Supplementary material

Proof of formula in footnote 17

Definition (Generalized version of Rescue 1 and 2)**.** Let $n, k, N \in \mathbb{N}$ s.t. n divides N and $k < n$. Let Ω with $|\Omega| = N$. Let $\mathcal{P} \subset 2^{\Omega}$ be any partition of Ω s.t. $|P| = n$, $\forall P \in \mathcal{P}$. Let $\mathcal{K} = \{A : A \in 2^{\Omega} \wedge |A| = k\}$ be the set of all subsets of Ω of size k. Finally let

$$\mathcal{O} = \mathcal{K} \cup \mathcal{P} \,. \tag{1}$$

The pair $(\Omega, \mathcal{O})$ defines a rescue dilemma, where Ω is the set of claimants and $\mathcal{O}$ are the possible outcome groups, i.e. the groups of claimants that can be rescued together.

Remarks.

- The case ($|\Omega| = 1000$, $n = 500$, $k = 2$) is Rescue 1 in the article. The case ($|\Omega| = 1000$, $n = 500$, $k = 3$) is Rescue 2.
- If N is prime, it follows that $n = N$. This does not yield an interesting rescue dilemma, since the group of size n contains all claimants. The theorem below still holds (trivially) in that case.

Theorem. *Consider the rescue dilemma from the definition above using the same notation. Let*

$$P^{\text{EXCS}} = \mathbb{P}\left(\text{Winning group of EXCS lottery is in } \mathcal{P}\right) \tag{2}$$

be the probability that any of the larger groups will win Vong's EXCS lottery. Then

$$P^{\text{EXCS}} = \frac{1 + \binom{n-1}{k-1}\frac{k-1}{n-1}}{1 + \binom{N-1}{k-1}\frac{k-1}{n-1}} \,. \tag{3}$$

Proof. Let Ω, $\mathcal{O}$, $\mathcal{P}$ and $\mathcal{K}$ be defined as in the definition.

Let $\mathcal{K}_{\mathcal{P}} = \{A \in \mathcal{K} : \exists P \in \mathcal{P} \text{ s.t. } A \subset P\}$ be the set of outcome groups of size k, which are completely contained in an outcome group of size n. It's easy to see that $|\mathcal{P}| = \frac{N}{n}$, $|\mathcal{K}| = \binom{N}{k}$ and $|\mathcal{K}_{\mathcal{P}}| = \binom{n}{k} \times \frac{N}{n}$.

Let $c \in \Omega$ be a particular claimant. According to the rules of the EXCS they get a baseline claim of $\frac{1}{N}$, which they distribute among all outcome groups, they belong to $\{O \in \mathcal{O} : c \in O\}$, weighted by how many distributionally relevant claimants there are in those outcome groups. Note that in this example *all* other claimants are distributionally relevant for c. Hence the unique $P \in \mathcal{P}$, which contains c, has $n-1$ distributionally relevant claimants, and each $K \in \mathcal{K}$, which contains c has $k-1$ distributionally relevant claimants. Since $|\{K \in \mathcal{K} : c \in K\}| = \binom{N-1}{k-1}$, we find that c distributes a claim of $\frac{1}{N} \times \frac{n-1}{n-1+(k-1)\binom{N-1}{k-1}}$ to the unique $P \in \mathcal{P}$ containing c and a claim $\frac{1}{N} \times \frac{k-1}{n-1+(k-1)\binom{N-1}{k-1}}$ to each of the groups in $\{K \in \mathcal{K} : c \in K\}$. Since this holds for each claimant, the total claim (and therefore their probability of being chosen in the first step of the lottery procedure) for each of the groups of size n (i.e. all groups in $\mathcal{P}$) is

$$P_n^{\text{EXCS}} = \frac{n}{N} \times \frac{n-1}{n-1+(k-1)\binom{N-1}{k-1}}\,, \tag{4}$$

while for each of the groups of size k (i.e. all groups in $\mathcal{K}$) it amounts to

$$P_k^{\text{EXCS}} = \frac{k}{N} \times \frac{k-1}{n-1+(k-1)\binom{N-1}{k-1}}\,. \tag{5}$$

If an outcome group is chosen, which is a subgroup of some other outcome groups, the rules of EXCS demand that the lottery is repeated only among said outcome groups, which contain the selected group. Let $A \in \mathcal{K}_{\mathcal{P}}$. By definition of $\mathcal{K}_{\mathcal{P}}$ there exists $P \in \mathcal{P}$ s.t. $A \subset P$. In fact P is unique, since $\mathcal{P}$ is a partition of Ω. Therefore, if A is selected initially, the repeated application of the lottery will necessarily result in the selection of P instead. There are thus two ways for groups in $\mathcal{P}$ to win the lottery. Either by direct selection or through the repeated application of the lottery if one of the groups in $\mathcal{K}_{\mathcal{P}}$ was selected first. Therefore the probability that one of the groups in $\mathcal{P}$ wins the lottery is given by

$$\begin{aligned} P^{\mathrm{EXCS}} &= |\mathcal{P}| \times P_n^{\mathrm{EXCS}} + |\mathcal{K}_{\mathcal{P}}| \times P_k^{\mathrm{EXCS}} \\ &= \frac{n-1}{n-1+(k-1)\binom{N-1}{k-1}} + \binom{n}{k}\frac{k}{n} \times \frac{k-1}{n-1+(k-1)\binom{N-1}{k-1}} \\ &= \frac{1+\binom{n-1}{k-1}\frac{k-1}{n-1}}{1+\binom{N-1}{k-1}\frac{k-1}{n-1}}, \end{aligned} \tag{6}$$

where in the last step we have used $\binom{n}{k}\frac{k}{n} = \binom{n-1}{k-1}$ and divided both the numerator and denominator by $(n-1)$. $\square$