A MAXIMUM LIKELIHOOD APPROACH FOR UNCERTAIN VOLUMES IN THE ADDITIVE RESERVING MODEL

BY

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Online Supplementary Material

Example 1 (Commercial Liability)

Table 1 contains the premiums v_i and incremental payments $S_{i,k}$ of a large Commercial Liability portfolio. The triangle contains much more volatility than the Motor example from Riegel (2018), discussed in Section 7 of the paper.

i	v_i	$S_{i,1}$	$S_{i,2}$	$S_{i,3}$	$S_{i,4}$	$S_{i,5}$	$S_{i,6}$	$S_{i,7}$	$S_{i,8}$	$S_{i,9}$	$S_{i,10}$
1	1.520,0	144,5	148,6	122,2	62,3	58,9	32,5	24,6	28,2	39,2	13,7
2	1.542,5	160,2	137,8	101,2	68,9	49,7	40,6	28,6	26,2	30,8	
3	1.580,3	163,4	153,1	94,6	57,2	87,7	54,9	43,8	29,0		
4	1.794,8	175,8	324,2	193,7	96,8	80,0	45,8	25,2			
5	1.830,4	173,4	161,2	105,9	86,8	73,1	62,9				
6	1.789,2	204,4	211,4	136,8	102,6	86,3					
7	1.903,2	194,1	203,9	134,0	83,3						
8	1.991,5	211,2	158,8	161,1							
9	2.099,1	203,8	215,9								
10	2.328,1	200,3									

Table 1: Triangle and volumes for Example 1

To get an idea, which variance function could be realistic, we calculate the unbiased estimators

$$\widehat{\mu}_k := \frac{\sum_{i=1}^{I(k)} S_{i,k}}{\sum_{i=1}^{I(k)} v_i} \quad \text{and} \quad \widehat{\sigma}_k^2 := \frac{1}{I(k)-1} \sum_{i=1}^{I(k)} v_i \left(\frac{S_{i,k}}{v_i} - \widehat{\mu}_k \right)^2$$

for the parameters μ_k and σ_k^2 of the additive model (see Mack (2002)). Assuming a variance function of the type $\psi(x) = x^p$, we use log-linear regression to estimate p (see Figure 1).

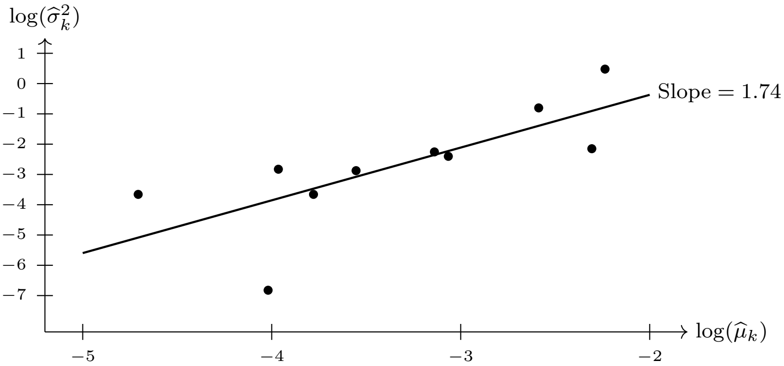

Figure 1: Log-linear regression to estimate the exponent p of the variance function $\psi(\mu) = \mu^p$ for Example 1

The resulting estimator $\widehat{p} = 1.74$ of p is obviously not very reliable, but inspired by this analysis, we include the results of the FR method with variance function $\psi(x) = x^{1.75}$ into our analysis. In the tables below, this method will be referred to as *FR 1.75*.

For such a volatile triangle, it makes sense to use an external estimator $\widehat{\varepsilon}^{\text{ext}}$ for ε. Table 2 contains the predicted reserves for various methods, using a very small estimator $\widehat{\varepsilon}^{\text{ext}} = 0.01\%$.

	Loss	Chain	MLE Approach ($\widehat{\varepsilon}^{\text{ext}} = 0.01\%$)					
i	Ratio	Ladder	dODP	FRF	Gaussian	FR ODP	FR Gamma	FR 1.75
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	13.9	13.4	12.1	13.9	13.9	13.9	13.9	13.9
3	50.4	53.7	46.8	50.4	50.4	50.4	50.4	50.4
4	89.5	120.2	83.3	89.5	89.5	89.5	89.5	89.5
5	126.0	119.1	118.8	126.0	126.0	126.0	126.0	126.0
6	174.4	200.3	166.1	174.4	174.4	174.4	174.4	174.4
7	267.9	266.7	258.1	267.9	267.9	267.9	267.9	267.9
8	373.2	353.5	361.6	373.2	373.2	373.2	373.2	373.2
9	551.3	530.1	539.8	551.3	551.3	551.3	551.3	551.3
10	860.1	729.8	853.5	860.1	860.1	860.1	860.1	860.1
Total	2,506.6	2,386.8	2,440.0	2,506.6	2,506.6	2,506.6	2,506.6	2,506.6

Table 2: Predicted reserves for Example 1 with $\widehat{\varepsilon}^{\text{ext}} = 0.01\%$

We observe that the methods with fixed relativities lead to the same predictions as the loss ratio method. We also see that the dODP model does not align well with the loss ratio method, which is not intuitive. This drawback of the dODP method was highlighted in Section 5 of the paper. The chain ladder reserves are substantially below the loss ratio reserves. Tables 3–5 provide the predicted reserves for various external estimators $\widehat{\varepsilon}^{\text{ext}}$. We observe that the reserves decrease for all variants of the MLE approach with increasing $\widehat{\varepsilon}^{\text{ext}}$. For large values of $\widehat{\varepsilon}^{\text{ext}}$ the reserves are closer to the chain ladder reserves than to the loss ratio predictions.

	Loss	Chain	MLE Approach ($\widehat{\varepsilon}^{\text{ext}} = 3\%$)					
i	Ratio	Ladder	dODP	FRF	Gaussian	FR ODP	FR Gamma	FR 1.75
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	13.9	13.4	12.2	13.8	14.0	14.0	14.0	14.0
3	50.4	53.7	47.9	51.4	51.3	51.4	51.8	51.7
4	89.5	120.2	86.6	92.2	92.9	93.0	92.9	92.9
5	126.0	119.1	121.0	129.2	127.7	128.3	129.2	129.0
6	174.4	200.3	168.6	180.8	176.0	177.1	178.6	178.2
7	267.9	266.7	257.1	272.4	265.8	267.6	270.1	269.4
8	373.2	353.5	355.1	373.9	364.7	367.7	372.1	370.9
9	551.3	530.1	526.0	539.4	534.9	538.9	545.5	543.6
10	860.1	729.8	827.8	824.6	830.9	836.8	846.9	844.0
Total	2,506.6	2,386.8	2,402.3	2,477.7	2,458.1	2,474.8	2,501.0	2,493.7

Table 3: Predicted reserves for Example 1 with $\widehat{\varepsilon}^{\text{ext}} = 3\%$

i	Loss Ratio	Chain Ladder	MLE Approach ($\widehat{\varepsilon}^{\text{ext}} = 10\%$) dODP	FRF	Gaussian	FR ODP	FR Gamma	FR 1.75
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	13.9	13.4	12.3	13.3	13.9	13.9	14.0	13.9
3	50.4	53.7	50.7	52.5	53.4	54.0	55.4	54.9
4	89.5	120.2	99.2	95.4	108.2	105.7	101.1	102.5
5	126.0	119.1	123.2	128.8	127.9	129.4	131.7	131.1
6	174.4	200.3	178.5	192.9	184.5	185.8	187.5	187.1
7	267.9	266.7	258.6	275.6	266.0	267.4	269.8	269.1
8	373.2	353.5	346.3	380.2	352.5	357.0	362.0	360.7
9	551.3	530.1	508.0	527.3	518.5	519.1	519.6	519.2
10	860.1	729.8	778.1	757.4	790.2	785.2	780.7	780.9
Total	2,506.6	2,386.8	2,354.9	2,423.4	2,415.1	2,417.4	2,421.6	2,419.4

TABLE 4: Predicted reserves for Example 1 with $\widehat{\varepsilon}^{\text{ext}} = 10\%$

i	Loss Ratio	Chain Ladder	MLE Approach ($\widehat{\varepsilon}^{\text{ext}} = 20\%$) dODP	FRF	Gaussian	FR ODP	FR Gamma	FR 1.75
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	13.9	13.4	12.1	13.2	13.5	13.6	13.7	13.7
3	50.4	53.7	51.0	52.7	52.7	54.1	56.8	56.0
4	89.5	120.2	106.7	96.7	118.7	113.4	104.9	107.3
5	126.0	119.1	118.9	126.2	120.7	124.0	128.9	127.7
6	174.4	200.3	184.9	198.7	189.4	190.9	193.1	192.5
7	267.9	266.7	258.2	272.5	264.1	265.2	267.6	266.9
8	373.2	353.5	342.9	384.9	345.1	351.5	357.8	356.3
9	551.3	530.1	505.5	528.2	517.0	515.4	509.4	511.2
10	860.1	729.8	729.0	727.9	752.3	736.3	711.1	717.0
Total	2,506.6	2,386.8	2,309.2	2,401.0	2,373.5	2,364.3	2,343.4	2,348.6

TABLE 5: Predicted reserves for Example 1 with $\widehat{\varepsilon}^{\text{ext}} = 20\%$

The estimated standard errors with $\widehat{\varepsilon}^{\text{ext}} = 3\%$ are provided in Table 6. The standard errors of the Gaussian, dODP and FR ODP methods are large, the reason being that the Gaussian and ODP variance structures do not fit the data well.

i	Loss Ratio (MLE)	Chain Ladder (ODP GLM)	MLE Approach ($\widehat{\varepsilon}^{\text{ext}} = 3\%$) dODP	FRF	Gaussian	FR ODP	FR Gamma	FR 1.75
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	6.3	9.6	9.6	6.2	31.9	10.1	4.0	4.9
3	8.5	18.4	18.2	8.5	43.1	17.6	9.0	11.0
4	9.4	29.1	32.3	9.3	57.2	38.8	9.4	14.8
5	14.0	26.9	32.7	14.5	63.4	36.2	12.9	17.3
6	17.0	36.0	43.7	16.7	69.3	52.2	13.2	20.9
7	22.6	42.2	53.4	22.8	77.8	62.8	20.9	27.9
8	26.8	51.0	57.3	27.4	83.9	65.2	28.1	34.2
9	41.4	71.2	64.1	41.3	91.3	73.4	40.3	44.7
10	75.9	116.3	89.4	72.9	104.5	102.9	57.5	61.1
Total	120.7	216.4	315.6	118.6	402.4	378.7	70.2	134.9

TABLE 6: Standard errors for Example 1 with $\widehat{\varepsilon}^{\text{ext}} = 3\%$

Example 2 (Professional Liability)

Table 7 presents the premiums v_i and incremental incurred losses $S_{i,k}$ for a Professional Liability portfolio. The observed loss ratios exhibit significant volatility and cyclicality.

i	v_i	$S_{i,1}$	$S_{i,2}$	$S_{i,3}$	$S_{i,4}$	$S_{i,5}$	$S_{i,6}$	$S_{i,7}$	$S_{i,8}$	$S_{i,9}$
1	3,551.6	48.3	409.9	1,275.1	385.3	174.6	606.2	847.3	178.1	347.8
2	3,808.4	1,535.4	749.2	1,052.4	185.9	1,673.5	1,966.5	248.9	248.9	
3	4,213.3	1,074.7	3,519.3	1,550.6	863.8	986.8	1,705.3	3,807.2		
4	4,461.3	657.4	406.4	532.0	718.6	1,021.8	680.9			
5	3,618.6	48.2	139.7	139.5	885.9	233.0				
6	4,106.0	16.9	127.3	113.0	112.8					
7	5,433.1	9.5	54.4	246.5						
8	7,571.8	19.9	418.4							
9	8,373.1	126.4								

Table 7: Triangle and volumes for Example 2

In the Motor Liability and Commercial Liability examples discussed before, uncertainty in volume measures primarily stems from questions about whether rate changes align with underlying loss trends, such as inflation and frequency shifts. This example exhibits markedly different loss dynamics. Certain accident years display extreme claims activity, likely influenced by an economic downturn or a financial crises. Since such external factors are not directly measurable, they can be regarded as hidden variables contributing to the high uncertainty inherent in the available volume measure. This dataset provides an opportunity to examine how the different reserving approaches handle such a challenging scenario.

Table 8 shows the predicted reserves, where an external estimator $\widehat{\varepsilon}^{\text{ext}} = 0.01\%$ is used in the MLE approaches. Again, the predictions of the dODP model deviate from those of the additive model. The variants with fixed relativities align with the loss ratio method.

	Loss	Chain	MLE Approach ($\widehat{\varepsilon}^{\text{ext}} = 0.01\%$)				
i	Ratio	Ladder	dODP	FRF	Gaussian	FR ODP	FR Gamma
1	0	0	0	0	0	0	0
2	373	679	147	373	373	373	373
3	657	1,760	229	657	657	657	657
4	2,586	1,650	2,357	2,586	2,586	2,586	2,586
5	3,217	1,132	2,871	3,217	3,217	3,217	3,217
6	4,504	456	3,965	4,504	4,504	4,504	4,504
7	6,681	546	5,716	6,681	6,681	6,681	6,681
8	10,584	1,445	9,083	10,584	10,584	10,584	10,584
9	13,031	1,345	11,929	13,031	13,031	13,031	13,031
Total	41,632	9,013	36,297	41,632	41,632	41,632	41,632

Table 8: Predicted reserves for Example 2 with $\widehat{\varepsilon}^{\text{ext}} = 0.01\%$

For this situation, where loss ratios in older years are systematically higher than for the more recent years, chain ladder seems to be much more adequate than the loss ratio method. The chain ladder predictions are substantially lower, which is plausible.

Given the extreme dynamics of this example, we consider relatively large external estimators for ε. Tables 9–12 show the results for external estimators for ε at 10%, 20%, 50%, and 100%. We observe that the predictions of the MLE variants become closer to those of the chain ladder method as $\widehat{\varepsilon}^{\text{ext}}$ increases.

i	Loss Ratio	Chain Ladder	MLE Approach ($\widehat{\varepsilon}^{\text{ext}} = 10\%$) dODP	FRF	Gaussian	FR ODP	FR Gamma
1	0	0	0	0	0	0	0
2	373	679	171	411	374	370	368
3	657	1,760	268	721	675	650	639
4	2,586	1,650	2,293	2,578	2,474	2,387	2,350
5	3,217	1,132	2,621	3,043	2,918	2,807	2,761
6	4,504	456	3,435	4,079	3,928	3,751	3,672
7	6,681	546	4,783	6,031	5,682	5,392	5,250
8	10,584	1,445	7,536	9,817	9,013	8,551	8,297
9	13,031	1,345	10,058	12,599	11,282	10,743	10,426
Total	41,632	9,013	31,164	39,280	36,346	34,651	33,763

Table 9: Predicted reserves for Example 2 with $\widehat{\varepsilon}^{\text{ext}} = 10\%$

i	Loss Ratio	Chain Ladder	MLE Approach ($\widehat{\varepsilon}^{\text{ext}} = 20\%$) dODP	FRF	Gaussian	FR ODP	FR Gamma
1	0	0	0	0	0	0	0
2	373	679	205	464	394	383	379
3	657	1,760	334	822	752	682	654
4	2,586	1,650	2,242	2,423	2,399	2,219	2,153
5	3,217	1,132	2,316	2,540	2,536	2,348	2,278
6	4,504	456	2,786	2,963	3,160	2,873	2,757
7	6,681	546	3,729	4,399	4,443	3,949	3,729
8	10,584	1,445	6,052	8,143	7,402	6,514	6,071
9	13,031	1,345	8,837	12,452	10,237	9,035	8,436
Total	41,632	9,013	26,502	34,206	31,325	28,005	26,459

Table 10: Predicted reserves for Example 2 with $\widehat{\varepsilon}^{\text{ext}} = 20\%$

i	Loss Ratio	Chain Ladder	MLE Approach ($\widehat{\varepsilon}^{\text{ext}} = 50\%$) dODP	FRF	Gaussian	FR ODP	FR Gamma
1	0	0	0	0	0	0	0
2	373	679	319	528	520	545	552
3	657	1,760	661	1,062	1,459	1,186	1,017
4	2,586	1,650	2,004	2,210	1,970	1,950	1,939
5	3,217	1,132	1,409	1,991	1,137	1,189	1,222
6	4,504	456	1,025	1,257	745	802	838
7	6,681	546	1,204	1,561	914	924	900
8	10,584	1,445	3,179	4,203	3,280	2,894	2,514
9	13,031	1,345	9,895	11,869	13,860	10,634	8,699
Total	41,632	9,013	19,696	24,681	23,884	20,125	17,681

Table 11: Predicted reserves for Example 2 with $\widehat{\varepsilon}^{\text{ext}} = 50\%$

i	Loss Ratio	Chain Ladder	MLE Approach ($\widehat{\varepsilon}^{\text{ext}} = 100\%$) dODP	FRF	Gaussian	FR ODP	FR Gamma
1	0	0	0	0	0	0	0
2	373	679	345	558	534	602	678
3	657	1,760	754	1,269	1,549	1,357	1,248
4	2,586	1,650	1,916	2,218	1,885	1,852	1,856
5	3,217	1,132	1,278	2,088	1,030	1,016	976
6	4,504	456	788	880	616	588	534
7	6,681	546	798	940	661	585	480
8	10,584	1,445	1,957	2,462	2,074	1,748	1,397
9	13,031	1,345	6,148	10,813	7,940	6,668	5,873
Total	41,632	9,013	13,983	21,227	16,289	14,417	13,043

Table 12: Predicted reserves for Example 2 with $\widehat{\varepsilon}^{\text{ext}} = 100\%$

Table 13 provides the estimated standard errors for $\widehat{\varepsilon}^{\text{ext}} = 100\%$. We observe that the estimated standard errors of all variants of the MLE approach are substantially lower than those of the loss ratio method.

i	Loss Ratio (MLE)	Chain Ladder (ODP GLM)	MLE Approach ($\widehat{\varepsilon}^{\text{ext}} = 100\%$) dODP	FRF	Gaussian	FR ODP	FR Gamma
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.8	909.9	375.6	32.2	914.1	754.9	691.4
3	37.5	1,697.5	537.4	174.9	1,864.3	1,212.3	931.5
4	1,805.3	1,139.6	986.3	1,460.3	1,315.7	1,244.4	1,678.0
5	1,710.4	937.7	1,179.6	1,583.0	1,345.4	1,350.1	1,778.0
6	1,947.5	633.7	1,501.7	1,890.4	1,658.5	1,643.4	2,064.8
7	2,362.0	768.9	1,873.9	2,477.3	2,045.5	1,996.7	2,447.3
8	3,077.8	1,583.9	2,525.2	3,668.8	2,770.7	2,624.9	3,075.2
9	3,715.3	2,486.6	2,043.6	2,618.0	2,377.9	2,325.0	3,015.9
Total	9,334.2	5,012.8	5,146.2	6,706.9	7,218.0	6,107.0	6,808.9

TABLE 13: Standard errors for Example 2 with $\widehat{\varepsilon}^{\text{ext}} = 100\%$

EXAMPLE 3 (ARTIFICIAL TRIANGLE)

As our last example, we use the artificial dataset provided in Table 14 to demonstrate the impact of the assumed variance function. The volumes are equal for all accident years. The first two development years have large incremental claims and exhibit a strong decreasing trend. The development years 3–8 have small increments and show a strong increasing trend.

i	v_i	$S_{i,1}$	$S_{i,2}$	$S_{i,3}$	$S_{i,4}$	$S_{i,5}$	$S_{i,6}$	$S_{i,7}$	$S_{i,8}$	$S_{i,9}$
1	300.0	100.0	100.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0
2	300.0	95.0	95.0	7.0	7.0	7.0	7.0	7.0	7.0	
3	300.0	90.0	90.0	10.0	10.0	10.0	10.0	10.0		
4	300.0	85.0	85.0	15.0	15.0	15.0	15.0			
5	300.0	80.0	80.0	20.0	20.0	20.0				
6	300.0	75.0	75.0	25.0	25.0					
7	300.0	70.0	70.0	30.0						
8	300.0	65.0	65.0							
9	300.0	60.0								

TABLE 14: Triangle Ex 4

In this example, we use $\widehat{\varepsilon}^{\text{ext}} = 10\%$ for all calculations. Table 15 displays the posterior means $\widehat{v}_i^{(\infty)}$ of the volumes V_i with different variance functions. The Gaussian model assumes the same variances for all increments in the triangle. Therefore, the first two development years (which have large increments) heavily influence the posterior distribution of $(V_1, \ldots, V_{m-1})$, resulting in posterior means $\widehat{v}_i^{(\infty)}$ that are substantially larger than the volumes v_i (for $i \leq 8$). The dODP and FR ODP approaches assume that the variances are linear in the μ_k and $\widehat{\mu}_k^{\text{MLE}}$, respectively. Therefore, lower variances are assumed for the development years 3–8, which results in a lower correction of the volumes. Finally, the FR Gamma approach assumes that the standard deviations are linear in the $\widehat{\mu}_k^{\text{MLE}}$. Here, the development years 3–8 with the low increments have a significant impact, resulting in posterior means $\widehat{v}_i^{(\infty)}$ that are lower than the original volumes v_i for $i \in \{1, 2, 3, 4\}$.

i	v_i	$\widehat{v}_i^{(\infty)}$ MLE Approach ($\widehat{\varepsilon}^{\text{ext}} = 10\%$) dODP	FRF	Gaussian	FR ODP	FR Gamma
1	300.0	372.9	398.1	459.8	374.9	182.2
2	300.0	374.2	408.2	445.3	376.4	208.4
3	300.0	376.6	406.8	427.7	378.9	246.1
4	300.0	378.3	397.6	409.8	380.4	287.4
5	300.0	373.1	381.1	390.0	374.8	315.7
6	300.0	359.0	359.2	367.5	360.1	325.7
7	300.0	336.8	334.5	342.2	337.3	317.9
8	300.0	312.3	311.6	316.1	312.3	301.5
9	300.0	300.0	300.0	300.0	300.0	300.0

TABLE 15: Posterior means $\widehat{v}_i^{(\infty)}$ for Example 3

Table 16 provides the reserve estimates. The FR Gamma model leads to a higher total reserve than the loss ratio method. For all other variants of the MLE approach, the total reserves are lower, since the $\widehat{v}_i^{(\infty)}$ are greater than the v_i (for $i \leq 8$).

i	Loss Ratio	Chain Ladder	MLE Approach ($\widehat{\varepsilon}^{\text{ext}} = 10\%$) dODP	FRF	Gaussian	FR ODP	FR Gamma
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	5.0	5.0	4.0	5.1	4.8	5.0	5.7
3	11.0	11.3	9.2	11.2	10.3	11.1	14.2
4	18.3	19.3	15.9	18.1	16.6	18.5	26.6
5	27.6	28.9	24.5	26.1	23.9	27.4	42.3
6	39.0	39.0	34.7	34.7	32.0	37.1	59.3
7	52.7	47.8	45.5	43.7	40.5	47.0	75.4
8	68.7	52.2	56.7	53.2	49.1	56.7	90.4
9	151.2	108.2	122.6	117.3	109.3	122.8	174.9
total	373.4	311.8	313.1	309.4	286.4	325.6	488.8

TABLE 16: Predicted reserves for Example 3

Table 17 evaluates, the accuracy of the approximation of the variance functions by the fixed relativities. The approximation is acceptable for the FR ODP method but not for the FR Gamma approach. In this extreme example, the FR Gamma approach leads to very different relativities between the estimators of the μ_k than maximum likelihood estimation in the additive model with normal distributed increments. We therefore follow the approach proposed in the remark at the end of Section 6 and iterate with adjusted relativities η_k. After 10 iterations, the variance function is approximated very well (see Table 17).

	Without Iterations		10 Iterations	
	FR ODP	FR Gamma	FR ODP	FR Gamma
k	$(\widehat{\sigma}_k^{(\infty)})^2/\widehat{\mu}_k^{(\infty)}$	$(\widehat{\sigma}_k^{(\infty)})^2/(\widehat{\mu}_k^{(\infty)})^2$	$(\widehat{\sigma}_k^{(10\vert\infty)})^2/\widehat{\mu}_k^{(10\vert\infty)}$	$(\widehat{\sigma}_k^{(10\vert\infty)})^2/(\widehat{\mu}_k^{(10\vert\infty)})^2$
1	2.71	27.6	2.85	26.3
2	2.77	27.2	2.85	26.3
3	2.89	21.0	2.85	26.3
4	2.89	19.8	2.85	26.3
5	2.89	18.1	2.85	26.3
6	2.89	16.1	2.85	26.3
7	2.89	14.2	2.85	26.3
8	2.88	12.3	2.85	26.3
9	2.88	10.9	2.85	26.3

TABLE 17: Accuracy of the approximation of the variance functions without and with iterations

Table 18 provides the posterior means $\widehat{v}_i^{(\infty)}$ for the FR ODP and the FR Gamma method after 10 iterations. The corrections for the FR ODP model are small, but the posterior means for the FR Gamma model change substantially.

		$\widehat{v}_i^{(\infty)}$ MLE Approach (10 Iterations)	
i	v_i	FR ODP	FR Gamma
1	300.0	371.2	215.7
2	300.0	373.2	237.6
3	300.0	376.5	269.9
4	300.0	378.8	305.0
5	300.0	373.9	328.1
6	300.0	359.7	334.0
7	300.0	337.1	323.1
8	300.0	312.2	304.4
9	300.0	300.0	300.0

Table 18: Posterior means $\widehat{v}_i^{(\infty)}$ for Example 3 after 10 iterations

Table 19 presents the corresponding reserve estimates after 10 iterations. The changes for the FR ODP approach are small, but the reserves for the FR Gamma approach are significantly reduced. As mentioned above, this is an extreme example. We would not expect such large effects for realistic situations.

	Loss	Chain	MLE Approach – 10 Iterations	
i	Ratio	Ladder	FR ODP	FR Gamma
1	0.0	0.0	0.0	0.0
2	5.0	5.0	5.0	5.4
3	11.0	11.3	11.1	13.3
4	18.3	19.3	18.6	24.4
5	27.6	28.9	27.5	38.4
6	39.0	39.0	37.4	53.7
7	52.7	47.8	47.3	68.3
8	68.7	52.2	57.2	82.0
9	151.2	108.2	123.5	162.7
total	373.4	311.8	327.6	448.2

Table 19: Predicted reserves for Example 3 after 10 Iterations

References

Mack, T. (2002). *Schadenversicherungsmathematik. 2. Auflage.* VVW, Karlsruhe.

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