Online Appendix

for "Analyzing Ballot Order Effects When Voters Rank Candidates"

A Identification and Estimation of Position Effects

Suppose that we consider candidate j ($j = \{1, \ldots, J\}$) and voter i ($i = \{1, \ldots, n\}$). Let $O_{i,j}$ be a random variable denoting the position where voter i sees candidate j on the ballot. For example, $O_{i,j} = 1$ means that voter i sees candidate j in the first position. Similarly, $O_{i,j} = J$ denotes that voter i sees candidate j in the last position. Let $\boldsymbol{O}_{i[-j]}$ be a random vector denoting the order of the remaining $J - 1$ candidates.

Suppose that we seek to study the causal effect of position t (treated position) compared to position t^* (control position) on voters' rankings. More generally, t can be defined as any combination of multiple positions (e.g., first and second positions combined). Suppose also that the remaining candidates have a specific order $\boldsymbol{o}$. Let $\mathbf{Y}_i(O_{ij} = t, \boldsymbol{O}_{i[-j]} = \boldsymbol{o})$ and $\mathbf{Y}_i(O_{ij} = t^*, \boldsymbol{O}_{i[-j]} = \boldsymbol{o})$ be the potential rankings that voter i submits under the treatment and the control with respect to candidate j when the remaining candidates have ordering $\boldsymbol{o}$. Let $\mathbf{Y}_i = \mathbf{Y}_i(O_{ij} = t, \boldsymbol{O}_{i[-j]} = \boldsymbol{o})\mathcal{I}(O_{ij} = t, \boldsymbol{O}_{i[-j]} = \boldsymbol{o})$ be the recorded ranking when voter i sees candidate j in position $t \in \mathcal{T} = \{1, \ldots, J\}$. Let $g()$ be a known function that maps ranking vectors into scalars (e.g., marginal rank and selection indicator function).

Given that, one causal quantity of interest is the average treatment effect of position t for candidate j on voters' rankings with respect to counterfactual position t^* and the order of the remaining candidates $\boldsymbol{o}$. I call it the *conditional position effect* and define it as follows:

$$\tau_{jt^*\boldsymbol{o}} = \mathbb{E}\Big[g\big(\mathbf{Y}_i(O_{ij} = t, \boldsymbol{O}_{i[-j]} = \boldsymbol{o})\big) - g\big(\mathbf{Y}_i(O_{ij} = t^*, \boldsymbol{O}_{i[-j]} = \boldsymbol{o})\big)\Big| O_{ij} = t^*, \boldsymbol{O}_{i[-j]} = \boldsymbol{o}\Big] \tag{A.1}$$

In principle, the conditional position effect allows researchers to study the most fine-grained effect of a particular position t relative to alternative position t^* for candidate j with the order of the remaining candidates $\boldsymbol{o}$. These effects can be motivated by substantive theory and practical needs.

In many applications, however, researchers are more interested in the overall effect of position t for candidate j on voters' rankings *averaged over* all possible counterfactual position $t^* \in \mathcal{T}$ *and* all possible orderings of the remaining candidates $\boldsymbol{o} \in \mathcal{O}$. I call it the *average position effect* of position t and define it as follows:

$$\begin{aligned}
\tau_j &= \mathbb{E}[\tau_{jt^*\boldsymbol{o}}] \\
&= \sum_{\boldsymbol{o}} \Bigg\{ \sum_{t^*} \mathbb{E}\Bigg[g\Big(\underbrace{\mathbf{Y}_i(O_{ij} = t, \boldsymbol{O}_{i[-j]} = \boldsymbol{o})}_{\text{potential ranking under the treatment}}\Big) - g\Big(\underbrace{\mathbf{Y}_i(O_{ij} = t^*, \boldsymbol{O}_{i[-j]} = \boldsymbol{o})}_{\text{potential ranking under the control}}\Big) \Bigg| O_{ij} = t^*, \boldsymbol{O}_{i[-j]} = \boldsymbol{o} \Bigg] \\
&\times \underbrace{\mathbb{P}(t, t^* | \boldsymbol{O}_{i[-j]} = \boldsymbol{o})}_{\text{conditional probability of candidate } j \text{ listed in } t\text{th and } t^*\text{th positions}} \Bigg\} \\
&\times \underbrace{\mathbb{P}(\boldsymbol{O}_{i[-j]} = \boldsymbol{o})}_{\text{probability of a particular ballot order } \boldsymbol{o}}
\end{aligned} \tag{A.2}$$

In other words, the average position effect is the probability-weighted average of all possible conditional position effects, where the joint probability $\mathbb{P}(O_{ij} = t^*, \boldsymbol{O}_{i[-j]} = \boldsymbol{o})$ is factorized into $\mathbb{P}(O_{ij} = t^* | \boldsymbol{O}_{i[-j]} = \boldsymbol{o})\mathbb{P}(\boldsymbol{O}_{i[-j]} = \boldsymbol{o})$.

To identify the average position effect, I consider four assumptions, including one assumption that is specific to the current study and three standard assumptions.

Assumption A.1 (*Ballot Order Randomization*) *Candidate order on the ballot is randomized at the voter level. Under ballot order randomization, the probability that the remaining candidates are ordered in a given way becomes* $\mathbb{P}(\boldsymbol{O}_{i[-j]=\boldsymbol{o}}) = \frac{1}{(J-1)!}$ *and the probability of choosing two positions becomes* $\mathbb{P}(t, t^* | \boldsymbol{O}_{i,-j} = \boldsymbol{o}) = \frac{1}{J(J-1)}$ *for all* $t = 1, ..., J$.

Assumption A.2 (*Random Assignment*) *The potential rankings are independent of the actual position of candidate* j. *Formally,* $\{\mathbf{Y}_i(O_{ij} = t, \boldsymbol{O}_{i[-j]} = \boldsymbol{o}), \mathbf{Y}_i(O_{ij} = t^*, \boldsymbol{O}_{i[-j]} = \boldsymbol{o})\} \perp O_{ij}$.

Assumption A.3 (*Positivity*) *There is a positive probability that candidate* j *appears in the* t*th position on the ballot. Formally,* $0 < \mathbb{P}(O_{ij} = t) < 1$.

Assumption A.4 (*Stable-unit-treatment-value-assumption (SUTVA)*) *Voter* i*'s ranking depends solely on the position where voter* i *sees, and not other voters see, candidate* j.

Assumptions A.2 and A.3 will be satisfied as long as ballot order randomization is implemented.

With the four assumptions, the average position effect can be nonparametrically estimated with the following estimator.

Proposition A1 (Nonparametric Estimator for the Average Position Effect). *Given Assumptions A.1-A.4, the average ballot order effect for candidate* j *is estimated by a doubly-averaged-difference-in-mean-ranks estimator:*

$$\begin{aligned}
\widehat{\tau}_j &= \widehat{\mathbb{E}}\left[g(\mathbf{Y}_i)|O_{ij} = t, \boldsymbol{O}_{i[-j]} = \boldsymbol{o}\right] - \widehat{\mathbb{E}}\left[g(\mathbf{Y}_i)|O_{ij} = t^*, \boldsymbol{O}_{i[-j]} = \boldsymbol{o}\right] \\
&= \underbrace{\frac{1}{(J-1)!}\sum_{\boldsymbol{o}}}_{\text{average over all ordering}} \underbrace{\frac{1}{J(J-1)}\sum_{t^*}}_{\text{average over all comparison}} \left\{\widehat{\mathbb{E}}\left[g(\mathbf{Y}_i)|O_j = t, \boldsymbol{O_i} = \boldsymbol{o}\right] - \widehat{\mathbb{E}}\left[g(\mathbf{Y}_i)|O_j = t^*, \boldsymbol{O_i} = \boldsymbol{o}\right]\right\}
\end{aligned} \tag{A.3}$$

In principle, researchers can estimate the average position effect either by the within-strata difference-in-mean estimator or by ordinary least square regression. In reality, however, the number of possible treatment-control comparisons grows quickly as the number of candidates increases. For example, with ten candidates, the number of all possible strata (i.e., comparisons) becomes $(J-1)!$ (# possible counterfactual control positions) $\times (J-1)$ (# permutations of the remaining candidates) $= 9 \times 9! = 3,265,920$. Evidently, the curse of dimensionality kicks in, and researchers run out of data points to estimate the above effect.

To regularize the number of strata, I introduce two additional assumptions that average position effects do not depend on the order of the remaining candidates $\boldsymbol{o}$ and counterfactual position t^*. In essence, the following two assumptions allow researchers to pool their data to estimate the average position effect.

Assumption A.5 (*Constant Average Effect by Remaining Ballot Orders*) *The condition position effect does not depend on the order of the remaining candidates. Formally,* $\tau_{jt^*\boldsymbol{o}} \perp \boldsymbol{O}_{i[-j]}$ *for all* $i, j, t,$ *and* t^*.

Assumption A.6 (*Constant Average Effect by Counterfactual Positions*) *The conditional position effect does not depend on the counterfactual position t^*. Formally, $\tau_{jt^*\boldsymbol{o}} \perp t^*$ for all i, j, t, and $\boldsymbol{O}$.*

With these additional assumptions, researchers can estimate the average position effect via the standard difference-in-means estimator or linear regression:

Proposition A2 (Average Position Effect via Ordering and Position Mean Independence). *$N_{jt} = \sum_{i=1}^{N} I(O_{ij} = t)$ be the number of treated units (who see candidate j in the tth position of the ballot) and $N_{jt^*} = \sum_{i=1}^{N} I(O_{ij} = t^*)$ be the number of control units (who see candidate j in the t^*th position of the ballot) such that $N = N_{jt} + \sum_{t=2}^{J} N_{jt^*}$. The average position effect for candidate j is estimated by the following difference-in-mean estimator:*

$$\widehat{\tau_j} = \frac{\sum_{i=1}^{n} g(\mathbf{Y}_i) I(O_{ij} = t)}{N_{j1}} - \frac{1}{J-1} \sum_{t^*} \left[\frac{\sum_{i=1}^{n} g(\mathbf{Y}_i) I(O_{ij} = t^*)}{N_{jt^*}} \right] \quad \text{(with Assumption A.5)} \tag{A.4}$$

$$= \frac{\sum_{i=1}^{n} g(\mathbf{Y}_i) I(O_{ij} = t)}{N_{jt}} - \frac{\sum_{i=1}^{n} g(\mathbf{Y}_i) I(O_{ij} \neq t)}{N - N_{jt}} \quad \text{(with Assumptions A.5-A.6)}, \tag{A.5}$$

B Uniformity Test of Recorded Rankings

Here, we provide the result and proof for the uniformity test under item order randomization.

Let A_i denote the candidate order that voter i sees on the ballot. Let π_i denote voter i's underlying ranking preference. Finally, let R_i denote voter i's recorded ranking.

First, under ballot order randomization (Assumption A.1), candidate order is statistically independent of voters' underlying preferences.

Lemma B.1 *Independence between Underlying Preference and Choice Set*

$$\pi_i \perp A_i$$

Next, suppose that all voters provide ranked ballots according to their underlying preferences π and presented candidate order a. Then, voters submit recorded ranking r with probability 1.

Definition 1 *Non-pattern Ranking*

$$\mathbb{P}(R_i = r | \pi_i = \pi, \mathcal{A}_i = a) = 1$$

With Assumption A.1, Lemma B.1, and Definition 1, I show that observed rankings among non-pattern rankers follow a uniform distribution.

Proposition 1 *Uniformity of Recorded Rankings by Pattern Rankers*

$$\mathbb{P}(R_i = r, \pi_i = \pi, \mathcal{A}_i = a) \sim \mathcal{U}_J = \left(\frac{1}{J!}, \frac{1}{J!}, \cdots, \frac{1}{J!} \right)$$

Proof.

$$\begin{aligned}
\mathbb{P}(\text{Recorded ranking } r) &= \mathbb{P}(R_i = r, \pi_i = \pi, \mathcal{A}_i = a) \\
&= \mathbb{P}(R_i = r | \pi_i = \pi, \mathcal{A}_i = a) \underbrace{\mathbb{P}(\pi_i = \pi, \mathcal{A}_i = a)}_{\text{Can be separated via Lemma B.1}} \\
&= \underbrace{\mathbb{P}(R_i = r | \pi_i = \pi, \mathcal{A}_i = a)}_{\text{1 via Definition 1}} \underbrace{\mathbb{P}(\pi_i = \pi)}_{\text{1 by definition}} \underbrace{\mathbb{P}(\mathcal{A}_i = a)}_{\frac{1}{J!}\text{via Assumption A.1}} \\
&= 1 \times 1 \times \frac{1}{J!} \\
&= \frac{1}{J!}
\end{aligned}$$

Suppose that all voters provide ranked ballots according to some geometric patterns *regardless of* their underlying preferences π and presented candidate order a. In other words, $R_i \perp \{\pi_i, \mathcal{A}_i\}$. Then, voters submit recorded ranking r with (some unknown) probability $\mathbb{P}(R_i = r)$.

Definition 2 *Pattern Ranking*

$$\mathbb{P}(R_i = r | \pi_i = \pi, \mathcal{A}_i = a) = \mathbb{P}(R_i = r)$$

With Assumption A.1, Lemma B.1, and Definition 2, I show that observed rankings among pattern rankers do not follow a uniform distribution.

Proposition 2 *Non-uniformity of Recorded Rankings by Pattern Rankers*

$$\mathbb{P}(R_i = r, \pi_i = \pi, \mathcal{A}_i = a) \sim \mathcal{R}_J \neq \left(\frac{1}{J!}, \frac{1}{J!}, \cdots, \frac{1}{J!}\right)$$

Proof.

$$\begin{aligned}
\mathbb{P}(\text{Recorded ranking } r) &= \mathbb{P}(R_i = r, \pi_i = \pi, \mathcal{A}_i = a) \\
&= \mathbb{P}(R_i = r | \pi_i = \pi, \mathcal{A}_i = a) \underbrace{\mathbb{P}(\pi_i = \pi, \mathcal{A}_i = a)}_{\text{Separated via Lemma B.1}} \\
&= \underbrace{\mathbb{P}(R_i = r | \pi_i = \pi, \mathcal{A}_i = a)}_{\text{Simplified via Definition 2}} \underbrace{\mathbb{P}(\pi_i = \pi)}_{\text{1 by definition}} \underbrace{\mathbb{P}(\mathcal{A}_i = a)}_{\frac{1}{J!}\text{via Assumption A.1}} \\
&= \mathbb{P}(R_i = r) \times 1 \times \frac{1}{J!} \\
&= \mathbb{P}(R_i = r) \frac{1}{J!} \\
&\neq \frac{1}{J!}
\end{aligned}$$

C Survey Design

The experiments were implemented via the Lucid Marketplace from October 10th to November 7th, 2022. I geo-targeted respondents by sampling *within* Oakland and Alaska, respectively. To obtain the largest possible sample (after consulting with the firm) while achieving demographic representativeness, I used the 2020 Census to set quotas based on gender, race, and ethnicity in each area. The total number of respondents was 258 (Oakland) and 354 (Alaska), respectively. The study was pre-registered (here). Online Appendix D discusses how the study adheres to the Principles and Guidance for Human Subjects Research by the American Political Science Association (APSA) Council.

The study showed respondents a list of actual candidates (with their occupations in Oakland and registered parties in Alaska). I designed the experimental questions by following official sample ballots in Oakland and Alaska.[1] Finally, I collected respondents' rankings based on (1) optional and (2) forced ranking questions via radio buttons. In the optional question, respondents were allowed to rank up to three (in Oakland) or four candidates (in Alaska), respectively, reflecting the election laws in the two areas. In the forced question, respondents were asked to rank all candidates.

The following presents the survey instructions and questions for the Oakland 2022 mayoral election. The same set of instructions and questions was used for the US House and US Senate elections in Alaska.

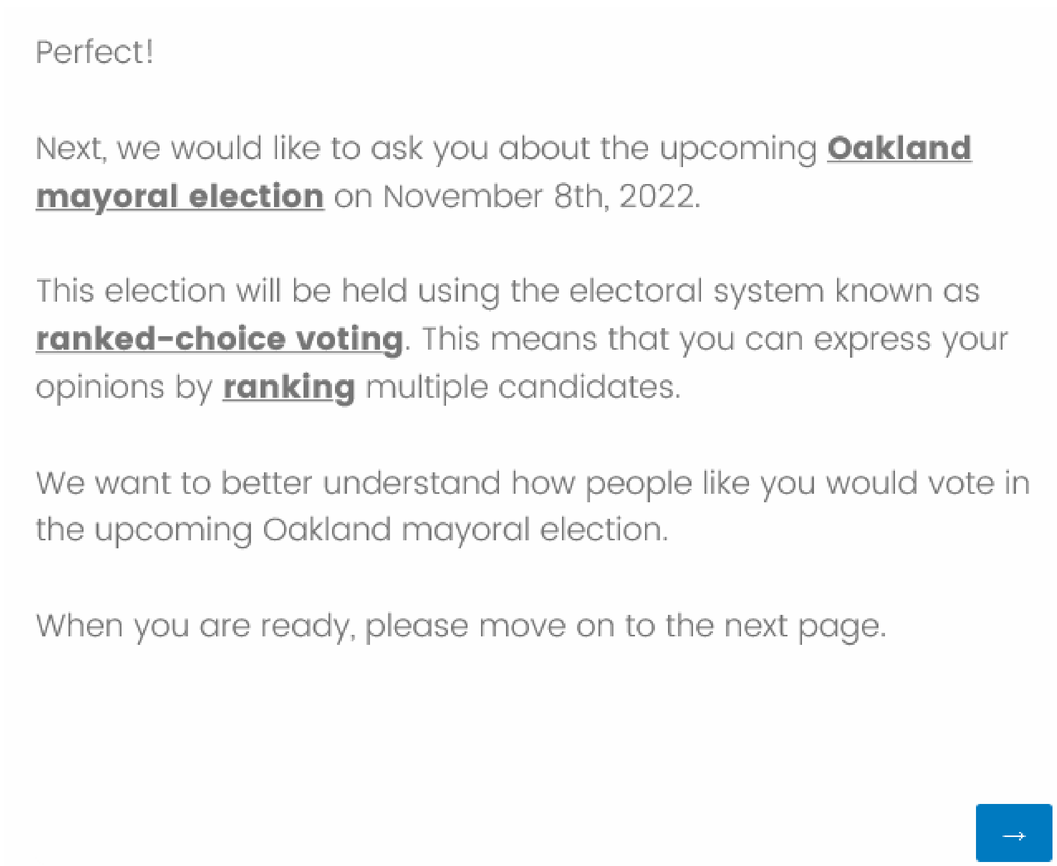
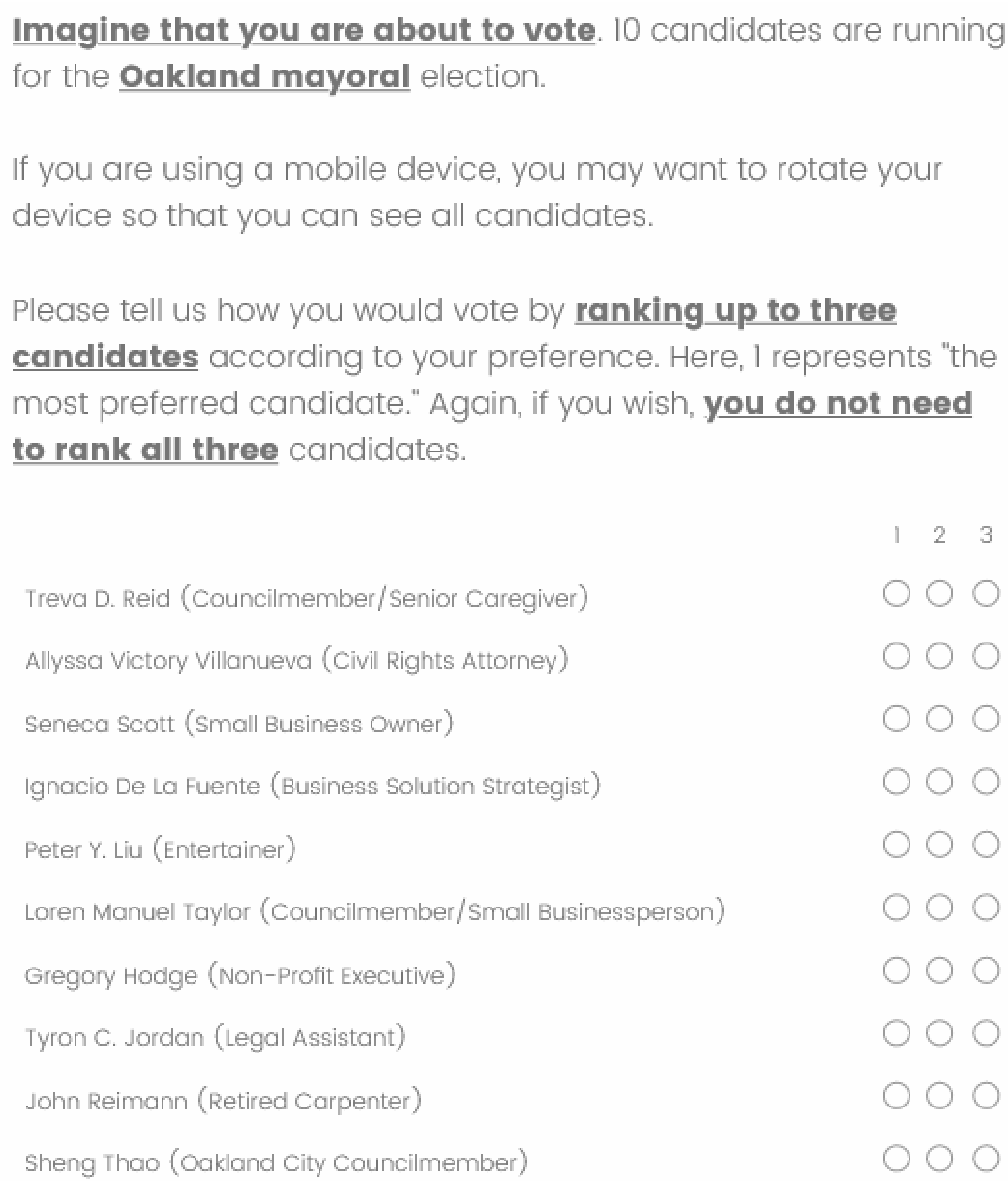


Figure C.1. Instruction for Experimental Questions

1. I obtained sample ballot for the last two elections from the City of Oakland website and the Alaska Division of Elections website.

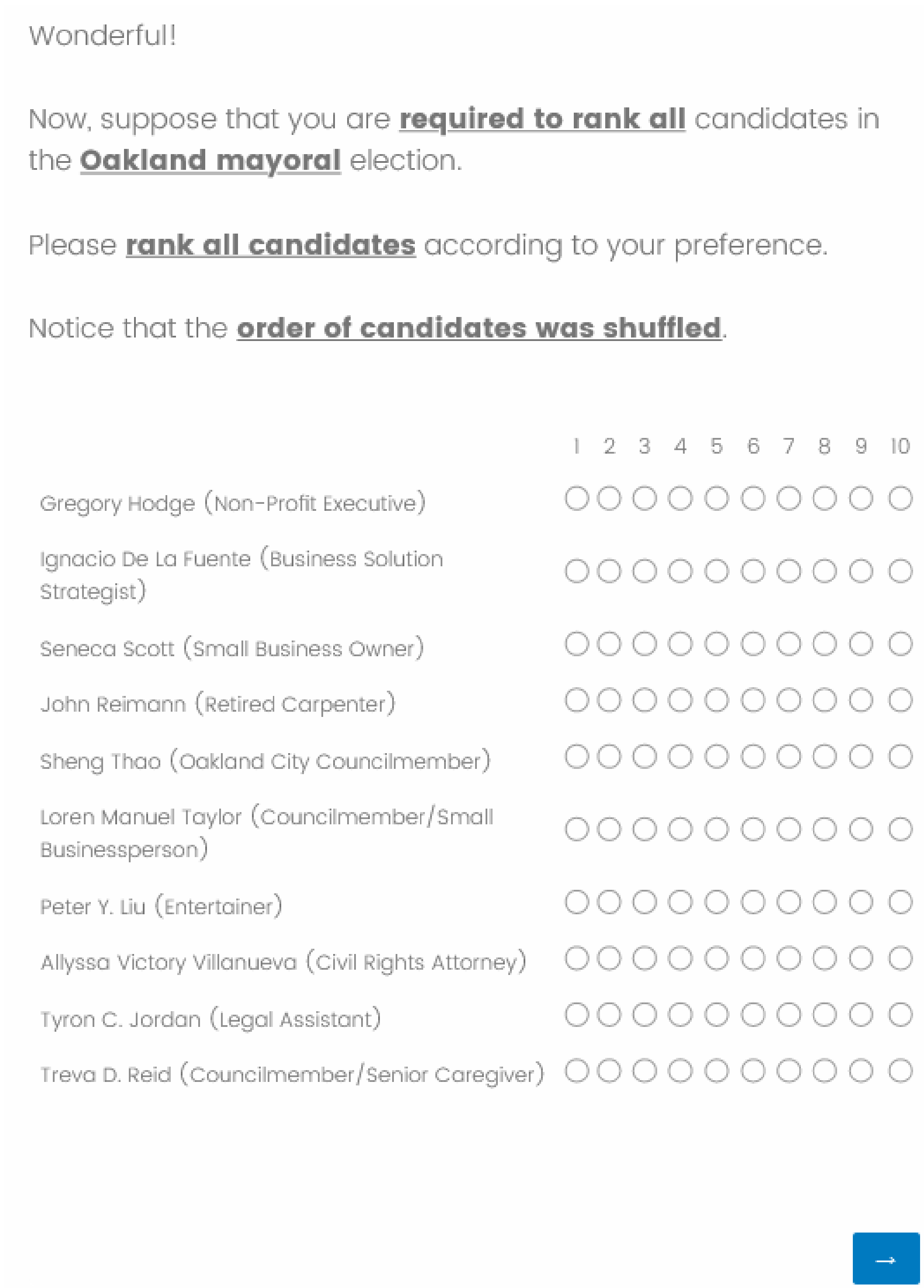

Imagine that you are about to vote. 10 candidates are running for the **Oakland mayoral** election.

If you are using a mobile device, you may want to rotate your device so that you can see all candidates.

Please tell us how you would vote by **ranking up to three candidates** according to your preference. Here, 1 represents "the most preferred candidate." Again, if you wish, **you do not need to rank all three** candidates.

	1	2	3
Treva D. Reid (Councilmember/Senior Caregiver)	○	○	○
Allyssa Victory Villanueva (Civil Rights Attorney)	○	○	○
Seneca Scott (Small Business Owner)	○	○	○
Ignacio De La Fuente (Business Solution Strategist)	○	○	○
Peter Y. Liu (Entertainer)	○	○	○
Loren Manuel Taylor (Councilmember/Small Businessperson)	○	○	○
Gregory Hodge (Non-Profit Executive)	○	○	○
Tyron C. Jordan (Legal Assistant)	○	○	○
John Reimann (Retired Carpenter)	○	○	○
Sheng Thao (Oakland City Councilmember)	○	○	○

Figure C.2. Optional Ranking Question



Wonderful!

Now, suppose that you are **required to rank all** candidates in the **Oakland mayoral** election.

Please **rank all candidates** according to your preference.

Notice that the **order of candidates was shuffled**.

	1	2	3	4	5	6	7	8	9	10
Gregory Hodge (Non-Profit Executive)										
Ignacio De La Fuente (Business Solution Strategist)										
Seneca Scott (Small Business Owner)										
John Reimann (Retired Carpenter)										
Sheng Thao (Oakland City Councilmember)										
Loren Manuel Taylor (Councilmember/Small Businessperson)										
Peter Y. Liu (Entertainer)										
Allyssa Victory Villanueva (Civil Rights Attorney)										
Tyron C. Jordan (Legal Assistant)										
Treva D. Reid (Councilmember/Senior Caregiver)										

→

Figure C.3. Full Ranking Question

D Adherence to Principles and Guidance for Human Subjects Research

This section discusses how the experimental studies presented in this paper adhere to the Principles and Guidance for Human Subjects Research approved by the APSA Council in 2020.

Voluntary Participation and Informed Consent

The three survey experiments were administered via the Lucid Marketplace from October 10th to November 7th, 2022. All survey respondents participated in the surveys *voluntarily* after reading a consent form (that was approved by the IRB of the author's institution) and explicitly agreeing to join the survey (by clicking "Yes"). The consent form explicitly states that "[respondents] are invited to take part in a research study." If survey workers do not agree to the consent form, they are sent back to the survey firms they belong to through a system at the Lucid Marketplace. The study also allowed respondents to exit the survey at any time they wished. More specifically, the consent form states, "Your participation is voluntary. Participation involves the completion of a survey. You may choose not to answer any or all questions. The information collected will be completely anonymous and no identifiable information (such as your name and address) will be asked during the study."

Absence of Deception

The consent form explicitly states that the study "aims to understand how ballot design matters to people who wish to express their preferences in ranked-choice voting elections. You will be shown a list of actual candidates running for the Oakland mayoral election (or the U.S. House and U.S. Senate elections in Alaska) on November 8th, 2022 in a particular format and asked to rank order a subset or all candidates." Thus, the study did not involve any deception of survey respondents.

Compensation

Respondents received $3 upon the completion of their participation. The author calculated the compensation amount through consultation with a consultant at Cint (formerly Lucid). Compensation is fair in the United States and slightly higher than conventional studies because (a) the study takes more than 5 minutes and (b) the study needs to collect as many responses as possible from geographically limited areas (Oakland and Alaska) for which the Lucid Marketplace holds limited numbers of potential respondents.

E Comparison of Survey Data and Official Election Results

This section compares the actual and estimated first-choice vote share for each candidate based on the survey data. The following figures illustrate that, overall, the survey data predict the relative popularity of each candidate in terms of their first-choice vote share. However, the survey estimates miss the relative popularity of a few candidates in the Oakland election. Additionally, due to pattern ranking, vote shares are underestimated for popular candidates and overestimated for fringe candidates.

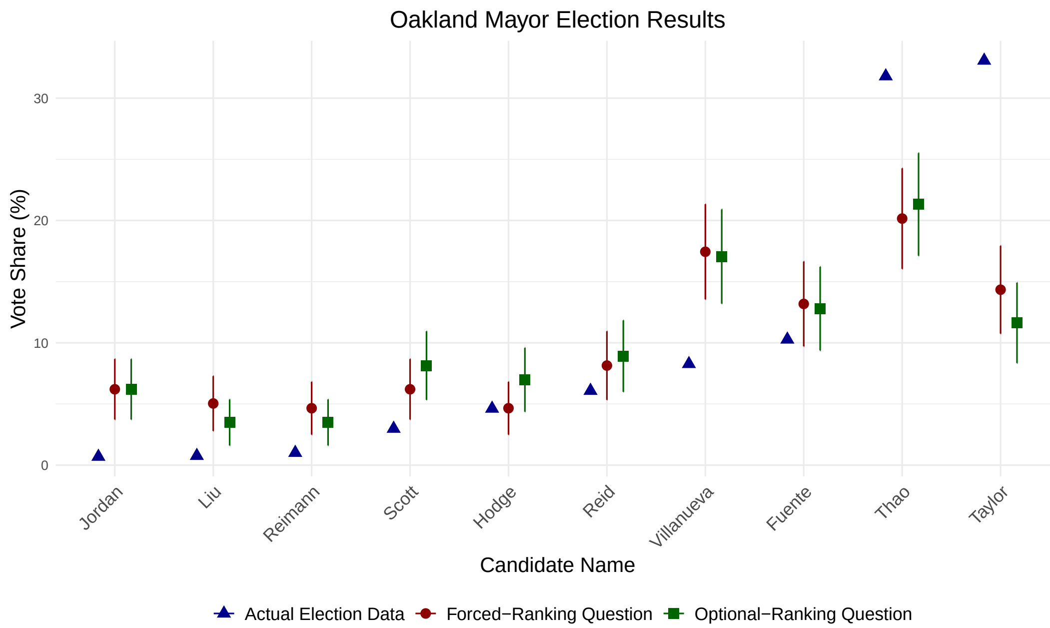


Figure E.1. **First-Choice Vote Shares in the Oakland Mayoral Election**

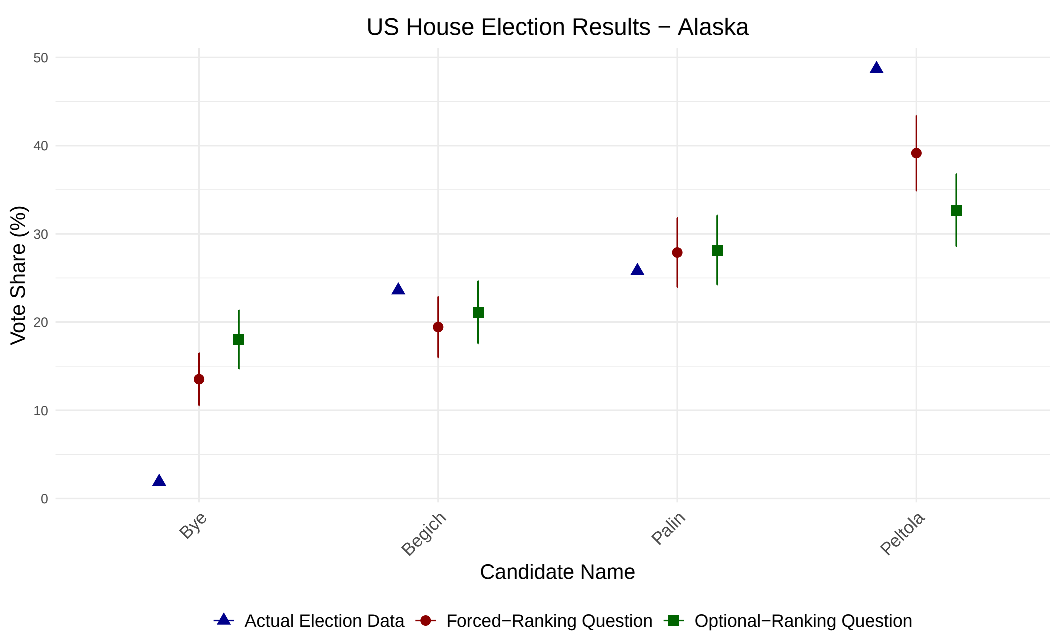


Figure E.2. **First-Choice Vote Shares in the U.S. House Election**

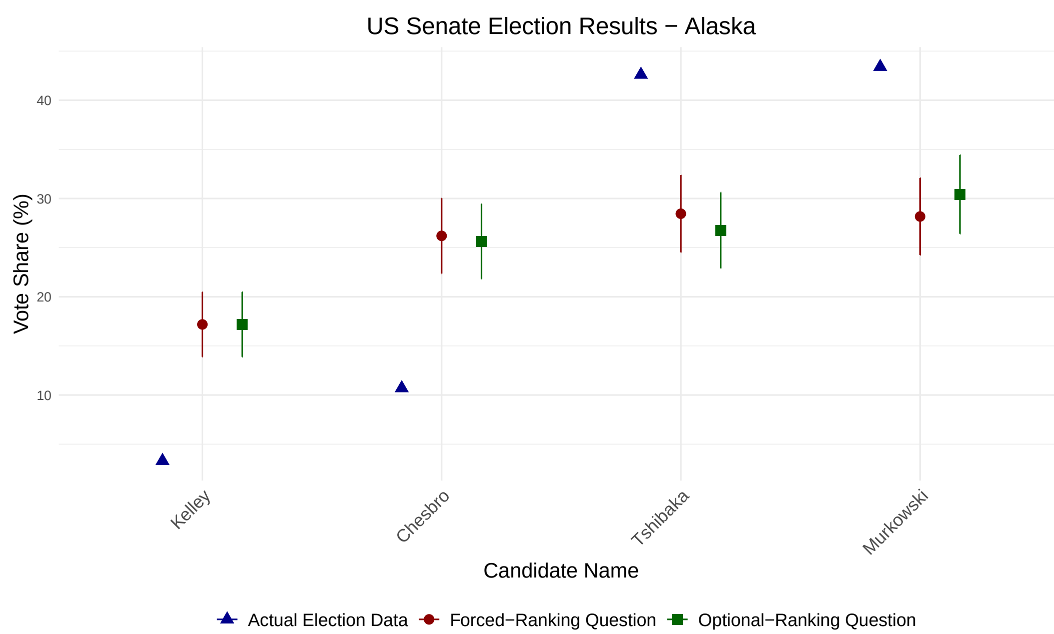


Figure E.3. **First-Choice Vote Shares in the U.S. Senate Electionlection**

F Additional Results from Survey Experiments

This section reports additional results from the survey experiments. Tables F.1 and F.2 present the average and total proportions of each type of pattern ranking by race and question format, respectively. The tables suggest that the dogleg vote is more prevalent than the zigzag vote in almost all elections, except for the forced question in the U.S. Senate race.

	Race	Format	Type	Average
1	U.S. House	forced	diagonal	0.18
2	U.S. House	forced	dogleg	0.03
3	U.S. House	forced	zigzag	0.03
4	U.S. House	option	diagonal	0.21
5	U.S. House	option	dogleg	0.03
6	U.S. House	option	zigzag	0.02
7	U.S. Senate	forced	diagonal	0.18
8	U.S. Senate	forced	dogleg	0.03
9	U.S. Senate	forced	zigzag	0.03
10	U.S. Senate	option	diagonal	0.21
11	U.S. Senate	option	dogleg	0.03
12	U.S. Senate	option	zigzag	0.02

Table F.1. Average Proportions of Pattern Ranking

	Race	Format	Type	Sum
1	U.S. House	forced	diagonal	0.36
2	U.S. House	forced	dogleg	0.38
3	U.S. House	forced	zigzag	0.25
4	U.S. House	option	diagonal	0.41
5	U.S. House	option	dogleg	0.36
6	U.S. House	option	zigzag	0.23
7	U.S. Senate	forced	diagonal	0.36
8	U.S. Senate	forced	dogleg	0.35
9	U.S. Senate	forced	zigzag	0.29
10	U.S. Senate	option	diagonal	0.43
11	U.S. Senate	option	dogleg	0.36
12	U.S. Senate	option	zigzag	0.21

Table F.2. Total Proportions of Pattern Ranking

G Additional Discussions and Results from Alaskan RCV Elections

G.1 Connection with Robson Rotation

Alaska's methods differ from the so-called Robson Rotation adopted in some Australian elections in several ways. Unlike under Robson Rotation, where candidate order is rotated across ballot papers, Alaska rotates candidates across its house districts. Secondly, while Robson Rotation often rotates candidates *within* each party, Alaska adopts rotation for all candidates. Moreover, the initial order is randomly determined under Robson Rotation, whereas Alaska's methods always use alphabetical order in District 1. Finally, despite its name, Robson Rotation does not actually rotate candidates because the relative order of available candidates can change (in Alaska, it does not change). Instead, Robson Rotation uses several different permutations of the candidates.

G.2 Additional Results from Natural Experiments

Figures G.1-G.2 visualize the analysis for twelve ranking profiles from the U.S. House and Alaska gubernatorial races in 2022.

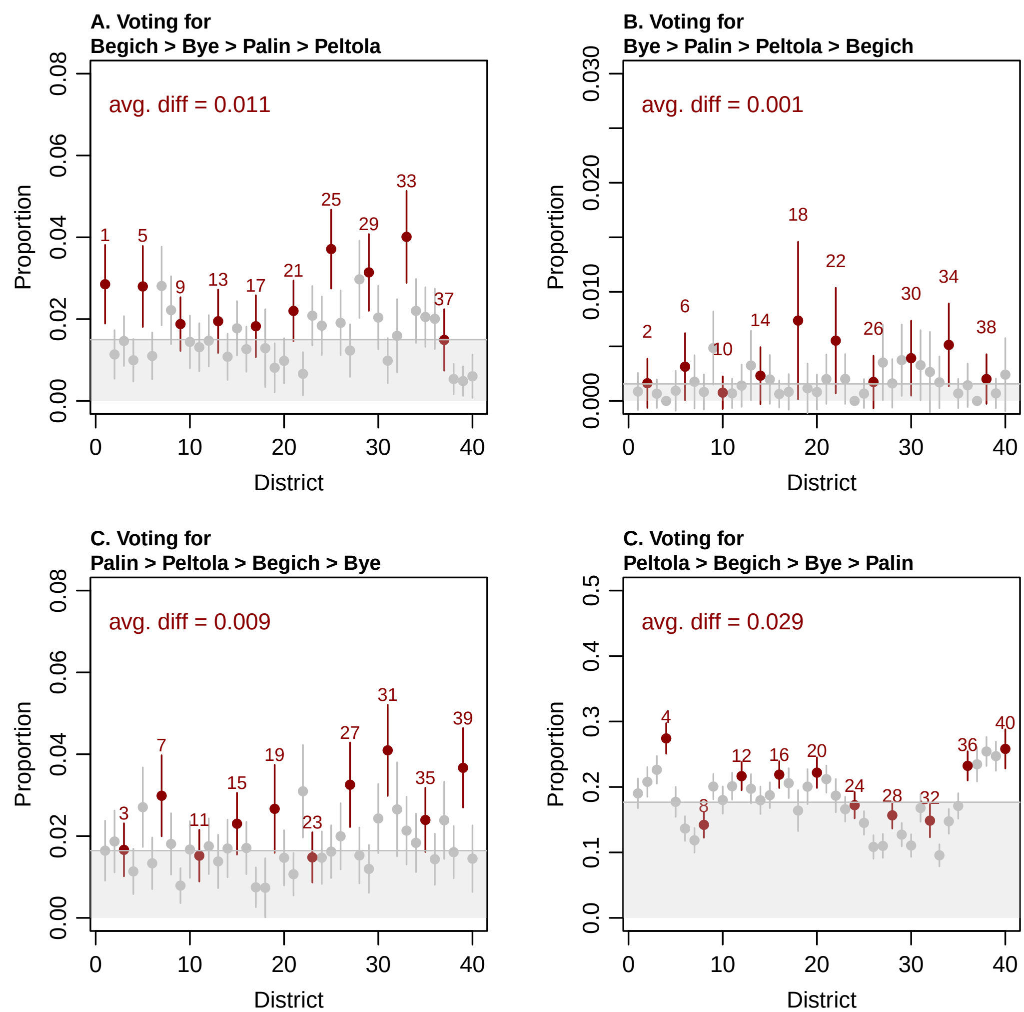


Figure G.1. **Proportions of Four Rankings in the U.S. House of Representatives Election**

H Additional Limitations of This Study

In this section, I discuss additional limitations of the current study.

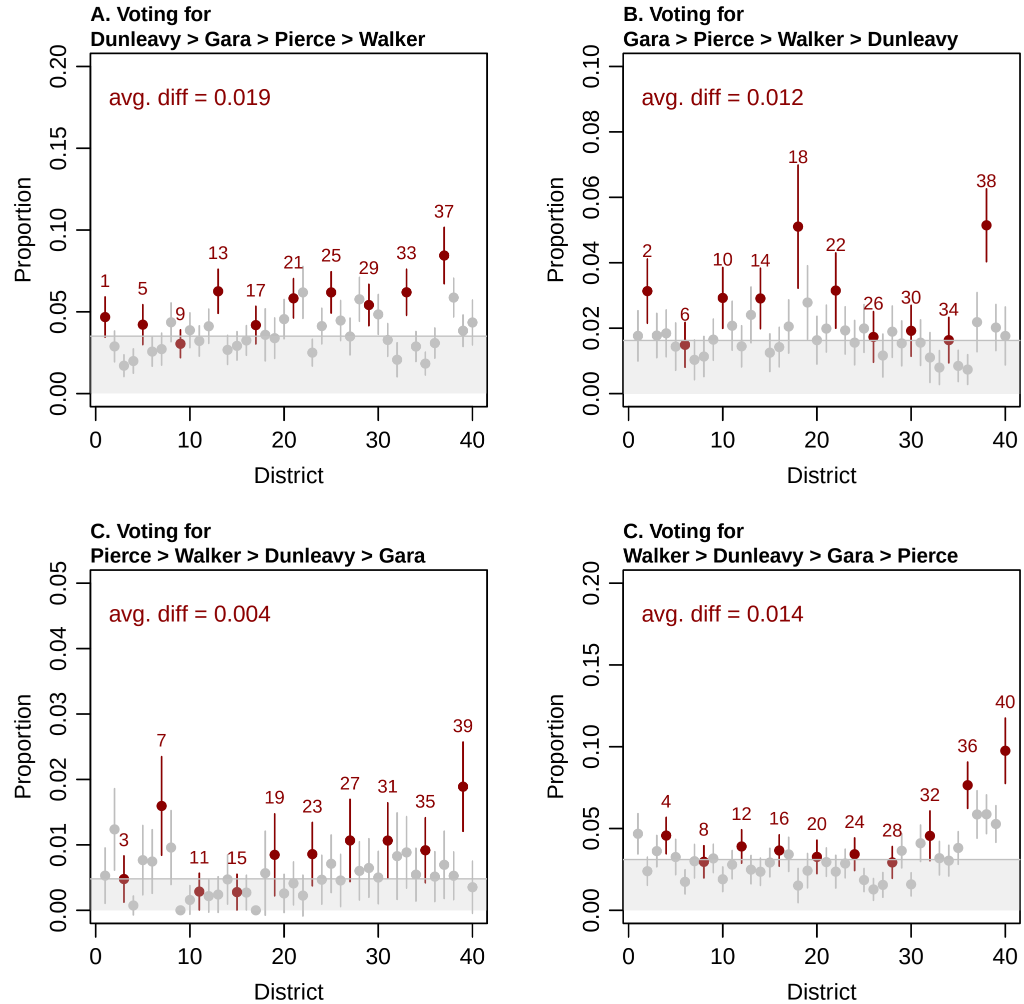


Figure G.2. **Proportions of Four Rankings in the Alaska Gubernatorial Election**

Another limitation is that this work does not directly assess the impact of ballot order effects on electoral results—who will be elected? To study the impact, scholars need to compare ballot order effects to the margin of victory, which quantifies the closeness of an electoral contest. However, computing the margin of victory in elections with ordinal ballots is challenging. Thus, future research must augment the presented results by newly computed margins of victory under RCV.

Finally, this study does not directly offer practical solutions to order effects in ballot design and survey research. Ultimately, researchers and experts would know how to engineer ballot designs and survey formats to mitigate order effects. Thus, more research will be needed to study how order randomization or rotation can be augmented with alternative methods to reduce order effects in elections and surveys. The analysis of the three Alaskan RCV elections (Section 4 and Appendix G) seems to suggest that the proportion of donkey voting varies. Thus, it may be possible to identify the ballot order that minimizes donkey voting if researchers can understand and explain the variation.