	0.2925	0.2858	0.2874	0.2828	0.2943	0.3221	0.2948	0.2892	0.2931	0.2840	0.2929	0.2915
WFE	**5**	**800**	**Mean**	0.0059	0.1132	0.0000	0.0000	0.0000				0.5862	0.5264	0.0116	0.0000
			Std	0.0271	0.1833	0.0000	0.0000	0.0000				0.2586	0.2758	0.0497	0.0000
TWFE	**5**	**800**	**Mean**	0.5034	0.4826	0.4959	0.5055	0.5081				0.5608	0.5165	0.4775	0.4983
			Std	0.2828	0.2825	0.2830	0.2950	0.2893				0.2696	0.2802	0.2895	0.2944
PWFE	**5**	**800**	**Mean**	0.4997	0.4854	0.4973	0.4963	0.4776	0.4606	0.4930	0.4914	0.4932	0.4855	0.4933	0.4912
			Std	0.2932	0.2882	0.2893	0.2916	0.2924	0.2938	0.2957	0.2968	0.2907	0.2893	0.2927	0.2928
				Negative exponential: scalar normalized											
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(-0.25)**	α_0**(2)**	α_1**(1.5)**	α_2**(3)**	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.0765	0.3300	0.0081	0.0004	0.0001				0.5818	0.5365	0.1893	0.0181
			Std	0.1436	0.2943	0.0503	0.0032	0.0010				0.2606	0.2706	0.2594	0.0712
TWFE	**5**	**200**	**Mean**	0.4878	0.4962	0.5070	0.5137	0.4977				0.5624	0.5250	0.4877	0.5111
			Std	0.2831	0.2829	0.2907	0.2892	0.2910				0.2702	0.2763	0.2924	0.2922
PWFE	**5**	**200**	**Mean**	0.4859	0.4919	0.4970	0.5037	0.6381	0.6465	0.5037	0.5071	0.4934	0.4929	0.4857	0.5008
			Std	0.2845	0.2800	0.2823	0.2849	0.2341	0.2356	0.2990	0.2890	0.2929	0.2848	0.2896	0.2896
WFE	**5**	**800**	**Mean**	0.0127	0.1358	0.0000	0.0000	0.0000				0.5822	0.5218	0.0117	0.0000
			Std	0.0464	0.2030	0.0000	0.0000	0.0000				0.2612	0.2776	0.0525	0.0000
TWFE	**5**	**800**	**Mean**	0.5045	0.4823	0.5029	0.4984	0.5074				0.5571	0.5150	0.4855	0.4911
			Std	0.2858	0.2825	0.2891	0.2913	0.2896				0.2712	0.2805	0.2931	0.2900
PWFE	**5**	**800**	**Mean**	0.5048	0.4841	0.5075	0.4890	0.6513	0.6556	0.4951	0.4915	0.4935	0.4853	0.5010	0.4900
			Std	0.2856	0.2825	0.2886	0.2870	0.2307	0.2298	0.2907	0.2950	0.2908	0.2892	0.2898	0.2880
				Inverse distance: row normalized											
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(-0.25)**	α_0**(2)**	α_1**(1.5)**	α_2**(3)**	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.2006	0.3041	0.2148	0.0033	0.0006				0.5290	0.5103	0.2968	0.0403
			Std	0.2616	0.2893	0.2746	0.0359	0.0120				0.2777	0.2806	0.3126	0.1171
TWFE	**5**	**200**	**Mean**	0.4896	0.5000	0.5013	0.5101	0.4978				0.5166	0.5155	0.4672	0.4996
			Std	0.2890	0.2807	0.2885	0.2855	0.2913				0.2856	0.2785	0.2932	0.2935
PWFE	**5**	**200**	**Mean**	0.4792	0.4890	0.4789	0.4959	0.3965	0.4296	0.4847	0.4855	0.4920	0.4930	0.4758	0.4673
			Std	0.2918	0.2811	0.2916	0.2922	0.2837	0.3546	0.3090	0.2982	0.2925	0.2824	0.3109	0.3024
WFE	**5**	**800**	**Mean**	0.3757	0.3589	0.0437	0.0000	0.0002				0.5177	0.4842	0.2168	0.0000
			Std	0.2938	0.3102	0.1502	0.0000	0.0027				0.2887	0.2871	0.2794	0.0000
TWFE	**5**	**800**	**Mean**	0.4992	0.4847	0.5113	0.5019	0.5043				0.5075	0.5033	0.4816	0.4904
			Std	0.2925	0.2860	0.2911	0.2928	0.2850				0.2879	0.2836	0.2983	0.2978
PWFE	**5**	**800**	**Mean**	0.4934	0.4836	0.4595	0.4920	0.4238	0.4430	0.4756	0.4920	0.4945	0.4862	0.4472	0.4700
			Std	0.2893	0.2884	0.2910	0.2928	0.2939	0.3176	0.3002	0.2926	0.2910	0.2886	0.3041	0.3077
				Inverse distance: scalar normalized											
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(-0.25)**	α_0**(2)**	α_1**(1.5)**	α_2**(3)**	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.2478	0.3562	0.2207	0.0153	0.0017				0.5231	0.5085	0.3083	0.0705
			Std	0.2828	0.2964	0.2827	0.0663	0.0212				0.2792	0.2807	0.3066	0.1708
TWFE	**5**	**200**	**Mean**	0.4902	0.4993	0.4986	0.5131	0.4986				0.5154	0.5123	0.4671	0.5045
			Std	0.2883	0.2816	0.2913	0.2884	0.2924				0.2857	0.2802	0.2956	0.2928
PWFE	**5**	**200**	**Mean**	0.4857	0.4911	0.5243	0.5095	0.6610	0.7079	0.5240	0.5154	0.4919	0.4937	0.4996	0.4925
			Std	0.2851	0.2784	0.2885	0.2920	0.2251	0.2314	0.2887	0.2833	0.2919	0.2834	0.3012	0.2976
WFE	**5**	**800**	**Mean**	0.4277	0.4014	0.0575	0.0000	0.0011				0.5069	0.4905	0.2132	0.0000
			Std	0.2901	0.3036	0.1736	0.0000	0.0161				0.2911	0.2852	0.2775	0.0000
TWFE	**5**	**800**	**Mean**	0.4998	0.4831	0.5102	0.4961	0.5093				0.5063	0.5015	0.4840	0.4872
			Std	0.2928	0.2848	0.2891	0.2899	0.2907				0.2885	0.2843	0.2995	0.2962
PWFE	**5**	**800**	**Mean**	0.4980	0.4826	0.4838	0.4936	0.7425	0.7977	0.4945	0.4895	0.4945	0.4858	0.4790	0.4972
			Std	0.2891	0.2866	0.2898	0.2959	0.1918	0.1508	0.2885	0.2836	0.2911	0.2887	0.2939	0.2959

Table B.13: Case III: Mean and standard deviation of the p-values of the parameters (Table B.2) and the direct (DE) and indirect (IE) effects of variables x_1 and x_2 (Table B.8, lower panel)

Settings	T	N		β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.01)	α_0(2)	α_1(1.5)	α_2(3)	DE_β_1	DE_β_2	IE_β_1	IE_β_2
				Negative exponential: row normalized											
WFE	**5**	**200**	**Mean**	0.1030	0.3359	0.0023	0.0001	0.0011				0.5659	0.5289	0.1224	0.0062
			Std	0.1687	0.2935	0.0206	0.0010	0.0083				0.2676	0.2756	0.2075	0.0277
TWFE	**5**	**200**	**Mean**	0.4891	0.4978	0.5108	0.5093	0.4943				0.5451	0.5202	0.4833	0.4993
			Std	0.2868	0.2816	0.2876	0.2848	0.2905				0.2749	0.2777	0.2977	0.2926
PWFE	**5**	**200**	**Mean**	0.4632	0.4926	0.4864	0.5036	0.4197	0.6395	0.4655	0.4915	0.4928	0.4935	0.5015	0.5090
			Std	0.2929	0.2897	0.2899	0.2863	0.2517	0.3809	0.2902	0.2844	0.2917	0.2844	0.2838	0.2907
WFE	**5**	**800**	**Mean**	0.0172	0.1409	0.0000	0.0000	0.0000				0.5596	0.5352	0.0008	0.0000
			Std	0.0486	0.2127	0.0000	0.0000	0.0000				0.2737	0.2698	0.0074	0.0000
TWFE	**5**	**800**	**Mean**	0.4785	0.5030	0.5096	0.5070	0.5057				0.5426	0.5252	0.4779	0.4942
			Std	0.2877	0.2869	0.2822	0.2845	0.2908				0.2808	0.2723	0.2913	0.2879
PWFE	**5**	**800**	**Mean**	0.4664	0.4948	0.4881	0.4843	0.4633	0.6637	0.4549	0.4989	0.4958	0.5011	0.5262	0.4961
			Std	0.2944	0.2878	0.2902	0.2904	0.2585	0.3637	0.2900	0.2987	0.2956	0.2783	0.2863	0.2851
				Negative exponential: scalar normalized											
	T	**N**		β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.01)	α_0(2)	α_1(1.5)	α_2(3)	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.1529	0.3794	0.0075	0.0004	0.0042				0.5546	0.5233	0.1437	0.0100
			Std	0.2154	0.2999	0.0474	0.0029	0.0317				0.2702	0.2758	0.2321	0.0475
TWFE	**5**	**200**	**Mean**	0.4882	0.4962	0.5074	0.5131	0.4957				0.5412	0.5174	0.4783	0.5051
			Std	0.2847	0.2821	0.2920	0.2888	0.2905				0.2765	0.2784	0.2966	0.2946
PWFE	**5**	**200**	**Mean**	0.4763	0.4934	0.5292	0.5090	0.5123	0.7418	0.5632	0.5200	0.4928	0.4934	0.5120	0.5134
			Std	0.2814	0.2839	0.2681	0.2858	0.2498	0.2739	0.2675	0.2766	0.2915	0.2845	0.2705	0.2907
WFE	**5**	**800**	**Mean**	0.0358	0.1633	0.0000	0.0000	0.0000				0.5558	0.5330	0.0020	0.0000
			Std	0.0897	0.2264	0.0000	0.0000	0.0001				0.2749	0.2698	0.0150	0.0000
TWFE	**5**	**800**	**Mean**	0.4813	0.5031	0.5091	0.5065	0.5034				0.5386	0.5241	0.4781	0.4918
			Std	0.2904	0.2862	0.2826	0.2871	0.2933				0.2821	0.2726	0.2900	0.2872
PWFE	**5**	**800**	**Mean**	0.4772	0.4974	0.5435	0.4894	0.5522	0.7590	0.5517	0.5184	0.4957	0.5010	0.5431	0.5003
			Std	0.2873	0.2867	0.2725	0.2897	0.2598	0.2517	0.2715	0.2953	0.2956	0.2782	0.2662	0.2869
				Inverse distance: row normalized											
	T	**N**		β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.01)	α_0(2)	α_1(1.5)	α_2(3)	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.2742	0.3333	0.2419	0.0037	0.0046				0.5171	0.5142	0.3119	0.0362
			Std	0.2903	0.2904	0.2801	0.0311	0.0361				0.2807	0.2779	0.3121	0.1019
TWFE	**5**	**200**	**Mean**	0.4898	0.4994	0.5015	0.5090	0.4970				0.5058	0.5117	0.4614	0.4983
			Std	0.2891	0.2804	0.2889	0.2843	0.2894				0.2876	0.2801	0.2943	0.2946
PWFE	**5**	**200**	**Mean**	0.4726	0.4891	0.4829	0.4873	0.3925	0.5858	0.4634	0.4852	0.4906	0.4942	0.4824	0.4790
			Std	0.2922	0.2846	0.2982	0.2955	0.2624	0.3637	0.3056	0.2991	0.2906	0.2828	0.3118	0.3047
WFE	**5**	**800**	**Mean**	0.3955	0.3994	0.0044	0.0000	0.0136				0.5128	0.5160	0.1153	0.0000
			Std	0.3005	0.3003	0.0487	0.0000	0.0859				0.2898	0.2748	0.2225	0.0000
TWFE	**5**	**800**	**Mean**	0.4871	0.5017	0.4992	0.5069	0.5069				0.5017	0.5145	0.4610	0.4842
			Std	0.2920	0.2844	0.2808	0.2881	0.2870				0.2936	0.2749	0.2957	0.2941
PWFE	**5**	**800**	**Mean**	0.4823	0.4983	0.4999	0.4893	0.4719	0.6169	0.4842	0.5007	0.4956	0.5012	0.4946	0.4849
			Std	0.2905	0.2856	0.2918	0.2865	0.2703	0.3495	0.2893	0.2924	0.2948	0.2782	0.2989	0.2961
				Inverse distance: scalar normalized											
	T	**N**		β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.01)	α_0(2)	α_1(1.5)	α_2(3)	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.3133	0.3798	0.2543	0.0176	0.0113				0.5129	0.5118	0.3229	0.0648
			Std	0.3002	0.2974	0.2863	0.0707	0.0628				0.2818	0.2795	0.3058	0.1476
TWFE	**5**	**200**	**Mean**	0.4902	0.4990	0.4981	0.5122	0.4990				0.5060	0.5086	0.4611	0.5007
			Std	0.2882	0.2817	0.2914	0.2880	0.2930				0.2877	0.2811	0.2960	0.2922
PWFE	**5**	**200**	**Mean**	0.4854	0.4947	0.5392	0.5096	0.5180	0.7269	0.5470	0.5196	0.4910	0.4944	0.5181	0.4927
			Std	0.2873	0.2840	0.2787	0.2942	0.2316	0.2462	0.2740	0.2845	0.2899	0.2836	0.2977	0.3014
WFE	**5**	**800**	**Mean**	0.4065	0.4035	0.0142	0.0000	0.0191				0.5114	0.5147	0.1512	0.0000
			Std	0.2996	0.3013	0.0797	0.0000	0.1081				0.2906	0.2751	0.2512	0.0000
TWFE	**5**	**800**	**Mean**	0.4874	0.5026	0.4969	0.5048	0.5006				0.5014	0.5134	0.4604	0.4819
			Std	0.2919	0.2846	0.2790	0.2904	0.2861				0.2937	0.2752	0.2952	0.2985
PWFE	**5**	**800**	**Mean**	0.4872	0.4990	0.5236	0.4866	0.5669	0.7440	0.5224	0.4972	0.4961	0.5012	0.5111	0.4927
			Std	0.2912	0.2862	0.2818	0.2852	0.2308	0.2515	0.2837	0.2857	0.2952	0.2782	0.2883	0.2898

Table B.14: Case IV: Mean and standard deviation of the p-values of the parameters (Table B.3) and the direct (DE) and indirect (IE) effects of variables x_1 and x_2 (Table B.10, upper panel)

Settings				Negative exponential: row normalized											
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(0.5)**	α_0**(2)**	α_1**(1.5)**	α_2**(1)**	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.3890	0.4813	0.0058	0.3948	0.3726				0.5144	0.4842	0.0137	0.4568
			Std	0.3007	0.2887	0.0327	0.3000	0.3066				0.2866	0.2866	0.0413	0.3040
TWFE	**5**	**200**	**Mean**	0.5075	0.4934	0.4915	0.4982	0.4965				0.5261	0.4900	0.4583	0.4965
			Std	0.2834	0.2915	0.2905	0.2961	0.2949				0.2794	0.2892	0.3013	0.3019
PWFE	**5**	**200**	**Mean**	0.5002	0.4943	0.4661	0.4787	0.4476	0.4709	0.4621	0.4797	0.5066	0.4895	0.4649	0.4726
			Std	0.2936	0.2936	0.2902	0.2951	0.3059	0.2886	0.3005	0.2959	0.2866	0.2903	0.2909	0.2965
WFE	**5**	**800**	**Mean**	0.2468	0.5029	0.0000	0.3491	0.1681				0.5026	0.5119	0.0000	0.4677
			Std	0.2776	0.2922	0.0002	0.2950	0.2405				0.2916	0.2914	0.0000	0.3043
TFE	**5**	**800**	**Mean**	0.4998	0.5119	0.5024	0.5131	0.4920				0.5123	0.5142	0.4540	0.5020
			Std	0.2942	0.2908	0.2977	0.2955	0.2874				0.2850	0.2873	0.2904	0.2943
PFE	**5**	**800**	**Mean**	0.4940	0.5107	0.4977	0.5035	0.4643	0.4885	0.5024	0.5125	0.4970	0.5124	0.4886	0.5083
			Std	0.2869	0.2911	0.2925	0.2932	0.3011	0.2955	0.2875	0.2991	0.2893	0.2868	0.2962	0.2967
				Negative exponential: scalar normalized											
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(0.5)**	α_0**(2)**	α_1**(1.5)**	α_2**(1)**	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.4065	0.4876	0.0107	0.3577	0.4072				0.5025	0.4951	0.0242	0.4222
			Std	0.3030	0.2850	0.0443	0.2948	0.3120				0.2915	0.2864	0.0640	0.3041
TWFE	**5**	**200**	**Mean**	0.4891	0.4955	0.5050	0.5214	0.4934				0.5075	0.4969	0.4716	0.5130
			Std	0.2875	0.2839	0.2931	0.2972	0.2956				0.2844	0.2853	0.2959	0.2997
PWFE	**5**	**200**	**Mean**	0.4820	0.4944	0.4984	0.4980	0.5494	0.5798	0.4895	0.4991	0.4927	0.4957	0.4758	0.4918
			Std	0.2872	0.2844	0.2860	0.2885	0.2711	0.2616	0.2947	0.2881	0.2901	0.2861	0.2889	0.2955
WFE	**5**	**800**	**Mean**	0.3285	0.4880	0.0000	0.2650	0.0746				0.5013	0.4859	0.0000	0.4354
			Std	0.2916	0.2866	0.0000	0.2773	0.1645				0.2880	0.2867	0.0000	0.3056
TFE	**5**	**800**	**Mean**	0.5002	0.4845	0.5013	0.4934	0.5037				0.5138	0.4867	0.4759	0.4865
			Std	0.2894	0.2865	0.2887	0.2910	0.2884				0.2946	0.2853	0.2939	0.2902
PFE	**5**	**800**	**Mean**	0.4993	0.4851	0.5088	0.4897	0.5780	0.5945	0.5004	0.5043	0.5001	0.4868	0.5053	0.4862
			Std	0.2906	0.2875	0.2849	0.2896	0.2632	0.2515	0.2930	0.2906	0.2985	0.2848	0.2936	0.2921
				Inverse distance: row normalized											
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(0.5)**	α_0**(2)**	α_1**(1.5)**	α_2**(1)**	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.4826	0.4931	0.1389	0.2277	0.3644				0.4750	0.4868	0.1607	0.2866
			Std	0.2950	0.2934	0.2200	0.2649	0.3187				0.2921	0.2962	0.1936	0.2815
TWFE	**5**	**200**	**Mean**	0.5086	0.4929	0.4904	0.4906	0.4818				0.5060	0.4918	0.4417	0.4995
			Std	0.2840	0.2906	0.2945	0.2881	0.2922				0.2838	0.2925	0.3039	0.2856
PWFE	**5**	**200**	**Mean**	0.4975	0.4904	0.4368	0.4498	0.3777	0.5165	0.4401	0.5226	0.4965	0.4899	0.4340	0.4484
			Std	0.2866	0.2906	0.3074	0.3060	0.3069	0.2619	0.3018	0.2701	0.2883	0.2918	0.3174	0.3149
WFE	**5**	**800**	**Mean**	0.4877	0.5086	0.0481	0.1867	0.1143				0.4893	0.5084	0.0468	0.2877
			Std	0.2942	0.2895	0.1163	0.2442	0.2097				0.2936	0.2908	0.0798	0.2832
TFE	**5**	**800**	**Mean**	0.5004	0.5138	0.4879	0.4880	0.4950				0.4990	0.5169	0.4366	0.4779
			Std	0.2931	0.2907	0.2957	0.2804	0.2915				0.2905	0.2906	0.3038	0.2817
PFE	**5**	**800**	**Mean**	0.5015	0.5136	0.4913	0.4528	0.4183	0.4791	0.4859	0.5014	0.4999	0.5161	0.4774	0.4486
			Std	0.2932	0.2904	0.2954	0.3107	0.3026	0.2843	0.2916	0.2869	0.2913	0.2903	0.3055	0.3186
				Inverse distance: scalar normalized											
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(0.5)**	α_0**(2)**	α_1**(1.5)**	α_2**(1)**	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.4645	0.4967	0.1438	0.2384	0.4263				0.4708	0.4954	0.1614	0.2901
			Std	0.2929	0.2822	0.2198	0.2599	0.3154				0.2930	0.2853	0.1930	0.2704
TWFE	**5**	**200**	**Mean**	0.4911	0.4950	0.5032	0.5054	0.5033				0.4955	0.4949	0.4672	0.5199
			Std	0.2891	0.2839	0.2906	0.2853	0.2949				0.2898	0.2850	0.2934	0.2849
PWFE	**5**	**200**	**Mean**	0.4898	0.4951	0.5047	0.5041	0.6102	0.6495	0.5008	0.5550	0.4913	0.4955	0.4866	0.4830
			Std	0.2886	0.2837	0.2849	0.2853	0.2399	0.2309	0.2859	0.2430	0.2888	0.2844	0.2931	0.2957
WFE	**5**	**800**	**Mean**	0.4739	0.4843	0.0918	0.2267	0.1570				0.4829	0.4840	0.0679	0.3303
			Std	0.2921	0.2879	0.1774	0.2611	0.2412				0.2948	0.2890	0.1046	0.2937
TFE	**5**	**800**	**Mean**	0.4973	0.4864	0.5041	0.4826	0.5082				0.4987	0.4858	0.4658	0.4781
			Std	0.2938	0.2889	0.2877	0.2853	0.2932				0.2963	0.2874	0.3038	0.2897
PFE	**5**	**800**	**Mean**	0.4960	0.4860	0.4873	0.4953	0.6594	0.7216	0.4959	0.5401	0.4970	0.4856	0.4781	0.4785
			Std	0.2918	0.2891	0.2866	0.2853	0.2292	0.1844	0.2847	0.2713	0.2941	0.2878	0.2954	0.2911

Table B.15: Case V: Mean and standard deviation of the p-values of the parameters (Table B.4) and the direct (DE) and indirect (IE) effects of variables x_1 and x_2 (Table B.9, upper panel)

Settings	T	N		β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.5)	α_0(2)	α_1(1.5)	α_2(0.5)	DE_β_1	DE_β_2	IE_β_1	IE_β_2
				Negative exponential: row normalized											
WFE	**5**	**200**	**Mean**	0.4260	0.4995	0.0038	0.0004	0.2547				0.5017	0.5004	0.0078	0.0018
			Std	0.3026	0.2812	0.0192	0.0031	0.2777				0.2892	0.2846	0.0217	0.0104
TWFE	**5**	**200**	**Mean**	0.4913	0.4940	0.5085	0.5170	0.4981				0.5075	0.4957	0.4839	0.5133
			Std	0.2897	0.2837	0.2882	0.2901	0.2942				0.2831	0.2856	0.2965	0.2941
textbfPWFE	**5**	**200**	**Mean**	0.4791	0.4932	0.4890	0.4799	0.4447	0.4970	0.4838	0.4769	0.4889	0.4957	0.4736	0.4797
			Std	0.2917	0.2848	0.2944	0.2999	0.2998	0.2785	0.2980	0.3026	0.2880	0.2860	0.2835	0.3028
TWFE	**5**	**800**	**Mean**	0.2768	0.5029	0.0000	0.0000	0.1014				0.5030	0.5072	0.0000	0.0000
			Std	0.2862	0.2822	0.0001	0.0000	0.1896				0.2859	0.2852	0.0000	0.0000
WFE	**5**	**800**	**Mean**	0.4809	0.5009	0.5107	0.4934	0.5085				0.5099	0.5033	0.4829	0.4871
			Std	0.2878	0.2783	0.2840	0.2913	0.2912				0.2846	0.2842	0.2901	0.2963
PWFE	**5**	**800**	**Mean**	0.4784	0.5016	0.4888	0.4971	0.4689	0.4863	0.4711	0.4962	0.4948	0.5054	0.4991	0.4974
			Std	0.2909	0.2791	0.2870	0.2896	0.2939	0.2976	0.2881	0.2997	0.2885	0.2861	0.2914	0.2894
				Negative exponential: scalar normalized											
	T	**N**		β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.5)	α_0(2)	α_1(1.5)	α_2(0.5)	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.4348	0.4989	0.0074	0.0012	0.2879				0.5009	0.4988	0.0138	0.0034
			Std	0.2997	0.2823	0.0326	0.0073	0.2963				0.2914	0.2857	0.0375	0.0156
TWFE	**5**	**200**	**Mean**	0.4899	0.4940	0.5049	0.5158	0.4946				0.5061	0.4950	0.4763	0.5106
			Std	0.2885	0.2841	0.2918	0.2908	0.2941				0.2844	0.2854	0.2977	0.2919
PWFE	**5**	**200**	**Mean**	0.4838	0.4934	0.5013	0.5095	0.5670	0.5957	0.4981	0.5351	0.4917	0.4947	0.4794	0.4986
			Std	0.2877	0.2849	0.2864	0.2935	0.2660	0.2512	0.2942	0.2846	0.2888	0.2859	0.2847	0.2983
WFE	**5**	**800**	**Mean**	0.3225	0.5000	0.0000	0.0000	0.0687				0.5014	0.5059	0.0000	0.0000
			Std	0.2973	0.2829	0.0001	0.0000	0.1569				0.2892	0.2850	0.0001	0.0000
TWFE	**5**	**800**	**Mean**	0.4833	0.5007	0.5106	0.4805	0.5104				0.5067	0.5027	0.4827	0.4711
			Std	0.2893	0.2781	0.2840	0.2868	0.2945				0.2857	0.2825	0.2950	0.2859
PWFE	**5**	**800**	**Mean**	0.4818	0.5013	0.5128	0.5133	0.5852	0.6160	0.4799	0.5086	0.4945	0.5045	0.4999	0.5083
			Std	0.2894	0.2788	0.2853	0.2958	0.2610	0.2523	0.2829	0.2921	0.2890	0.2843	0.2884	0.2952
				Inverse distance: row normalized											
	T	**N**		β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.5)	α_0(2)	α_1(1.5)	α_2(0.5)	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.4666	0.4952	0.0728	0.0410	0.1874				0.4271	0.4913	0.0793	0.0776
			Std	0.2916	0.2846	0.1563	0.1115	0.2619				0.2929	0.2888	0.1220	0.1442
TWFE	**5**	**200**	**Mean**	0.4906	0.4947	0.5091	0.5015	0.5028				0.4940	0.4949	0.4700	0.5218
			Std	0.2890	0.2839	0.2868	0.2867	0.2906				0.2901	0.2847	0.2972	0.2810
PWFE	**5**	**200**	**Mean**	0.4889	0.4937	0.4579	0.4524	0.3896	0.5119	0.4724	0.6004	0.4900	0.4962	0.4496	0.4510
			Std	0.2920	0.2840	0.2968	0.2963	0.3076	0.2535	0.2977	0.2257	0.2892	0.2849	0.3096	0.3014
WFE	**5**	**800**	**Mean**	0.4753	0.4999	0.0343	0.0145	0.0816				0.4748	0.4955	0.0309	0.0415
			Std	0.2948	0.2836	0.0937	0.0517	0.1876				0.2988	0.2856	0.0509	0.0978
TWFE	**5**	**800**	**Mean**	0.4919	0.5007	0.4881	0.4931	0.4999				0.4952	0.5012	0.4437	0.4920
			Std	0.2942	0.2782	0.2770	0.2934	0.2853				0.2926	0.2808	0.2942	0.2925
PWFE	**5**	**800**	**Mean**	0.4902	0.5003	0.4811	0.4507	0.4360	0.4771	0.4867	0.5501	0.4949	0.5010	0.4738	0.4485
			Std	0.2934	0.2787	0.2922	0.3002	0.3032	0.2818	0.2948	0.2597	0.2917	0.2806	0.3017	0.3076
				Inverse distance: scalar normalized											
	T	**N**		β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.5)	α_0(2)	α_1(1.5)	α_2(0.5)	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.4696	0.4952	0.0960	0.0718	0.2504				0.4422	0.4939	0.0934	0.1126
			Std	0.2913	0.2836	0.1777	0.1544	0.2947				0.2920	0.2874	0.1287	0.1785
TWFE	**5**	**200**	**Mean**	0.4907	0.4945	0.5053	0.4989	0.5025				0.4955	0.4946	0.4653	0.5212
			Std	0.2885	0.2840	0.2899	0.2865	0.2949				0.2901	0.2847	0.2939	0.2791
PWFE	**5**	**200**	**Mean**	0.4916	0.4950	0.5056	0.5025	0.6088	0.6770	0.5040	0.6110	0.4923	0.4972	0.4868	0.4878
			Std	0.2889	0.2833	0.2871	0.2875	0.2448	0.2210	0.2853	0.2423	0.2897	0.2842	0.2958	0.2963
WFE	**5**	**800**	**Mean**	0.4792	0.5005	0.0568	0.0265	0.1092				0.4800	0.4973	0.0454	0.0616
			Std	0.2941	0.2821	0.1298	0.0806	0.2137				0.2999	0.2841	0.0736	0.1265
TWFE	**5**	**800**	**Mean**	0.4926	0.5005	0.4903	0.4920	0.5002				0.4953	0.5007	0.4410	0.4912
			Std	0.2945	0.2782	0.2813	0.2929	0.2860				0.2930	0.2805	0.2910	0.2905
PWFE	**5**	**800**	**Mean**	0.4928	0.5004	0.5047	0.4778	0.6748	0.7297	0.5095	0.5678	0.4959	0.4996	0.4983	0.4668
			Std	0.2945	0.2783	0.2845	0.2954	0.2188	0.1899	0.2893	0.2527	0.2925	0.2795	0.2886	0.3048

Table B.16: Case VI: Mean and standard deviation of the p-values of the parameters (Table B.5) and the direct (DE) and indirect (IE) effects of variables x_1 and x_2 (Table B.9, lower panel)

Settings				Negative exponential: row normalized											
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(0.5)**	α_0**(2)**	α_1**(10)**	α_2**(3)**	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.0000	0.0902	0.0000	0.0068	0.0000				0.6292	0.5651	0.0001	0.1873
			Std	0.0007	0.1538	0.0000	0.0524	0.0000				0.2400	0.2712	0.0012	0.2589
TWFE	**5**	**200**	**Mean**	0.5030	0.4936	0.4898	0.5015	0.4887				0.5752	0.5049	0.4524	0.4960
			Std	0.2883	0.2909	0.2832	0.2881	0.2935				0.2618	0.2859	0.2947	0.2901
PWFE	**5**	**200**	**Mean**	0.4984	0.4897	0.4966	0.4988	0.4602	0.4914	0.6584	0.5005	0.5079	0.4912	0.4912	0.4947
			Std	0.2937	0.2887	0.2902	0.2897	0.2980	0.2857	0.3677	0.2892	0.2850	0.2903	0.2919	0.2931
WFE	**5**	**800**	**Mean**	0.0000	0.0038	0.0000	0.0000	0.0000				0.3912	0.5419	0.0000	0.0031
			Std	0.0000	0.0163	0.0000	0.0000	0.0000				0.2745	0.2762	0.0000	0.0349
TWFE	**5**	**800**	**Mean**	0.4967	0.5049	0.5063	0.5052	0.4894				0.5640	0.5239	0.4679	0.4993
			Std	0.2921	0.2856	0.2867	0.2913	0.2869				0.2677	0.2856	0.2961	0.2932
PWFE	**5**	**800**	**Mean**	0.4854	0.5059	0.5040	0.5053	0.4647	0.4883	0.6272	0.5049	0.4954	0.5139	0.4783	0.5000
			Std	0.2897	0.2892	0.2869	0.2916	0.2973	0.2865	0.3100	0.2919	0.2882	0.2875	0.2923	0.2888
				Negative exponential: scalar normalized											
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(0.5)**	α_0**(2)**	α_1**(10)**	α_2**(3)**	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.0002	0.1309	0.0000	0.0112	0.0000				0.6233	0.5620	0.0002	0.1962
			Std	0.0024	0.1851	0.0000	0.0576	0.0000				0.2439	0.2742	0.0018	0.2629
TWFE	**5**	**200**	**Mean**	0.5077	0.4936	0.4929	0.5020	0.4881				0.5720	0.5023	0.4533	0.4933
			Std	0.2896	0.2928	0.2837	0.2865	0.2880				0.2614	0.2856	0.2964	0.2900
PWFE	**5**	**200**	**Mean**	0.5075	0.4915	0.5422	0.4967	0.5517	0.6579	0.6274	0.4951	0.5064	0.4906	0.5084	0.4931
			Std	0.2914	0.2909	0.2773	0.2817	0.2664	0.2400	0.3536	0.2924	0.2842	0.2891	0.2859	0.2850
WFE	**5**	**800**	**Mean**	0.0000	0.0053	0.0000	0.0000	0.0000				0.3986	0.5600	0.0000	0.0029
			Std	0.0000	0.0220	0.0000	0.0000	0.0000				0.2755	0.2741	0.0000	0.0263
TWFE	**5**	**800**	**Mean**	0.5007	0.5051	0.5073	0.5039	0.4818				0.5618	0.5236	0.4635	0.4989
			Std	0.2938	0.2860	0.2869	0.2878	0.2799				0.2681	0.2851	0.2985	0.2913
PWFE	**5**	**800**	**Mean**	0.5015	0.5069	0.5411	0.5027	0.5619	0.6859	0.6023	0.5028	0.4955	0.5145	0.4996	0.5001
			Std	0.2886	0.2889	0.2762	0.2929	0.2709	0.2150	0.2922	0.2898	0.2876	0.2872	0.2828	0.2864
				Inverse distance: row normalized											
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(0.5)**	α_0**(2)**	α_1**(10)**	α_2**(3)**	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.0002	0.1161	0.0000	0.1236	0.0000				0.5550	0.5506	0.0000	0.2198
			Std	0.0012	0.1802	0.0000	0.2394	0.0000				0.2630	0.2688	0.0004	0.2804
TWFE	**5**	**200**	**Mean**	0.5015	0.4951	0.4901	0.5005	0.4869				0.5757	0.5021	0.4603	0.4925
			Std	0.2858	0.2931	0.2831	0.2902	0.2967				0.2589	0.2893	0.3012	0.2901
PWFE	**5**	**200**	**Mean**	0.4968	0.4914	0.4945	0.4865	0.4422	0.5220	0.6074	0.4870	0.5050	0.4932	0.4421	0.4794
			Std	0.2928	0.2904	0.2922	0.2994	0.3067	0.2690	0.3154	0.2926	0.2833	0.2936	0.3081	0.3012
WFE	**5**	**800**	**Mean**	0.0000	0.0798	0.0000	0.0000	0.0000				0.0494	0.5424	0.0000	0.0300
			Std	0.0000	0.1527	0.0000	0.0001	0.0000				0.0829	0.2624	0.0000	0.1230
TWFE	**5**	**800**	**Mean**	0.4968	0.5070	0.5062	0.5062	0.4937				0.5724	0.5208	0.4681	0.4843
			Std	0.2916	0.2869	0.2884	0.2917	0.2946				0.2671	0.2883	0.2930	0.2969
PWFE	**5**	**800**	**Mean**	0.4804	0.5060	0.4984	0.5078	0.4565	0.4892	0.5284	0.5017	0.4975	0.5144	0.4655	0.5016
			Std	0.2880	0.2897	0.2883	0.2917	0.2939	0.2843	0.2701	0.2870	0.2897	0.2897	0.3058	0.2942
				Inverse distance: scalar normalized											
	T	**N**		β_1**(-1)**	β_2**(0.2)**	γ_1**(1.5)**	γ_2**(-0.3)**	ρ**(0.5)**	α_0**(2)**	α_1**(10)**	α_2**(3)**	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.0004	0.1559	0.0000	0.1549	0.0000				0.5174	0.5605	0.0000	0.2167
			Std	0.0031	0.2087	0.0000	0.2723	0.0000				0.2716	0.2646	0.0000	0.2821
TWFE	**5**	**200**	**Mean**	0.5064	0.4943	0.4935	0.4993	0.4891				0.5705	0.4995	0.4587	0.4890
			Std	0.2888	0.2936	0.2837	0.2871	0.2941				0.2604	0.2888	0.3006	0.2899
PWFE	**5**	**200**	**Mean**	0.5061	0.4921	0.5235	0.5156	0.5728	0.6738	0.5717	0.5154	0.5047	0.4926	0.5316	0.4904
			Std	0.2916	0.2925	0.2826	0.2892	0.2635	0.2338	0.3065	0.2863	0.2833	0.2916	0.2846	0.2993
WFE	**5**	**800**	**Mean**	0.0000	0.0878	0.0000	0.0005	0.0000				0.0903	0.5851	0.0000	0.0284
			Std	0.0000	0.1628	0.0000	0.0116	0.0000				0.1289	0.2571	0.0000	0.1264
TWFE	**5**	**800**	**Mean**	0.4971	0.5075	0.5072	0.5063	0.4880				0.5706	0.5206	0.4671	0.4854
			Std	0.2896	0.2870	0.2894	0.2912	0.2884				0.2677	0.2883	0.2985	0.2955
PWFE	**5**	**800**	**Mean**	0.5073	0.5092	0.5106	0.5041	0.6526	0.7727	0.5314	0.5017	0.4981	0.5152	0.6119	0.5161
			Std	0.2869	0.2898	0.2831	0.2901	0.2355	0.1688	0.2723	0.2878	0.2899	0.2901	0.2495	0.2854

Table B.17: Case VII: Mean and standard deviation of the p-values of the parameters (Table B.6) and the direct (DE) and indirect (IE) effects of variables x_1 and x_2 (Table B.10, lower panel)

Settings	T	N		β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.01)	α_0(2)	α_1(10)	α_2(3)	DE_β_1	DE_β_2	IE_β_1	IE_β_2
				Negative exponential: row normalized											
WFE	**5**	**200**	**Mean**	0.0000	0.0098	0.0000	0.0089	0.0000				0.6471	0.6124	0.0002	0.2848
			Std	0.0000	0.0322	0.0000	0.0475	0.0000				0.2274	0.2390	0.0039	0.2939
TWFE	**5**	**200**	**Mean**	0.4888	0.4984	0.5191	0.5116	0.4949				0.6135	0.5227	0.4710	0.5009
			Std	0.2850	0.2813	0.2929	0.2850	0.2885				0.2483	0.2774	0.3037	0.2898
PWFE	**5**	**200**	**Mean**	0.4553	0.4947	0.5230	0.5037	0.4537	0.6507	0.6726	0.5066	0.4933	0.4933	0.4950	0.5146
			Std	0.2883	0.2846	0.2895	0.2860	0.2506	0.3856	0.3607	0.2844	0.2915	0.2840	0.2834	0.2883
WFE	**5**	**800**	**Mean**	0.0000	0.0000	0.0000	0.0000	0.0000				0.3785	0.5986	0.0000	0.0215
			Std	0.0000	0.0001	0.0000	0.0000	0.0000				0.2544	0.2509	0.0000	0.0823
TWFE	**5**	**800**	**Mean**	0.5124	0.4814	0.5107	0.5047	0.5108				0.6126	0.5100	0.4555	0.4960
			Std	0.2856	0.2793	0.2879	0.2926	0.2830				0.2501	0.2821	0.2969	0.2936
PWFE	**5**	**800**	**Mean**	0.4843	0.4746	0.5113	0.4970	0.4938	0.6470	0.6521	0.4984	0.4952	0.4864	0.5169	0.5225
			Std	0.2979	0.2850	0.2869	0.2928	0.2581	0.3790	0.3063	0.2930	0.2926	0.2886	0.2733	0.2935
				Negative exponential: scalar normalized											
	T	**N**		β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.01)	α_0(2)	α_1(10)	α_2(3)	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.0000	0.0201	0.0000	0.0198	0.0000				0.6049	0.6075	0.0005	0.2644
			Std	0.0000	0.0601	0.0000	0.0876	0.0000				0.2433	0.2439	0.0078	0.2862
TWFE	**5**	**200**	**Mean**	0.4882	0.4954	0.5157	0.5148	0.5015				0.6141	0.5202	0.4672	0.5070
			Std	0.2776	0.2816	0.2949	0.2889	0.2874				0.2483	0.2777	0.3043	0.2944
PWFE	**5**	**200**	**Mean**	0.4658	0.4904	0.5213	0.5104	0.5121	0.7733	0.6687	0.5253	0.4930	0.4934	0.5145	0.5152
			Std	0.2835	0.2834	0.2854	0.2813	0.2464	0.2661	0.3305	0.2741	0.2910	0.2840	0.2851	0.2885
WFE	**5**	**800**	**Mean**	0.0000	0.0000	0.0000	0.0000	0.0000				0.3874	0.5921	0.0000	0.0348
			Std	0.0000	0.0001	0.0000	0.0000	0.0000				0.2578	0.2516	0.0000	0.1180
TWFE	**5**	**800**	**Mean**	0.5099	0.4808	0.5110	0.4992	0.5100				0.6109	0.5090	0.4588	0.4907
			Std	0.2837	0.2801	0.2873	0.2910	0.2826				0.2512	0.2822	0.2987	0.2920
PWFE	**5**	**800**	**Mean**	0.4903	0.4774	0.5166	0.4969	0.5469	0.7763	0.6438	0.5121	0.4955	0.4862	0.5317	0.5093
			Std	0.2964	0.2851	0.2853	0.2865	0.2516	0.2497	0.2767	0.2888	0.2925	0.2885	0.2721	0.2863
				Inverse distance: row normalized											
	T	**N**		β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.01)	α_0(2)	α_1(10)	α_2(3)	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.0001	0.0348	0.0000	0.1196	0.0000				0.3865	0.4658	0.0000	0.2088
			Std	0.0025	0.0949	0.0000	0.2376	0.0000				0.2649	0.2807	0.0000	0.2836
TWFE	**5**	**200**	**Mean**	0.4931	0.4985	0.5157	0.5130	0.4959				0.6035	0.5120	0.4504	0.5014
			Std	0.2850	0.2803	0.2912	0.2859	0.2873				0.2528	0.2801	0.3024	0.2951
PWFE	**5**	**200**	**Mean**	0.4426	0.4916	0.5109	0.4758	0.4126	0.6216	0.6116	0.4799	0.4939	0.4940	0.4492	0.4783
			Std	0.2882	0.2828	0.2903	0.2882	0.2571	0.3505	0.3193	0.2961	0.2914	0.2832	0.3017	0.2967
WFE	**5**	**800**	**Mean**	0.0001	0.0327	0.0000	0.0014	0.0000				0.1089	0.5835	0.0000	0.0710
			Std	0.0023	0.1074	0.0000	0.0293	0.0000				0.1610	0.2525	0.0000	0.1897
TWFE	**5**	**800**	**Mean**	0.5083	0.4815	0.5132	0.5020	0.5130				0.5998	0.4983	0.4558	0.4890
			Std	0.2857	0.2805	0.2879	0.2920	0.2816				0.2558	0.2853	0.2927	0.2926
PWFE	**5**	**800**	**Mean**	0.4735	0.4797	0.4961	0.4923	0.4613	0.6123	0.5526	0.4974	0.4951	0.4867	0.4848	0.4866
			Std	0.2959	0.2860	0.2899	0.2963	0.2734	0.3701	0.2673	0.2950	0.2924	0.2890	0.2916	0.2946
				Inverse distance: scalar normalized											
	T	**N**		β_1(-1)	β_2(0.2)	γ_1(1.5)	γ_2(-0.3)	ρ(0.01)	α_0(2)	α_1(10)	α_2(3)	DE_β_1	DE_β_2	IE_β_1	IE_β_2
WFE	**5**	**200**	**Mean**	0.0002	0.0559	0.0000	0.1568	0.0000				0.3774	0.4998	0.0000	0.2103
			Std	0.0045	0.1262	0.0000	0.2670	0.0000				0.2679	0.2816	0.0001	0.2899
TWFE	**5**	**200**	**Mean**	0.4918	0.4964	0.5108	0.5155	0.4961				0.6007	0.5094	0.4439	0.5057
			Std	0.2818	0.2819	0.2940	0.2898	0.2852				0.2541	0.2807	0.3018	0.2975
PWFE	**5**	**200**	**Mean**	0.4640	0.4923	0.5174	0.5144	0.5234	0.7627	0.6103	0.5131	0.4931	0.4942	0.5233	0.4974
			Std	0.2839	0.2843	0.2841	0.2877	0.2360	0.2389	0.2899	0.2722	0.2912	0.2842	0.2806	0.2976
WFE	**5**	**800**	**Mean**	0.0004	0.0551	0.0000	0.0036	0.0000				0.1651	0.5748	0.0000	0.0763
			Std	0.0107	0.1410	0.0000	0.0441	0.0000				0.2048	0.2544	0.0000	0.1908
TWFE	**5**	**800**	**Mean**	0.5101	0.4802	0.5127	0.4971	0.5108				0.5983	0.4974	0.4541	0.4885
			Std	0.2870	0.2804	0.2873	0.2908	0.2815				0.2567	0.2855	0.2945	0.2973
PWFE	**5**	**800**	**Mean**	0.4854	0.4771	0.5063	0.4964	0.5848	0.7724	0.5525	0.4934	0.4955	0.4866	0.5848	0.4953
			Std	0.2914	0.2836	0.2852	0.2977	0.2276	0.2589	0.2587	0.2826	0.2927	0.2891	0.2537	0.2913

Table B.18: Case VIII: Mean and standard deviation of the p-values of the parameters (Table B.7) and the direct (DE) and indirect (IE) effects of variables x_1 and x_2 (Table B.11)

		One common decay parameter α			
	1	2	3	4	5
Determinants/W	YE	Ex_rn	Ex_sn	ID_rn	ID_sn
ρ	0.241***	0.257***	0.555***	0.231***	0.454***
	(8.39)	(5.74)	(8.40)	(3.77)	(5.04)
β_1 GDP	-0.530***	-0.513***	-0.506***	-0.534***	-0.553***
	(-5.57)	(-5.33)	(-5.30)	(-5.55)	(-5.70)
β_2 Population	1.184***	1.213***	0.552	0.816**	0.875**
	(3.18)	(2.93)	(1.40)	(2.07)	(2.27)
β_3 International War	0.073*	0.085**	0.079*	0.074*	0.068
	(1.72)	(1.99)	(1.83)	(1.74)	(1.59)
β_4 Civil War	0.006	0.007	0.010	0.006	0.002
	(0.37)	(0.47)	(0.65)	(0.37)	(0.14)
β_5 Political Regime	-0.016***	-0.018***	-0.017***	-0.017***	-0.018***
	(-3.28)	(-3.55)	(-3.42)	(-3.50)	(-3.46)
γ_1 W(α)*GDP	0.085	-0.041	0.116	0.219	0.712***
	(0.53)	(-0.20)	(0.53)	(1.09)	(2.68)
γ_2 W(α)*Population	-0.517	-1.170	3.930*	0.544	-1.399
	(-0.84)	(-1.55)	(1.74)	(0.63)	(-0.31)
γ_3 W(α)*International War	-0.055	-0.109	-0.289	-0.080	-0.221
	(-0.70)	(-1.14)	(-1.15)	(-0.92)	(-0.43)
γ_4 W(α)*Civil War	-0.029	-0.081*	-0.097	-0.050	0.045
	(-1.03)	(-1.81)	(-0.89)	(-1.39)	(0.46)
γ_5 W(α)*Political Regime	-0.019**	0.000	-0.011	-0.016	-0.022
	(-2.06)	(0.02)	(-0.36)	(-1.37)	(-0.63)
α (Distance decay)		2.022***	2.305***	2.113***	0.766***
		(4.03)	(5.35)	(4.48)	(7.89)
DE_1 GDP	-0.535***	-0.522***	-0.510***	-0.529***	-0.548***
	(-5.37)	(-5.41)	(-5.28)	(-5.50)	(-5.62)
DE_2 Population	1.164***	1.171***	0.649*	0.852**	0.863**
	(3.11)	(2.91)	(1.69)	(2.22)	(2.30)
DE_3 International War	0.072*	0.080*	0.073*	0.071*	0.066
	(1.64)	(1.89)	(1.72)	(1.66)	(1.55)
DE_4 Civil War	0.004	0.003	0.008	0.003	0.003
	(0.26)	(0.21)	(0.50)	(0.21)	(0.17)
DE_5 Political Regime	-0.017***	-0.018***	-0.018***	-0.018***	-0.018***
	(-3.44)	(-3.52)	(-3.43)	(-3.61)	(-3.48)
IE_1 GDP	-0.048	-0.224	-0.119	0.120	0.729*
	(-0.24)	(-0.86)	(-0.72)	(0.49)	(1.60)
IE_2 Population	-0.273	-1.113	3.064*	0.916	-1.584
	(-0.36)	(-1.21)	(1.89)	(0.86)	(-0.23)
IE_3 International War	-0.050	-0.113	-0.177	-0.078	-0.300
	(-0.50)	(-0.92)	(-0.98)	(-0.73)	(-0.37)
IE_4 Civil War	-0.034	-0.103*	-0.066	-0.061	0.073
	(-0.97)	(-1.72)	(-0.83)	(-1.31)	(0.46)
IE_5 Political Regime	-0.029**	-0.006	-0.015	-0.025*	-0.047
	(-2.50)	(-0.35)	(-0.66)	(-1.57)	(-0.81)
Observations	2160	2160	2160	2160	2160
Log-likelihood function value	-1311.39	-1313.66	-1316.37	-1319.19	-1333.49
R-squared	0.702	0.702	0.701	0.700	0.694
H0 (SAR): γ=0 (p-value)	7.85 (0.16)	7.23 (0.20)	6.77 (0.24)	4.84 (0.44)	7.70 (0.17)
H0 (SEM): $\rho\beta_k \boldsymbol{W} + \gamma_k \boldsymbol{W} = \boldsymbol{0}_N$ (p-value)	7.39 (0.19)	5.79 (0.33)	5.37 (0.37)	4.29 (0.51)	3.27 (0.66)

Notes: YE=Yesilurt and Elhorst (2017), Ex=Negative exponential distance decay, ID=Inverse distance decay, rn=row normalized, sn=scalar normalized; *,**,*** significant at respectively 10%, 5% and 1%.

Table C.1: Military expenditures according to one common binary contiguity matrix (YE) or one common parameterized weight matrix (R results)

Determinants/W	1 YE	2 PWFE	3 EWFE	4 EWFEBC
ρ	0.241***	0.251***	0.251***	0.247***
	(8.69)	(5.29)	(7.93)	(7.80)
β_1 GDP	-0.530***	-0.487***	-0.487***	-0.486***
	(-5.57)	(-5.04)	(-5.07)	(-5.08)
β_2 Population	1.184***	1.204***	1.205***	1.171***
	(3.18)	(2.86)	(2.98)	(2.90)
β_3 International War	0.073*	0.084**	0.084**	0.077*
	(1.72)	(1.98)	(1.99)	(1.81)
β_4 Civil War	0.006	0.010	0.010	0.007
	(0.37)	(0.66)	(0.67)	(0.47)
β_5 Political Regime	-0.016***	-0.018***	-0.018***	-0.017***
	(-3.28)	(-3.58)	(-3.61)	(-3.50)
γ_1 W(α_1)*GDP	0.085	-0.944	-0.944*	-0.951*
	(0.53)	(-0.76)	(-1.77)	(-1.78)
γ_2 W(α_2)*Population	-0.517	-1.466	-1.466*	-1.389*
	(-0.84)	(-1.43)	(-1.81)	(-1.71)
γ_3 W(α_3)*International War	-0.055	-0.066	-0.066	-0.032
	(-0.70)	(-1.17)	(-1.46)	(-0.41)
γ_4 W(α_4)*Civil War	-0.029	-0.078	-0.078**	-0.091**
	(-1.03)	(-1.27)	(-2.06)	(-2.42)
γ_5 W(α_5)*Political Regime	-0.019**	-0.005	-0.005	-0.023**
	(-2.06)	(-0.59)	(-0.77)	(-2.44)
α_0 (Spatially lagged dependent variable)		2.003***		
		(3.56)		
α_1		0.410		
		(0.74)		
α_2		1.485		
		(1.01)		
α_3		10.00		
		(0.45)		
α_4		2.321		
		(1.12)		
α_5		10.00		
		(0.25)		
DE_1 GDP	-0.537***	-0.505***	-0.503***	-0.497***
	(-5.59)	(-5.22)	(-5.44)	(-5.22)
DE_2 Population	1.167***	1.160***	1.149***	1.136***
	(3.31)	(2.84)	(2.82)	(2.97)
DE_3 International War	0.071*	0.080*	0.081*	0.076*
	(1.76)	(1.89)	(1.96)	(1.76)
DE_4 Civil War	0.004	0.006	0.006	0.002
	(0.25)	(0.40)	(0.43)	(0.16)
DE_5 Political Regime	-0.018***	-0.019***	-0.018***	-0.019***
	(-3.42)	(-3.61)	(-3.65)	(-3.87)
IE_1 GDP	-0.039	-1.406	-1.400**	-1.404**
	(-0.19)	(-0.85)	(-1.98)	(-2.01)
IE_2 Population	-0.282	-1.510	-1.540	-1.400
	(-0.37)	(-1.14)	(-1.53)	(-1.46)
IE_3 International War	-0.046	-0.057	-0.057	-0.017
	(-0.50)	(-0.80)	(-0.99)	(-0.17)
IE_4 Civil War	-0.033	-0.097	-0.098**	-0.116**
	(-0.95)	(-1.20)	(-2.03)	(-2.36)
IE_5 Political Regime	-0.029**	-0.012	-0.012*	-0.035***
	(-2.46)	(-1.10)	(-1.39)	(-2.95)
Observations	2160	2160	2160	2160
Log-likelihood function value	-1311.39	-1311.02	-1311.02	-1309.23
R-squared	0.702	0.702	0.702	0.702
H0 (SAR): γ=0 (p-value)	7.85 (0.16)	7.79 (0.17)	13.21 (0.02)	16.53 (0.01)
H0 (SEM): $\rho\beta_k \boldsymbol{W} + \gamma_k \boldsymbol{W} = \boldsymbol{0}_N$ (p-value)	7.41 (0.19)	21.22 (0.00)	12.49 (0.03)	17.86 (0.00)

Notes: YE=one common binary contiguity matrix based on Yesilurt and Elhorst (2017), PWFE=Parameterized W matrices and fixed effects (FE), EWFE=Estimated W matrices and FE, EWFEBC=Estimated W or binary contiguity (BC) matrices and FE; *,**,*** significant at respectively 10%, 5% and 1%.

Table D.1: Military expenditures according to one common binary contiguity matrix and different distance decay matrices (Matlab2022a results)

		One common decay parameter α			
	1	2	3	4	5
Determinants/W	YE	Ex_rn	Ex_sn	ID_rn	ID_sn
ρ	0.241***	0.257***	0.555***	0.231***	0.454***
	(8.69)	(5.68)	(8.41)	(3.74)	(5.03)
β_1 GDP	-0.530***	-0.514***	-0.506***	-0.534***	-0.553***
	(-5.57)	(-5.33)	(-5.30)	(-5.55)	(-5.70)
β_2 Population	1.184***	1.213***	0.552	0.816**	0.875**
	(3.18)	(2.93)	(1.40)	(2.07)	(2.27)
β_3 International War	0.073*	0.085**	0.079*	0.074*	0.068
	(1.72)	(1.99)	(1.83)	(1.74)	(1.59)
β_4 Civil War	0.006	0.007	0.010	0.006	0.002
	(0.37)	(0.47)	(0.65)	(0.37)	(0.14)
β_5 Political Regime	-0.016***	-0.018***	-0.017***	-0.017***	-0.018***
	(-3.28)	(-3.55)	(-3.42)	(-3.50)	(-3.46)
γ_1 W(α)*GDP	0.085	-0.041	0.116	0.219	0.712***
	(0.53)	(-0.20)	(0.53)	(1.09)	(2.67)
γ_2 W(α)*Population	-0.517	-1.170	3.930*	0.544	-1.399
	(-0.84)	(-1.55)	(1.74)	(0.63)	(-0.31)
γ_3 W(α)*International War	-0.055	-0.109	-0.289	-0.080	-0.221
	(-0.70)	(-1.13)	(-1.15)	(-0.92)	(-0.43)
γ_4 W(α)*Civil War	-0.029	-0.081*	-0.097	-0.050	0.045
	(-1.03)	(-1.81)	(-0.89)	(-1.39)	(0.46)
γ_5 W(α)*Political Regime	-0.019**	0.000	-0.011	-0.016	-0.022
	(-2.06)	(0.02)	(-0.36)	(-1.37)	(-0.63)
α (Distance decay)		2.022***	2.305***	2.113***	0.766***
		(3.94)	(5.36)	(4.44)	(7.72)
DE_1 GDP	-0.537***	-0.522***	-0.510***	-0.529***	-0.548***
	(-5.59)	(-5.41)	(-5.28)	(-5.50)	(-5.62)
DE_2 Population	1.167***	1.172***	0.649*	0.852**	0.863**
	(3.31)	(2.91)	(1.69)	(2.22)	(2.30)
DE_3 International War	0.071*	0.080*	0.073*	0.071*	0.066
	(1.76)	(1.89)	(1.72)	(1.66)	(1.55)
DE_4 Civil War	0.004	0.003	0.008	0.003	0.003
	(0.25)	(0.21)	(0.50)	(0.21)	(0.17)
DE_5 Political Regime	-0.018***	-0.018***	-0.018***	-0.018***	-0.018***
	(-3.42)	(-3.52)	(-3.43)	(-3.61)	(-3.48)
IE_1 GDP	-0.039	-0.224	-0.119	0.120	0.729*
	(-0.19)	(-0.86)	(-0.72)	(0.49)	(1.59)
IE_2 Population	-0.282	-1.113	3.064*	0.916	-1.585
	(-0.37)	(-1.21)	(1.89)	(0.86)	(-0.23)
IE_3 International War	-0.046	-0.113	-0.177	-0.078	-0.300
	(-0.50)	(-0.92)	(-0.98)	(-0.73)	(-0.37)
IE_4 Civil War	-0.033	-0.103*	-0.066	-0.061	0.073
	(-0.95)	(-1.71)	(-0.83)	(-1.31)	(0.46)
IE_5 Political Regime	-0.029**	-0.006	-0.015	-0.025*	-0.047
	(-2.46)	(-0.35)	(-0.66)	(-1.56)	(-0.80)
Observations	2160	2160	2160	2160	2160
Log-likelihood function value	-1311.39	-1313.66	-1316.37	-1319.19	-1333.49
R-squared	0.702	0.702	0.701	0.700	0.694
H0 (SAR): γ=0 (p-value)	7.85 (0.16)	7.19 (0.21)	6.77 (0.24)	4.81 (0.44)	7.67 (0.18)
H0 (SEM): $\rho\beta_k\boldsymbol{W} + \gamma_k\boldsymbol{W} = \mathbf{0}_N$ (p-value)	7.41 (0.19)	5.76 (0.33)	5.37 (0.37)	4.27 (0.51)	3.25 (0.66)

Notes: YE=Yesilurt and Elhorst (2017), Ex=Negative exponential distance decay, ID=Inverse distance decay, rn=row normalized, sn=scalar normalized; *,**,*** significant at respectively 10%, 5% and 1%.

Table D.2: Military expenditures according to one common binary contiguity matrix (YE) or one common parameterized weight matrix (Matlab2022a results)