

Appendix: Coefficients of the Main Intermediate Equations

$$f_0 = -L_1^2 L_2^2 + L_2^4 - 2L_1^2 m^2 + 6L_2^2 m^2 + 8m^4 - 8m^2 m_1^2$$

$$f_1 = -3L_1^2 L_2 m + 7L_2^3 m + 20L_2 m^3 - 4L_2 m m_1^2$$

$$f_2 = 12L_2^2 m^2$$

$$f_3 = -L_2^2 m^2 \left(L_1^4 + (L_2^2 + 4m^2 - 4m_1^2)^2 - 2L_1^2 (L_2^2 + 4(m^2 + m_1^2)) \right)$$

$$f_4 = 8L_2^3 m^3 (L_1^2 - L_2^2 - 4m^2 + 4m_1^2)$$

$$f_5 = L_2^2 m^2 \left(\begin{array}{l} L_1^4 + L_2^4 + 16(m^2 - m_1^2)^2 - 8L_2^2 (m^2 + m_1^2) \\ -2L_1^2 (L_2^2 + 4(m^2 + m_1^2)) \end{array} \right)$$

$$f_6 = -f_4$$

$$f_7 = -f_3 - f_5$$

$$h_0 = (f_0 + f_1 + f_2)^2 - f_3 - f_4 - f_5 - f_6 - f_7$$

$$h_1 = 0$$

$$h_2 = 2(2(f_0 - f_2)(f_0 + f_1 + f_2) - 2f_3 - f_4 + f_6 + 2f_7)$$

$$h_3 = 0$$

$$h_4 = 2(3f_0^2 - f_1^2 - 2f_0 f_2 + 3f_2^2 - 3f_3 + f_5 - 3f_7)$$

$$h_5 = 0$$

$$h_6 = 2(2(f_0 - f_2)(f_0 - f_1 + f_2) - 2f_3 + f_4 - f_6 + 2f_7)$$

$$h_7 = 0$$

$$h_8 = ((f_0 - f_1 + f_2)^2 - f_3 + f_4 - f_5 + f_6 - f_7)$$

$$f_{10} = \left(\begin{array}{l} L_1^4 + L_2^4 + 8c\beta L_2^3 m + 32c\beta L_2 m(m^2 - m_1^2) \\ -L_2^2 m^2 \left(\begin{array}{l} +16(m^2 - m_1^2)^2 + 8L_2^2 ((2 + c2\beta)m^2 - m_1^2) \\ -2L_1^2 (L_2^2 + 4c\beta L_2 m + 4(m^2 + m_1^2)) \end{array} \right) \end{array} \right) (s\beta)^2 \right)^{\frac{1}{2}}$$

$$f_{11} = \frac{\left(\begin{array}{l} L_1^2 L_2^3 m + L_2^5 m - 2L_2^3 m(L_3^2 - 3m^2 + m_1^2 - n^2 + n_1^2) \\ +2L_2 m^3 (L_1^2 - 4(L_3^2 - n^2 + n_1^2)) \\ +L_2^2 m^2 (3L_1^2 + 5L_2^2 - 8L_3^2 + 4m^2 - 4m_1^2 + 8n^2 - 8n_1^2) c\beta \\ +2L_2^3 m^3 (c\beta)^2 - 2L_2^3 m^3 (s\beta)^2 - K_A L_2 m f_{10} \end{array} \right)}{2L_2 m (L_2^2 + 4m^2 + 4L_2 m c\beta)}$$

$$f_{12} = \frac{\left(-L_2^2 m (L_1^2 + 3L_2^2 + 12m^2 - 4m_1^2) ns\beta \right. \\ \left. + K_A n (2m + L_2 c\beta) \frac{1}{s\beta} f_{10} - 6L_2^3 m^2 ns 2\beta \right)}{2L_2 m (L_2^2 + 4m^2 + 4L_2 mc\beta)}$$

$$j_0 = f_1$$

$$j_1 = 2f_2$$

$$j_2 = f_1$$

$$d_1 = 2L_{02}^2 + 8m_0^2$$

$$d_2 = 8L_{02}m_0$$

$$d_3 = L_{02}^4 + 2m_0^2 \left(-L_{01}^2 + 4(m_0 - m_{01})(m_0 + m_{01}) \right) - L_{02}^2 L_{01}^2 + 12L_{02}^2 m_0^2 - 6L_{02}^2 m_0^2$$

$$d_4 = L_{02}m_0 \left(-3L_{01}^2 + 20m_0^2 - 4m_{01}^2 \right) + 7L_{02}^3 m_0$$

$$d_5 = 12L_{02}^2 m_0^2$$

$$d_6 = 0$$

$$d_7 = L_{02} \left(L_{01}^2 - 4(m_0^2 + m_{01}^2) \right) n_0 - n_0 L_{02}^3$$

$$d_8 = -4L_{02}^2 m_0 n_0$$

$$d_9 = -2m_0 n_0$$

$$d_{10} = L_{02}m_0$$

$$d_{11} = -L_{02}n_0$$

$$d_{12} = -L_{01}^4 - L_{02}^4 - 16(m_0^2 - m_{01}^2)^2 + 8L_{02}^2 (-2m_0^2 + m_{01}^2) + 2L_{01}^2 (L_{02}^2 + 4(m_0^2 + m_{01}^2)) + 8L_{02}^2 m_0^2$$

$$d_{13} = 8L_{02}m_0 (L_{01}^2 - L_{02}^2 - 4m_0^2 + 4m_{01}^2)$$

$$d_{14} = -16L_{02}^2 m_0^2$$

$$D_1 = L_{02}^4 + L_{01}^2 (L_{02}^2 + 2m_0^2) + 8m_0^2 (-L_{03}^2 + n_0^2 + n_{01}^2)$$

$$+ 2L_{02}^2 (-L_{03}^2 + 3m_0^2 - m_{01}^2 + n_0^2 + n_{01}^2) - 2L_{02}^2 m_0^2$$

$$D_2 = 3L_{01}^2 L_{02}m_0 + 5L_{02}^3 m_0 - 8L_{02}L_{03}^2 m_0 + 4L_{02}m_0^3 - 4L_{02}m_0m_{01}^2 + 8L_{02}m_0n_0^2 + 8L_{02}m_0n_{01}^2$$

$$D_3 = -\frac{1}{4}d_{14}$$

$$D_4 = -4L_{02}^2 n_0 n_{01} - 16m_0^2 n_0 n_{01}$$

$$D_5 = d_6$$

$$D_6 = -16L_{02}m_0 n_0 n_{01}$$

$$D_7 = -L_{01}^2 L_{02}n_0 - 3L_{02}^3 n_0 - 12L_{02}m_0^2 n_0 + 4L_{02}m_{01}^2 n_0$$

$$D_8 = 3d_8$$

$$\begin{aligned}
t_1 &= (L_{02} + 2m_0)(L_{02}^3 + 5L_{02}^2m_0 + 8L_{02}m_0^2 + 4m_0^3 - L_{01}^2(L_{02} + m_0) - 4m_0m_{01}^2) \\
t_2 &= -2(-L_{02}^4 + 6L_{02}^2m_0^2 - 8m_0^4 + L_{01}^2(L_{02}^2 + 2m_0^2) + 8m_0^2m_{01}^2) \\
t_3 &= (L_{02} - 2m_0)(L_{02}^3 - 5L_{02}^2m_0 + 8L_{02}m_0^2 - 4m_0^3 + L_{01}^2(-L_{02} + m_0) + 4m_0m_{01}^2) \\
t_4 &= 2L_{02}m_0 \\
t_5 &= -4L_{02}(-L_{01}^2 + L_{02}^2 + 4L_{02}m_0 + 4(m_0^2 + m_{01}^2))n_0 \\
t_6 &= 4L_{02}(L_{01}^2 - L_{02}^2 + 4L_{02}m_0 - 4(m_0^2 + m_{01}^2))n_0 \\
t_7 &= -2(L_{02} + 2m_0)n_0 \\
t_8 &= -8m_0n_0 \\
t_9 &= 2(L_{02} - 2m_0)n_0 \\
r_1 &= (L_{02} + 2m_0)\left(L_{02}^3 + 3L_{02}^2m_0 + L_{01}^2(L_{02} + m_0) + 4m_0(-L_{03}^2 + n_0^2 + n_{01}^2) + 2L_{02}(-L_{03}^2 + m_0^2 - m_{01}^2 + n_0^2 + n_{01}^2)\right) \\
r_2 &= 2\left(L_{02}^4 + L_{01}^2(L_{02}^2 + 2m_0^2) + 8m_0^2(-L_{03}^2 + n_0^2 + n_{01}^2) + 2L_{02}^2(-L_{03}^2 - m_{01}^2 + n_0^2 + n_{01}^2)\right) \\
r_3 &= (L_{02} - 2m_0)\left(L_{02}^3 + L_{01}^2(L_{02} - m_0) - 3L_{02}^2m_0 + 4m_0(L_{03}^2 - n_0^2 - n_{01}^2) + 2L_{02}(-L_{03}^2 + m_0^2 - m_{01}^2 + n_0^2 + n_{01}^2)\right) \\
r_4 &= -4(L_{02} + 2m_0)^2n_0n_{01} \\
r_5 &= -8(L_{02}^2 + 4m_0^2)n_0n_{01} \\
r_6 &= -4(L_{02} - 2m_0)^2n_0n_{01} \\
r_7 &= -t_4 \\
r_8 &= -4L_{02}(L_{01}^2 + 3(L_{02} + 2m_0)^2 - 4m_{01}^2)n_0 \\
r_9 &= -4L_{02}(L_{01}^2 + 3(L_{02} - 2m_0)^2 - 4m_{01}^2)n_0 \\
r_{10} &= -t_7 \\
r_{11} &= -t_8 \\
r_{12} &= -t_9 \\
r_{13} &= -2r_4 \\
r_{14} &= -2r_5 \\
r_{15} &= -2r_6
\end{aligned}$$

$$T(y_0) = \left(\frac{\begin{aligned} & \left((L_{01} + L_{02} + 2m_0 - 2m_{01})(L_{01} - L_{02} - 2m_0 + 2m_{01}) \right. \\ & \left. (L_{01} - L_{02} - 2(m_0 + m_{01}))(L_{01} + L_{02} + 2(m_0 + m_{01})) \right. \\ & + 2 \left(L_{01}^4 + L_{02}^4 + 16(m_0^2 - m_{01}^2)^2 - 8L_{02}^2(m_0^2 + m_{01}^2) \right) y^2 \\ & \left. - 2L_{01}^2(L_{02}^2 + 4(m_0^2 + m_{01}^2)) \right) \\ & + (L_{01} - L_{02} + 2m_0 - 2m_{01})(L_{01} + L_{02} - 2m_0 + 2m_{01}) \\ & \left. (L_{01} + L_{02} - 2(m_0 + m_{01}))(L_{01} - L_{02} + 2(m_0 + m_{01})) y^4 \right) \\ & - (1 + y^2)^2 \end{aligned}}{\begin{aligned} & \left((L_{01} + L_{02} + 2m_0 - 2m_{01})(L_{01} - L_{02} - 2m_0 + 2m_{01}) \right. \\ & \left. (L_{01} - L_{02} - 2(m_0 + m_{01}))(L_{01} + L_{02} + 2(m_0 + m_{01})) \right) \\ & + 2 \left(L_{01}^4 + L_{02}^4 + 16(m_0^2 - m_{01}^2)^2 - 8L_{02}^2(m_0^2 + m_{01}^2) \right) y^2 \\ & \left. - 2L_{01}^2(L_{02}^2 + 4(m_0^2 + m_{01}^2)) \right) \\ & + (L_{01} - L_{02} + 2m_0 - 2m_{01})(L_{01} + L_{02} - 2m_0 + 2m_{01}) \\ & \left. (L_{01} + L_{02} - 2(m_0 + m_{01}))(L_{01} - L_{02} + 2(m_0 + m_{01})) y^4 \right) \\ & - (1 + y^2)^2 \end{aligned}} \right)^{\frac{1}{2}}$$