The formulas (4)–(7) used to calculate $P(tie)$, $P(1)$, and $P(2)$ and the likelihood in (13) must be evaluated at each iteration of the MCMC chain for each respondent. The most time-consuming part of this calculation, computation of the many coefficients in the sums, is repeated many times across respondents and over iterations. We recommend that the technique of pre-computation be used to streamline the MCMC routine. The idea of precomputation is to handle expensive, repetitive calculations a single time (typically before the iterative algorithm begins), storing the needed results for future, quick use. This appendix describes such a simplification.

For any given value $K_i = K$, there are $K(K+2)$ possible combinations of hits on the shared counter and the two unique counters. Exactly K combinations correspond to ties, with k hits on the shared counter and $K-k$ hits on each unique counter. There are $K+(K-1)+\cdots+1$ combinations that lead to the counter for a_1 having reached K first and the same number of combinations that lead to the counter for a_2 having reached K first. Thus there are a total of $K(K+2)$ cases.

Now let $\tilde{N} = \{\tilde{N}_s, \tilde{N}_1, \tilde{N}_2\}$ be a matrix of size $K(K+2)\times 3$, with $\tilde{N}_s$, $\tilde{N}_1$, and $\tilde{N}_2$ denoting vectors of possible hits at the time when the threshold is reached. The jth row of $\tilde{N}$ is denoted by $\{\tilde{N}_s(j), \tilde{N}_1(j), \tilde{N}_2(j)\}$. Additionally, a matrix $\tilde{I} = \{\tilde{I}_s, \tilde{I}_1, \tilde{I}_2\}$ of indicators is used to record whether a row of $\tilde{N}$ corresponds to a tie or a chosen alternative. For example, if a row j in $\tilde{N}$ corresponds to a tie, then the jth row of $\tilde{I}$ is $\tilde{I}_s(j) = 1$, $\tilde{I}_1(j) = 0$, and $\tilde{I}_2(j) = 0$.

The hits on the three counters that precede the threshold-reaching hit could have arrived in any sequence. The number of different sequences is given by the multiplicative coefficients within the summation terms in Theorem 2.1. A vector $\tilde{C}$ of length $K(K+2)$ is used to store these coefficients for every combination of hits. The jth element of $\tilde{C}$ is set to be

$$\tilde{C}(j) = \begin{cases} \dfrac{(\tilde{N}_s(j)+\tilde{N}_1(j)+\tilde{N}_2(j)-1)!}{(\tilde{N}_s(j)-1)!\tilde{N}_1(j)!\tilde{N}_2(j)!} \quad \text{if } \tilde{I}_s(j) = 1, \\ \dfrac{(\tilde{N}_s(j)+\tilde{N}_1(j)+\tilde{N}_2(j)-1)!}{\tilde{N}_s(j)!(\tilde{N}_1(j)-1)!\tilde{N}_2(j)!} + \dfrac{(\tilde{N}_s(j)+\tilde{N}_1(j)+\tilde{N}_2(j)-1)!}{(\tilde{N}_s(j)-1)!\tilde{N}_1(j)!\tilde{N}_2(j)!} I(\tilde{N}_s(j) \neq 0) \\ \quad \text{if } \tilde{I}_1(j) = 1, \\ \dfrac{(\tilde{N}_s(j)+\tilde{N}_1(j)+\tilde{N}_2(j)-1)!}{\tilde{N}_s(j)!\tilde{N}_1(j)!(\tilde{N}_2(j)-1)!} + \dfrac{(\tilde{N}_s(j)+\tilde{N}_1(j)+\tilde{N}_2(j)-1)!}{(\tilde{N}_s(j)-1)!\tilde{N}_1(j)!\tilde{N}_2(j)!} I(\tilde{N}_s(j) \neq 0) \\ \quad \text{if } \tilde{I}_2(j) = 1, \end{cases}$$

where $j = 1, \ldots, K(K+2)$ and where $I(\tilde{N}_s(j) \neq 0) = 1$ if $\tilde{N}_s(j) \neq 0$ and 0 otherwise. For any given K, the vectors $\tilde{C}$, $\tilde{N}$, $\tilde{I}$ are the same. Thus, they can be calculated in advance and stored in a multidimensional array.

In each MCMC iteration, for each respondent, when K, p_s, p_1, and p_2 are specified, a vector $\tilde{p} = \{p_s^{\tilde{N}_s} p_1^{\tilde{N}_1} p_2^{\tilde{N}_2}\}$ of length $K(K+2)$ is computed whose jth element is $\{p_s^{\tilde{N}_s(j)} p_1^{\tilde{N}_1(j)} p_2^{\tilde{N}_2(j)}\}$.

Then the expression in Theorem 2.1 can be written as

$$P(tie) = \tilde{C}'[\tilde{p} \bullet \tilde{I}_s], \tag{SM.1}$$

$$P(1) = \tilde{C}'[\tilde{p} \bullet \tilde{I}_1], \tag{SM.2}$$

$$P(2) = \tilde{C}'[\tilde{p} \bullet \tilde{I}_2], \tag{SM.3}$$

where the operator "•" is defined as the elementwise product of two vectors or matrices.

A similar technique can be used for set size three and larger. Pursuing the notion of precomputation, a set of matrices is created and stored which speed evaluation of the formulas presented in Appendix D. Details are available in Ruan (unpublished PhD dissertation, The Ohio State University, 2007).