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Bragg scattering of surface-gravity waves by an ice shelf with rolling surface morphology

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Supplemental material

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Yuri V. Konovalov

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ORCID: 0000-0002-8469-9706

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Department of Mathematics, MIREA - Russian Technological University, Vernadsky avenue 78,

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Moscow, Russian Federation, 119454

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Numerical experiments

Numerical experiments were carried out using three models with different combinations of parameters α_1 and α_2 in equation (10). The results presented below were obtained using **Model 1** for (i) $\alpha_1 = 1$; $\alpha_2 = 0$ and (ii) $\alpha_1 = 0.2$; $\alpha_2 = 0.8$; using **Model 2** for $\alpha_1 = 0$; $\alpha_2 = 1$ (the results obtained for $\alpha_1 = 1$; $\alpha_2 = 0$ are presented in the main manuscript); and using **Model 3** for $\alpha_1 = 1$; $\alpha_2 = 0$.

1. Results obtained by Model 1 (case of $\alpha_1 = 1.0$, $\alpha_2 = 0.0$)

Experiments with ice shelves that having a rolling surface morphology have revealed the following. There is a threshold value of the amplitude of ice thickness oscillations (A_H), at which band gaps appear in the dispersion spectra (Figure 1) (Konovalov, 2023a). Essentially, the amplitude of ice thickness oscillations (A_H) determines the depth of the ice shelf cavities that result from the “rolling” morphology. These cavities are the analogous to crevasses in the base of an ice shelf (Freed-Brown et al., 2012).

In the considered experiments this threshold value depends on the value of the *Bragg wavenumber*. In particular, the first band gap, corresponding to the first *Bragg wavenumber* $k_b^{(1)} \approx 6.28 \text{ km}^{-1}$, appears in the spectrum at $A_H > 16 \text{ m}$ (Fig. 1a), i.e. the first threshold value $(A_H)_{th}^{(1)} \approx 16 \text{ m}$. Curves 4 and 5 in Figure 1 show that the typical dependence of wavenumber vs periodicity/frequency, similar to that observed in curves 1, 2 and 3, is absent for $A_H \geq 16 \text{ m}$. The algorithm used to determine the wavenumber (Konovalov, 2021b) in the general case provides the value of the wavenumber. However, the absence of a typical wavenumber dependence (as in curves 1, 2, 3 in Fig. 1a) should essentially be treated as the absence of a wavenumber. Essentially in the range (of periodicity of the forcing) 2..190 s the superposition of band gaps for two Bragg's

68 wavenumbers ($k_b^{(1)} \approx 6.28 \text{ km}^{-1}$, $k_b^{(2)} \approx 12.57 \text{ km}^{-1}$) is observed in Curves 4 and 5 in
69 Figure 1. Respectively, in the range of 5..190 s there are no resonances in the amplitude
70 spectrum (Curve 3 in Fig. 4a).

71 The second band gap (Fig. 1b), which, accordingly, corresponds to the second *Bragg*
72 *wavenumber* $k_b^{(2)} \approx 12.57 \text{ km}^{-1}$, appears in the spectrum at $A_H \geq 1 \text{ m}$, i.e. the second
73 threshold value $(A_H)_{th}^{(2)} \leq 1 \text{ m}$. Similarly, the third and fourth band gaps (Fig. 1c) which
74 respectively correspond to the *Bragg wavenumbers* $k_b^{(3)} \approx 19.04 \text{ km}^{-1}$ and $k_b^{(4)} \approx$
75 25.13 km^{-1} , also appear in the spectrum at $A_H \geq 1 \text{ m}$, so that $(A_H)_{th}^{(3)} \leq 1 \text{ m}$ and $(A_H)_{th}^{(4)} \leq$
76 1 m .

77

78 If the amplitude of ice thickness oscillations (A_H) is less than the threshold value, then the
79 dominant effect in the model is resonance (Figure 2, Curves 1 and 2) and the dispersion
80 spectra reveal areas of the expected appearance of band gaps (Curves 1, 2 and 3 in Figure
81 1a). These areas are located in the vicinity of the Bragg value (for the considered
82 periodicity of the rolls the first Bragg wavenumber is equal to 6.28 km^{-1}).

83 When the amplitude of ice thickness oscillations (A_H) exceeds the threshold value, the
84 band gaps become the dominant effect and abate the resonances in the amplitude spectra
85 (Figure 2, Curve 3).

86 Thus, we can say that the abatement of the incident wave by the ice shelf with a “rolling”
87 surface/base morphology protects the ice shelf on the resonant impact.

88

89 Figure 3 shows superpositions of the dispersion spectrum and the amplitude spectrum in
90 the periodicity range containing the region of the expected first band gap. These
91 superpositions reveal that in the region of the expected band gap, the resonance peak
92 coincides with the part of the dispersion spectrum, where the wavenumbers are close to
93 the first *Bragg wavenumber* $k_b^{(1)} \approx 6.28 \text{ km}^{-1}$. In particular, with the amplitude of ice

94 thickness fluctuations $A_H = 5 \text{ m}$, the resonance peak is observed at the periodicity $T_n \approx$
95 32.68 s (i.e. $T_n \approx 32.68 \text{ s}$ is one of the eigenvalues), at which the wavenumber in the
96 dispersion spectrum is about 5.86 km^{-1} (i.e. $k_n \approx 5.86 \text{ km}^{-1}$) (Fig. 3a). Similarly, with the
97 amplitude of ice thickness fluctuations $A_H = 10 \text{ m}$, the wavenumber k_n is about
98 6.23 km^{-1} (Fig. 3b) and, with the amplitude of ice thickness fluctuation is $A_H = 12 \text{ m}$, the
99 wavenumber k_n is about 6.43 km^{-1} (Fig. 3c). Thus, the relative deviation of k_n from the
100 first Bragg wavenumber $k_b^{(1)}$ does not exceed 7%.

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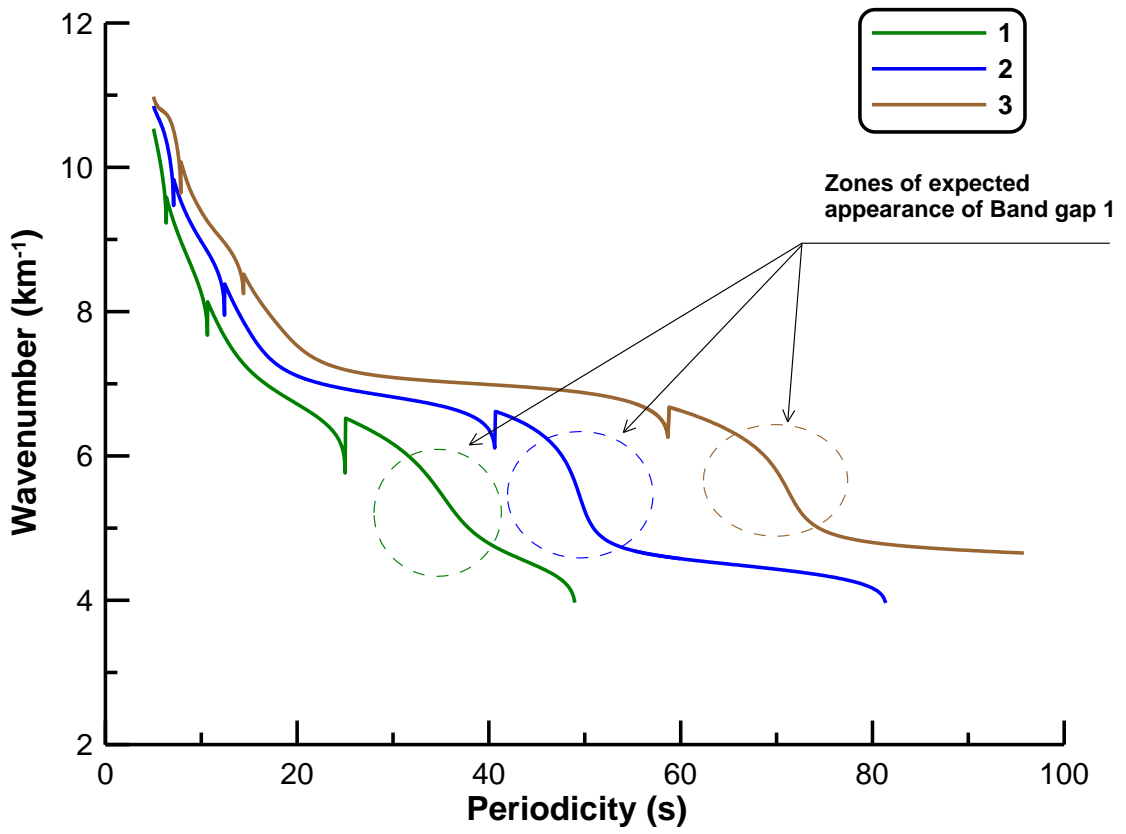


Fig. 1a

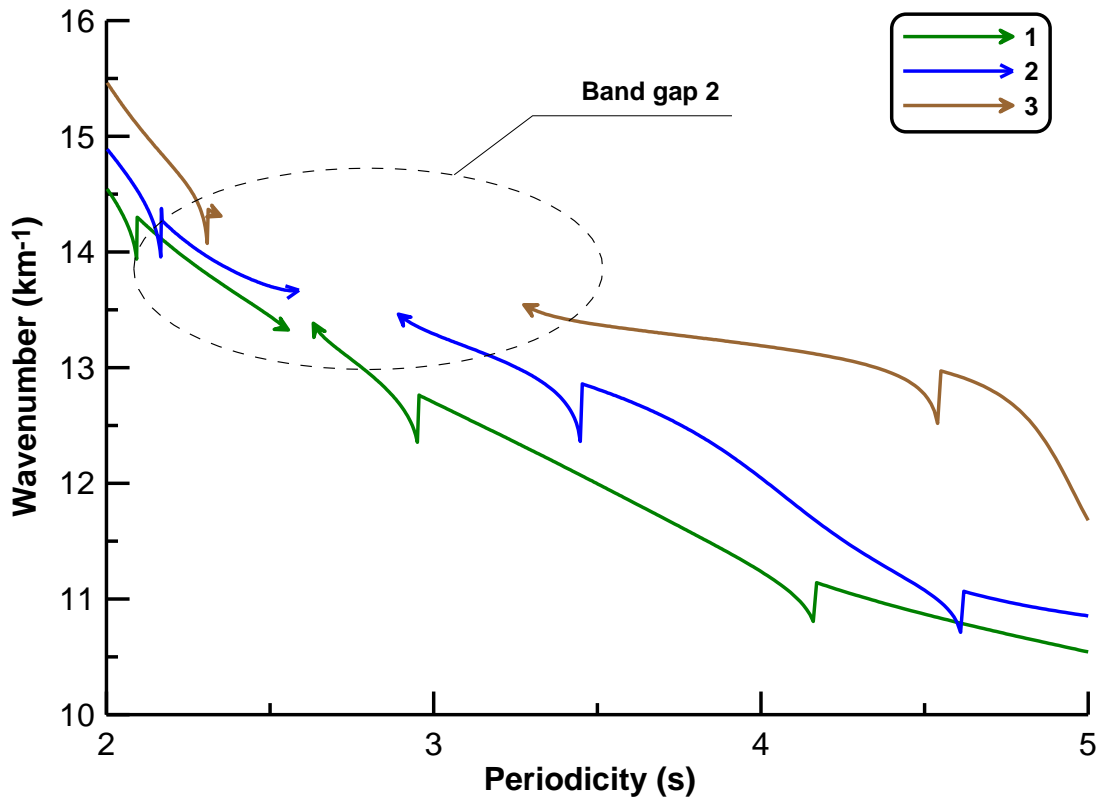


Fig. 1b

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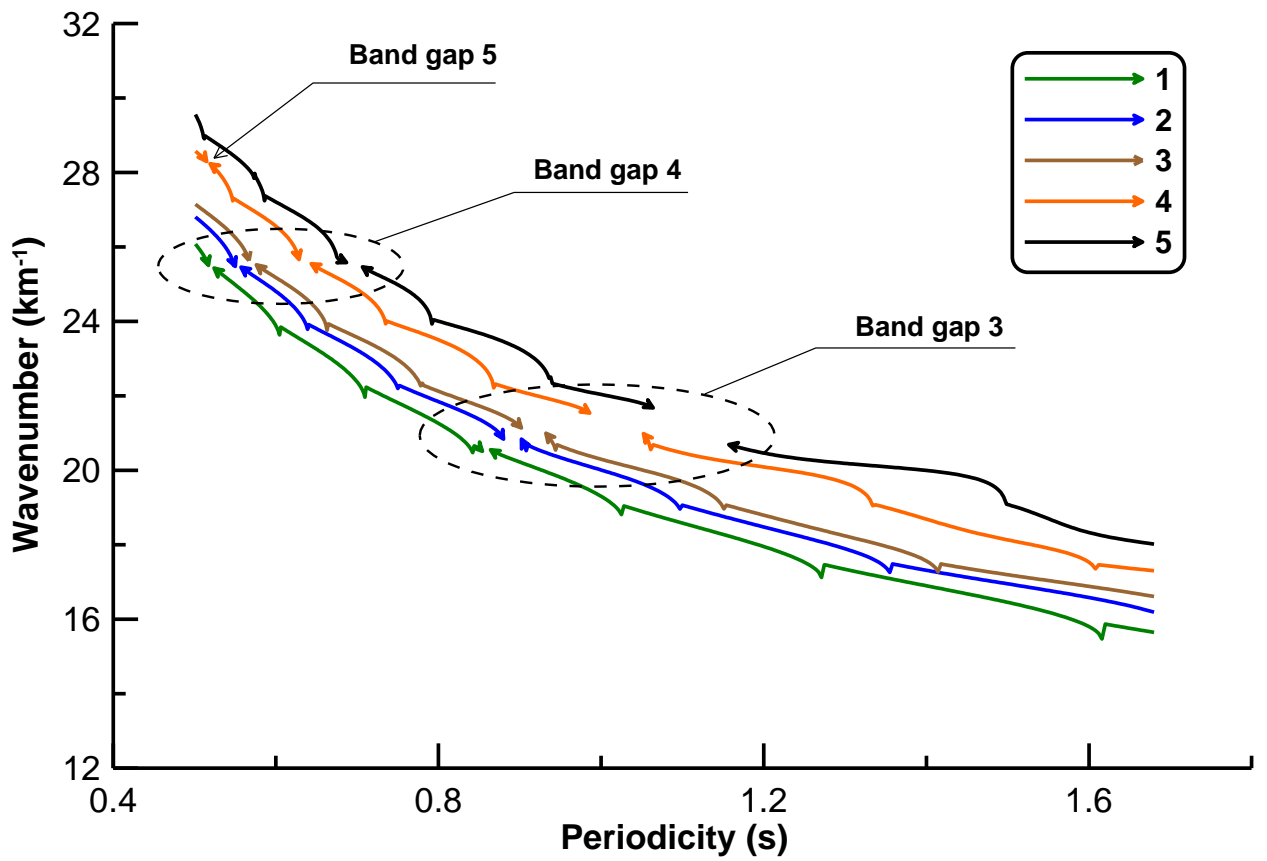


Fig. 1c

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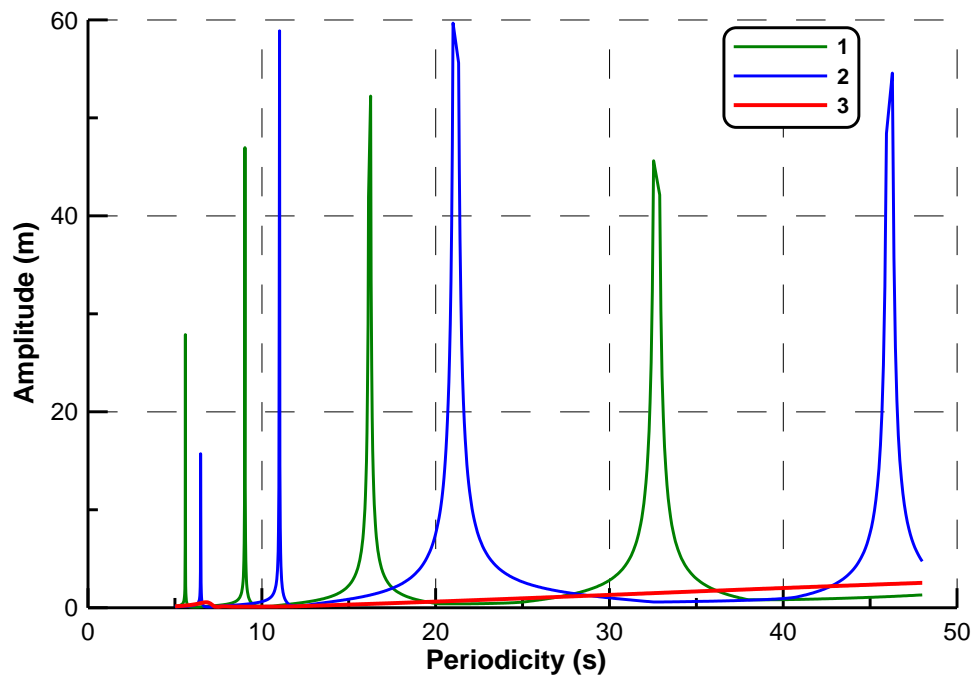
114 **Figure 1.** Dispersion spectra obtained using **Model 1** with $\alpha_1 = 1$, $\alpha_2 = 0$ for ice shelf geometries

115 differing in the amplitude of ice thickness oscillations A_H : **1** $-A_H = 5$ m; **2** $-A_H = 10$ m; **3** $-A_H =$

116 12 m; **4** $-A_H = 16$ m; **5** $-A_H = 18$ m. (a) area of the expected first band gap; (b) area of the

117 expected second band gap; (c) area of expected third and fourth band gaps.

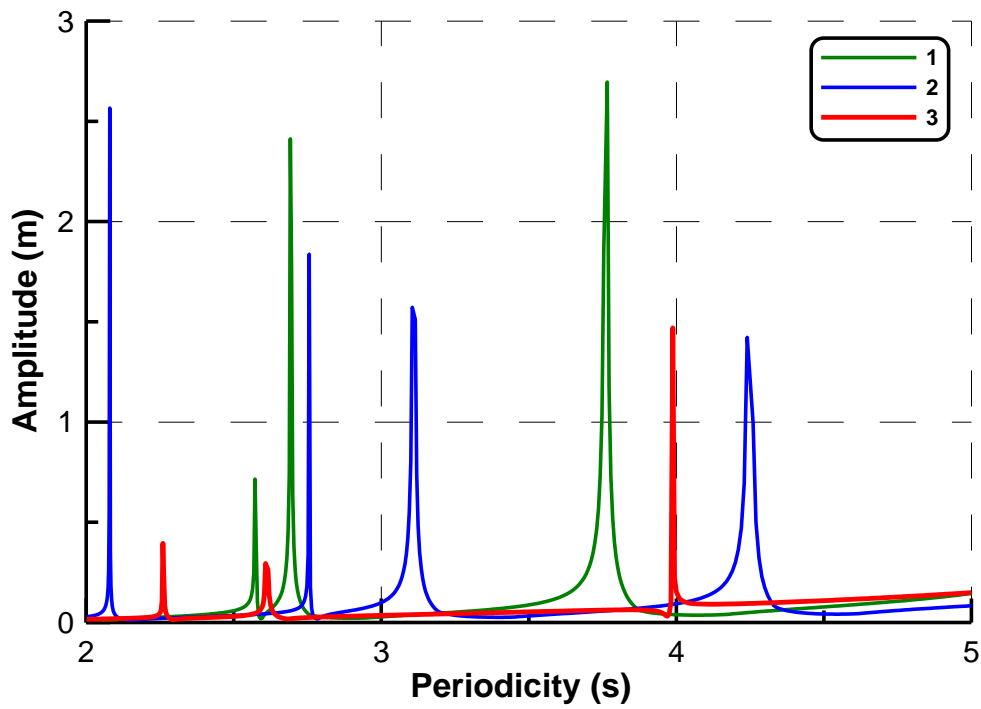
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Fig. 2a



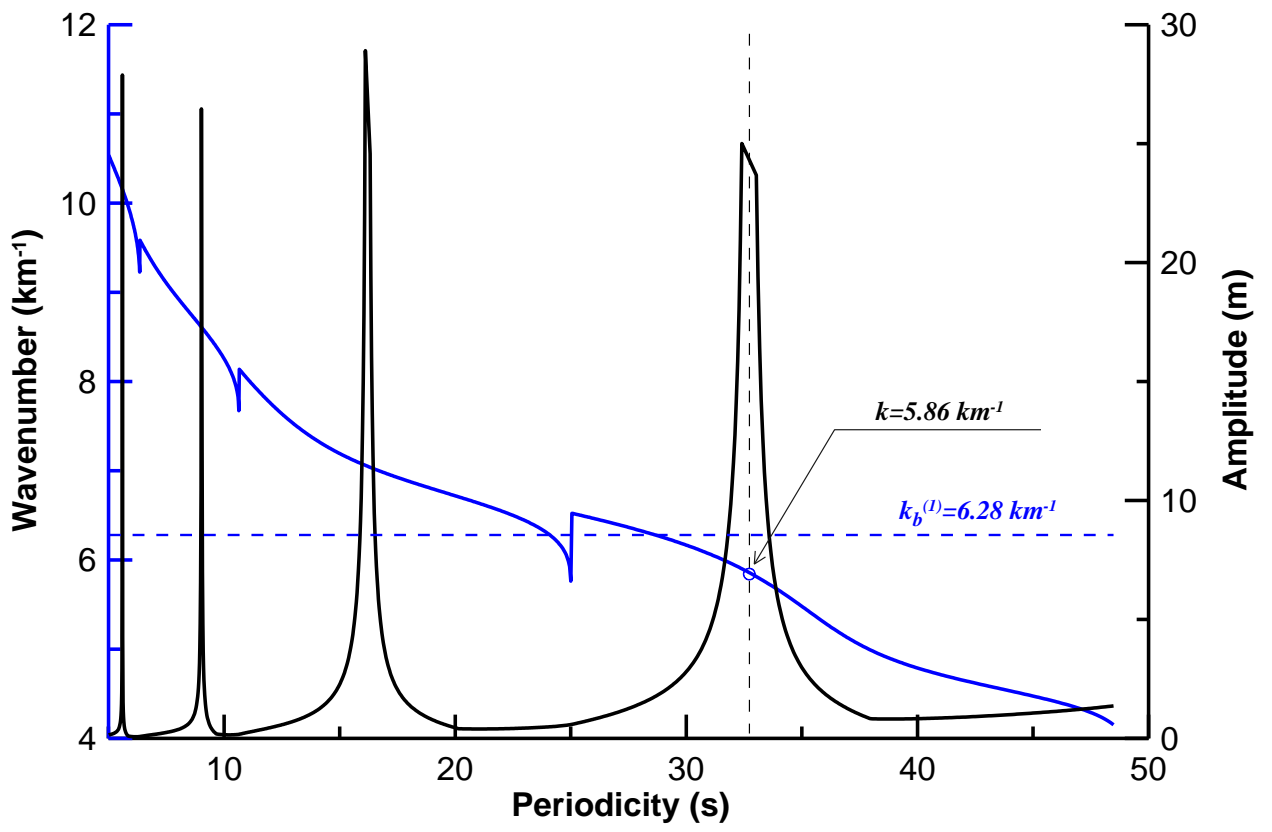
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Fig. 2b

123 **Figure 2.** Amplitude spectra obtained using **Model 1** with $\alpha_1 = 1$, $\alpha_2 = 0$ for ice shelf geometries
 124 differing in the amplitude of ice thickness oscillations A_H : **1** $-A_H = 5$ m; **2** $-A_H = 10$ m; **3** $-A_H =$
 125 18 m. **(a)** area of the expected first band gap (Fig 1a); **(b)** area of the expected second band gap
 126 (Fig. 1b).

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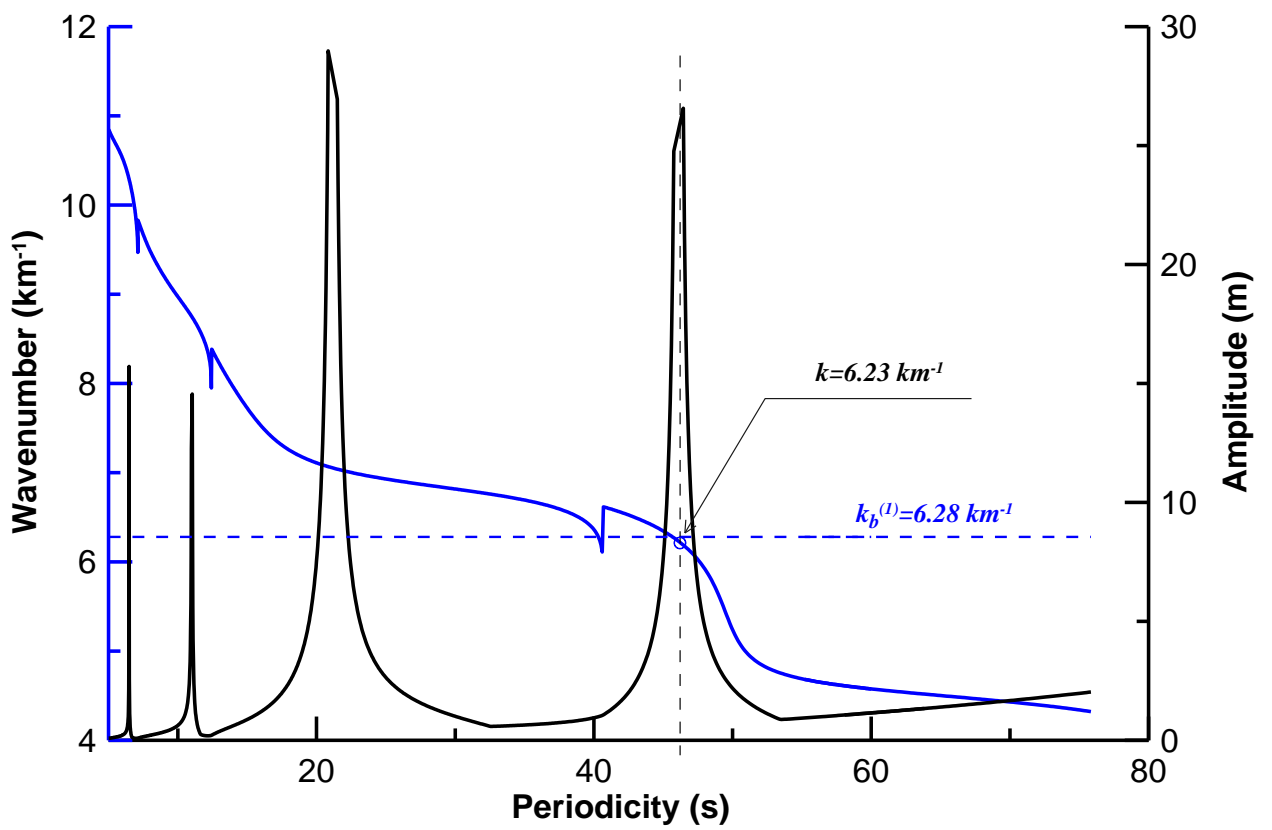


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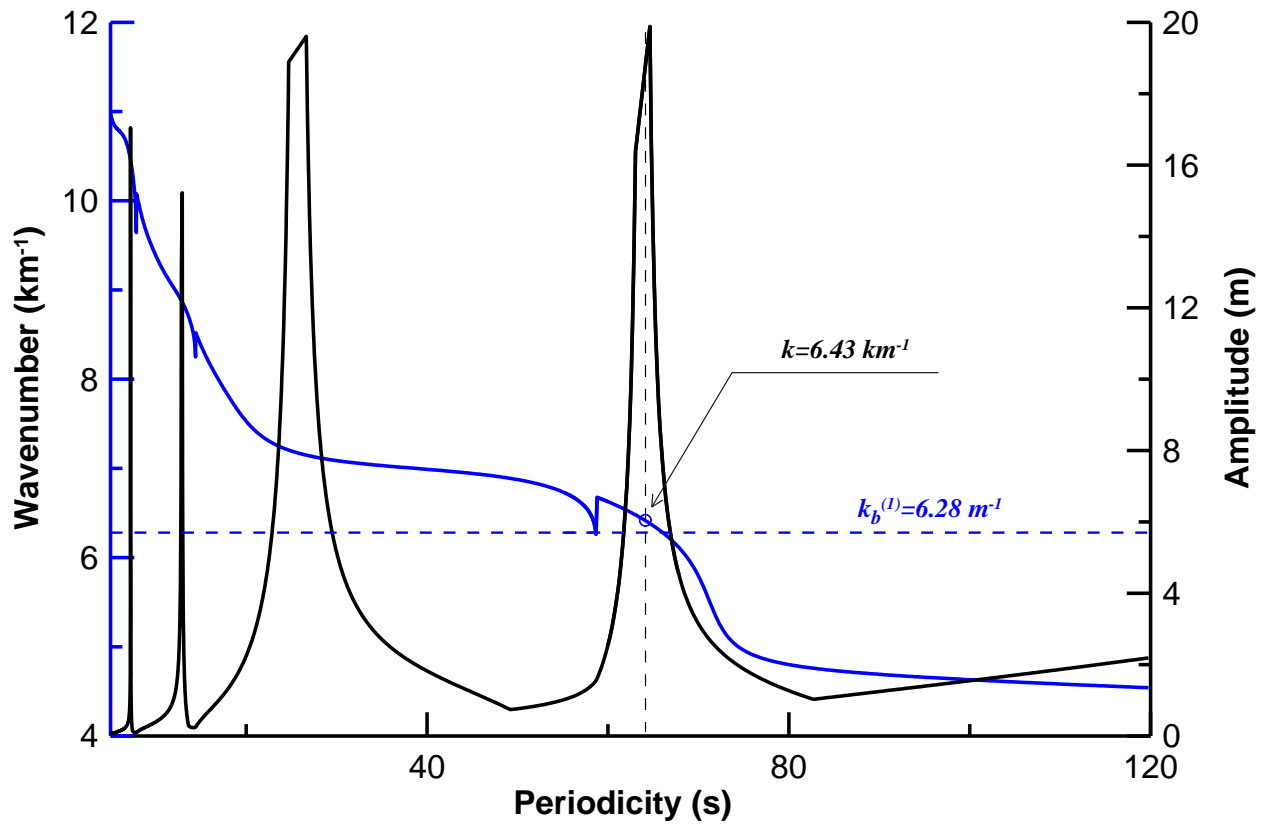
Fig. 3a



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Fig. 3b



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136

137 **Figure 3.** Dispersion spectrum and amplitude spectrum, including the area of the expected first
 138 band gap, obtained using **Model 1** with $\alpha_1 = 1$, $\alpha_2 = 0$ for ice shelf geometries differing in the
 139 amplitude of ice thickness fluctuations A_H : (a) $A_H = 5$ m; (b) $A_H = 10$ m; (c) $A_H = 12$ m.

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143 **2. Results obtained by Model 1 (case of $\alpha_1 = 0.2, \alpha_2 = 0.8$)**

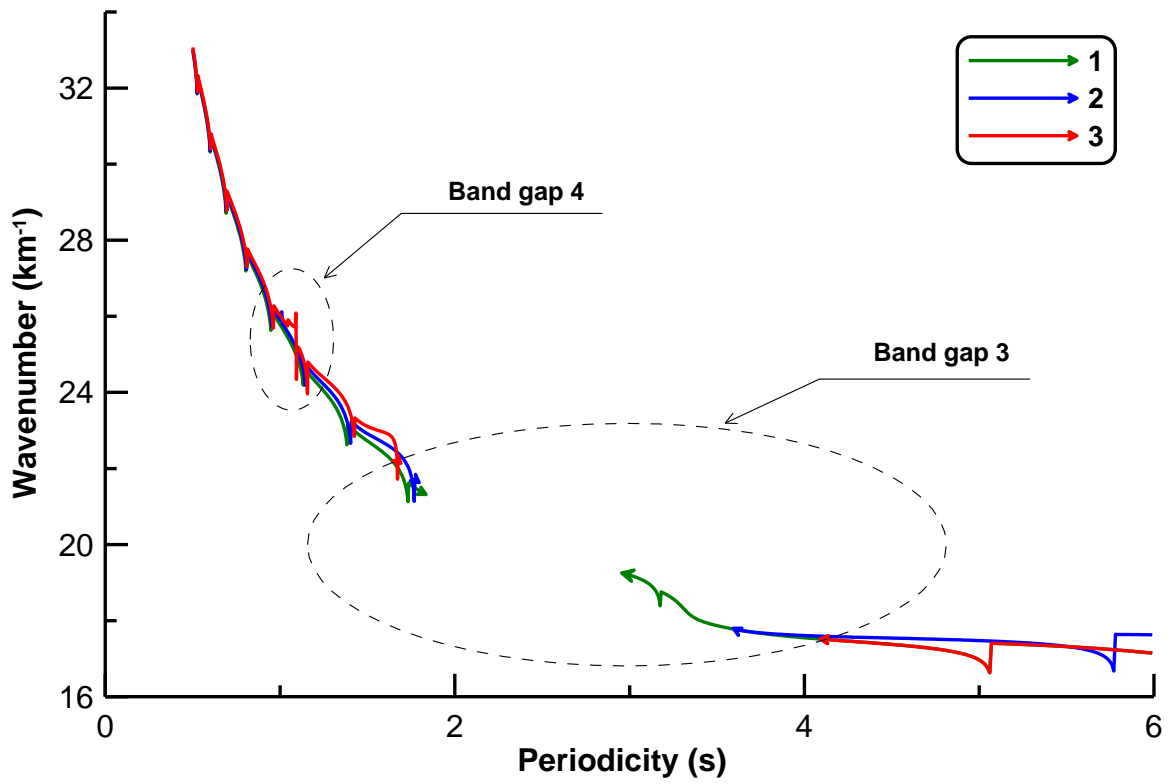
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145 In Konovalov (2023) it was found that the dominance of the second type of boundary
146 conditions in equation (10) (i.e. $\alpha_2 > \alpha_1$ in equation (10)) provides higher sensitivity in
147 the appearance of band gaps. Similar results are observed for an ice shelf with a rolling
148 surface morphology (Figure 4). The transition from a typical dispersion spectrum with
149 intermode spaces to a dispersion spectrum, containing bad gaps, occurs at $A_H = 1 m$, i.e.
150 the threshold value $(A_H)_{th} \approx 1 m$ (Fig. 4a).

151 In this case, in contrast to the case of $\alpha_1 = 1, \alpha_2 = 0$ in Model 1, the widths of the third
152 and fourth bad gaps increase relatively quickly in the range of A_H from $1 m$ to $2 m$ (Fig
153 4a). Essentially, at $A_H > 2 m$, two band gaps merge into a ban gap located in a wide part
154 of the dispersion spectrum (for example, curve 2 in Fig. 4b).

155 Nevertheless, more significant degradation of the amplitude spectrum is observed at
156 higher values of A_H , than considered in Figure 4(as in Figure 2 at $A_H \approx 18 m$). That is, the
157 amplitude spectra obtained at $A_H = 2.3 m$ reveal approximately the same resonance
158 peaks as the spectrum obtained at $A_H = 1 m$ (Figure 5), although we observe a thinning
159 of the resonance peaks obtained at $A_H = 3 m$, compared the peaks obtained at $A_H = 1 m$
160 (Fig. 5a).

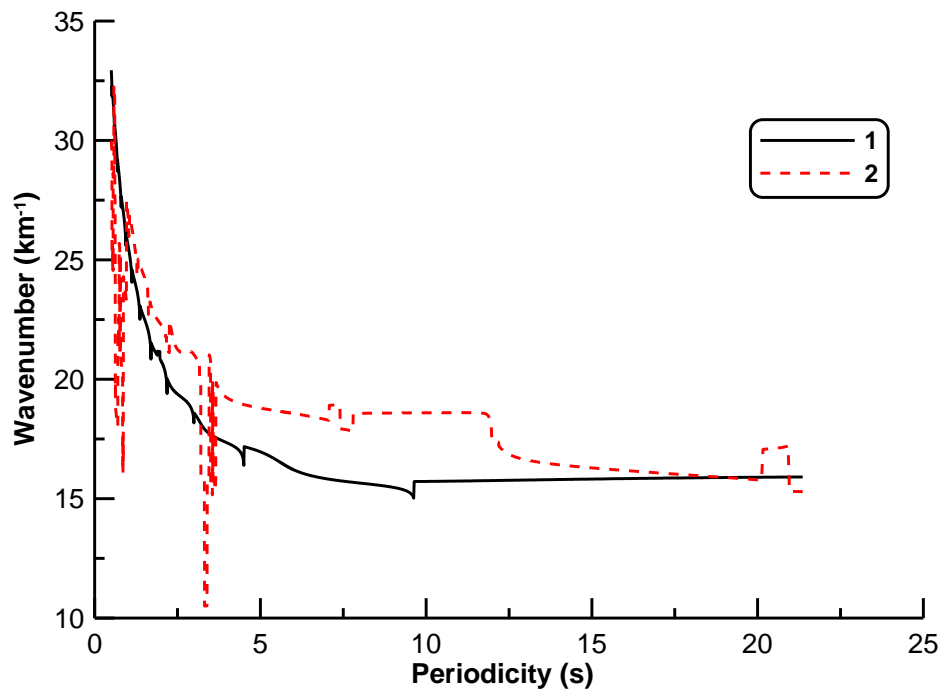
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Fig. 4a



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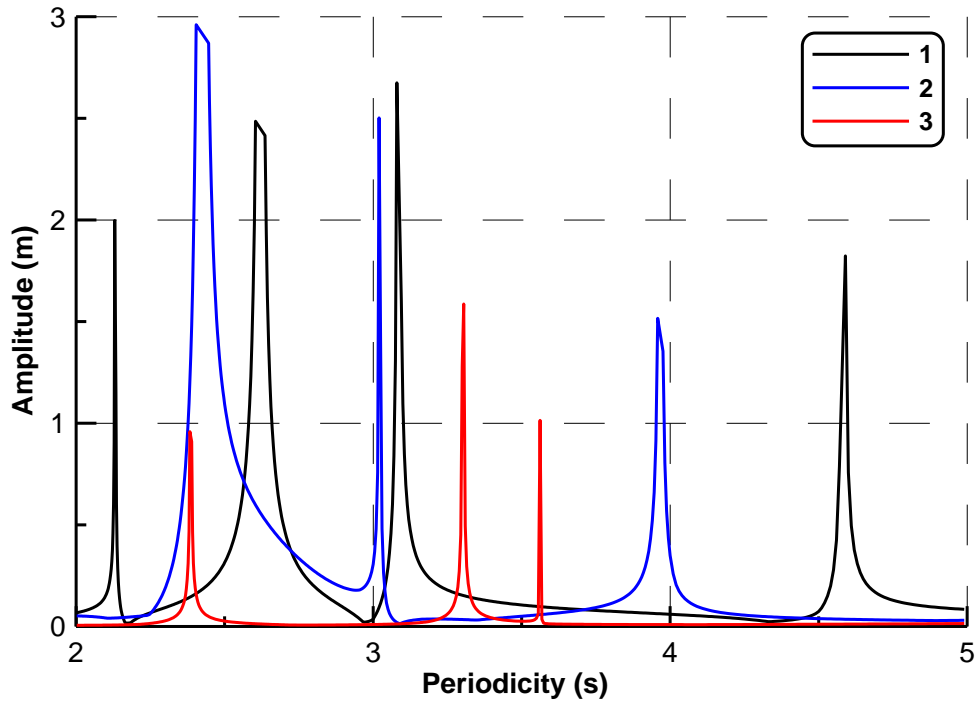
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Fig. 4b

166 **Figure 4.** Dispersion spectra obtained using **Model 1** with $\alpha_1 = 0.2, \alpha_2 = 0.8$ for ice shelf
 167 geometries differing in the amplitude of ice thickness oscillations A_H : **(a)** 1 – $A_H = 1.2$ m; 2 –
 168 $A_H = 1.3$ m; 3 – $A_H = 1.4$ m; **(b)** 1 – $A_H = 1.1$ m; 2 – $A_H = 3$ m.

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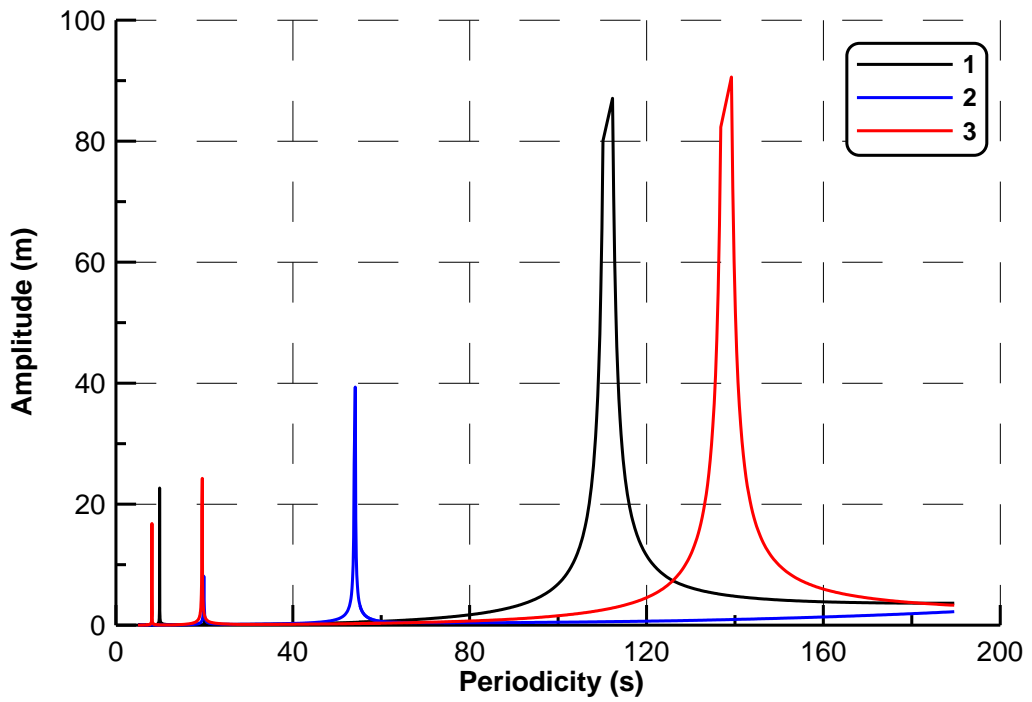
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Fig. 5a



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Fig. 5b

175 **Figure 5.** Amplitude spectra obtained using **Model 1** with $\alpha_1 = 0.2, \alpha_2 = 0.8$ for ice shelf
176 geometries differing in the amplitude of ice thickness oscillations A_H : **1** $-A_H = 1\text{ m}$; **2** $-A_H =$
177 2 m ; **3** $-A_H = 3\text{ m}$.

178

179

180 **3. Results obtained by Model 2 (case of $\alpha_1 = 0, \alpha_2 = 1$)**

181

182 In the case of $\alpha_1 = 0, \alpha_2 = 1$ in Model 2 the first band gap ($k_b^{(1)} \approx 6.28 \text{ km}^{-1}$) appears in
183 the spectrum at $A_H > 15 \text{ m}$ (Fig. 6a), i.e. the first threshold value is about the same as in
184 the case of $\alpha_1 = 1, \alpha_2 = 0$: $(A_H)_{th}^{(1)} \approx 15 \text{ m}$.

185 Second band gap ($k_b^{(2)} \approx 12.57 \text{ km}^{-1}$, Fig. 6b) appears in the spectrum at $A_H > 2 \text{ m}$, i.e.
186 corresponding threshold value $(A_H)_{th}^{(2)} \approx 2 \text{ m}$.

187 Third band gap ($k_b^{(3)} \approx 19.04 \text{ km}^{-1}$, Fig. 6c) appears in the spectrum at $A_H > 1 \text{ m}$, i.e.
188 corresponding threshold value $(A_H)_{th}^{(3)} \leq 1 \text{ m}$.

189 Fourth band gap ($k_b^{(4)} \approx 25.13 \text{ km}^{-1}$, Fig. 6c) is observed in the spectrum at $A_H > 2 \text{ m}$, i.e.
190 corresponding threshold values are also $(A_H)_{th}^{(4)} \approx 2 \text{ m}$.

191

192 Comparing Fig.2a or Fig. 5a from the main manuscript with Fig. 7a, in the case of $\alpha_1 =$
193 $0, \alpha_2 = 1$ we also observe a decline of the amplitude spectrum at the highest values of A_H
194 from the considered range.

195

196 Similarly Figure 6 from the main manuscript, Figure 8 shows the combination of two
197 spectra: dispersion spectrum and amplitude spectrum in the area where the first band
198 gap is expected to appear ($k_b^{(1)} \approx 6.28 \text{ km}^{-1}$).

199 In particular,

200 **(a)** when the amplitude of ice thickness fluctuations A_H is equal to **5 m** (Fig. 8a), the
201 resonance peak is observed at the periodicity $T_n \approx 29.28 \text{ s}$ (i.e. $T_n \approx 29.28 \text{ s}$ is one of the
202 eigenvalues), at which the wavenumber in the dispersion spectrum is about 5.77 km^{-1}
203 (i.e. $k_n \approx 5.77 \text{ km}^{-1}$);

204 **(b)** when the amplitude of ice thickness fluctuations A_H is equal to **10 m** (Fig. 8b), the
205 resonance peak is observed at the periodicity $T_n \approx 32.68$ s, at which the wavenumber in
206 the dispersion spectrum is about 5.74 km^{-1} (i.e. $k_n \approx 5.74 \text{ km}^{-1}$);

207 and

208 **(c)** when the amplitude of ice thickness fluctuations A_H is equal to **12 m** (Fig. 8c), the
209 resonance peak is observed at the periodicity $T_n \approx 36.18$ s, at which the wavenumber in
210 the dispersion spectrum is about 5.1 km^{-1} (i.e. $k_n \approx 5.1 \text{ km}^{-1}$).

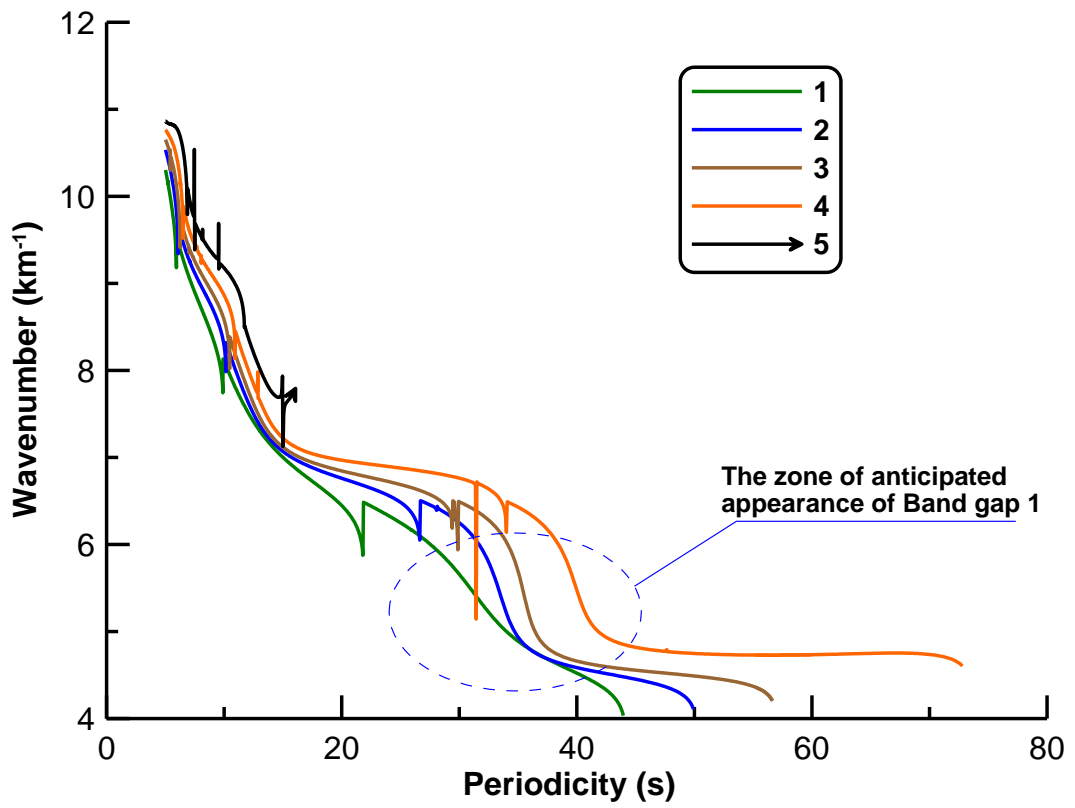
211 Respectively, the relative deviation of k_n from the first *Bragg wavenumber* $k_b^{(1)}$ doesn't
212 exceed 20%.

213 The main difference of the case $\alpha_1 = 0, \alpha_2 = 1$ and the previous one ($\alpha_1 = 0, \alpha_2 = 1$) is the
214 presence of a torsional component of deformations in the deformations of the ice shelf
215 (Konovalov, 2023c). The presence of a torsional component of deformation (Figure 2b
216 from the main manuscript) yields the appearance of additional inter-mode spaces in the
217 dispersion spectra, accompanying the transitions between torsional components of
218 deformation (torsion eigenmodes). These inter-mode spaces appear most significantly in
219 Model 3 (see next paragraph).

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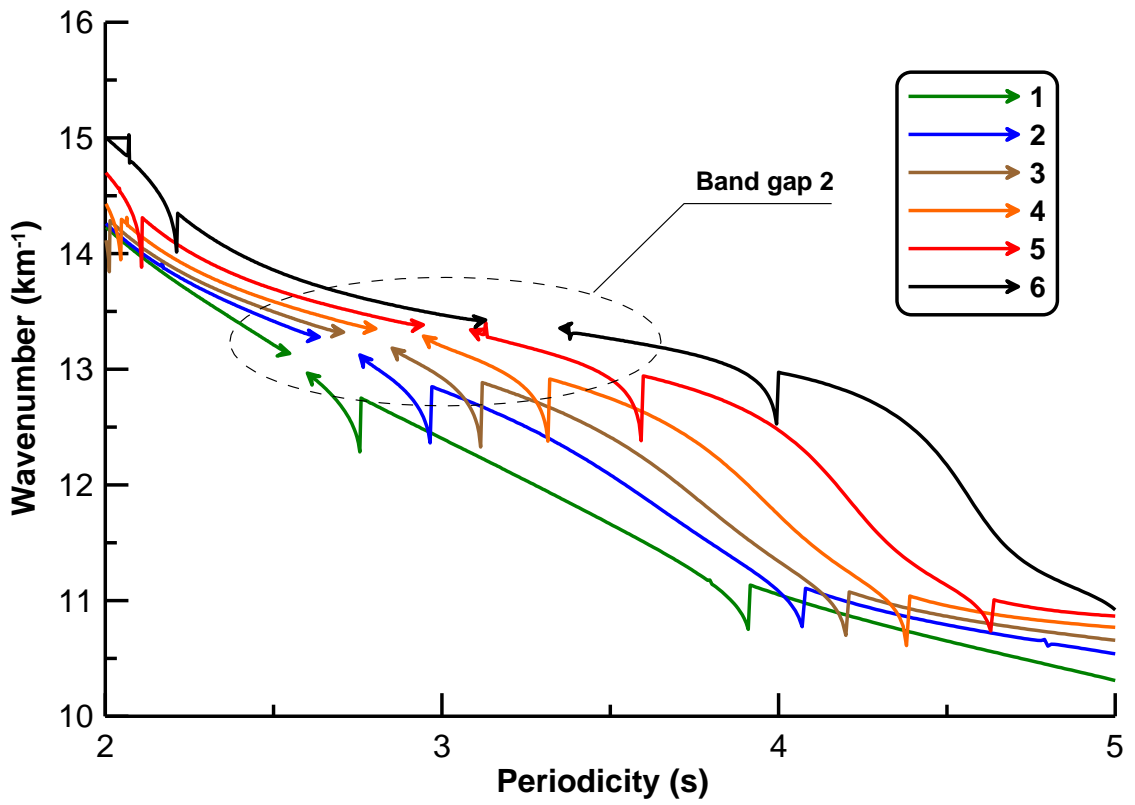


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Fig. 6a



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Fig. 6b

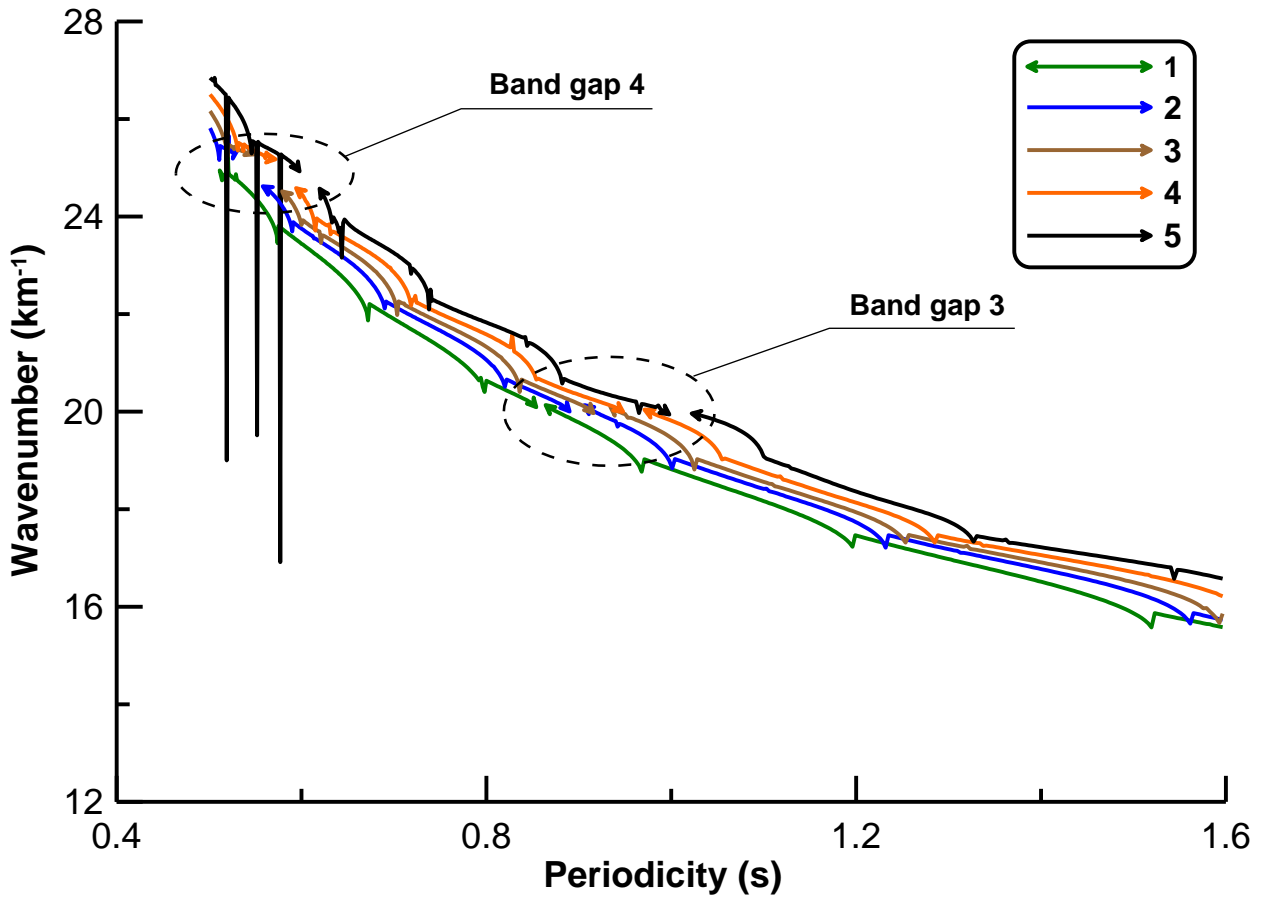


Fig. 6c

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232 **Figure 6.** Dispersion spectra obtained using **Model 2** with $\alpha_1 = 0$, $\alpha_2 = 1$ for ice shelf geometries

233 differing in the amplitude of ice thickness oscillations A_H .

234 (a) area of the expected first band gap; **1** $-A_H = 5\text{ m}$; **2** $-A_H = 10\text{ m}$; **3** $-A_H = 12\text{ m}$; **4** $-A_H =$

235 14 m ; **5** $-A_H = 16\text{ m}$;

236 (b) area of the expected second band gap; **1** $-A_H = 5\text{ m}$; **2** $-A_H = 10\text{ m}$; **3** $-A_H = 12\text{ m}$; **4** $-A_H =$

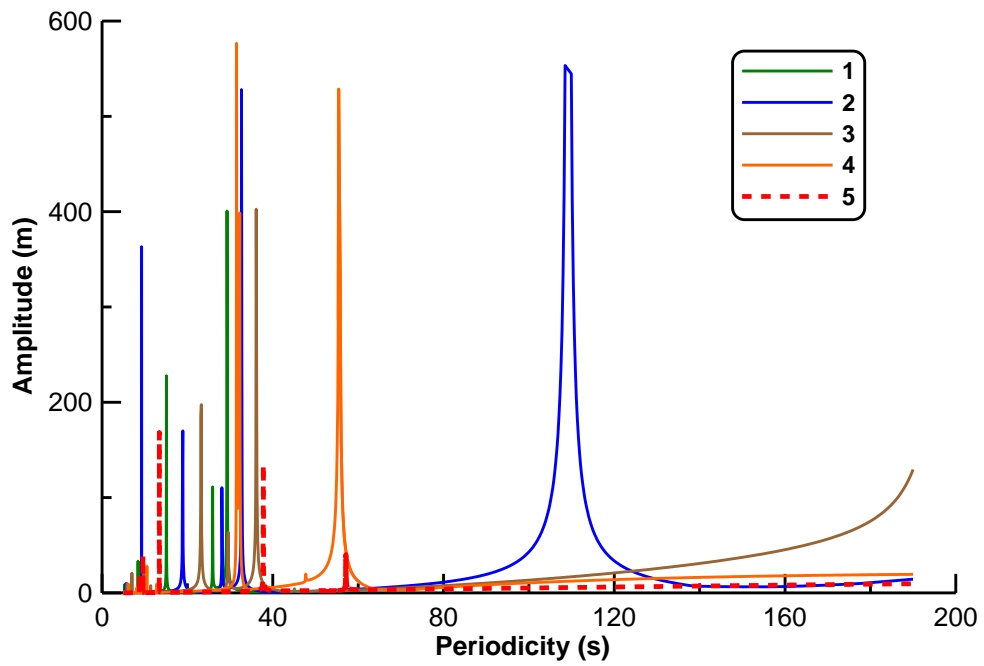
237 14 m ; **5** $-A_H = 16\text{ m}$; **6** $-A_H = 18\text{ m}$;

238 (c) area of expected third and fourth band gaps; **1** $-A_H = 5\text{ m}$; **2** $-A_H = 10\text{ m}$; **3** $-A_H = 12\text{ m}$; **4**

239 $-A_H = 14\text{ m}$; **5** $-A_H = 16\text{ m}$.

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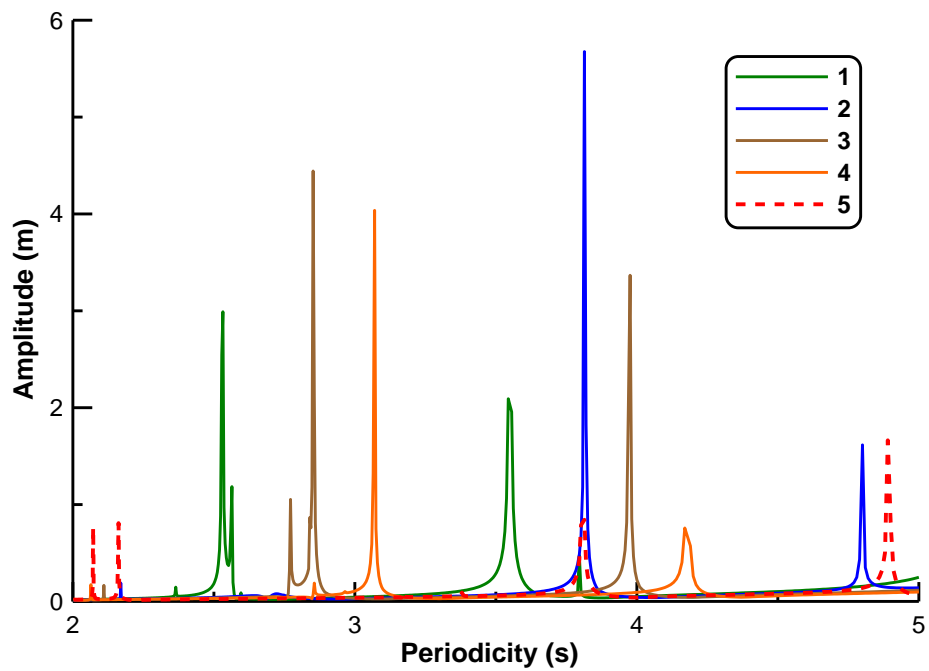
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Fig. 7a

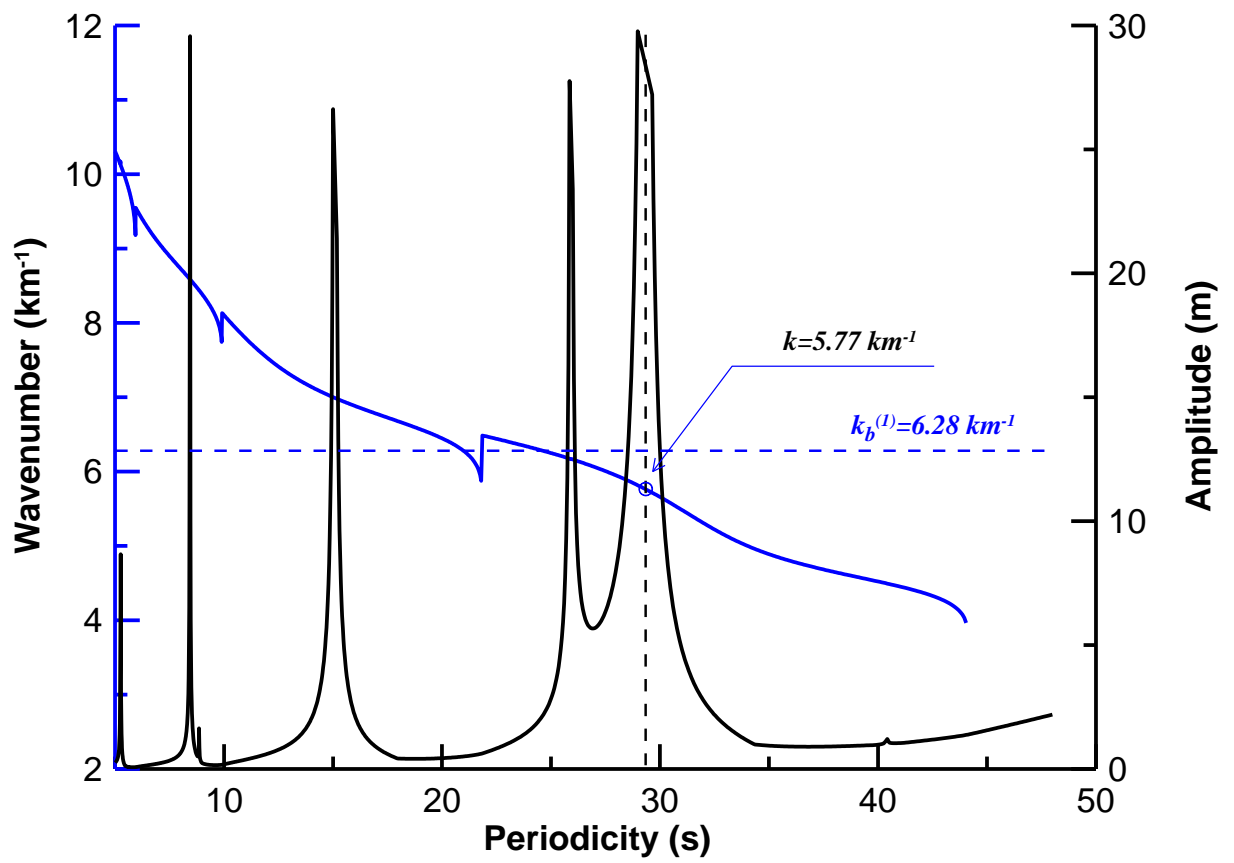


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Fig. 7b

246 **Figure 7.** Amplitude spectra obtained using **Model 2** with $\alpha_1 = 0$, $\alpha_2 = 1$ for ice shelf geometries
 247 differing in the amplitude of ice thickness oscillations A_H : **1** $-A_H = 5$ m; **2** $-A_H = 10$ m; **3** $-A_H =$
 248 12 m; **4** $-A_H = 14$ m; **5** $-A_H = 18$ m;
 249 **(a)** area of the expected first band gap (Fig 6a); **(b)** area of the expected second band gap (Fig. 6b)

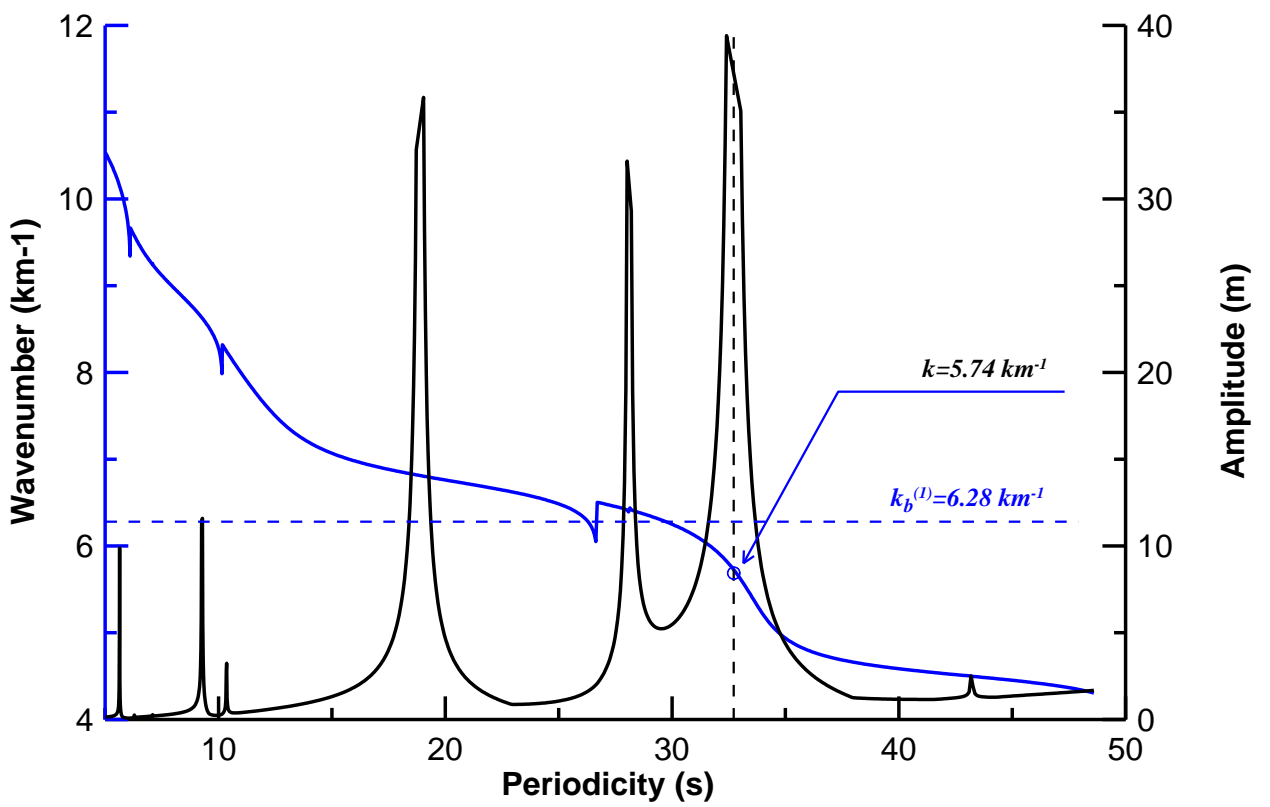


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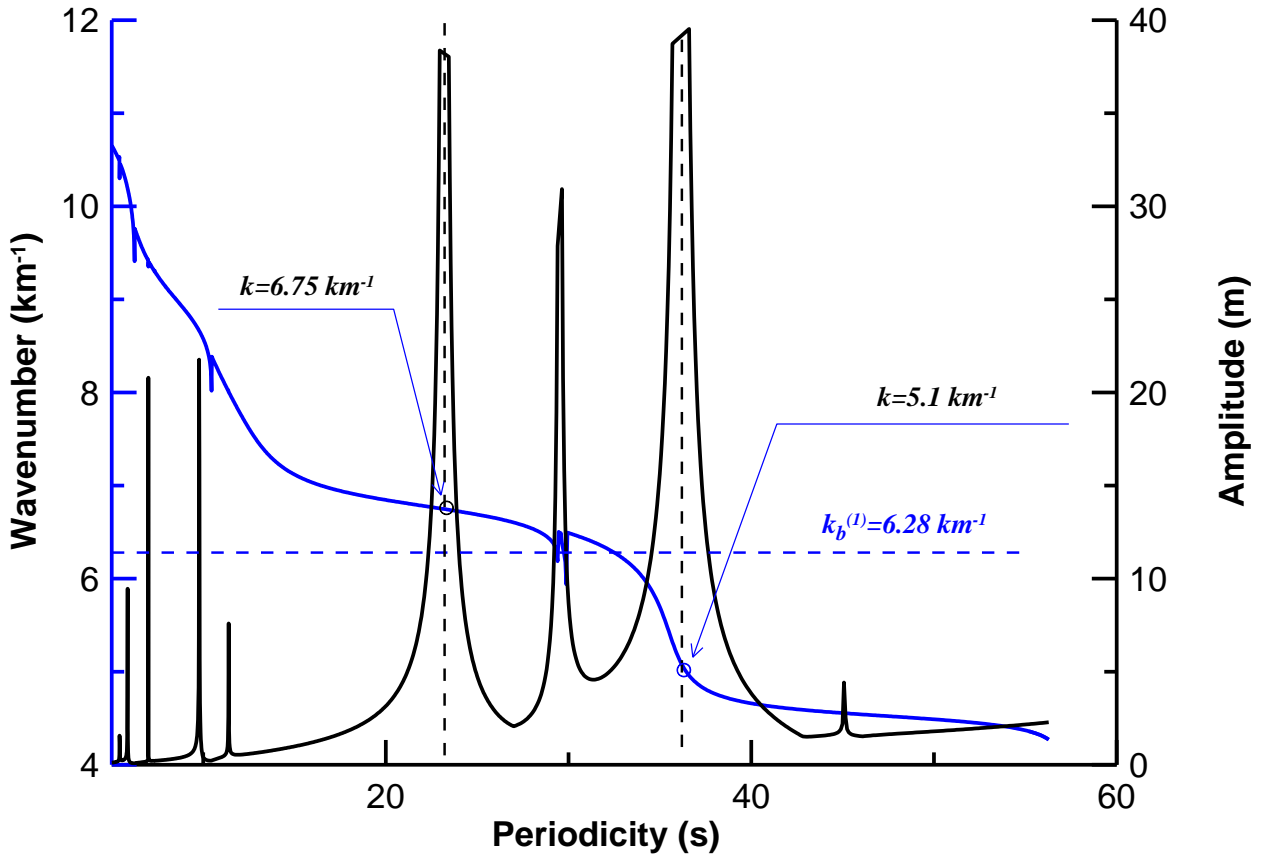
Fig. 8a



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Fig. 8b



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257

258

259 **Figure 8.** Dispersion spectrum and amplitude spectrum, including the area of the expected first
 260 band gap, obtained using **Model 2** with $\alpha_1 = 0$, $\alpha_2 = 1$ for ice shelf geometries differing in the
 261 amplitude of ice thickness fluctuations A_H : (a) $A_H = 5$ m; (b) $A_H = 10$ m; (c) $A_H = 12$ m.

262

263

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265 **4. Results obtained by Model 3 (case of $\alpha_1 = 1, \alpha_2 = 0$)**

266

267 Dispersion spectra obtained using Model 3 reveal many areas of discontinuity in the
268 curves (Figure 9). Firstly, there are discontinuities in the dispersion spectra, which as was
269 previously established (e.g. Konovalov, 2021a), represent inter-mode spaces
270 accompanying changes in the flexural component of ice shelf deformations (transitions
271 between bending eigenmodes) (Fig. 9b; Fig. 9c). Secondly, discontinuities are observed
272 in the dispersion spectra, which look like band gaps corresponding to Bragg scattering of
273 the incident wave. However, not all of these discontinuity regions coincide with the values
274 of the Bragg wavenumbers, which are determined by the equation (13), and they cannot
275 be associated with the band gaps corresponding to the Bragg scattering of the incident
276 wave.

277 The combination of the dispersion spectrum with the amplitude spectrum (Figure 10 and
278 Figure 11) allows us to establish the following. The discontinuities in the dispersion
279 spectra, which have the form of band gaps, but do not correspond to the Bragg scattering
280 of the incident wave, coincide with resonance peaks in the amplitude spectra (Figure 10
281 and Figure 11). In other words, these discontinuities are accompanied by a transition
282 through resonances, while the band gaps corresponding to Bragg scattering are not
283 accompanied by the same transition. Moreover, investigations of these resonances reveal
284 that they accompany changes in the torsional component of ice shelf deformations
285 (transitions between torsion eigenmodes) (Figure 12 and Figure 13). Thus, changes in the
286 torsional component of ice shelf deformations (transitions between torsion eigenmodes)
287 are accompanied by resonances in the amplitude spectra and are reflected in the
288 dispersion spectra in the form of discontinuities. While changes in the flexural component
289 of ice shelf deformations (transitions between bending Lamb-type eigenmodes) are not
290 accompanied by resonances in the amplitude spectra but, on the contrary, coincide with

291 the amplitude minima in the spectrum (Figure 10 and Figure 11, and, also, Figure 3,
292 Figure 6 from the main manuscript).

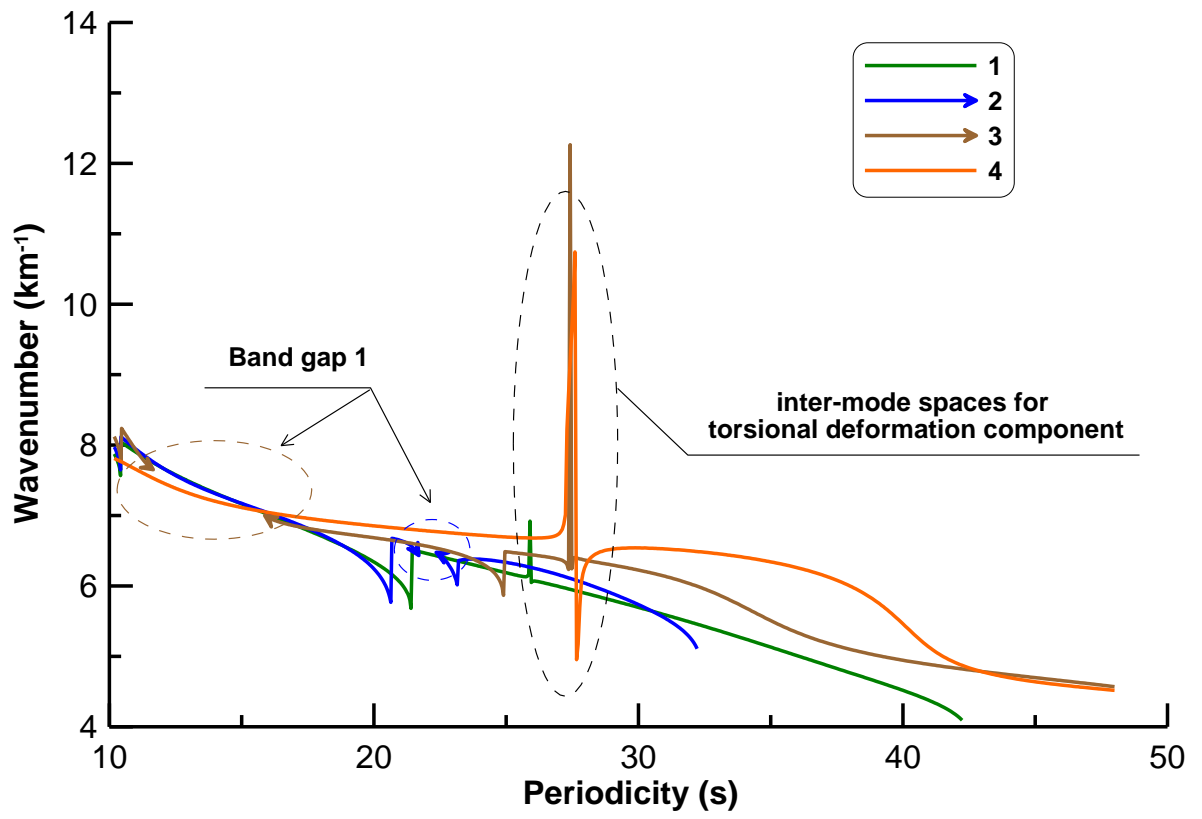
293 Therefore, the combination of the dispersion and amplitude spectra allows us to establish
294 the type of observed discontinuity in the dispersion spectra in Model 3 (Figure 10 and
295 Figure 11).

296 In the case of $\alpha_1 = 1, \alpha_2 = 0$ in Model 3 the first band gap ($k_b^{(1)} \approx 6.28 \text{ km}^{-1}$), appears in
297 the spectrum at $A_H > 3 \text{ m}$ (Figure 9a), i.e. the first threshold value $(A_H)_{th}^{(1)} \approx 3 \text{ m}$.

298 However, in particular, at $A_H = 8 \text{ m}$ the first band gap in the dispersion spectrum
299 disappears due to the alignment of the areas of the expected band gap with the resonance
300 peak in the corresponding regions of the dispersion spectrum. Then, at $A_H \geq 10 \text{ m}$ the
301 first band gap appears again in the dispersion spectrum.

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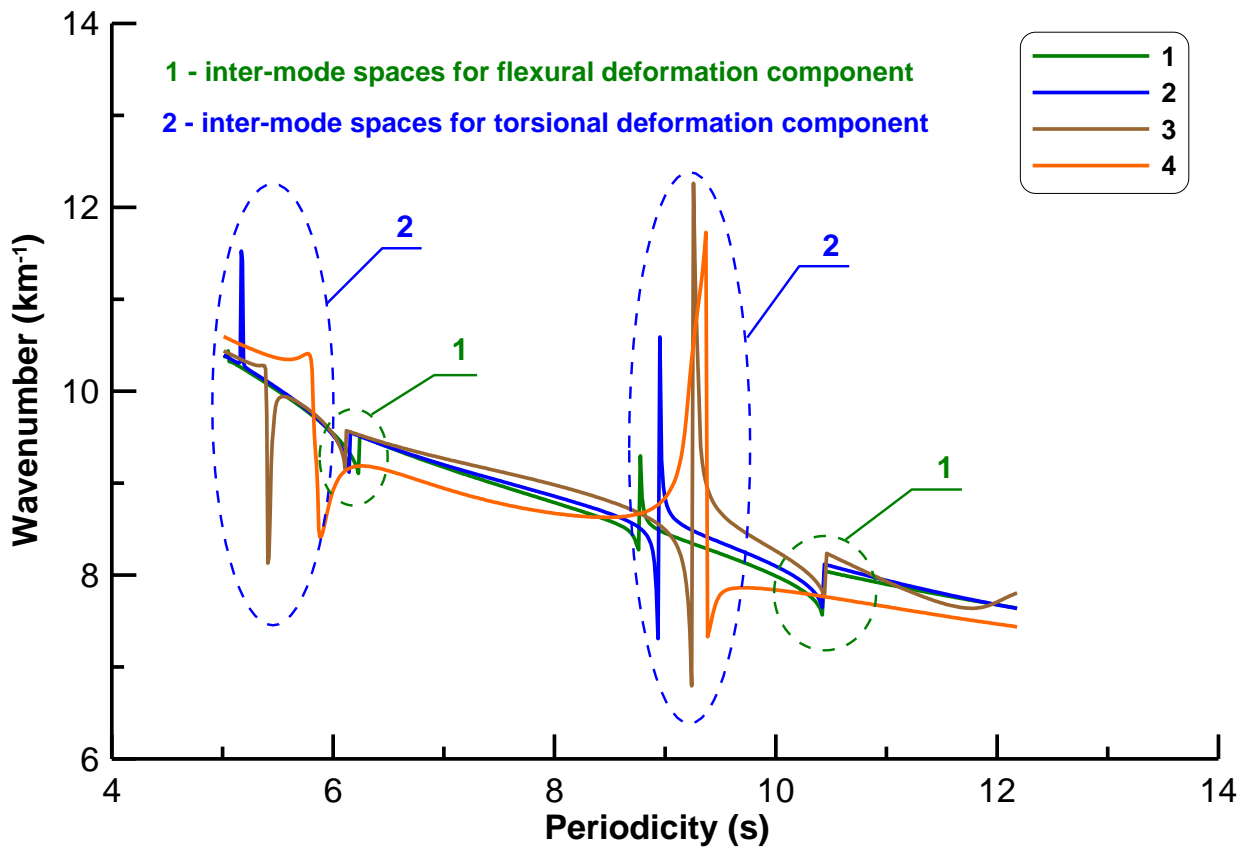


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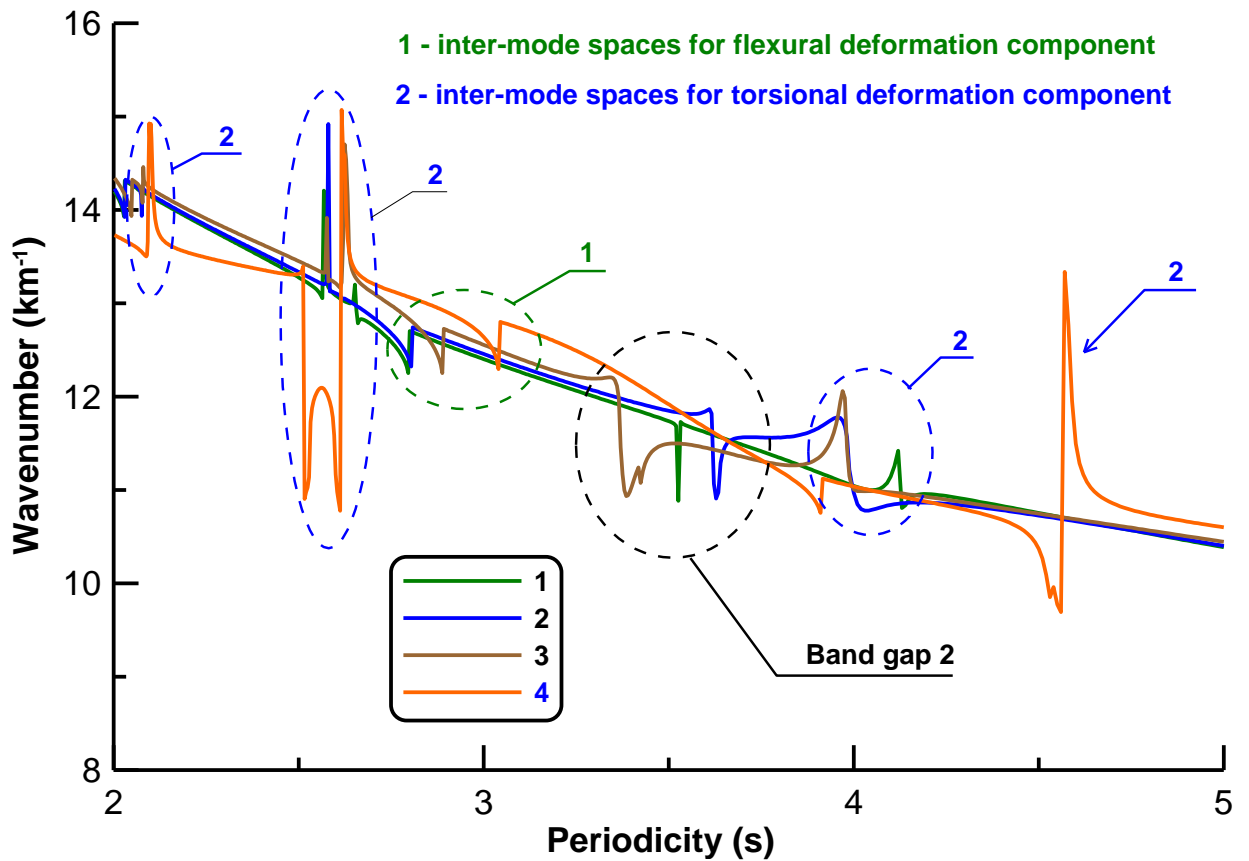
Fig. 9a



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Fig. 9b



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Fig. 9c

313 **Figure 9.** Dispersion spectra obtained using **Model 3** with $\alpha_1 = 1$, $\alpha_2 = 0$ for ice shelf geometries
 314 differing in the amplitude of ice thickness oscillations A_H .

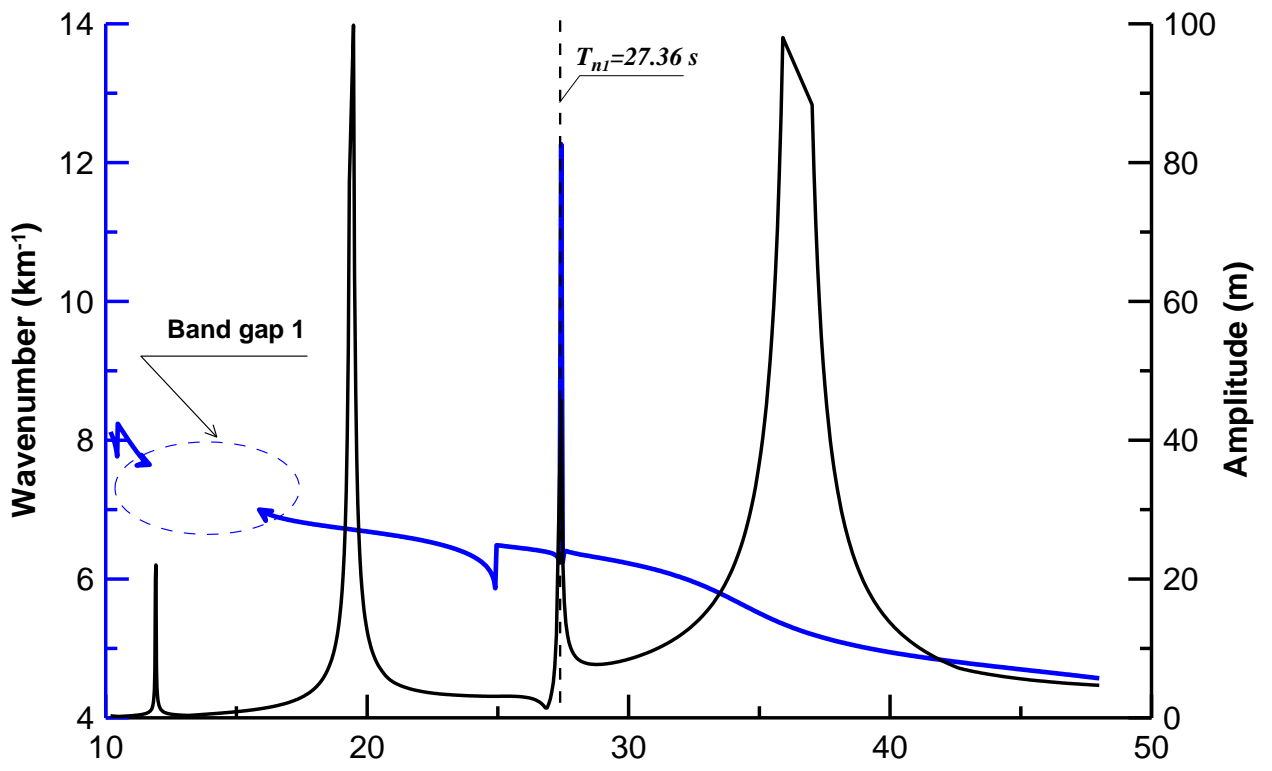
315 **(a)** area of the expected first band gap; **1** $-A_H = 1\text{ m}$; **2** $-A_H = 3\text{ m}$; **3** $-A_H = 5\text{ m}$; **4** $-A_H = 8\text{ m}$;

316 **(b) and (c)** areas of the expected second band gap; **1** $-A_H = 1\text{ m}$; **2** $-A_H = 3\text{ m}$; **3** $-A_H = 5\text{ m}$; **4**

317 $-A_H = 8\text{ m}$;

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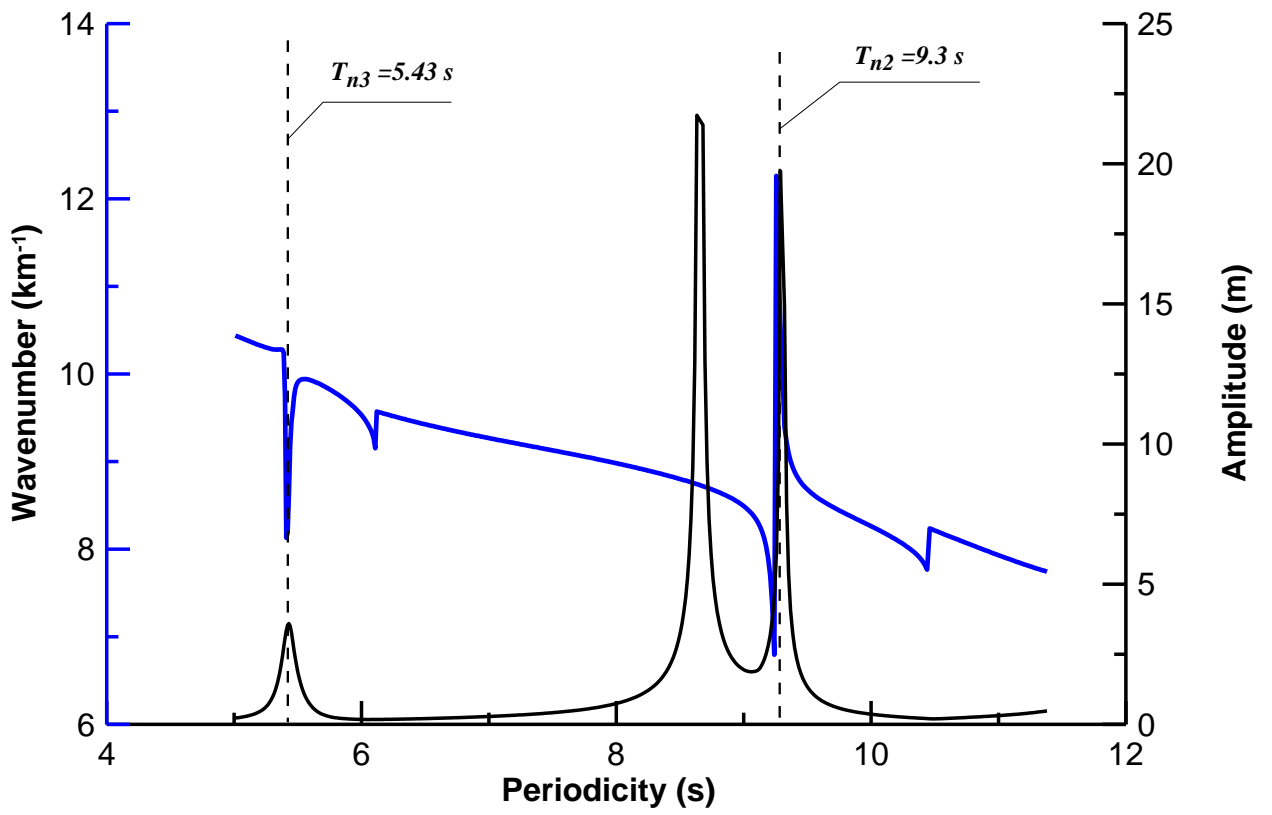


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Fig. 10a

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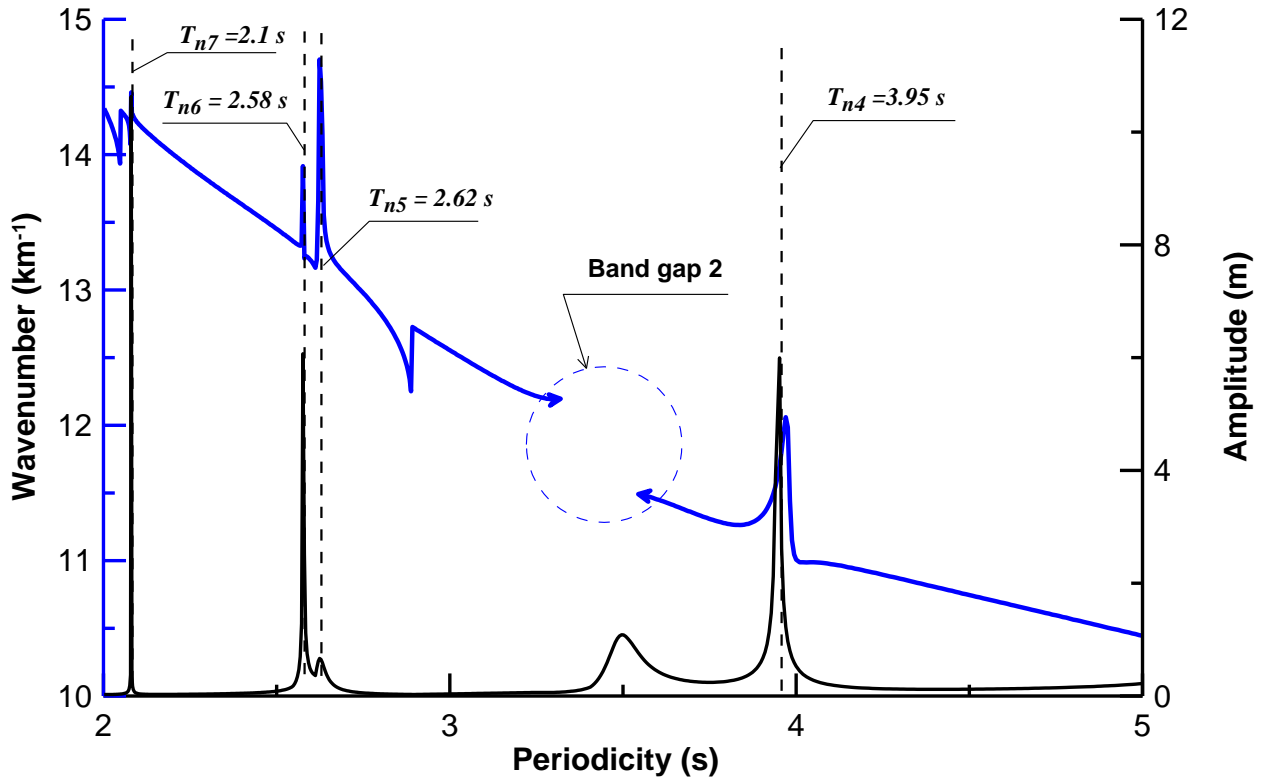
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Fig. 10b

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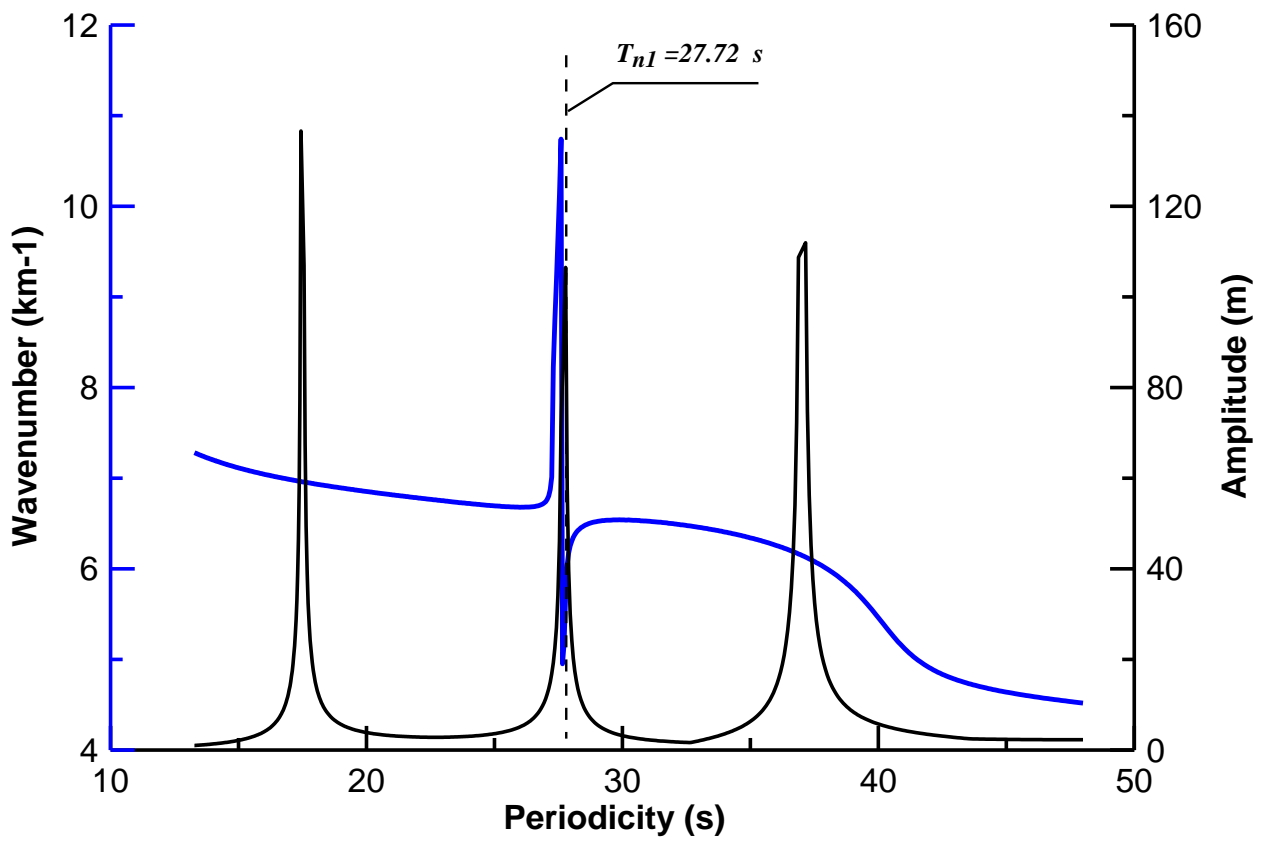
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Fig. 10c

Figure 10. Dispersion spectrum and amplitude spectrum, including (a) and (b) the area of the expected first band gap, (c) the area of the expected second band gap, obtained using **Model 3** with $\alpha_1 = 1, \alpha_2 = 0$ for the amplitude of ice thickness fluctuations $A_H = 5 m$.

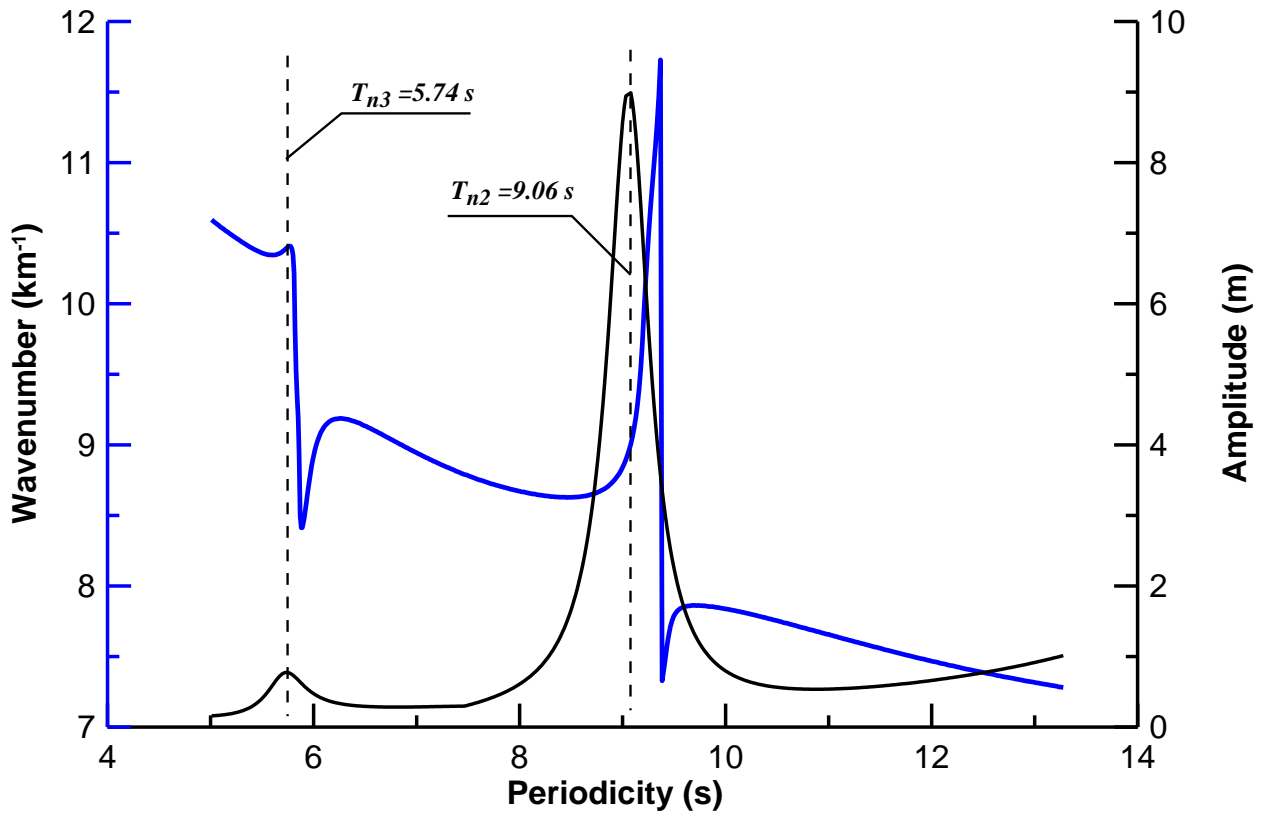


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Fig. 11a

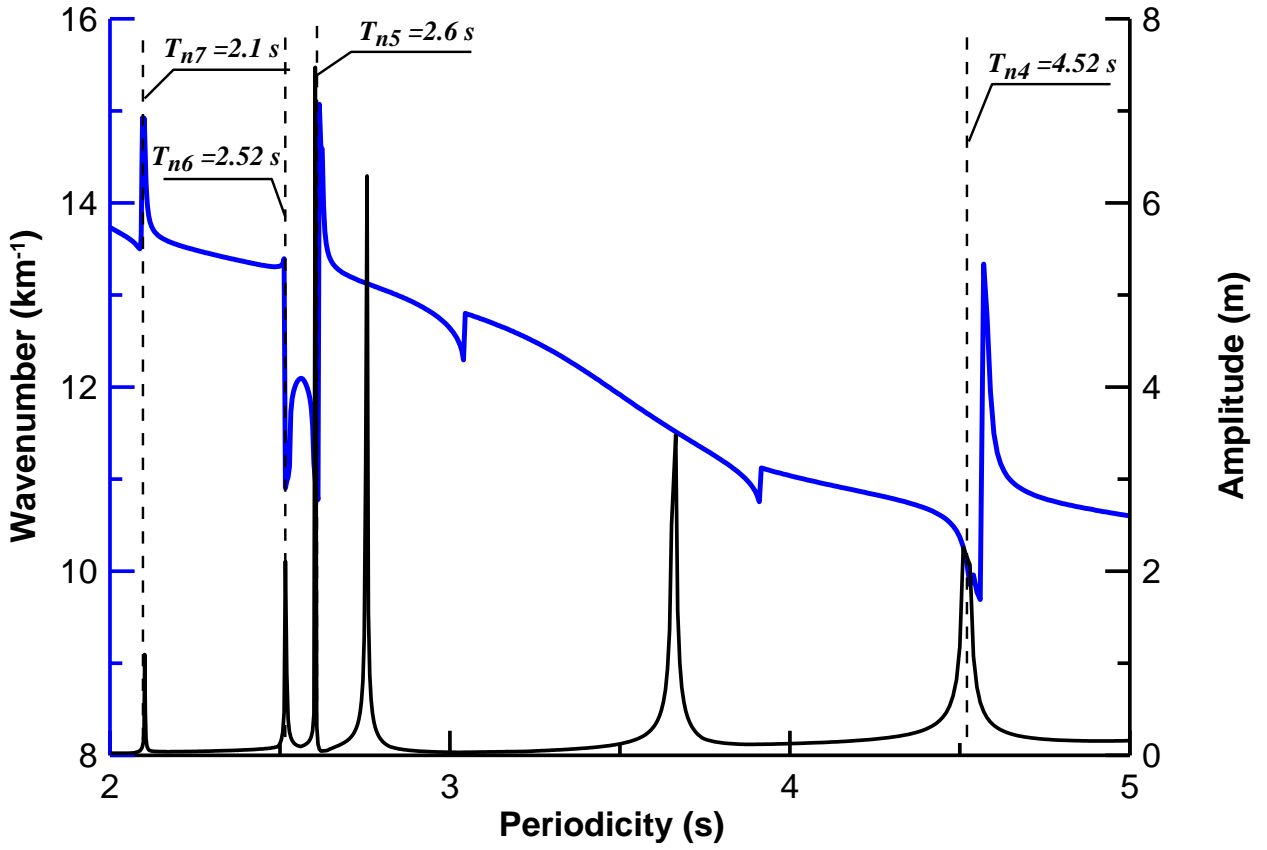


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Fig. 11b

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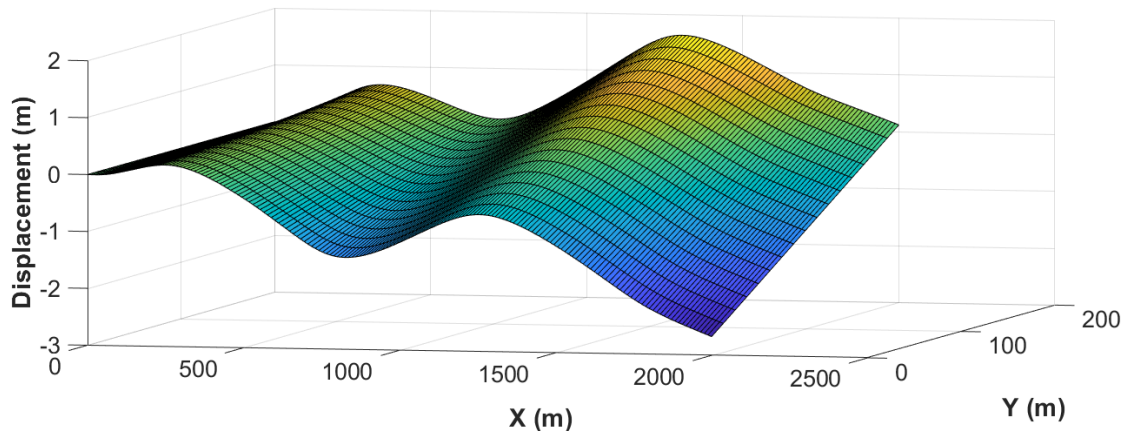


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Fig. 11c

Figure 11. Dispersion spectrum and amplitude spectrum, including (a) and (b) the area of the expected first band gap, (c) the area of the expected second band gap, obtained using **Model 3** with $\alpha_1 = 1, \alpha_2 = 0$ for the amplitude of ice thickness fluctuations $A_H = 8 m$.

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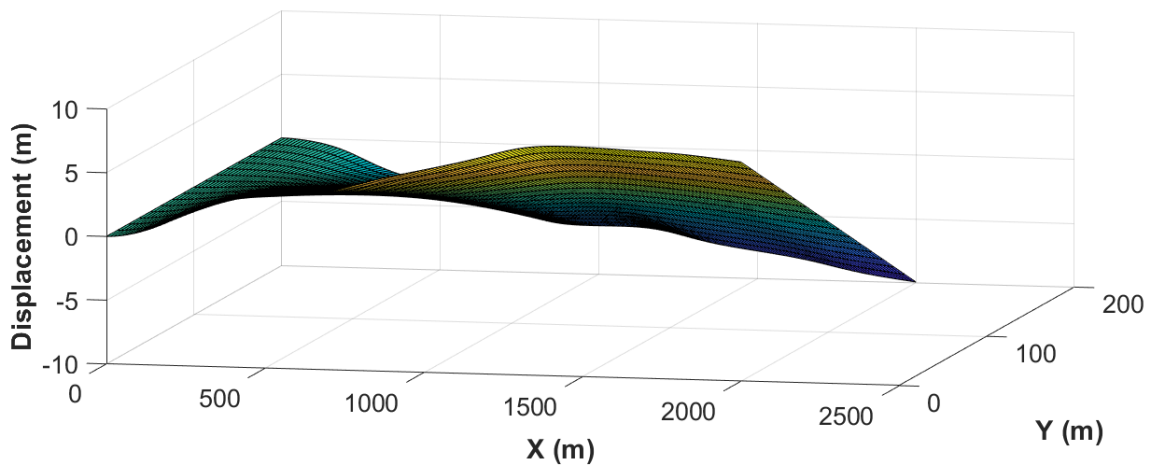


356

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Fig. 12.1a

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Fig. 12.1b

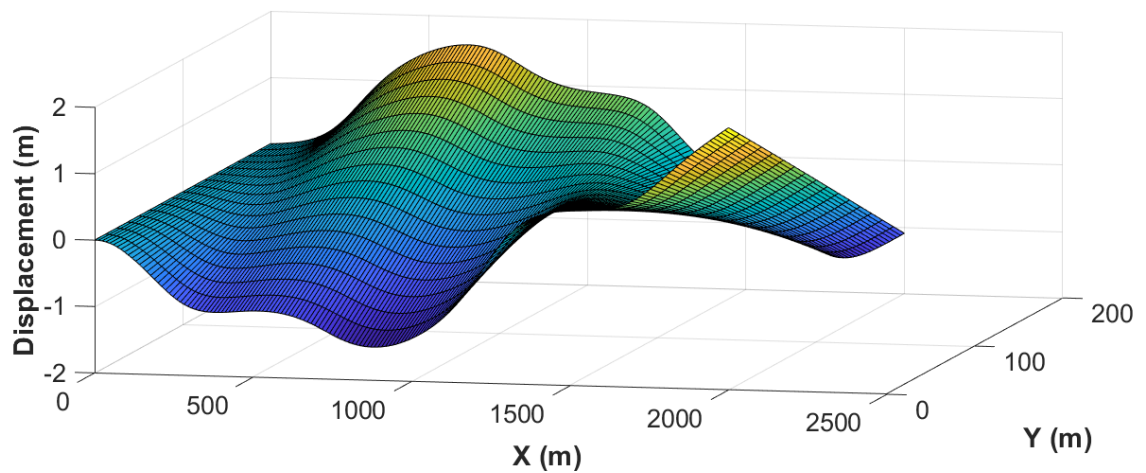
361

362 **Figure 12.1.** The vertical deflections of the ice shelf surface resulting from the impact of the frontal
363 incident wave were obtained from **Model 3** with the forcing periodicity of **(a) $T = 27.0$ s** and **(b)**
364 **$T = 27.8$ s**. These periodicities are located, respectively, to the left and to the right of the
365 resonance peak observed at **$T_{n_1} = 27.36$ s** (in **Fig. 10a**). The amplitude of ice thickness
366 fluctuations **$A_H = 5$ m**.

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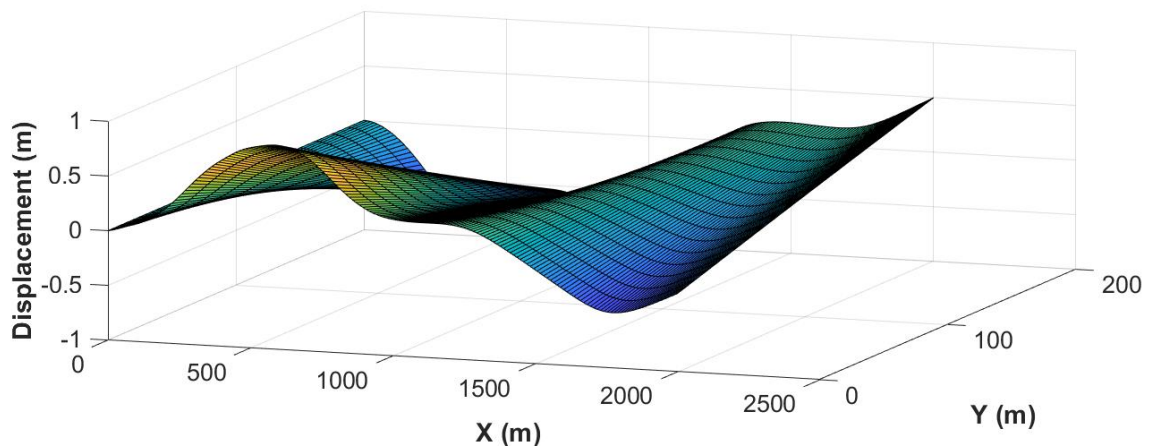


370

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Fig. 12.2a

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374

Fig. 12.2b

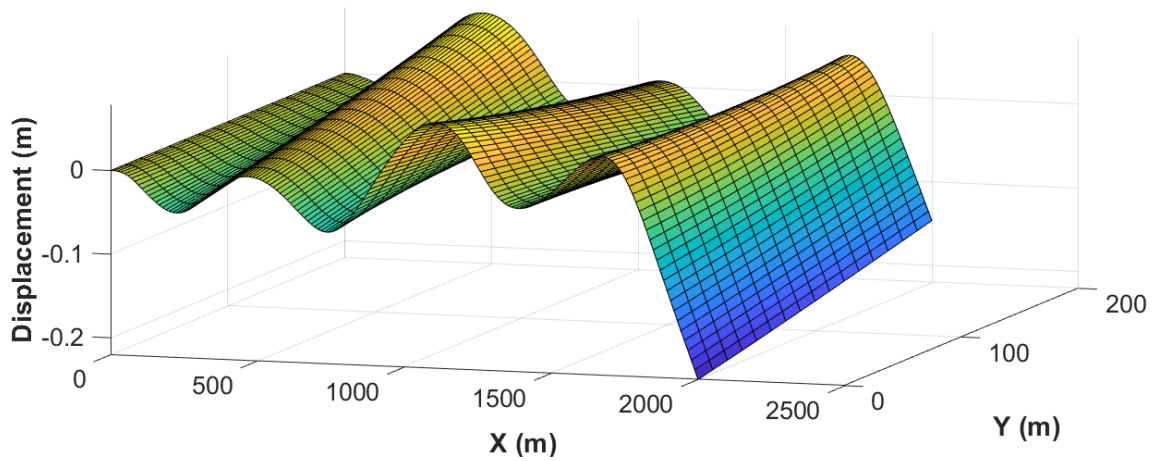
375

376 **Figure 12.2.** The vertical deflections of the ice shelf surface resulting from the impact of the frontal
377 incident wave were obtained from **Model 3** with the forcing periodicity of **(a) $T = 9.0$ s** and **(b)**
378 **$T = 9.6$ s**. These periodicities are located, respectively, to the left and to the right of the resonance
379 peak observed at $T_{n_2} = 9.3$ s (in **Fig. 10b**). The amplitude of ice thickness fluctuations $A_H =$
380 **5 m.**

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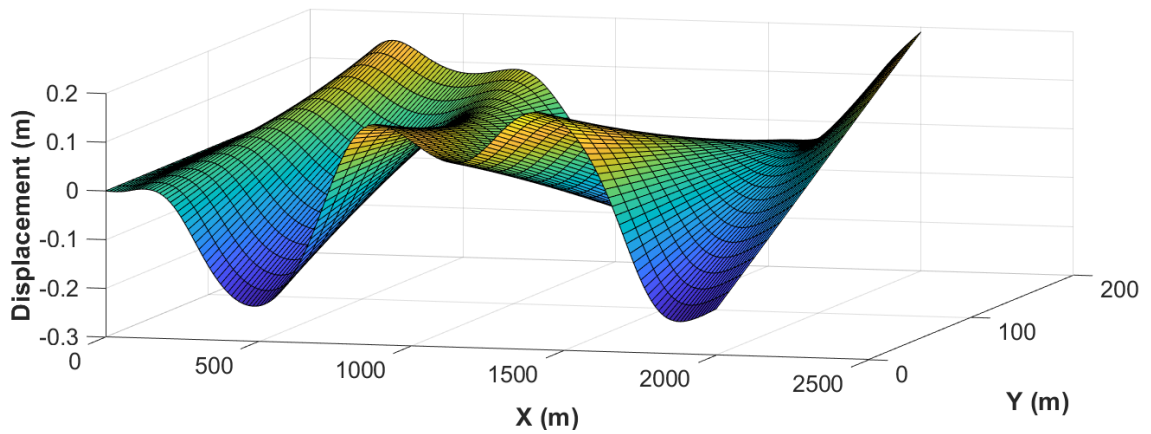


384

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Fig. 12.3a

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Fig. 12.3b

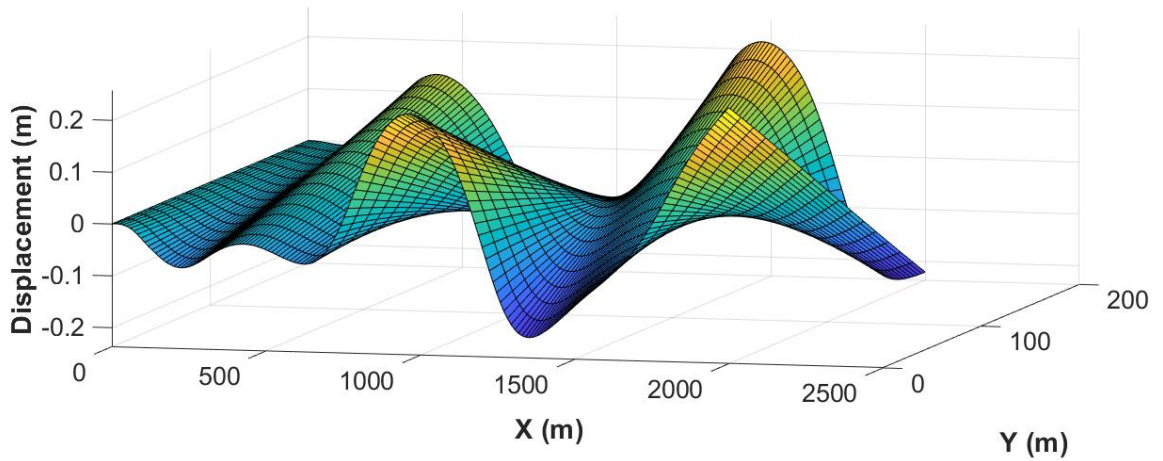
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391 **Figure 12.3.** The vertical deflections of the ice shelf surface resulting from the impact of the frontal
392 incident wave were obtained from **Model 3** with the forcing periodicity of **(a) $T = 5.0$ s** and **(b)**
393 **$T = 5.8$ s**. These periodicities are located, respectively, to the left and to the right of the resonance
394 peak observed at $T_{n_3} = 5.43$ s (in **Fig. 10b**). The amplitude of ice thickness fluctuations $A_H =$
395 **5 m.**

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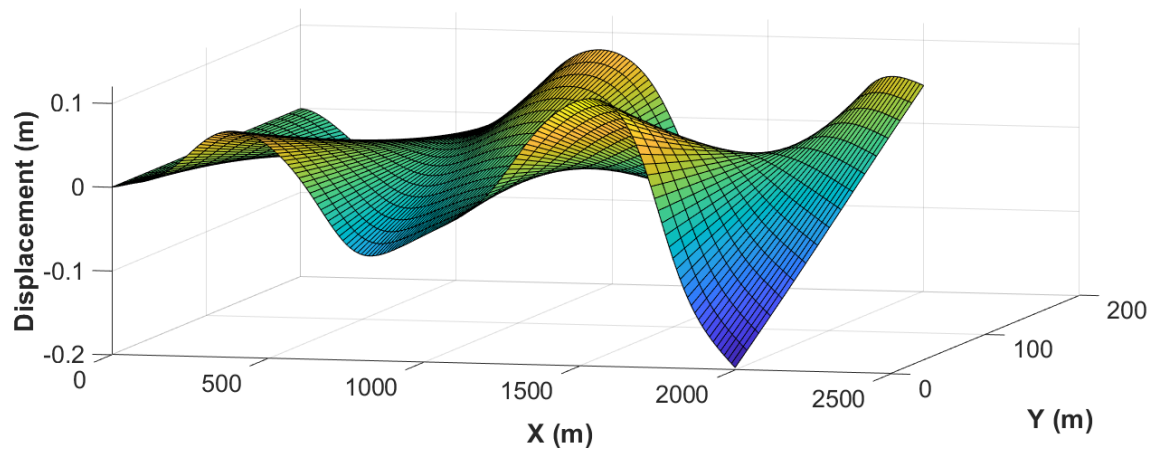


398

Fig. 12.4a

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Fig. 12.4b

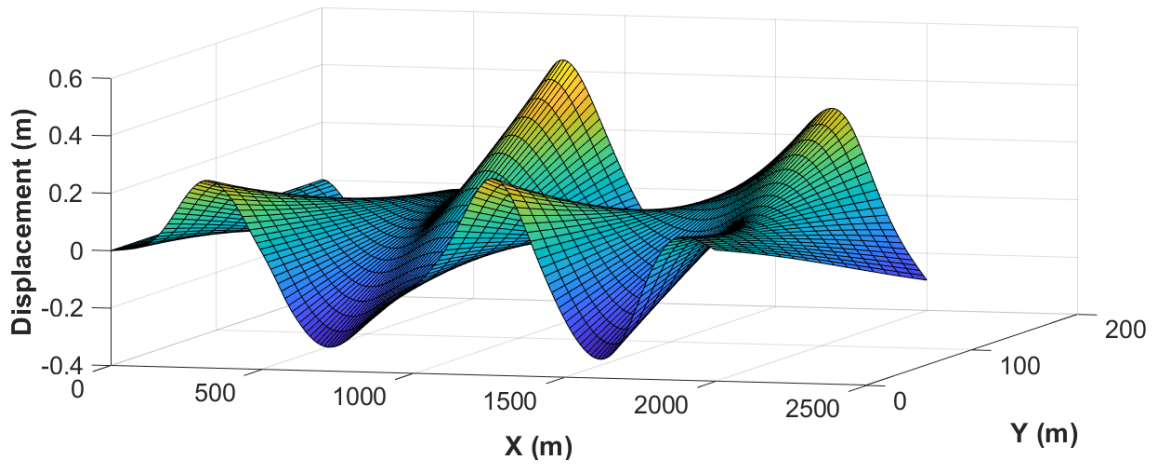
402

403

404 **Figure 12.4.** The vertical deflections of the ice shelf surface resulting from the impact of the frontal
405 incident wave were obtained from **Model 3** with the forcing periodicity of **(a) $T = 3.7$ s** and **(b)**
406 **$T = 4.1$ s**. These periodicities are located, respectively, to the left and to the right of the resonance
407 peak observed at $T_{n_4} = 3.95$ s (in **Fig. 10c**). The amplitude of ice thickness fluctuations $A_H =$
408 **5 m.**

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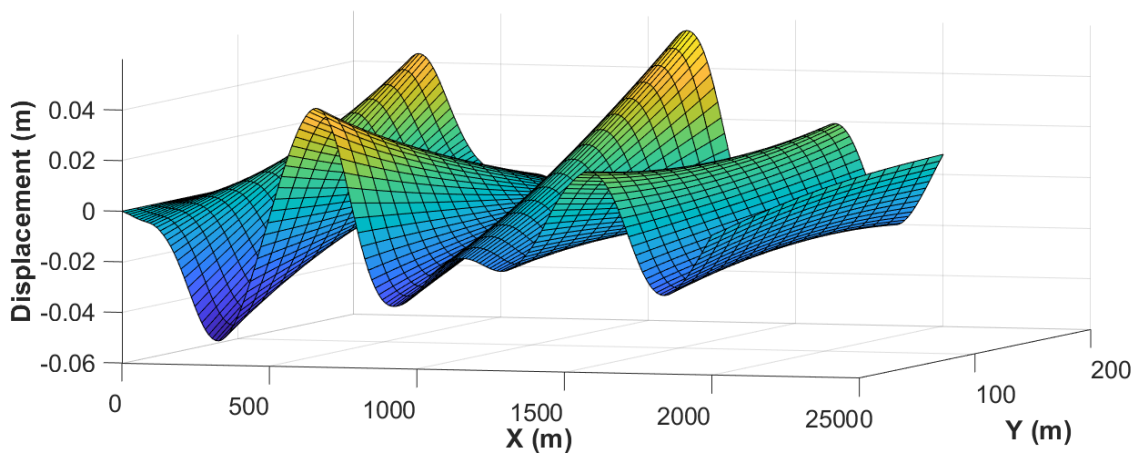


411

Fig. 12.5a

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Fig. 12.5b

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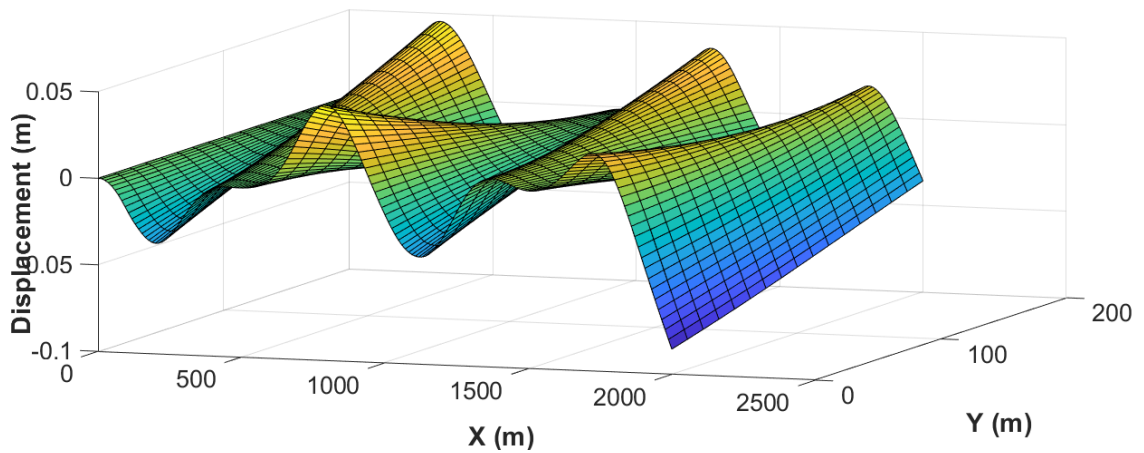
416

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418 **Figure 12.5.** The vertical deflections of the ice shelf surface resulting from the impact of the frontal
419 incident wave were obtained from **Model 3** with the forcing periodicity of **(a) $T = 2.6$ s** and **(b)**
420 **$T = 2.75$ s**. These periodicities are located, respectively, to the left and to the right of the
421 resonance peak observed at $T_{n_5} = 2.62$ s (in **Fig. 10c**). The amplitude of ice thickness
422 fluctuations $A_H = 5$ m.

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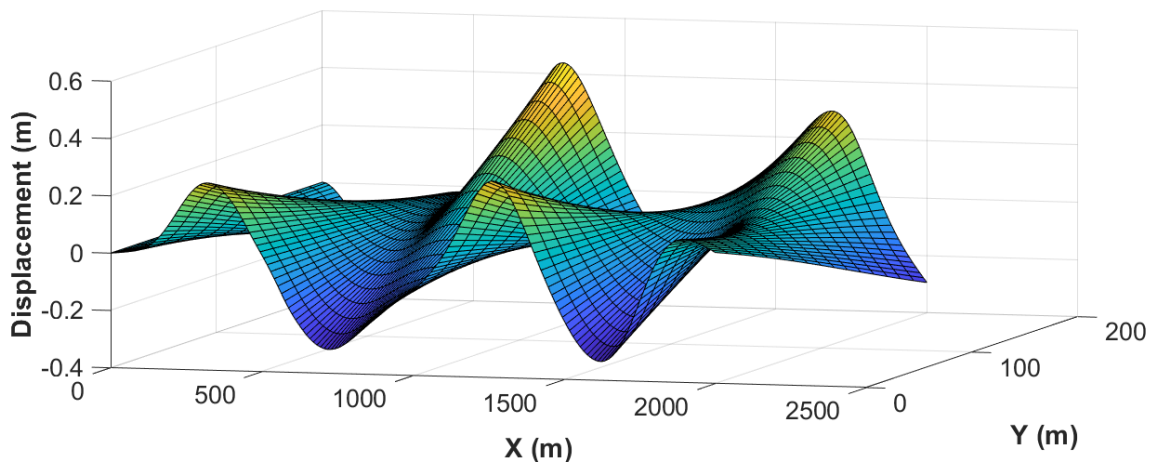


425

Fig. 12.6a

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Fig. 12.6b

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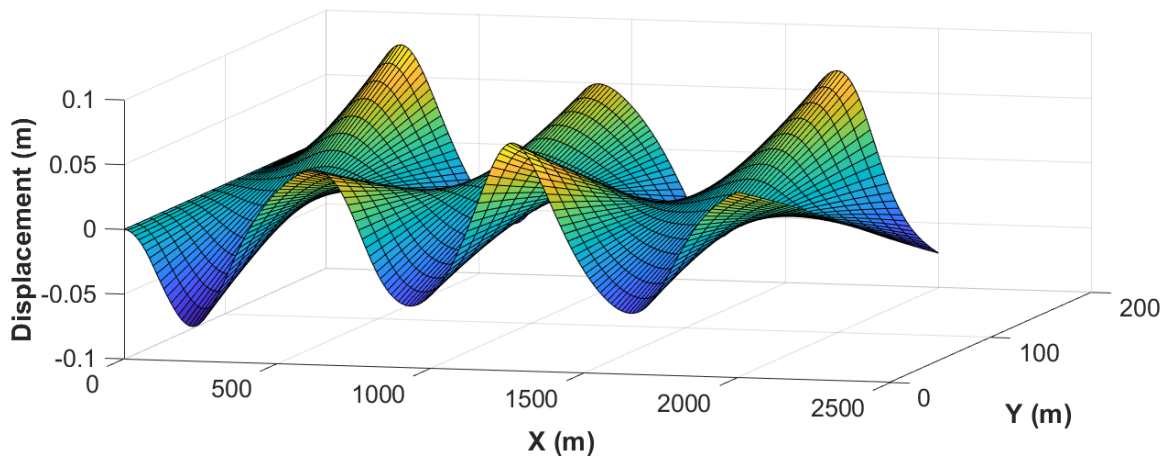
430

431 **Figure 12.6.** The vertical deflections of the ice shelf surface resulting from the impact of the frontal
432 incident wave were obtained from **Model 3** with the forcing periodicity of **(a) $T = 2.5$ s** and **(b)**
433 **$T = 2.6$ s**. These periodicities are located, respectively, to the left and to the right of the resonance
434 peak observed at $T_{n_6} = 2.58$ s (in **Fig. 10c**). The amplitude of ice thickness fluctuations $A_H =$
435 **5 m.**

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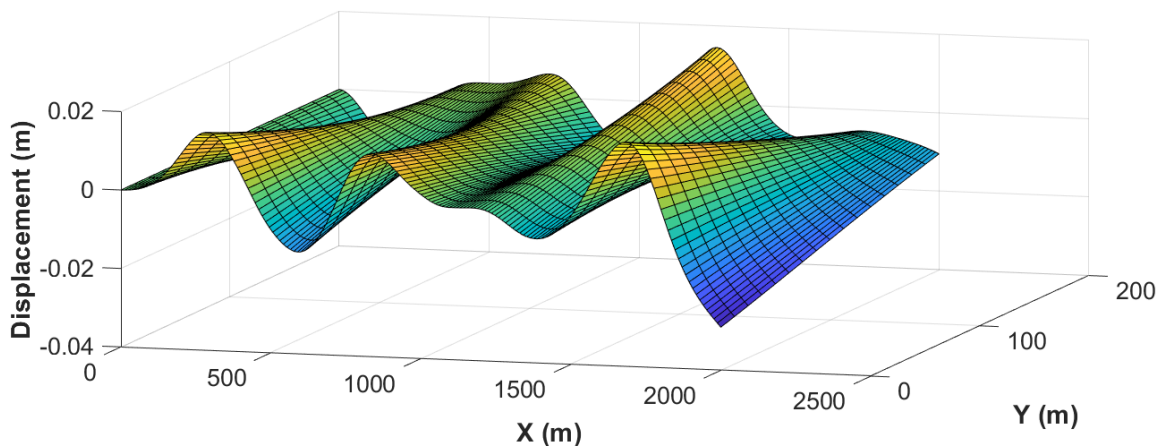


439

Fig. 12.7a

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441



442

Fig. 12.7b

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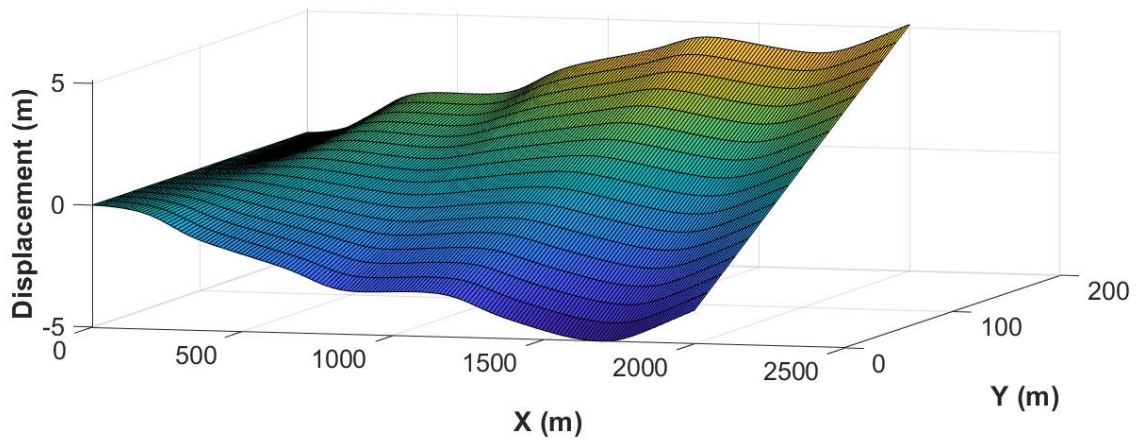
445 **Figure 12.7.** The vertical deflections of the ice shelf surface resulting from the impact of the frontal
446 incident wave were obtained from **Model 3** with the forcing periodicity of **(a) $T = 2.07$ s** and **(b)**
447 **$T = 2.13$ s**. These periodicities are located, respectively, to the left and to the right of the
448 resonance peak observed at $T_{n_7} = 2.1$ s (in **Fig. 10c**). The amplitude of ice thickness fluctuations

449 **$A_H = 5$ m.**

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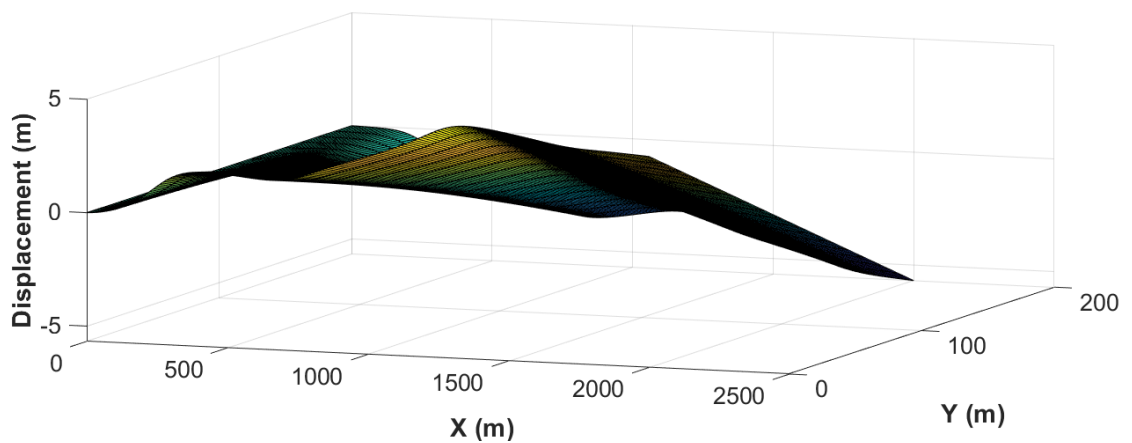


453

Fig. 13.1a

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456

Fig. 13.1b

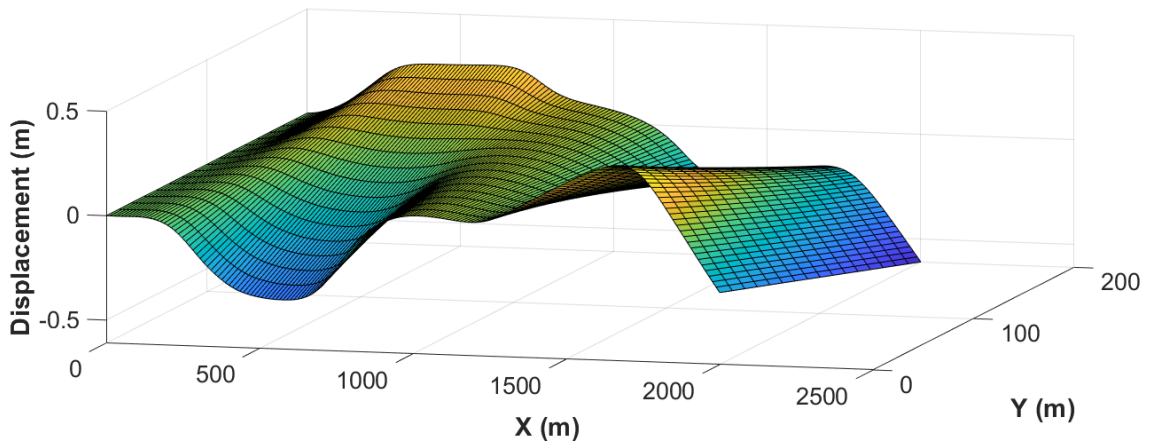
457

458

459 **Figure 13.1.** The vertical deflections of the ice shelf surface resulting from the impact of the frontal
460 incident wave were obtained from **Model 3** with the forcing periodicity of (a) $T = 26.0$ s and (b)
461 $T = 29.0$ s. These periodicities are located, respectively, to the left and to the right of the
462 resonance peak observed at $T_{n_1} = 27.72$ s (in **Fig. 11a**). The amplitude of ice thickness
463 fluctuations $A_H = 8$ m.

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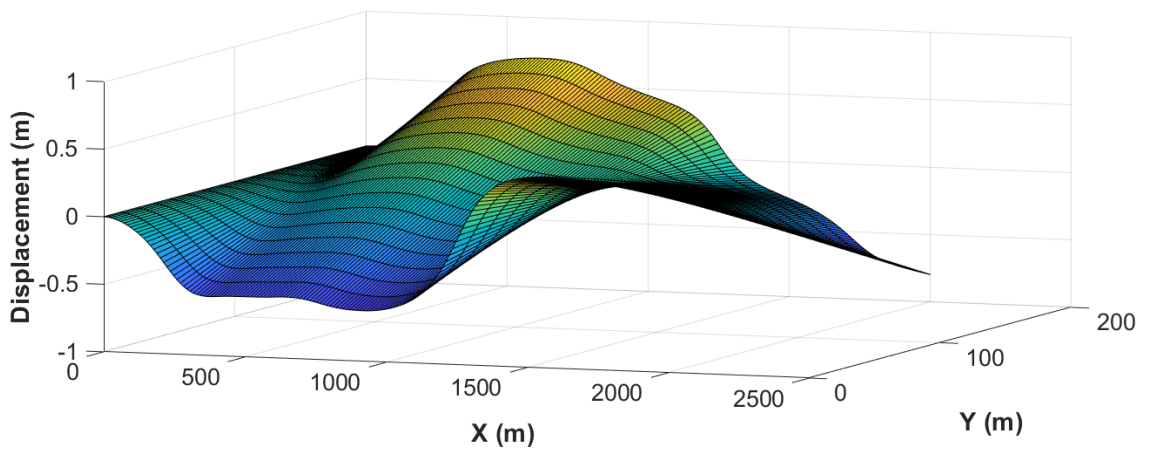


466

467

Fig. 13.2a

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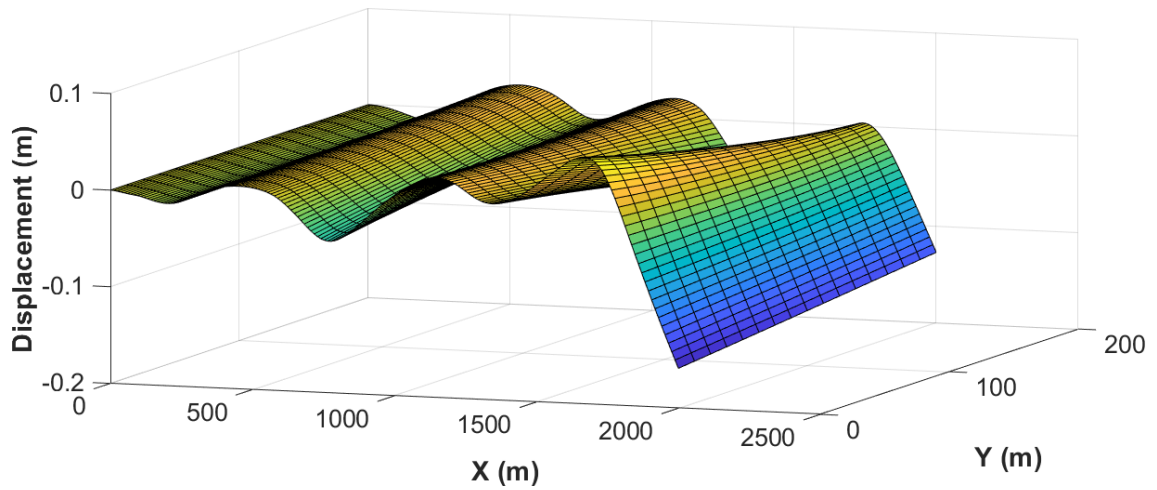
Fig. 13.2b

471

472 **Figure 13.2.** The vertical deflections of the ice shelf surface resulting from the impact of the frontal
473 incident wave were obtained from **Model 3** with the forcing periodicity of **(a) $T = 8.0$ s** and **(b)**
474 **$T = 10.0$ s**. These periodicities are located, respectively, to the left and to the right of the
475 resonance peak observed at **$T_{n_2} = 9.06$ s** (in **Fig. 11b**). The amplitude of ice thickness
476 fluctuations **$A_H = 8$ m**.

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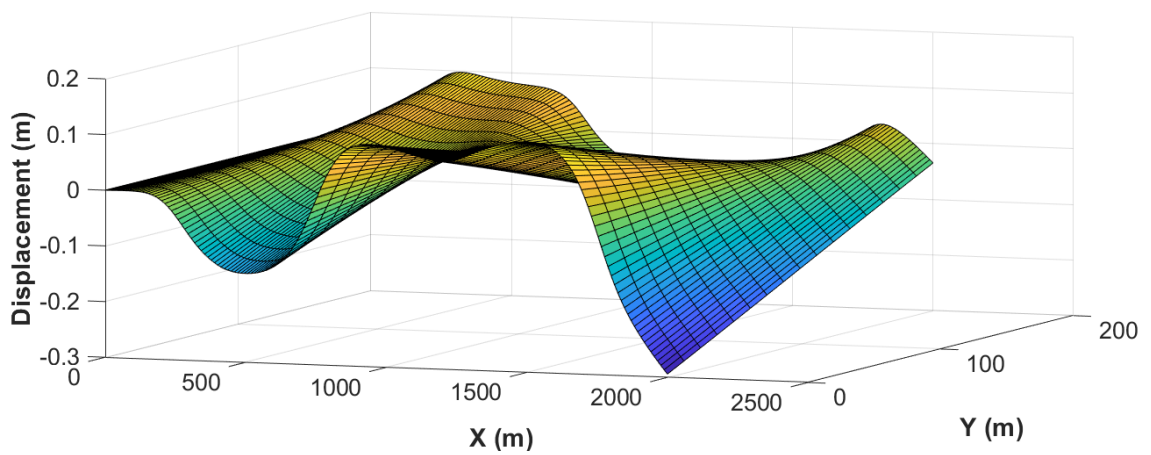


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Fig. 13.3a

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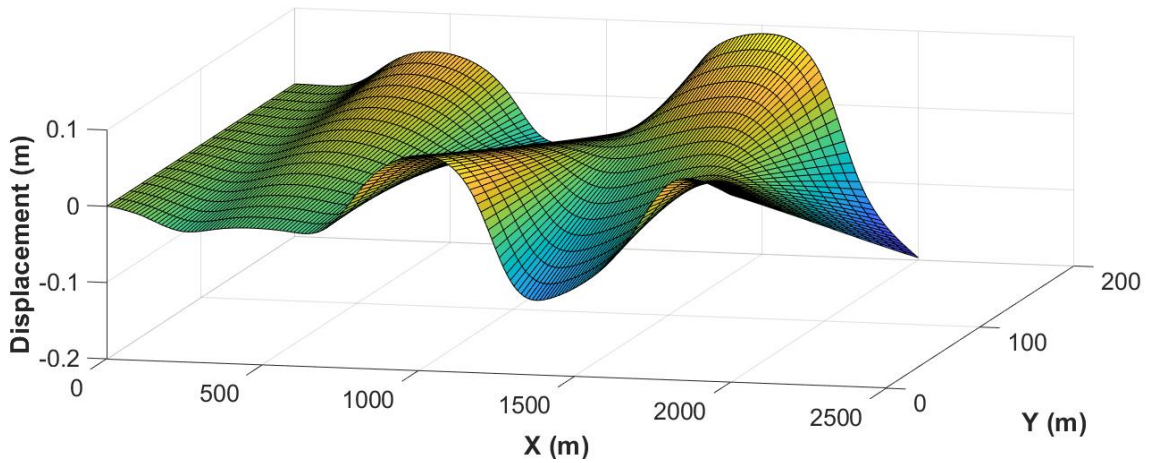
Fig. 13.3b

484

485 **Figure 13.3.** The vertical deflections of the ice shelf surface resulting from the impact of the frontal
486 incident wave were obtained from **Model 3** with the forcing periodicity of (a) $T = 5.0$ s and (b)
487 $T = 6.5$ s. These periodicities are located, respectively, to the left and to the right of the resonance
488 peak observed at $T_{n_3} = 5.74$ s (in **Fig. 11b**). The amplitude of ice thickness fluctuations $A_H =$
489 **8 m.**

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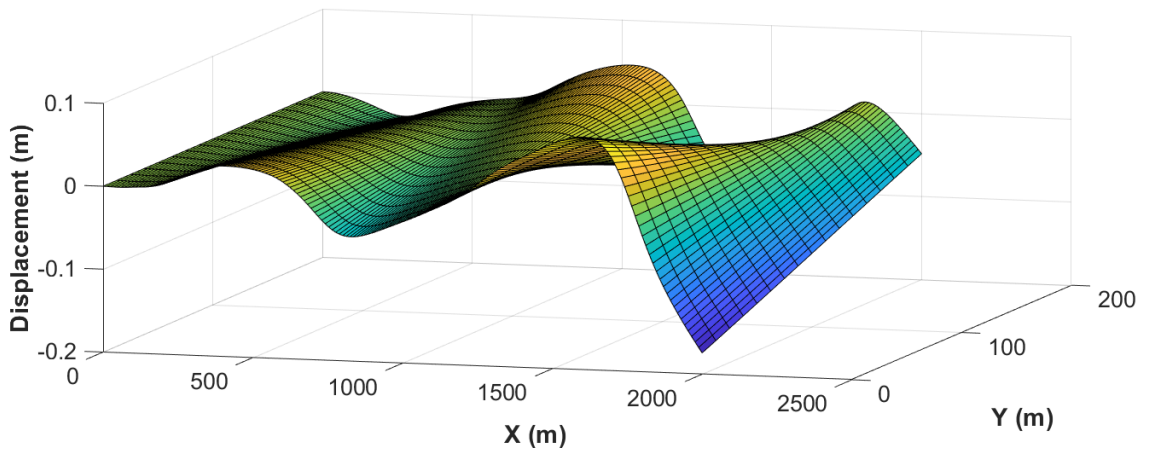


492

Fig. 13.4a

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Fig. 13.4b

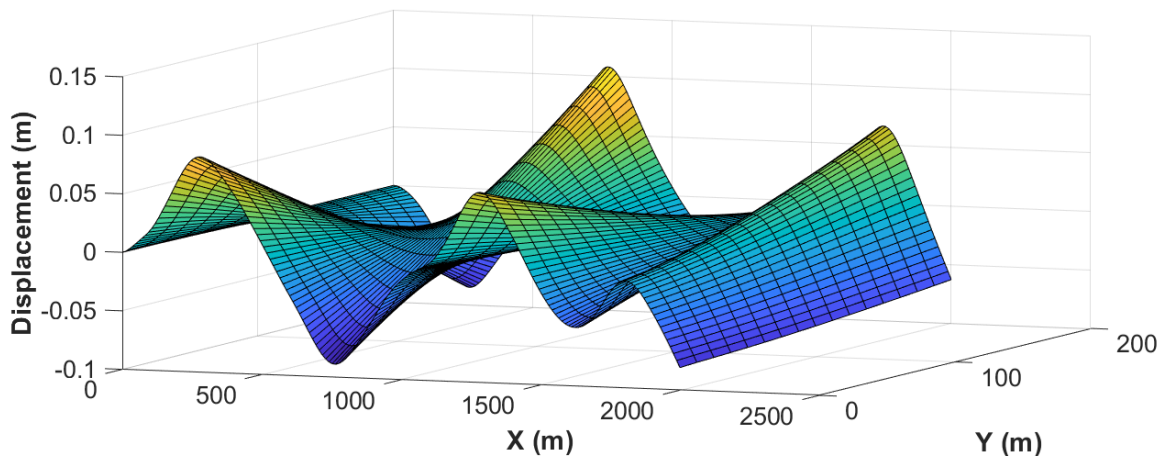
496

497

498 **Figure 13.4.** The vertical deflections of the ice shelf surface resulting from the impact of the frontal
499 incident wave were obtained from **Model 3** with the forcing periodicity of **(a) $T = 4.3$ s** and **(b)**
500 **$T = 4.75$ s**. These periodicities are located, respectively, to the left and to the right of the
501 resonance peak observed at $T_{n_4} = 4.52$ s (in **Fig. 11c**). The amplitude of ice thickness
502 fluctuations $A_H = 8$ m.

503

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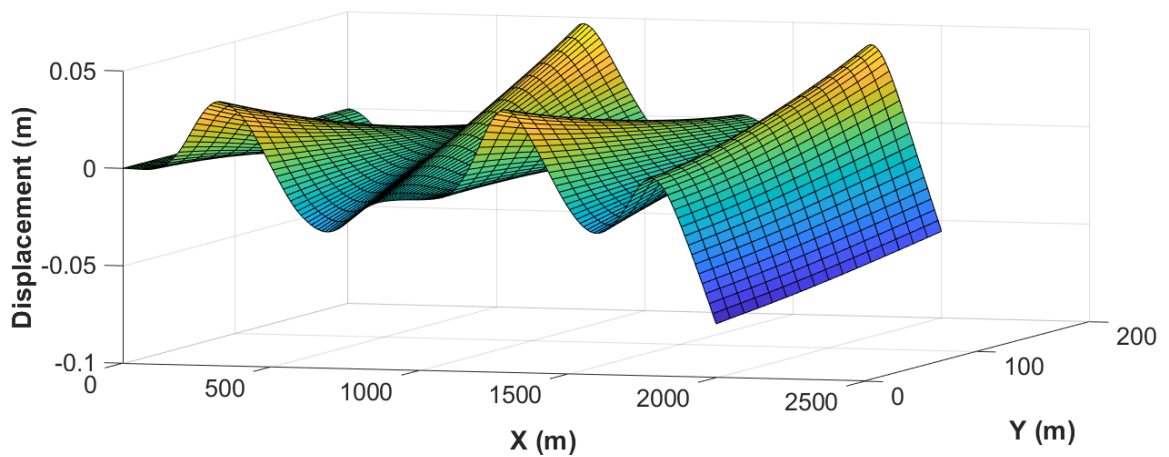


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Fig. 13.5a

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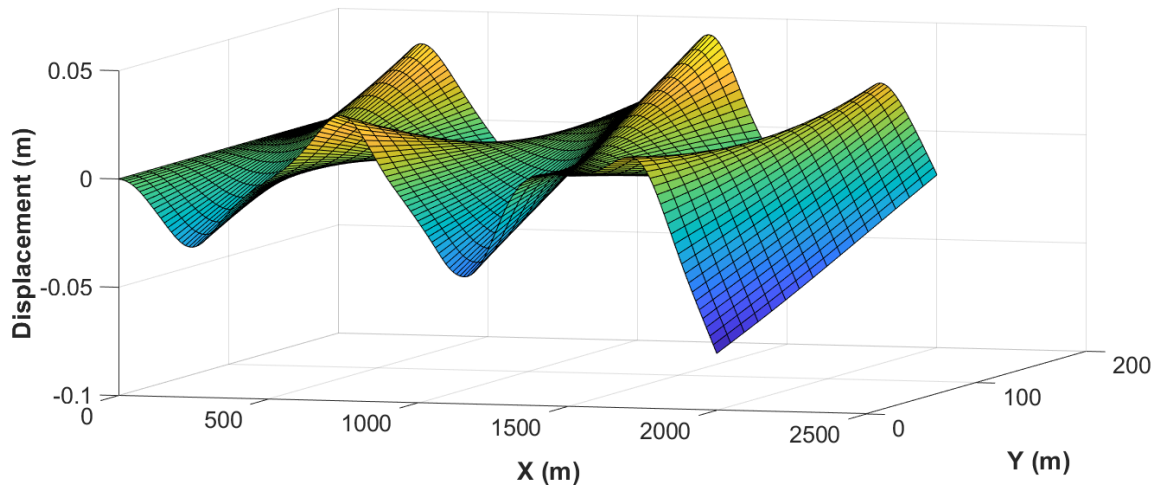
Fig. 13.5b

510

511 **Figure 13.5.** The vertical deflections of the ice shelf surface resulting from the impact of the frontal
512 incident wave were obtained from **Model 3** with the forcing periodicity of (a) $T = 2.55$ s and (b)
513 $T = 2.65$ s. These periodicities are located, respectively, to the left and to the right of the
514 resonance peak observed at $T_{n_5} = 2.6$ s (in Fig. 11c). The amplitude of ice thickness fluctuations
515 $A_H = 8$ m.

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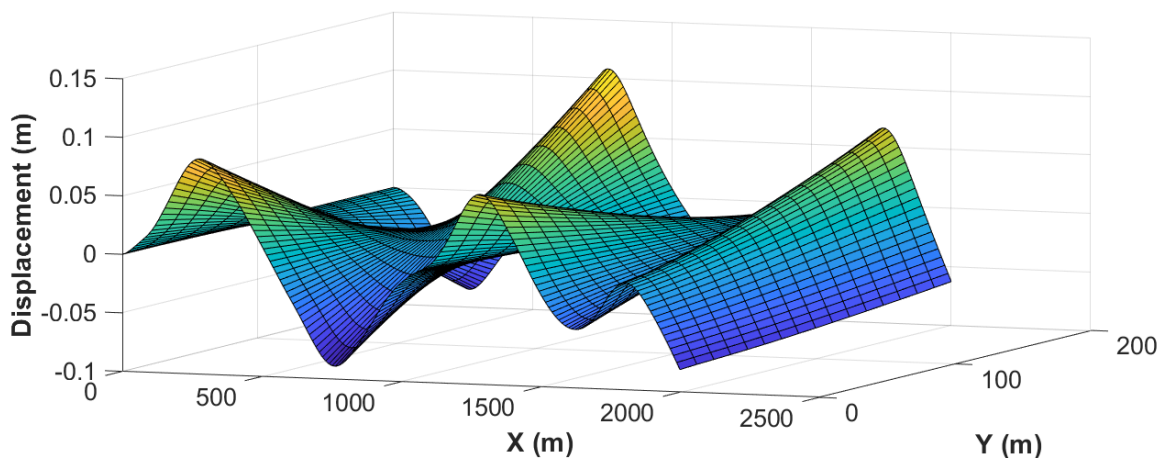


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Fig. 13.6a

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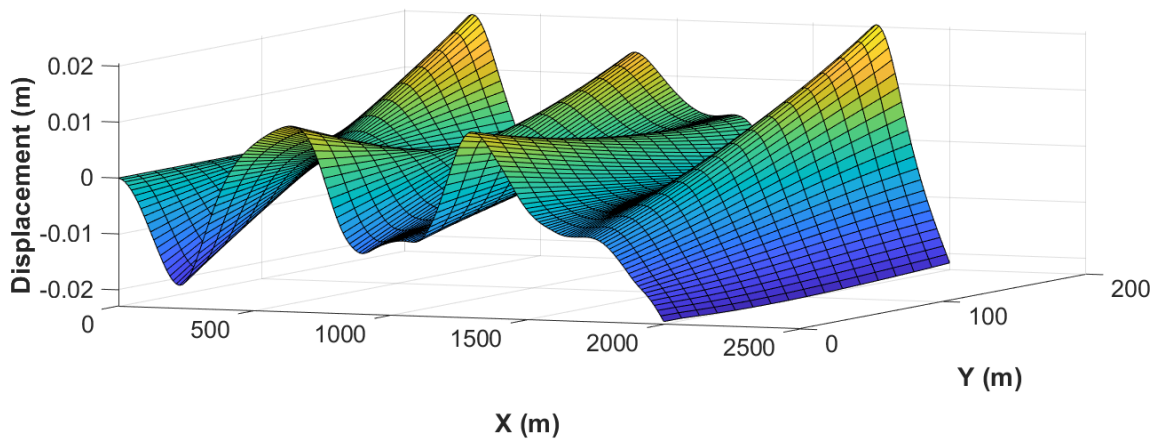
Fig. 13.6b

523

524 **Figure 13.6.** The vertical deflections of the ice shelf surface resulting from the impact of the frontal
525 incident wave were obtained from **Model 3** with the forcing periodicity of (a) $T = 2.46$ s and (b)
526 $T = 2.55$ s. These periodicities are located, respectively, to the left and to the right of the
527 resonance peak observed at $T_{n_6} = 2.52$ s (in **Fig. 11c**). The amplitude of ice thickness
528 fluctuations $A_H = 8$ m.

529

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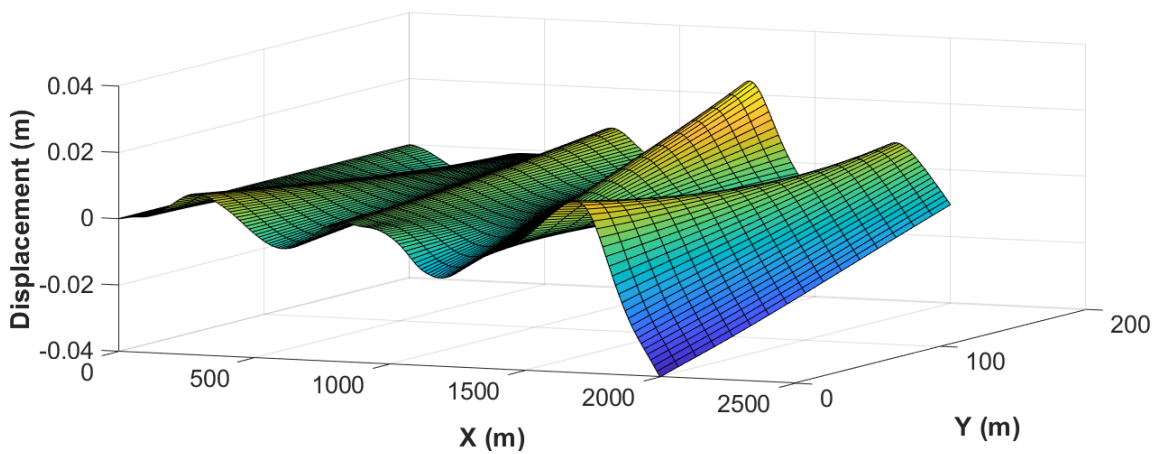


531

Fig. 13.7a

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Fig. 13.7b

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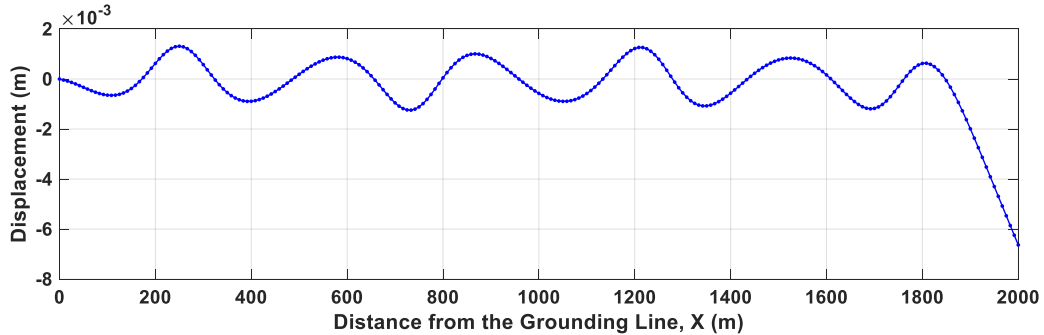
536

537 **Figure 13.7.** The vertical deflections of the ice shelf surface resulting from the impact of the frontal
538 incident wave were obtained from **Model 3** with the forcing periodicity of (a) $T = 2.05 \text{ s}$ and (b)
539 $T = 2.15 \text{ s}$. These periodicities are located, respectively, to the left and to the right of the
540 resonance peak observed at $T_{n7} = 2.1 \text{ s}$ (in **Fig. 11c**). The amplitude of ice thickness fluctuations
541 $A_H = 8 \text{ m}$.

542

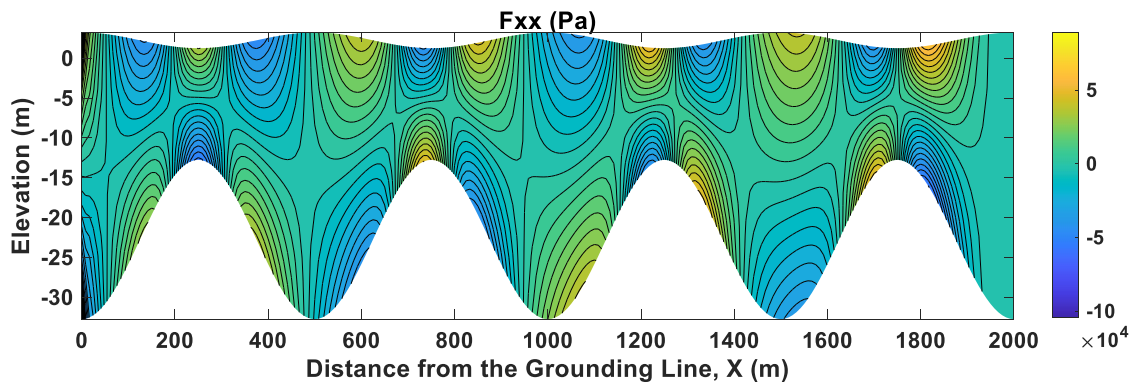
543 **Appendix A. Ice stress distributions along the ice shelf center-line**
 544 **profile obtained by Model 1**

546 **1) The amplitude of ice thickness oscillations in Eq. (3) $A_H = 11\text{ m}$**



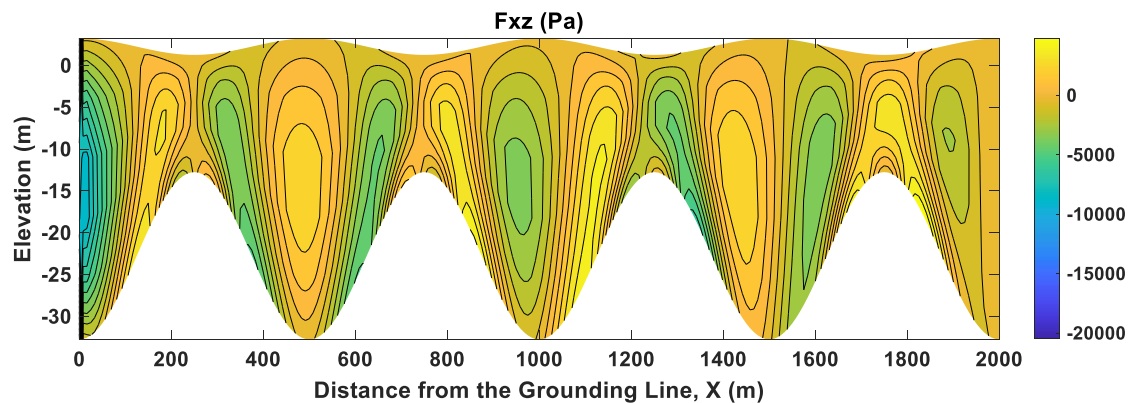
547

548 **Fig. A1a**



549

550 **Fig. A1b**

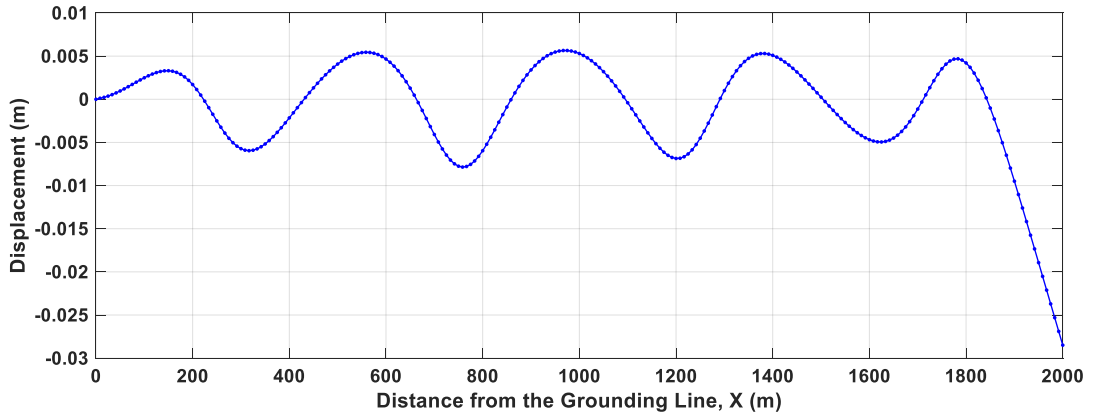


551

552 **Fig. A1c**

553 **Figure A1. (a)** Vertical displacement of ice W along the centerline due to the impact of the frontal
 554 incident wave. **(b)** Distribution of longitudinal stress (σ_{xx}) in a vertical cross-section of the ice
 555 shelf along the centerline. **(c)** Distribution of shear stress (σ_{xz}) in a vertical cross-section of the ice
 556 shelf along the centerline. The periodicity of the forcing $T = 1\text{ s}$. The amplitude of ice thickness
 557 oscillations $A_H = 11\text{ m}$.

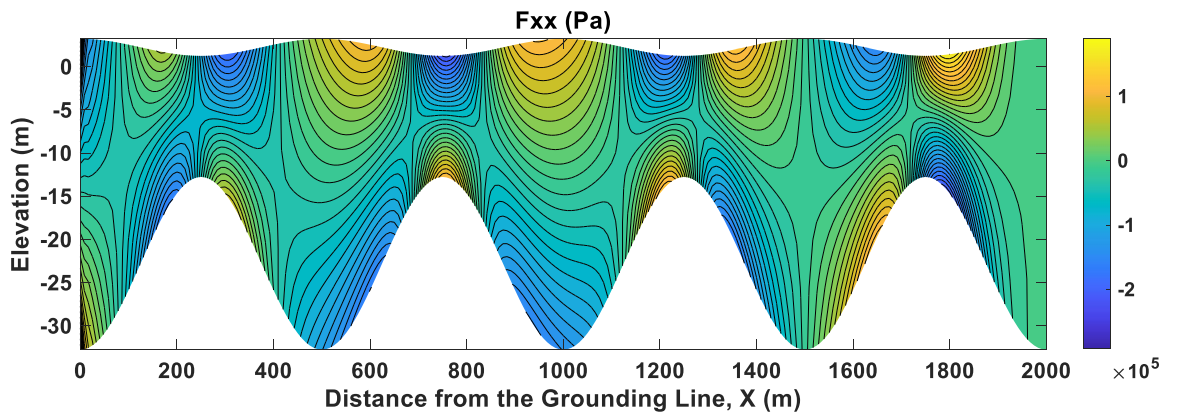
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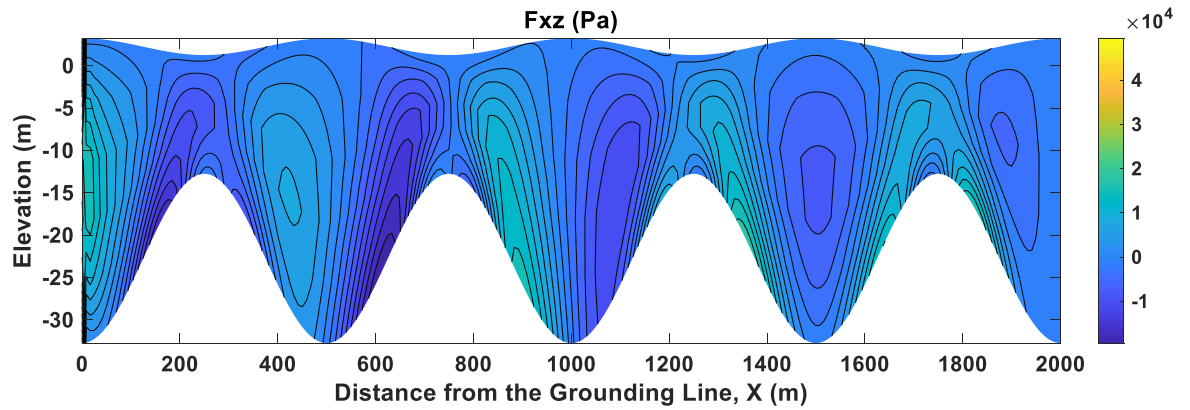
Fig. A2a



561

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Fig. A2b



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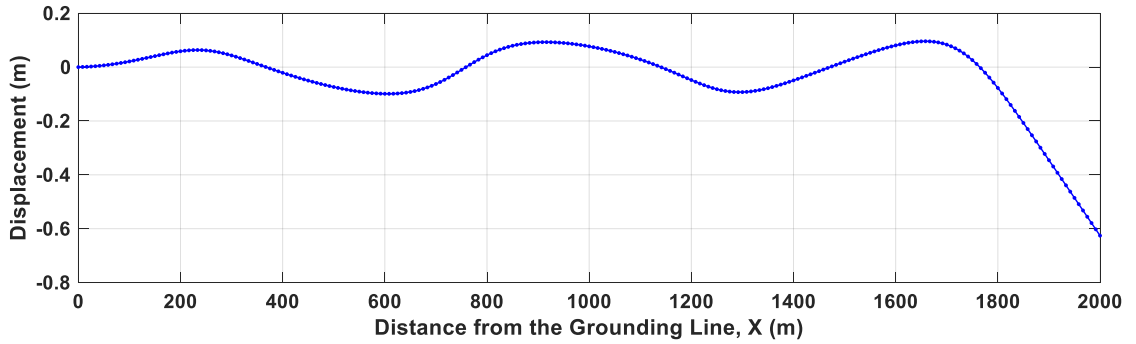
564

Fig. A2c

565 **Figure A2.** (a) Vertical displacement of ice W along the centerline due to the impact of the frontal
 566 incident wave. (b) Distribution of longitudinal stress (σ_{xx}) in a vertical cross-section of the ice
 567 shelf along the centerline. (c) Distribution of shear stress (σ_{xz}) in a vertical cross-section of the ice
 568 shelf along the centerline. The periodicity of the forcing $T = 2s$. The amplitude of ice thickness
 569 oscillations $A_H = 11m$.

570

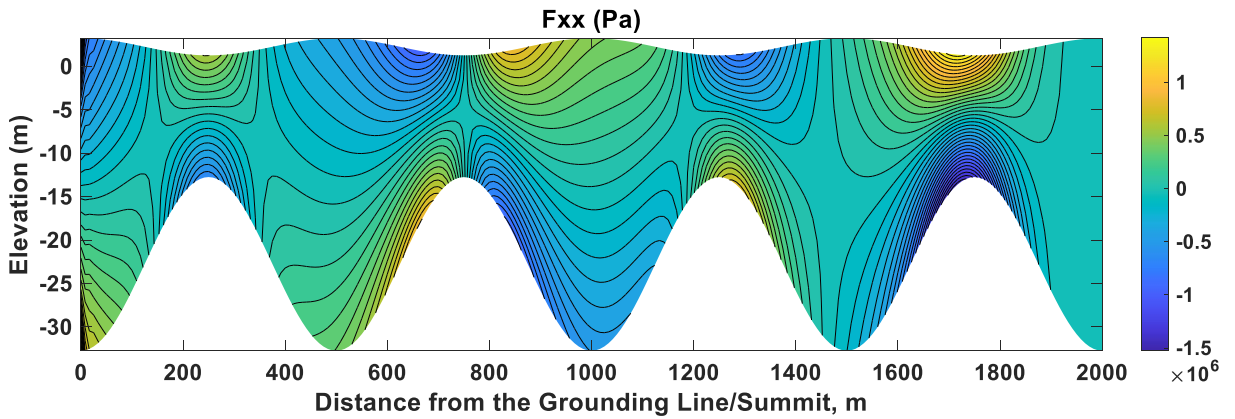
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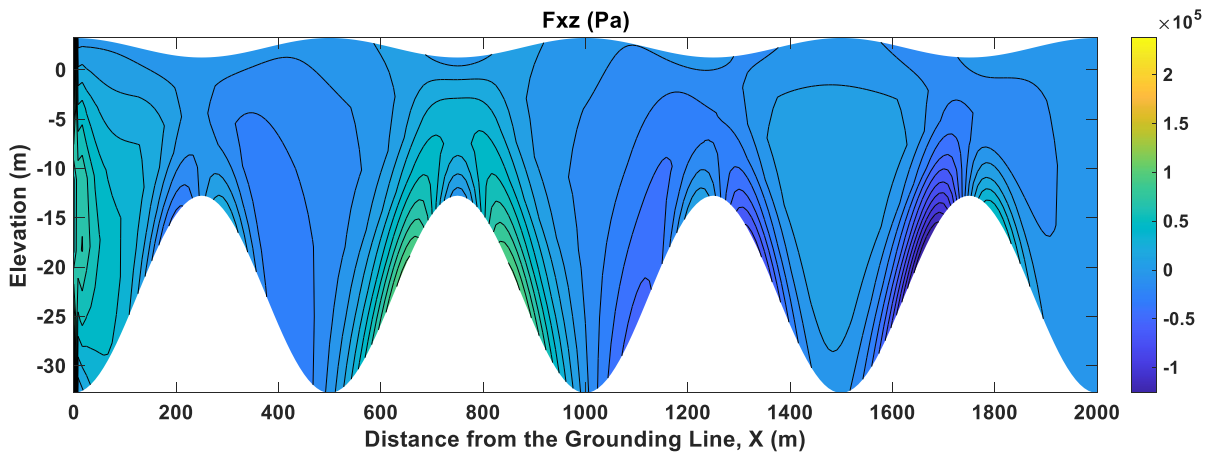
Fig. A3a



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Fig. A3b



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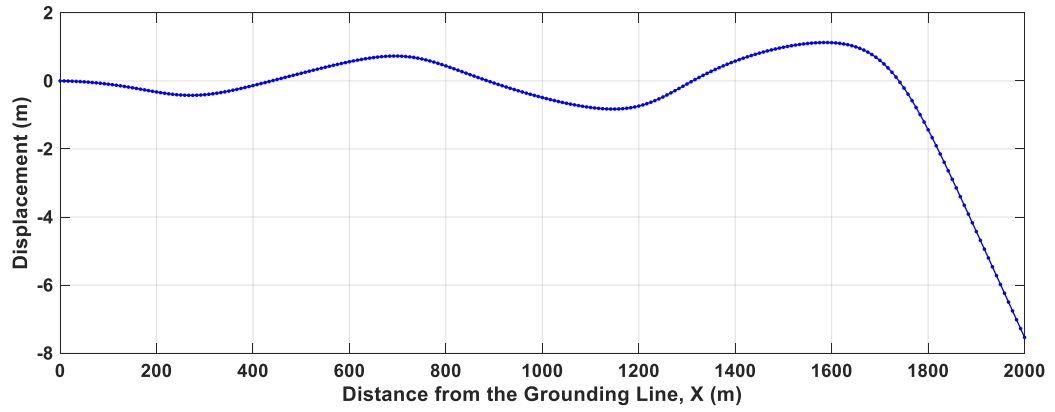
577

578

Fig. A3c

579 **Figure A3.** (a) Vertical displacement of ice W along the centerline due to the impact of the frontal
 580 incident wave. (b) Distribution of longitudinal stress (σ_{xx}) in a vertical cross-section of the ice
 581 shelf along the centerline. (c) Distribution of shear stress (σ_{xz}) in a vertical cross-section of the ice
 582 shelf along the centerline. The periodicity of the forcing $T = 10s$. The amplitude of ice thickness
 583 oscillations $A_H = 11m$.

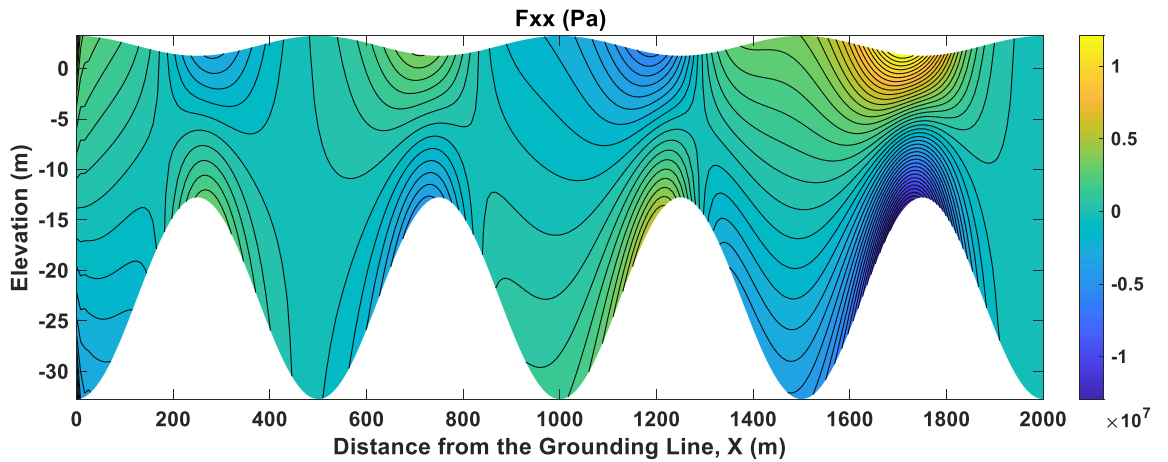
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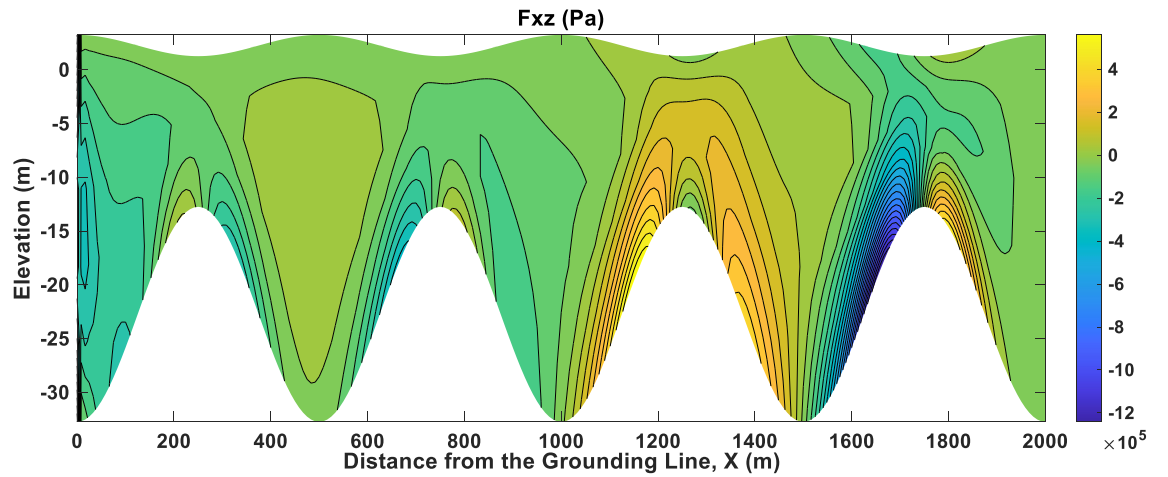
Fig. A4a



587

588

Fig. A4b



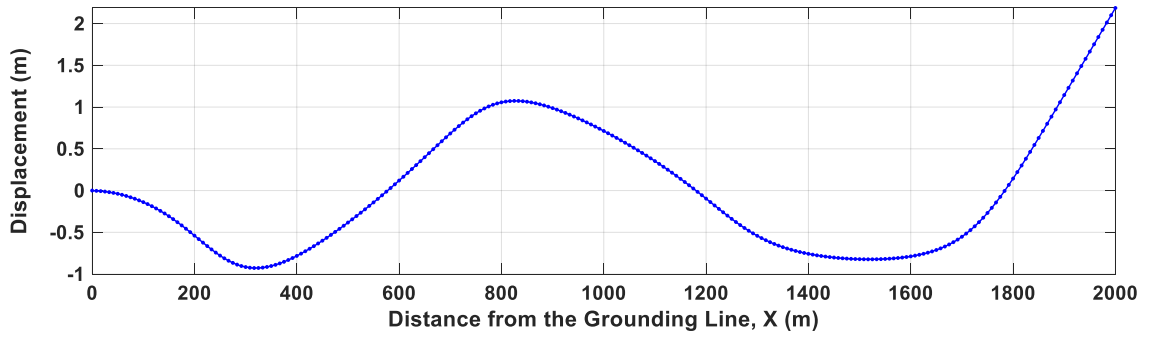
589

590

Fig. A4c

591 **Figure A4.** (a) Vertical displacement of ice W along the centerline due to the impact of the frontal
 592 incident wave. (b) Distribution of longitudinal stress (σ_{xx}) in a vertical cross-section of the ice
 593 shelf along the centerline. (c) Distribution of shear stress (σ_{xz}) in a vertical cross-section of the ice
 594 shelf along the centerline. The periodicity of the forcing $T = 20\text{s}$. The amplitude of ice thickness
 595 oscillations $A_H = 11\text{m}$.

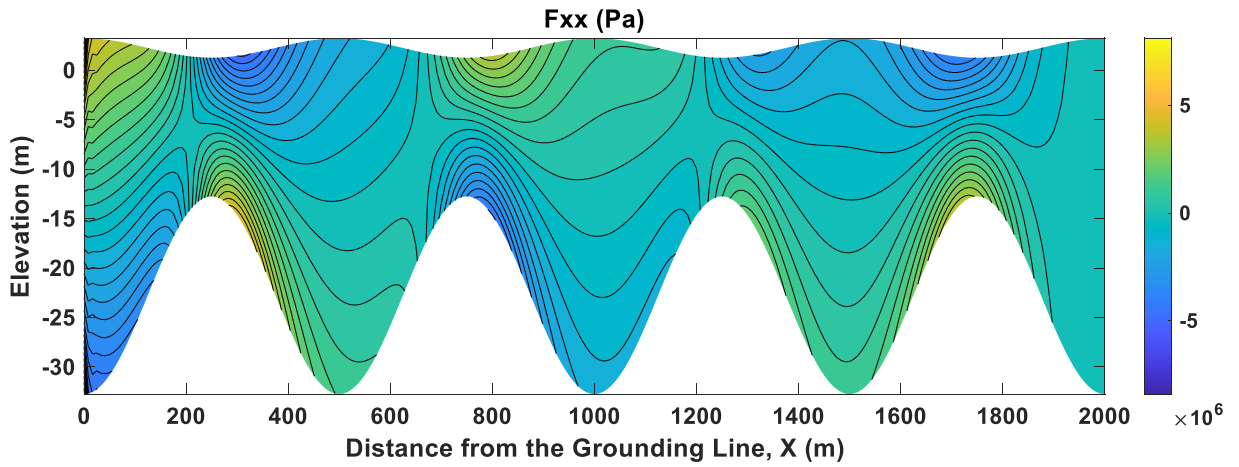
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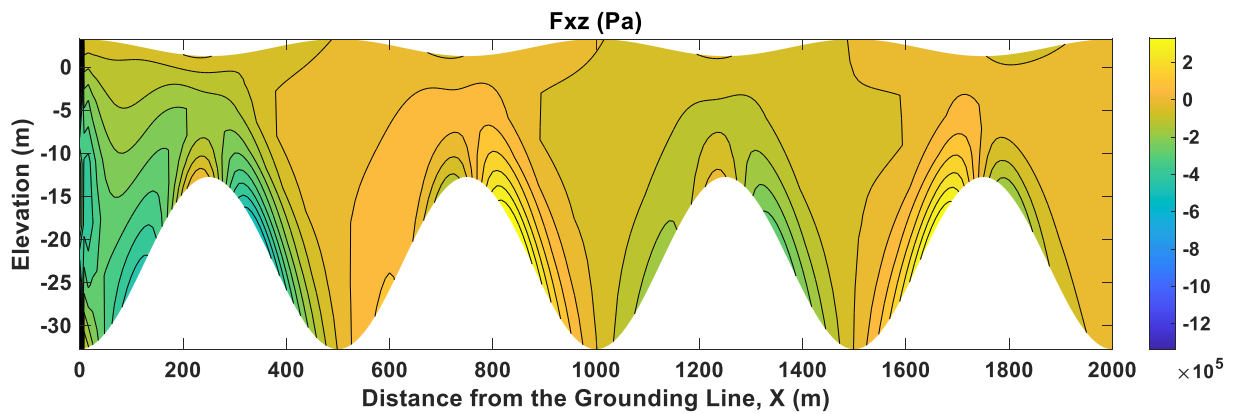
Fig. A5a



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Fig. A5b



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602

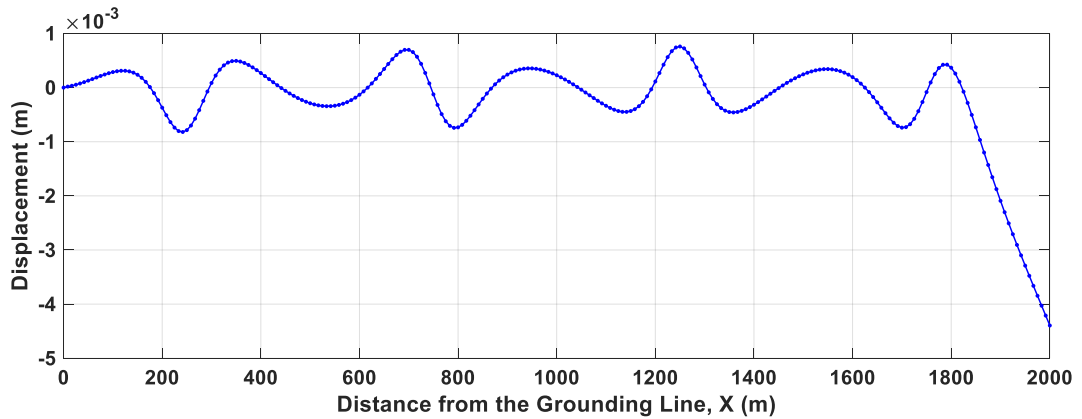
Fig. A5c

603 **Figure A5. (a)** Vertical displacement of ice W along the centerline due to the impact of the frontal
 604 incident wave. **(b)** Distribution of longitudinal stress (σ_{xx}) in a vertical cross-section of the ice
 605 shelf along the centerline. **(c)** Distribution of shear stress (σ_{xz}) in a vertical cross-section of the ice
 606 shelf along the centerline. The periodicity of the forcing $T = 50s$. The amplitude of ice thickness
 607 oscillations $A_H = 11m$.

608

2) The amplitude of ice thickness oscillations in Eq. (3) $A_H = 20\text{ m}$

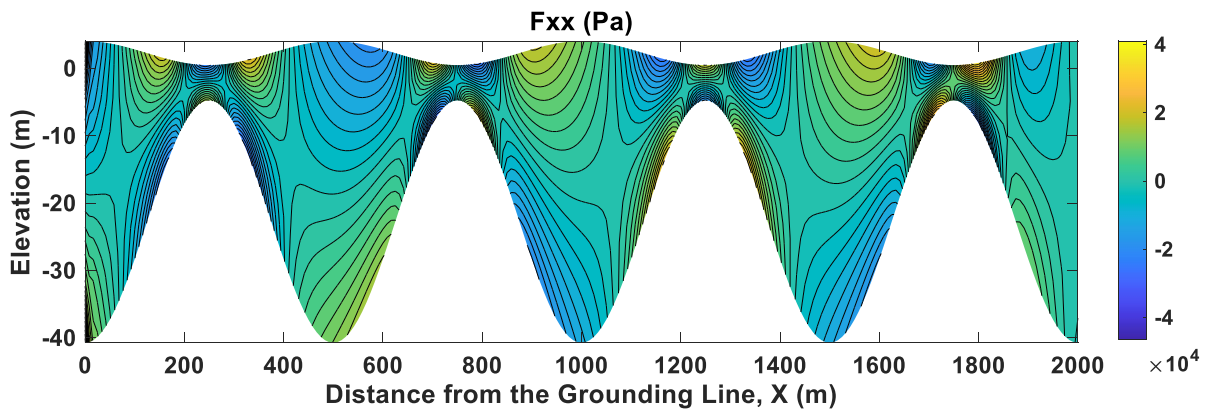
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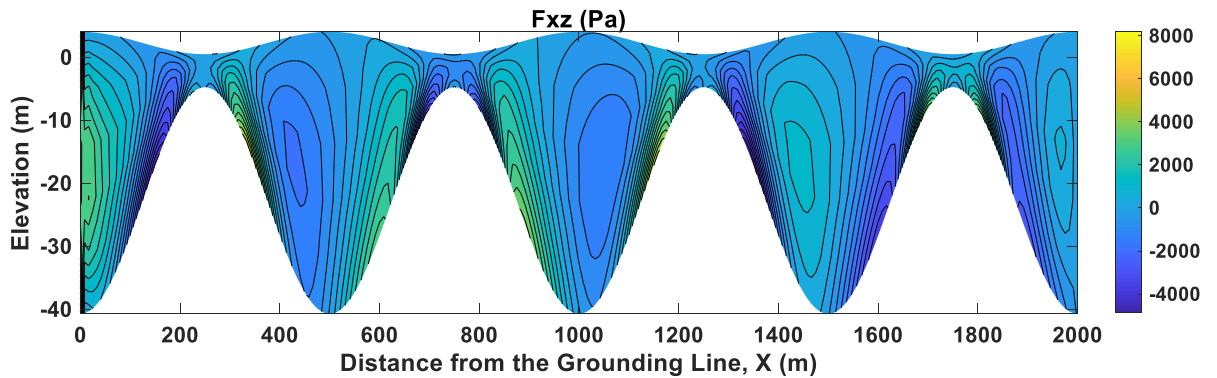
Fig. A6a



612

613

Fig. A6b

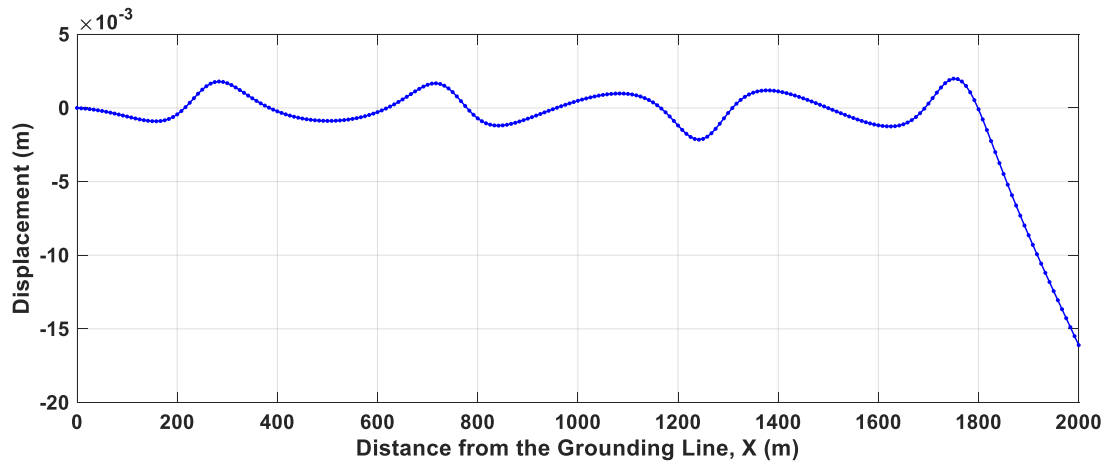


614

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Fig. A6c

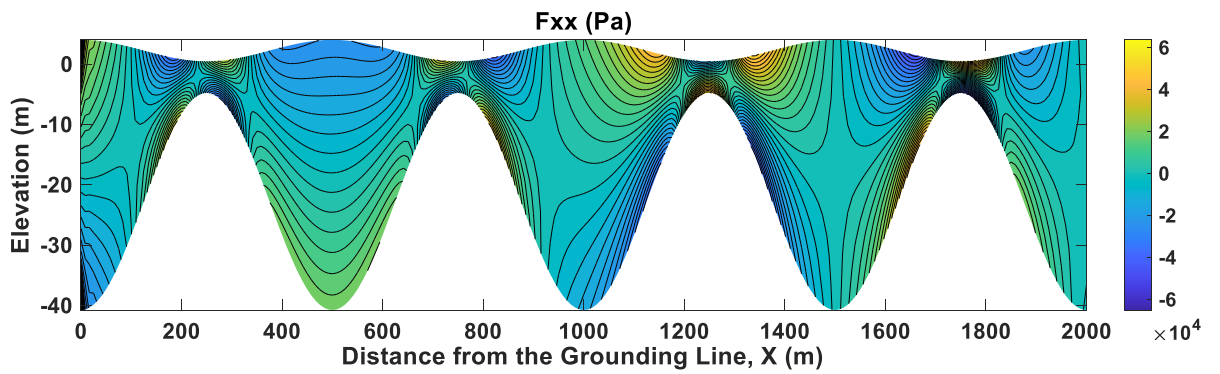
616 **Figure A6. (a)** Vertical displacement of ice W along the centerline due to the impact of the frontal
 617 incident wave. **(b)** Distribution of longitudinal stress (σ_{xx}) in a vertical cross-section of the ice
 618 shelf along the centerline. **(c)** Distribution of shear stress (σ_{xz}) in a vertical cross-section of the ice
 619 shelf along the centerline. The periodicity of the forcing $T = 1\text{ s}$. The amplitude of ice thickness
 620 oscillations $A_H = 20\text{ m}$.



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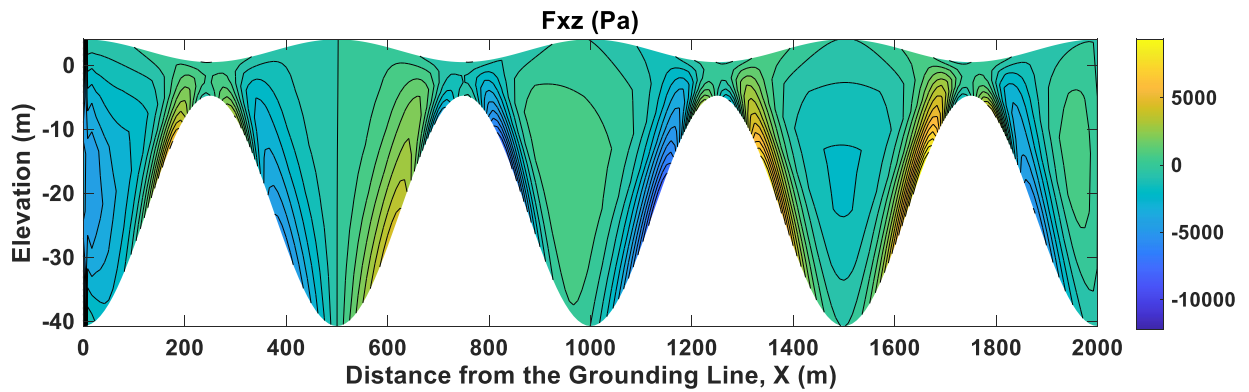
Fig. A7a



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624

Fig. A7b



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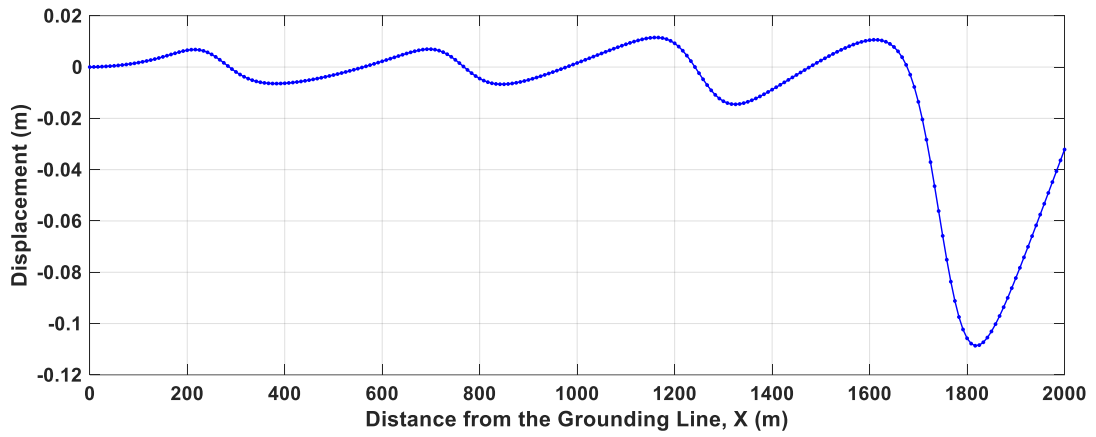
626

627

Fig. A7c

628 **Figure A7.** (a) Vertical displacement of ice W along the centerline due to the impact of the frontal
 629 incident wave. (b) Distribution of longitudinal stress (σ_{xx}) in a vertical cross-section of the ice
 630 shelf along the centerline. (c) Distribution of shear stress (σ_{xz}) in a vertical cross-section of the ice
 631 shelf along the centerline. The periodicity of the forcing $T = 2s$. The amplitude of ice thickness
 632 oscillations $A_H = 20m$.

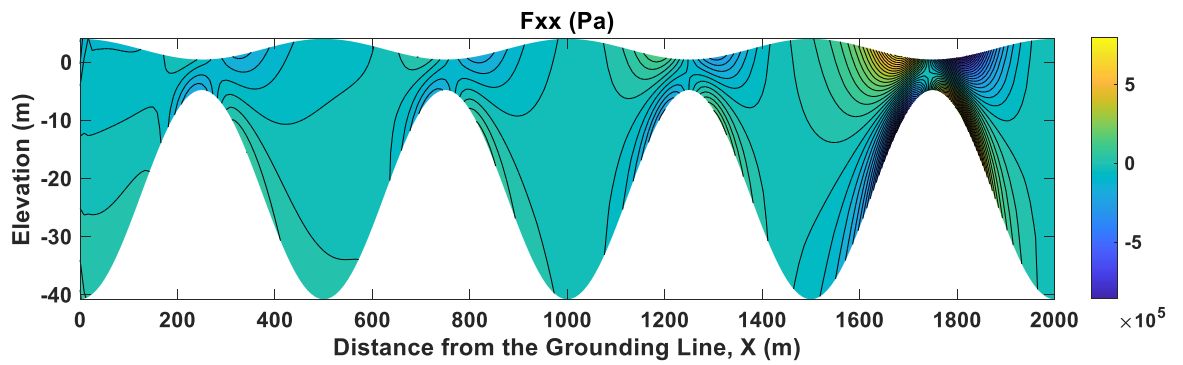
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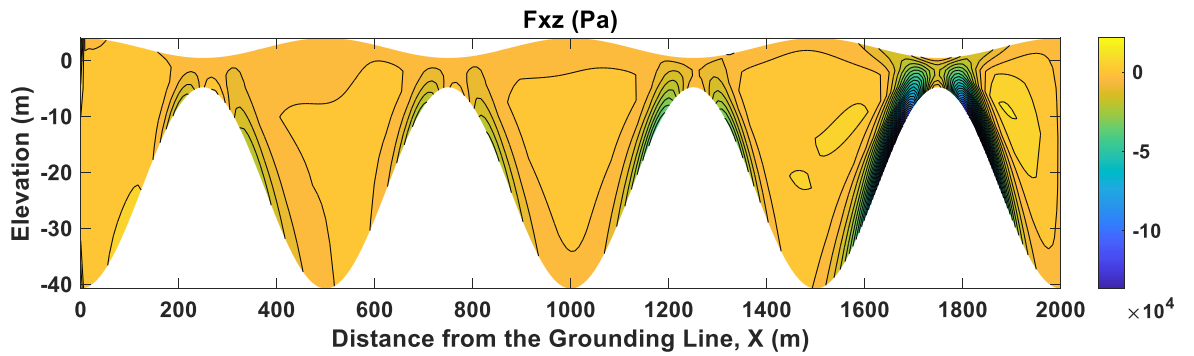
Fig. A8a



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637

Fig. A8b



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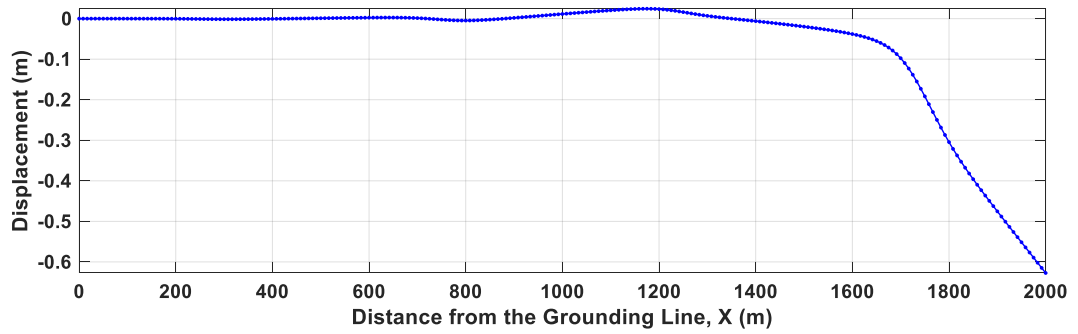
639

640

Fig. A8c

641 **Figure A8.** (a) Vertical displacement of ice W along the centerline due to the impact of the frontal
 642 incident wave. (b) Distribution of longitudinal stress (σ_{xx}) in a vertical cross-section of the ice
 643 shelf along the centerline. (c) Distribution of shear stress (σ_{xz}) in a vertical cross-section of the ice
 644 shelf along the centerline. The periodicity of the forcing $T = 10s$. The amplitude of ice thickness
 645 oscillations $A_H = 20m$.

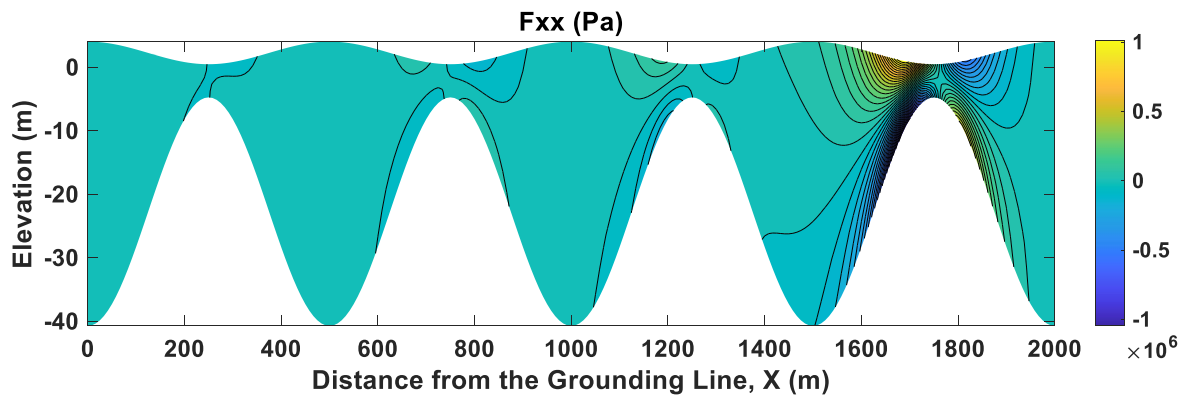
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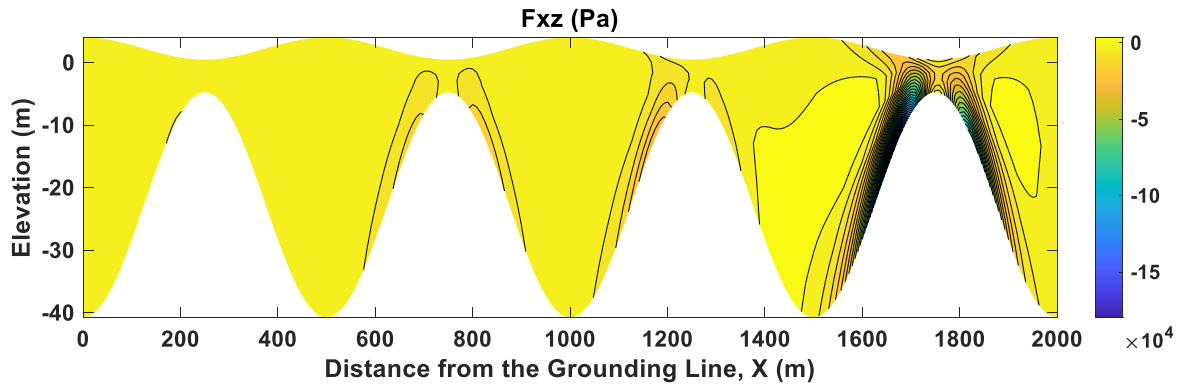
Fig. A9a



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Fig. A9b



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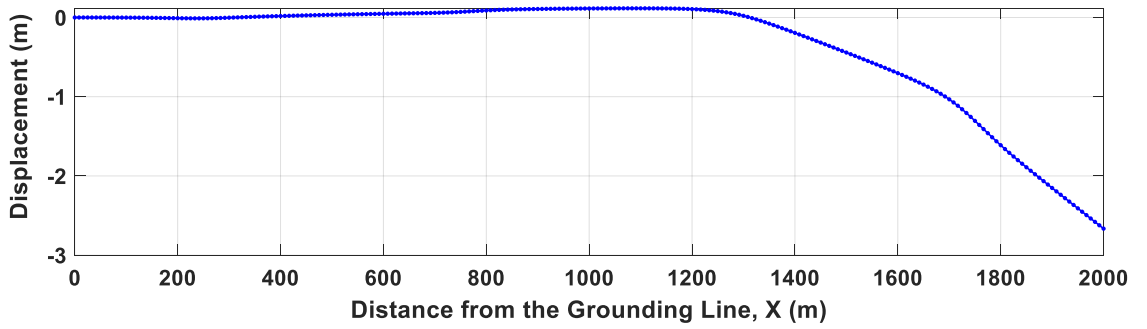
653

Fig. A9c

654 **Figure A9(a)** Vertical displacement of ice W along the centerline due to the impact of the frontal
 655 incident wave. **(b)** Distribution of longitudinal stress (σ_{xx}) in a vertical cross-section of the ice
 656 shelf along the centerline. **(c)** Distribution of shear stress (σ_{xz}) in a vertical cross-section of the ice
 657 shelf along the centerline. The periodicity of the forcing $T = 20s$. The amplitude of ice thickness
 658 oscillations $A_H = 20m$.

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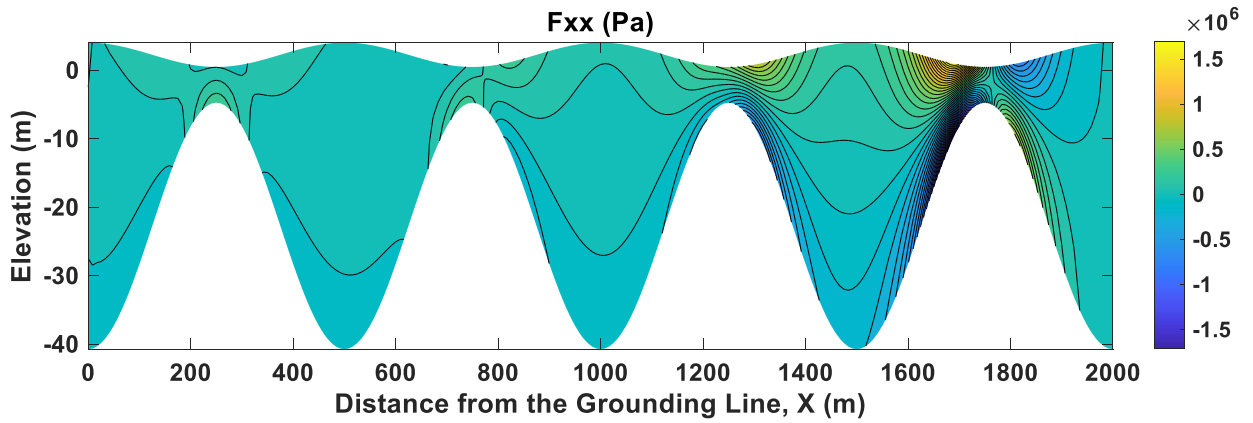
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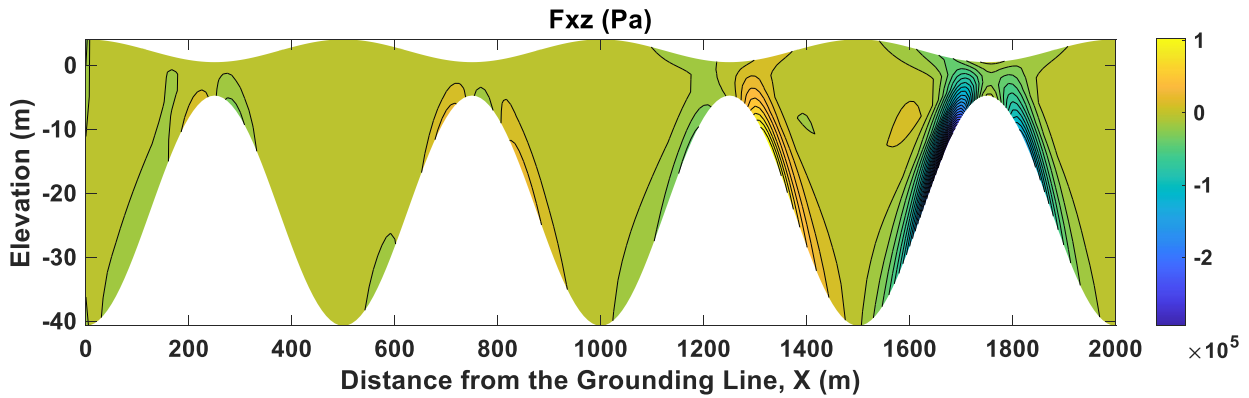
Fig. A10a



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Fig. A10b



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Fig. A10c

669 **Figure A10.** (a) Vertical displacement of ice W along the centerline due to the impact of the frontal
 670 incident wave. (b) Distribution of longitudinal stress (σ_{xx}) in a vertical cross-section of the ice
 671 shelf along the centerline. (c) Distribution of shear stress (σ_{xz}) in a vertical cross-section of the ice
 672 shelf along the centerline. The periodicity of the forcing $T = 50s$. The amplitude of ice thickness
 673 oscillations $A_H = 20m$.

674

675 **Appendix B. Amplitude spectra and free energy spectra obtained**
 676 **using Model 1**

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678 The elastic free energy of the ice plate (the free energy of the elastic deformations of the
 679 ice plate) is expressed as (e.g. Landau & Lifshitz, 1986)

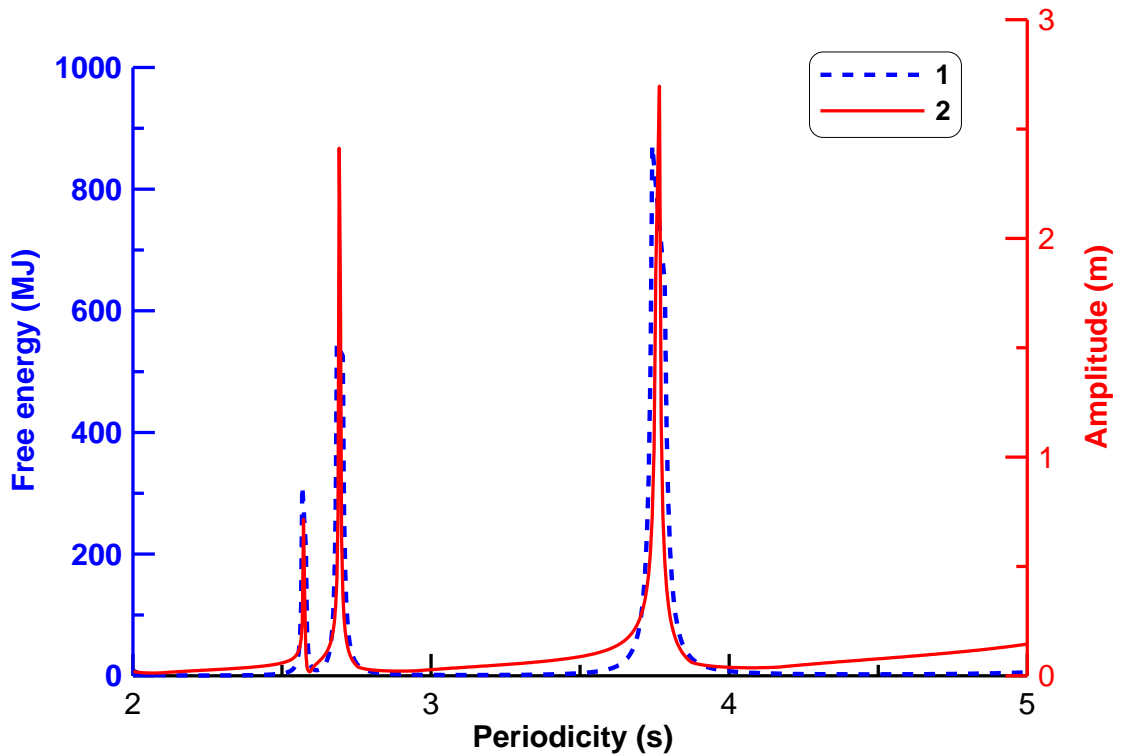
680

$$681 \quad F_{full} = \frac{E}{2(1+\nu)} \int_V \left(u_{ik}^2 + \frac{\nu}{1-2\nu} u_{ii}^2 \right) dV, \quad (B1)$$

682

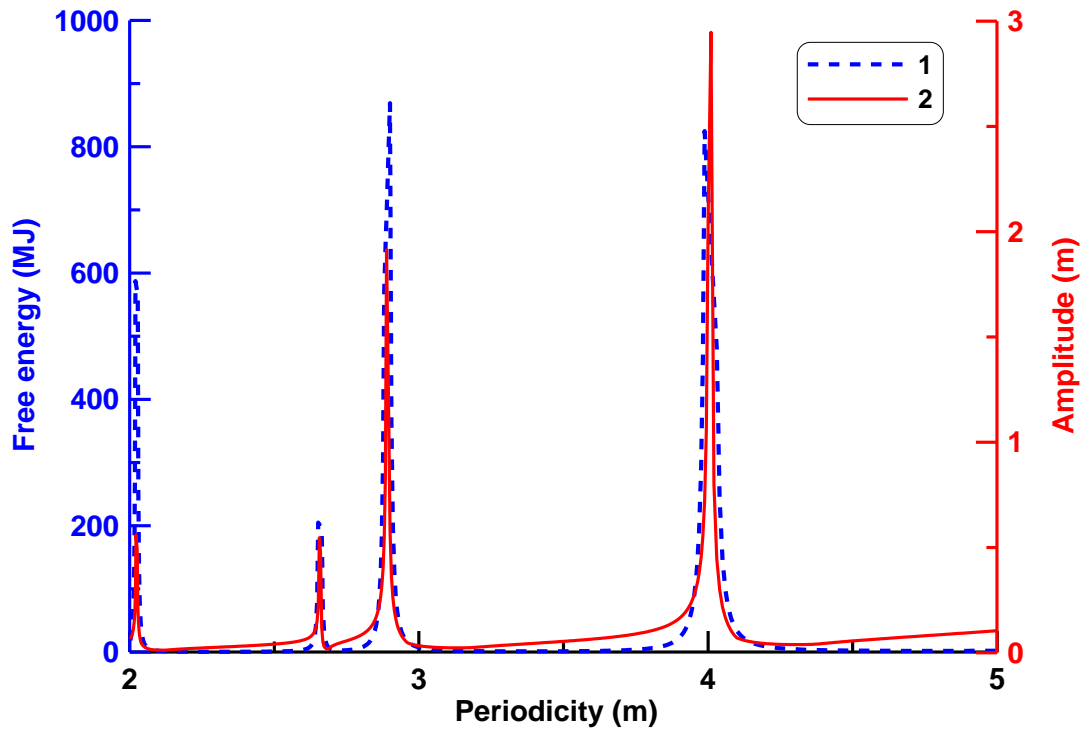
683 where \mathbf{u} is the strain tensor, E is Young's modulus, ν is Poisson's ratio, V is volume of
 684 undeformed ice plate.

685 Figures B1-B4 show the free energy spectra combined with amplitude spectra obtained
 686 using Model 1 for some values of the parameter A_H . In the experiments performed, no
 687 differences were revealed in the location of the resonance peaks in the two types of
 688 spectrum. This, in particular, confirms that we can essentially use the amplitude spectra
 689 (as a simpler case) to analyze the vibrations (the possibility of resonant motion) of the ice
 690 shelf.



691

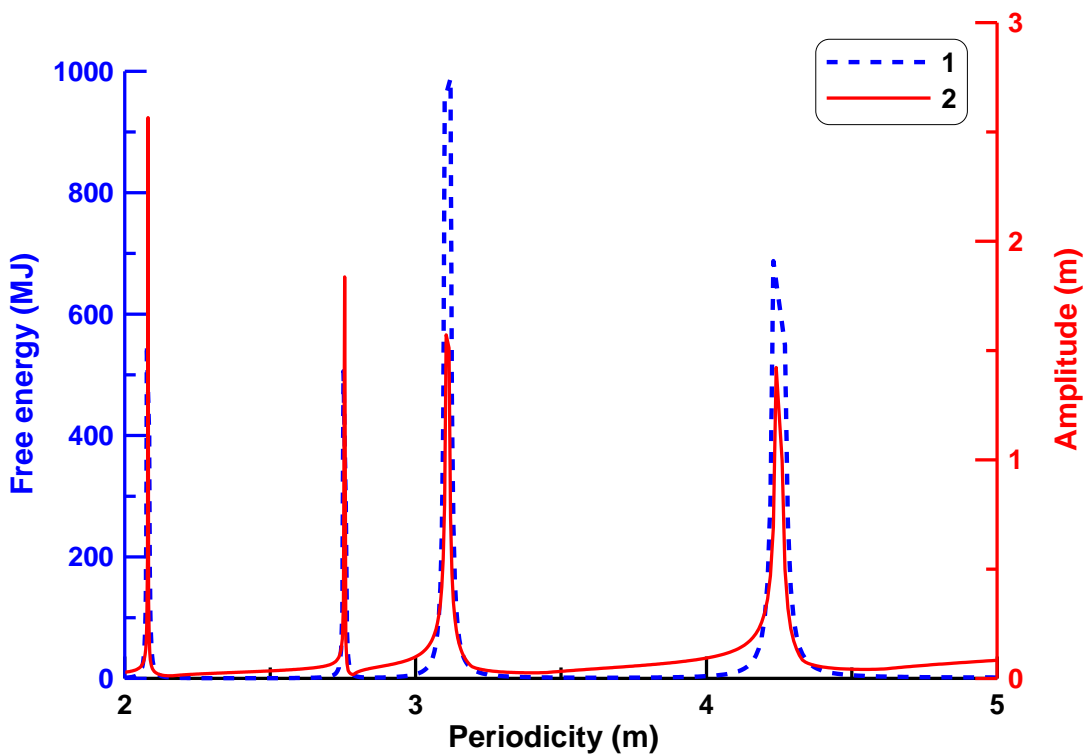
692 **Figure B1.** Free energy spectrum (1) and amplitude spectrum (2) obtained using **Model 1** for the
 693 ice thickness oscillation amplitude $A_H = 5 \text{ m}$.



695

696 **Figure B2.** Free energy spectrum (1) and amplitude spectrum (2) obtained using **Model 1** for the
 697 ice thickness oscillation amplitude $A_H = 9 \text{ m}$.

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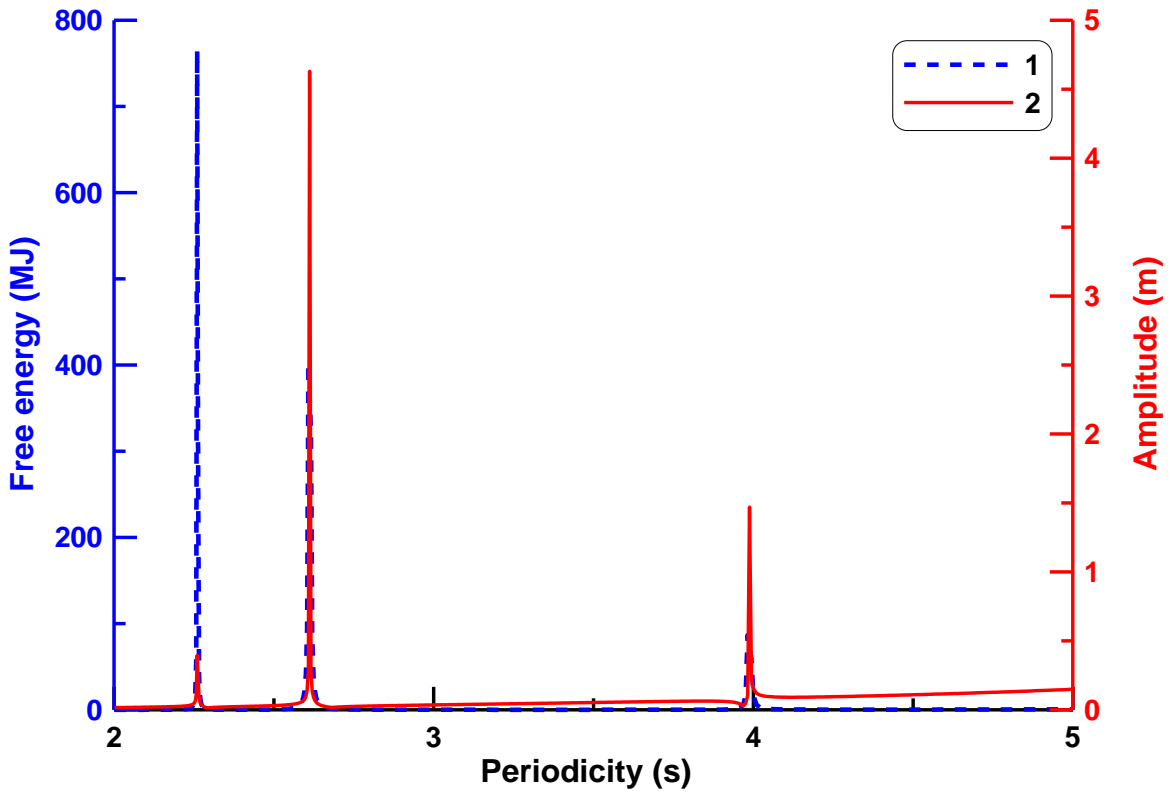


699

700 **Figure B3.** Free energy spectrum (1) and amplitude spectrum (2) obtained by **Model 1** for ice
 701 thickness oscillation amplitude $A_H = 11 \text{ m}$

702

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705

706 **Figure B4.** Free energy spectrum (1) and amplitude spectrum (2) obtained by **Model 1** for the
707 ice thickness oscillation amplitude $A_H = 20 \text{ m}$

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