# Objective Bayesian Comparison of Constrained Analysis of Variance Models

# Supplementary material

## Comparison of the methodology presented in the paper with the prior adjusted default Bayes factors of Mulder (2014)

To better evaluate the features of our method we analyzed Example 2 in Mulder (2014) (henceforth Mulder). This is a one-way ANOVA, and for illustrative purposes Mulder considered only 2 groups. Let $(\theta_1, \theta_2)$ be the two group means. The hypotheses under consideration are: $H_u : (\theta_1, \theta_2) \in$

$\mathbb{R}^2$ (unrestricted model); $H_1 : \theta_1 < \theta_2$ (constrained model); $H_0 : \theta_1 = \theta_2$ (null model, notice that Mulder uses $H_2$). Mulder argues that the standard Fractional Bayes factor (FBF) is inadequate in the context of constrained models, because it does not support $H_1$ strongly enough even when evidence is clearly in favor of $H_1$. To illustrate this point, Mulder considers several different datasets with $\bar{x}_1 = -\bar{x}_2$, where $\bar{x}_1 \in (-2, 1)$, $n_1 = n_2 = 20$ and $s_1^2 = s_2^2 = 20$. Next he considers the posterior probability of the three hypotheses $H_t$ under three different scenarios used to compute the Bayes factor: i) the FBF and ii) two Prior adjusted default BFs (Type I and Type II); see Mulder (2014, Fig. 3). While the behavior of the posterior probability of each of the $H_t$ is similar across the three scenarios when $(-\bar{x}_2) = \bar{x}_1 > x_0$ ($x_0$ being equal to a small negative value in the neighborhood of zero), the behaviors of the posterior probability of $H_u$ and $H_1$ are markedly different between the FBF on the one hand, and the two prior adjusted default BFs on the other hand, when $\bar{x}_1$ moves away from 0 towards negative values. With regard to FBF: when $\bar{x}_1 = -1$, the posterior probability of $H_1$ is somewhat higher than that of $H_u$; however when $\bar{x}_1 = -2$ the two probabilities are essentially equal: this shows that the standard FBF is unable to support in a strong way $H_1$ when it should. On the other hand for $\bar{x}_1 < -0.5$, the whole posterior probability mass under each of the two prior adjusted default BFs is concentrated on $H_u$ and $H_1$, with the latter accounting for a probability of about 2/3; this shows that Mulder's prior adjusted default BFs exhibit a better performance than the FBF.

We conducted the same analysis using our methodology. The results are reported in Figure 1. When $\bar{x}_1 < -0.5$ the posterior probability of $H_1$ is essentially 0.70, and that of $H_u$ about 0.3; thus our approach performs basically as Mulder's. On the other hand the posterior probabilities of the three hypotheses behave as expected for values of $\bar{x}_1$ in the neighborhood of zero or above a certain threshold. (Notice that no analytical expression is available for the BF in our approach, which requires a simulation step that may explain the somewhat irregular appearance of the curves).

## References

Mulder, J. (2014). Prior adjusted default Bayes factors for testing (in)equality constrained hypotheses. *Computational Statistics and Data Analysis* 71 448–463.

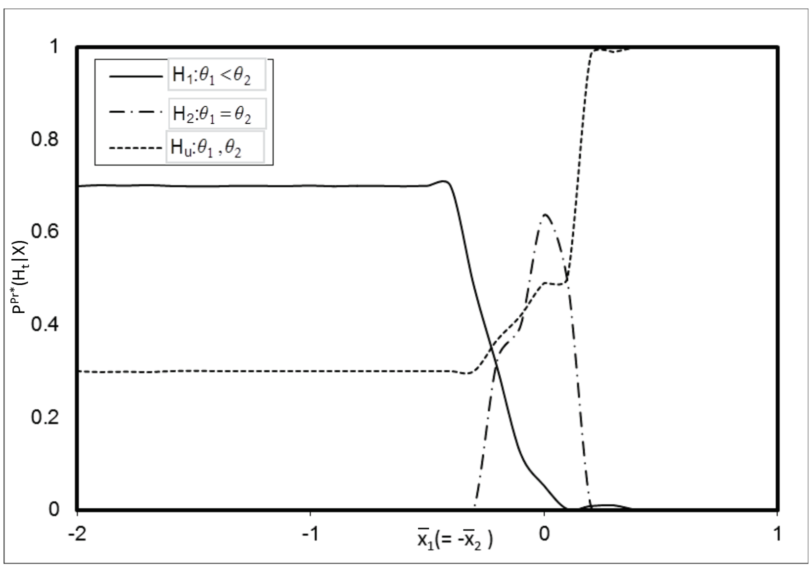


Figure 1: Posterior model probabilities of $H_u : (\theta_1, \theta_2) \in \mathbb{R}^2$; $H_1 : \theta_1 < \theta_2$; $H_0 : \theta_1 = \theta_2$