Supplemental Material S9: R scripts to conduct analysis and calculation of indices using three-level random intercept models

```
DATA<-read.csv("ExampleDATA.csv") #read "ExampleDATA.csv" file included in Supplemental Material
```

Y and X indicates responses about satisfaction with school and self-worth, respectively. And XC and XS indicate class- and school means of X.

```
library(nlme)
CLASSID<-factor(DATA[,2])
SCHOOLID<-factor(DATA[,3])
attach(DATA)
Result1<- lme(Y ~X+XC+XS,random = ~1 |SCHOOLID/CLASSID,data=DATA);summary(Result1)#fit the model (1)
Result2<- lme(Y ~X+XC,random = ~1 |SCHOOLID/CLASSID,data=DATA);summary(Result2)#fit the model (2)
Result3<- lme(Y ~X+XS,random = ~1 |SCHOOLID/CLASSID,data=DATA);summary(Result3)#fit the model (3)
Result4<- lme(Y ~XC+XS,random = ~1 |SCHOOLID/CLASSID,data=DATA);summary(Result4)#fit the model (4)
Result5<- lme(Y ~X+XC,random = ~1 |CLASSID,data=DATA);summary(Result5)#fit the model (5)
```

From analysis results and the relations (8)-(10) in the manuscript, the following table can be obtained. To calculate η_3^2 and η_2^2 from DATA, the following script can be used:

 # Calculation of η_3^2 defined as Equation (12)

```
SQ111<-sum( (XS-mean(DATA$X))^2)
SQ112<-sum( (X-XS)^2)
Eta3SQ<-SQ111/(SQ111+SQ112) #Eta3SQ corresponds to η3^2.
```

 # Calculation of η_2^2 defined as Equation (14)

```
SQ131<-sum((XC-XS)^2)
SQ132<-sum((X-XC)^2)
Eta2SQ<-SQ131/(SQ131+SQ132) #Eta2SQ corresponds to η2^2.
```

 # Calculation of η_2^2 defined as Equation (14) for two-level model (i.e., Equation (5)).

```
Eta22SQ<- sum( (XC-mean(DATA$X))^2)/( sum( (XC-mean(DATA$X))^2)+SQ132) #Eta22SQ corresponds to η2^2.
```

Table 1: Summary of estimation and calculation results from five kinds of multilevel models

Model	γ_0	γ_1 $(\gamma_1^{12},\gamma_1^{13})$	γ_2 $(\gamma_2^{12},\gamma_2^{23})$	γ_3 $(\gamma_3^{13},\gamma_3^{23})$	σ_1^2	σ_2^2	σ_3^2	Δ_2	Δ_3	ρ_1	ρ_2	η_3^2	η_2^2
Eq.1	0.787	0.243	0.262	0.574	4.576	0.404	0.000	0.117	0.257	0.081	0.000	0.010	0.031
Eq.2	3.208	0.243	0.402	-	4.576	0.408	0.000	0.180	-	0.082	0.000	0.010	0.031
Eq.3	0.772	0.246	-	0.836	4.576	0.408	0.000	-	0.374	0.082	0.000	0.010	0.031
Eq.4	0.783	-	0.506	0.574	4.818	0.395	0.000	0.222	0.251	0.076	0.000	0.010	0.031
Eq.5	3.208	0.243	0.402	-	4.576	0.408	-	0.180	-	0.082	-	-	0.041

Note: "-" denotes that the parameter or indice can not be either estimated or calculated from the corresponding model.

For example, an output of "summary(Result1)" in R for model1 (i.e., Equation (1)) shows:

```
Linear mixed-effects model fit by REML
 Data: DATA
       AIC      BIC    logLik
  10871.72 10912.38 -5428.859

Random effects:
 Formula: ~1 | SCHOOLID
         (Intercept)
StdDev: 0.0001807822

 Formula: ~1 | CLASSID %in% SCHOOLID
        (Intercept) Residual
StdDev:   0.6357088 2.139126

Fixed effects: Y ~ X + XC + XS
                Value Std.Error   DF   t-value p-value
(Intercept) 0.7870766 2.2325301 2379  0.352549  0.7245
X           0.2434088 0.0215947 2379 11.271706  0.0000
XC          0.2619692 0.2278690   63  1.149648  0.2546
XS          0.5738456 0.4606394   19  1.245759  0.2280
 Correlation:
   (Intr) X      XC
X   0.000
XC  0.006 -0.095
XS -0.870  0.000 -0.495

Standardized Within-Group Residuals:
       Min         Q1        Med         Q3        Max
-2.8355676 -0.6824663  0.1490082  0.7546435  2.2761466

Number of Observations: 2465
Number of Groups:
             SCHOOLID CLASSID %in% SCHOOLID
                   21                    85
```

Figure: An output of R by lme function for model1 (i.e., equation (1))

So, estimates of $\sigma_1^2, \sigma_2^2, \sigma_3^2$ can be evaluated as $2.139126^2 = 4.576, 0.357088^2 = 0.404, 0.0001807822^2 = 0.000$, respectively.

Δ_2 and Δ_3 can be calculated as $0.2619/\sqrt{0.000+0.404+4.576} = 0.117$ and $0.5738/\sqrt{0.000+0.404+4.576} = 0.257$, respectively (equation (8)).

And ρ_1 and ρ_2 can be calculated as

$$(0.000+0.404)/(0.000+0.404+4.576) = 0.081$$

and

$$(0.000)/(0.000+0.404+4.576) = 0.000,$$

respectively (equation (9)). Finally, η_3^2 and η_2^2 can be calculated using the above script as:

```
> Eta3SQ
[1] 0.01032805

> Eta2SQ
[1] 0.0313948
```

using the relation of Equation (10). Thus we obtain the same results of the first row in the above table.

Supplemental Material S10: R functions to estimate the required sample sizes

<statistical power>

```
#Equation (22) in the manuscript: For estimating I based on desired statistical power for γ2
Samplepowergamma2I<-function(J,K,eta2sq,eta3sq,rho1,rho2,D2,psi,alpha,varx){
I<-((1-rho1)*(qnorm(psi)+qnorm(1-alpha/2))^2)/((1-eta2sq)*(eta2sq*(1-eta3sq)*D2^2*J*K*varx-(qnorm(psi)+qnorm(1-alpha/2))^2*(rho1-rho2)))
return(I)
}
```

```
#Equation (23) in the manuscript: For estimating J based on desired statistical power for γ2
Samplepowergamma2J<-function(I,K,eta2sq,eta3sq,rho1,rho2,D2,psi,alpha,varx){
J<-(((1-eta2sq)*(rho1-rho2)*I+(1-rho1))*(qnorm(psi)+qnorm(1-alpha/2))^2)/((1-eta2sq)*eta2sq*(1-eta3sq)*D2^2*I*K*varx)
return(J)
}
```

```
#Equation (24) in the manuscript: For estimating K based on desired statistical power for γ2
Samplepowergamma2K<-function(I,J,eta2sq,eta3sq,rho1,rho2,D2,psi,alpha,varx){
K<-(((1-eta2sq)*(rho1-rho2)*I+(1-rho1))*(qnorm(psi)+qnorm(1-alpha/2))^2)/((1-eta2sq)*eta2sq*(1-eta3sq)*D2^2*I*J*varx)
return(K)
}
```

```
#Equation (25) in the manuscript: For estimating I based on desired statistical power for γ3
Samplepowergamma3I<-function(J,K,eta2sq,eta3sq,rho1,rho2,D3,psi,alpha,varx){
I<-((eta2sq+eta3sq-eta2sq*eta3sq)*(1-rho1)*(qnorm(psi)+qnorm(1-alpha/2))^2)/(eta2sq*(1-eta3sq)*J*(eta3sq*D3^2*K*varx-rho2*(qnorm(psi)+qnorm(1-alpha/2))^2)-(eta2sq+eta3sq-eta2sqeta3sq)*(rho1-rho2)*(qnorm(psi)+qnorm(1-alpha/2))^2)
return(I)
}
```

```
#Equation (26) in the manuscript: For estimating J based on desired statistical power for γ3
Samplepowergamma3J<-function(I,K,eta2sq,eta3sq,rho1,rho2,D3,psi,alpha,varx){
J<-((eta2sq+eta3sq-eta2sqeta3sq)*(I*(rho1-rho2)+(1-rho1))*(qnorm(psi)+qnorm(1-alpha/2))^2)/(eta2sq*(1-eta3sq)*I*(eta3sq*D3^2*K*varx-rho2*(qnorm(psi)+qnorm(1-alpha/2))^2))
return(J)
}
```

```
#Equation (27) in the manuscript: For estimating K based on desired statistical power for γ3
Samplepowergamma3K<-function(I,J,eta2sq,eta3sq,rho1,rho2,D3,psi,alpha,varx){
K<-((eta2sq*(1-eta3sq)*rho2*I*J+(eta2sq+eta3sq-eta2sq*eta3sq)*((rho1-rho2)*I+(1-rho1)))*(qnorm(psi)+qnorm(1-alpha/2))^2)/(eta2sq*(1-eta3sq)*eta3sq*D3^2*I*J*varx)
return(K)
}
```

As in the example of manuscript, if required sample size of K is needed to be estimated based on desired statistical power, we can use **Samplepowergamma2K** and **Samplepowergamma3K** functions. Under the settings of parameters/indices described in the example, in this case, each function returns:

```
Samplepowergamma2K(I=28.311,J=3.482,eta2sq=0.031,eta3sq=0.010,rho1=0.081,rho2=0,D2=0.117,psi=0.80,alpha=0.05,varx=4.1543)
[1] 147.8821
```

```
Samplepowergamma3K(I=28.311,J=3.482,eta2sq=0.031,eta3sq=0.010,rho1=0.081,rho2=0,D3=0.257,psi=0.80,alpha=0.05,varx=4.1543)
[1] 123.581
```

Thus K=148 and K=124 are required in testing γ_2 and γ_3 in the specified conditions, respectively.

#Supplemental Material S2: For estimating I based on desired statistical power for γ_2^{12}

```
Samplepowergamma212I<-function(J,K,eta2sq,eta3sq,rho1,rho2,D2,psi,alpha,varx){
I1<-((1-eta3sq)*rho2*J*((1-eta3sq)*rho2*J+2*(eta2sq*eta3sq-eta3sq+eta2sq)*(rho1-rho2))+(eta2sq*eta3sq-e
ta3sq-eta2sq)^2*(rho1-rho2)^2)*(qnorm(psi)+qnorm(1-alpha/2))^4
I2<-2*(1-eta2sq)*(1-eta3sq)*D2^2*J*K*varx*((1-eta3sq)*(eta2sq*eta3sq-eta3sq+eta2sq)
*rho2*J+(eta2sq+eta3sq-eta2sq*eta3sq)^2* (rho1-rho2))*(qnorm(psi)+qnorm(1-alpha/2))^2
I3<-(1-eta2sq)^2*(1-eta3sq)^2*(eta2sq+eta3sq-eta2sq*eta3sq)^2*D2^4*J^2*K^2*varx^4
I4<-(eta2sq*eta3sq-eta3sq-eta2sq+2)*(rho1-rho2)*(qnorm(psi)+qnorm(1-alpha/2))^2+(1-eta3sq)*J*(rho2*
(qnorm(psi)+qnorm(1-alpha/2))^2-(1-eta2sq)*(eta2sq+eta3sq-eta2sq*eta3sq)*D2^2*K*varx)
I<-((1-rho1)*(sqrt(I1+I2+I3)+I4))/(2*(1-eta2sq)*(1-eta3sq)*(-(rho1-rho2)*(rho2*(J-1)+rho1)*(qnorm(psi)+qno
rm(1-alpha/2))^2+D2^2*J*K*varx*(eta2sq*(1-eta3sq)*rho2*J+(eta2sq+eta3sq-eta2sq*eta3sq)*(rho1-rho2)))
)
return(I)
}
```

#Supplemental Material S2: For estimating J based on desired statistical power for γ_2^{12}

```
Samplepowergamma212J<-function(I,K,eta2sq,eta3sq,rho1,rho2,D2,psi,alpha,varx){
J1<-rho2^2*((1-eta2sq)*(rho1-rho2)*I+(1-rho1))^2*(qnorm(psi)+qnorm(1-alpha/2))^4
J2<-(-2*(1-eta2sq)*rho2*D2^2*((rho1-rho2)*I+(1-rho1))*((1-eta2sq)*(eta2sq*eta3sq+eta3sq-eta2sq)*(rho1-r
ho2)*I-(eta2sq*eta3sq-eta3sq+eta2sq)*(1-rho1)))*K*varx*(qnorm(psi)+qnorm(1-alpha/2))^2
J3<-(1-eta2sq)^2*(eta2sq+eta3sq-eta2sq*eta3sq)^2*D2^4*((rho1-rho2)*I+(1-rho1))^2*K^2*varx^4
J4<-rho2*((1-eta2sq)*(rho1-rho2)*I+(1-rho1))*(qnorm(psi)+qnorm(1-alpha/2))^2-(1-eta2sq)*(eta2sq+eta3sq-
eta2sq*eta3sq)*D2^2*((rho1-rho2)*I+(1-rho1))*K*varx
J<-(sqrt(J1+J2+J3)+J4)/(2*(1-eta2sq)*eta2sq*(1-eta3sq)*rho2*D2^2*I*K*varx)
return(J)
}
```

#Supplemental Material S2: For estimating K based on desired statistical power for γ_2^{12}

```
Samplepowergamma212K<-function(I,J,eta2sq,eta3sq,rho1,rho2,D2,psi,alpha,varx){
K<-(((1-eta3sq)*rho2*I*J*((1-eta2sq)*(rho1-rho2)*I+(1-rho1))+(I*(rho1-rho2)+(1-rho1))*((1-eta2sq)*
(1-eta3sq)*(rho1-rho2)*I+(1-rho1)))*(qnorm(psi)+qnorm(1-alpha/2))^2)/((1-eta2sq)*(1-eta3sq)*D2^2*I*J*var
x*(eta2sq*(1-eta3sq)*rho2*I*J+(eta2sq+eta3sq-eta2sq*eta3sq)*(I*(rho1-rho2)+(1-rho1))))
return(K)
}
```

#Supplemental Material S2: For estimating I based on desired statistical power for γ_3^{13}

```
Samplepowergamma313I<-function(J,K,eta2sq,eta3sq,rho1,rho2,D3,psi,alpha,varx){
I1<-((1-eta3sq)^2*rho2^2*J^2+(2*(1-eta3sq)*(eta2sq*eta3sq-eta3sq+eta2sq)*rho2*(rho1-rho2))*J+
(eta2sq+eta3sq-eta2sq*eta3sq)^2*(rho1-rho2)^2)*(qnorm(psi)+qnorm(1-alpha/2))^4
I2<-(-2*(1-eta3sq)^2*eta3sq*rho2*D3^2*J^2-2*(1-eta3sq)*eta3sq*(eta2sq*eta3sq-eta3sq+eta2sq)*(rho1-rho
2)*D3^2*J)*K*varx*(qnorm(psi)+qnorm(1-alpha/2))^2
I3<-(1-eta3sq)^2*eta3sq^4*D3^4*J^2*K^2*varx^4
I4<--((1-eta3sq)*rho2*J+(eta2sq*eta3sq-eta2sq-eta3sq+2)*(rho1-rho2))*(qnorm(psi)+qnorm(1-alpha/2))^2+(
1-eta3sq)*eta3sq*D3^2*J*K*varx
I<-((1-rho1)*(sqrt(I1+I2+I3)+I4))/(2*(1-eta2sq)*(1-eta3sq)*(rho1-rho2)*((rho2*(J-1)+rho1)*(qnorm(psi)+qnor
m(1-alpha/2))^2-eta3sq*D3^2*J*K*varx))
return(I)
}
```

#Supplemental Material S2: For estimating J based on desired statistical power for γ_3^{13}

```
Samplepowergamma313J<-function(I,K,eta2sq,eta3sq,rho1,rho2,D3,psi,alpha,varx){
J<-((I*(rho1-rho2)+(1-rho1))*((1-eta2sq)*(1-eta3sq)*(rho1-rho2)*I+(1-rho1))*(qnorm(psi)+qnorm(1-alpha/2))
^2)/((1-eta3sq)*I*((1-eta2sq)*(rho1-rho2)*I+(1-rho1))*(eta3sq*D3^2*K*varx-rho2*(qnorm(psi)+qnorm(1-alp
ha/2))^2))
return(J)
}
```

#Supplemental Material S2: For estimating K based on desired statistical power for γ_3^{13}

```
Samplepowergamma313K<-function(I,J,eta2sq,eta3sq,rho1,rho2,D3,psi,alpha,varx){
K<-(((1-eta3sq)*rho2*I*((1-eta2sq)*(rho1-rho2)*I+(1-rho1))*J+(rho1-rho2)*I*((1-eta2sq)*(1-eta3sq)*(rho1-rh
o2)*I-(eta2sq+eta3sq-eta2sq*eta3sq-2)*(1-rho1))+(1-rho1)^2)*(qnorm(psi)+qnorm(1-alpha/2))^2)/((1-eta3sq)
*eta3sq*D3^2*I*J*varx*((1-eta2sq)*(rho1-rho2)*I+(1-rho1)))
return(K)
}
```

If required sample size of K is needed to be estimated based on desired statistical power for testing γ_2^{12} and γ_3^{13}, we can use **Samplepowergamma212K** and **Samplepowergamma313K** functions. Under the settings of parameters/indices described in the example, in this case, each function returns:

```
Samplepowergamma212K(I=28.311,J=3.482,eta2sq=0.031,eta3sq=0.010,rho1=0.082,rho2=0,D2=0.180,psi=0.80,alpha=0.05,va
rx=4.1543)
[1] 47.66069
Samplepowergamma313K(I=28.311,J=3.482,eta2sq=0.031,eta3sq=0.010,rho1=0.082,rho2=0,D3=0.374,psi=0.80,alpha=0.05,va
rx=4.1543)
[1] 44.51746
```

Thus K=48 and K=45 are required in testing γ_2^{12} and γ_3^{13} in the specified conditions, respectively.

#Supplemental Material S2: For estimating I based on desired statistical power for γ_3^{23}

```
Samplepowergamma323I<-function(J,K,eta2sq,eta3sq,rho1,rho2,D3,psi,alpha,varx){
I<-((eta2sq+eta3sq-eta2sq*eta3sq)*(1-rho1)*(qnorm(psi)+qnorm(1-alpha/2))^2)/(eta2sq*(1-eta3sq)*eta3sq*D3^2*J*K*varx-(qnorm(psi)+qnorm(1-alpha/2))^2*(eta2sq*(1-eta3sq)*rho2*J+(eta2sq+eta3sq-eta2sq*eta3sq)* (rho1-rho2)))
return(I)
}
```

#Supplemental Material S2: For estimating J based on desired statistical power for γ_3^{23}

```
Samplepowergamma323J<-function(I,K,eta2sq,eta3sq,rho1,rho2,D3,psi,alpha,varx){
J<-((eta2sq+eta3sq-eta2sq*eta3sq)*(I*(rho1-rho2)+(1-rho1))*(qnorm(psi)+qnorm(1-alpha/2))^2)/(eta2sq*(1-eta3sq)*I*(eta3sq*D3^2*K*varx-rho2*(qnorm(psi)+qnorm(1-alpha/2))^2))
return(J)
}
```

#Supplemental Material S2: For estimating K based on desired statistical power for γ_3^{23}

```
Samplepowergamma323K<-function(I,J,eta2sq,eta3sq,rho1,rho2,D3,psi,alpha,varx){
K<-((eta2sq*(1-eta3sq)*rho2*I*J+(eta2sq+eta3sq-eta2sq*eta3sq)*(I*(rho1-rho2)+(1-rho1)))*(qnorm(psi)+qnorm(1-alpha/2))^2)/(eta2sq*(1-eta3sq)*eta3sq*D3^2*I*J*varx)
return(K)
}
```

#Supplemental Material S2: For estimating I based on desired statistical power for $\gamma_{2|2}$

```
Samplepowergamma22I<-function(J,eta2sq,rho1,D2,psi,alpha,varx){
I<-((1-rho1)*(qnorm(psi)+qnorm(1-alpha/2))^2)/((1-eta2sq)*(eta2sq*D2^2*J*varx-rho1*(qnorm(psi)+qnorm(1-alpha/2))^2))
return(I)
}
```

#Supplemental Material S2: For estimating J based on desired statistical power for $\gamma_{2|2}$

```
Samplepowergamma22J<-function(I,eta2sq,rho1,D2,psi,alpha,varx){
J<-((rho1*I* (1-eta2sq)+1-rho1) * (qnorm(psi)+qnorm(1-alpha/2))^2)/((1-eta2sq) *eta2sq*D2^2*I*varx)
return(J)
}
```

If required sample size of K is needed to be estimated based on desired statistical power for testing γ_3^{23}, we can use **Samplepowergamma323K** function. Under the settings of parameters/indices described in the example, in this case, the function returns:

```
Samplepowergamma323K(I=28.311,J=3.482,eta2sq=0.031,eta3sq=0.010,rho1=0.076,rho2=0,D3=0.251,psi=0.80,alpha=0.05,varx=4.1543)
[1] 124.0521
```

Thus K=125 in testing γ_3^{23} in the specified condition.

If required sample size of J is needed to be estimated based on desired statistical power for testing $\gamma_{2|2}$, we can use **Samplepowergamma22J** function. Under the settings of parameters/indices described in the example, in this case, the function returns:

```
Samplepowergamma22J (I=28.311,eta2sq=0.041,rho1=0.082,D2=0.180,psi=0.80,alpha=0.05,varx=4.1543)
[1] 164.7153
```

Thus J=165 in testing $\gamma_{2|2}$ in the specified condition.

<confidence interval>

```
#Supplemental Material S3: For estimating I based on desired confidence interval width for γ2
SampleCIgamma2I<-function(J,K,eta2sq,eta3sq,rho1,rho2,L,alpha,varx){
I<-(4*(1-rho1)*qnorm(1-alpha/2)^2)/((1-eta2sq)*(eta2sq*(1-eta3sq)*J*K*varx*L^2-2*qnorm(1-alpha/2)^2*(r
ho1-rho2)))
return(I)
}
```

```
#Supplemental Material S3: For estimating J based on desired confidence interval width for γ2
SampleCIgamma2J<-function(I,K,eta2sq,eta3sq,rho1,rho2,L,alpha,varx){
J<-(4*qnorm(1-alpha/2)^2*((1-eta2sq)*(rho1-rho2)*I+(1-rho1)))/((1-eta2sq)*eta2sq*(1-eta3sq)*I*K*varx
*L^2)
return(J)
}
```

```
#Supplemental Material S3: For estimating K based on desired confidence interval width for γ2
SampleCIgamma2K<-function(I,J,eta2sq,eta3sq,rho1,rho2,L,alpha,varx){
K<-(4*qnorm(1-alpha/2)^2*((1-eta2sq)*(rho1-rho2)*I+(1-rho1)))/((1-eta2sq)*eta2sq*(1-eta3sq)*I*J*varx
*L^2)
return(K)
}
```

```
#Supplemental Material S3: For estimating I based on desired confidence interval width for γ3
SampleCIgamma3I<-function(J,K,eta2sq,eta3sq,rho1,rho2,L,alpha,varx){
I<-(4*(eta2sq+eta3sq-eta2sq*eta3sq)*(1-rho1)*qnorm(1-alpha/2)^2)/(
eta2sq*(1-eta3sq)*(-eta3sq*J*K*varx*L^2-2*qnorm(1-alpha/2)^2*(rho2(J-1)+rho1))-2*eta3sq*(rho1-rho2)*q
norm(1-alpha/2)^2)
return(I)
}
```

```
#Supplemental Material S3: For estimating J based on desired confidence interval width for γ3
SampleCIgamma3J<-function(I,K,eta2sq,eta3sq,rho1,rho2,L,alpha,varx) {
J<-(4*(eta2sq+eta3sq-eta2sq*eta3sq)*qnorm(1-alpha/2)^2*(I*(rho1-rho2)+(1-rho1)))/(eta2sq*(1-eta3sq)*I*(e
ta3sq*K*varx *L^2-2*rho2*qnorm(1-alpha/2)^2))
return(J)
}
```

```
#Supplemental Material S3: For estimating K based on desired confidence interval width for γ3
SampleCIgamma3K<-function(I,J,eta2sq,eta3sq,rho1,rho2,L,alpha,varx){
K<-(4*qnorm(1-alpha/2)^2*(eta2sq*(1-eta3sq)*rho2*I*J+(eta2sq+eta3sq-eta2sq*eta3sq)*(I*(rho1-rho2)+(1-r
ho1))))/(eta2sq*(1-eta3sq)*eta3sq*I*J*varx *L^2)
return(K)
}
```

If required sample size of K is needed to be estimated based on desired confidence interval width L=0.40, we can use **SampleCIgamma2K** and **SampleCIgamma3K** functions. Under the settings of parameters/indices described in the example, in this case, each function returns:

```
SampleCIgamma2K(I=28.311,J=3.482,eta2sq=0.031,eta3sq=0.010,rho1=0.081,rho2=0,L=0.40,alpha=0.05,varx=4.
1543)
[1] 24.76941
```

```
SampleCIgamma3K(I=28.311,J=3.482,eta2sq=0.031,eta3sq=0.010,rho1=0.081,rho2=0,L=0.40,alpha=0.05,varx=4.
1543)
[1] 99.87264
```

Thus K=25 and K=100 are required in estimating γ_2 and γ_3 in the specified conditions, respectively.

#Supplemental Material S3: For estimating I based on desired statistical power for γ_2^{12}

```
SampleCIgamma212I<-function(J,K,eta2sq,eta3sq,rho1,rho2,L,alpha,varx){
I1<-(1-eta2sq)*(1-eta3sq)*J*K*varx*L^2*((1-eta2sq)*(1-eta3sq)*(eta2sq*eta3sq-eta3sq-eta2sq)^2*J*K*varx
*L^2+8*qnorm(1-alpha/2)^2*((1-eta3sq)*(eta2sq*eta3sq-eta3sq+eta2sq)*rho2*J+(eta2sq*eta3sq-eta3sq-eta
2sq)^2*(rho1-rho2)))
I2<-16*qnorm(1-alpha/2)^4*((1-eta3sq)*rho2*J*((1-eta3sq)*rho2*J+2*(eta2sq*eta3sq-eta3sq+eta2sq)*(rho1
-rho2))+(eta2sq+eta3sq-eta2sq*eta3sq)^2*(rho1-rho2)^2)
I3<--(1-eta2sq)*(1-eta3sq)*(eta2sq*eta3sq-eta3sq-eta2sq)*J*K*varx*L^2-4*qnorm(1-alpha/2)^2*((1-eta3sq)
*rho2*J+(eta2sq*eta3sq-eta3sq-eta2sq+2)*(rho1-rho2))
I<-((1-rho1)*(sqrt(I1+I2)+I3))/(2*(1-eta2sq)*(1-eta3sq)*(-J*(eta2sq*(1-eta3sq)*rho2*J-(eta2sq*eta3sq-eta3sq
-eta2sq)*(rho1-rho2))*K*varx*L^2+4*(rho1-rho2)*qnorm(1-alpha/2)^2*(rho2*J-rho2+rho1)))
return(I)
}
```

#Supplemental Material S3: For estimating J based on desired statistical power for γ_2^{12}

```
SampleCIgamma212J<-function(I,K,eta2sq,eta3sq,rho1,rho2,L,alpha,varx){
J1<-(1-eta2sq)^2*(eta2sq+eta3sq-eta2sq*eta3sq)^2*(I*(rho1-rho2)+(1-rho1))^2*K^2*varx^4L^4
J2<--8*(1-eta2sq)*rho2*qnorm(1-alpha/2)^2*(I*(rho1-rho2)+(1-rho1))*((1-eta2sq)*(eta2sq*eta3sq+eta3sq-et
a2sq)*(rho1-rho2)*I-(eta2sq*eta3sq-eta3sq+eta2sq)*(1-rho1))*K*varx*L^2
J3<-16*rho2^2*qnorm(1-alpha/2)^4*((1-eta2sq)*(rho1-rho2)*I+(1-rho1))^2
J4<-(1-eta2sq)*(eta2sq+eta3sq-eta2sq*eta3sq)*(I*(rho1-rho2)+(1-rho1))*K*varx*L^2-4*rho2*qnorm(1-alph
a/2)^2*((1-eta2sq)*(rho1-rho2)*I+(1-rho1))
J<--(sqrt(J1+J2+J3)+J4)/(2*(1-eta2sq)*eta2sq*(1-eta3sq)*rho2*I*K*varx*L^2)
return(J)
}
```

#Supplemental Material S3: For estimating K based on desired statistical power for γ_2^{12}

```
SampleCIgamma212K<-function(I,J,eta2sq,eta3sq,rho1,rho2,L,alpha,varx){
K<-(4*qnorm(1-alpha/2)^2*((1-eta3sq)*rho2*I*((1-eta2sq)*(rho1-rho2)*I+(1-rho1))*J+(rho1-rho2)*I*((1-eta2
sq)*(1-eta3sq)*(rho1-rho2)*I+(eta2sq*eta3sq-eta3sq-eta2sq+2)*(1-rho1))+(1-rho1)^2))/((1-eta2sq)*(1-eta3sq)
*I*J*varx*(eta2sq*(1-eta3sq)*rho2*I*J+(eta2sq+eta3sq-eta2sq*eta3sq)*(I*(rho1-rho2)+(1-rho1)))*L^2)
return(K)
}
```

#Supplemental Material S3: For estimating I based on desired statistical power for γ_3^{13}

```
SampleCIgamma313I<-function(J,K,eta2sq,eta3sq,rho1,rho2,L,alpha,varx){
I1<-(1-eta3sq)*J*(eta3sq*K*varx*L^2-4*rho2*qnorm(1-alpha/2)^2)*((1-eta3sq)*J*(eta3sq*K*varx*L^2-4*r
ho2*qnorm(1-alpha/2)^2)-8*(eta2sq*eta3sq-eta3sq+eta2sq)*(rho1-rho2)*qnorm(1-alpha/2)^2)
I2<-16*(eta2sq+eta3sq-eta2sq*eta3sq)^2*(rho1-rho2)^2*qnorm(1-alpha/2)^4
I3<-(1-eta3sq)*J*(4*rho2*qnorm(1-alpha/2)^2-eta3sq*K*varx*L^2)+4*(eta2sq*eta3sq-eta3sq-eta2sq+2)*(rh
o1-rho2)*qnorm(1-alpha/2)^2
I<-((1-rho1)*(sqrt(I1+I2)+I3))/(2*(1-eta2sq)*(1-eta3sq)*(rho1-rho2)*(eta3sq*J*K*varx*L^2-4*qnorm(1-alpha
/2)^2*(rho2*(J-1)+rho1)))
return(I)
}
```

#Supplemental Material S3: For estimating J based on desired statistical power for γ_3^{13}

```
SampleCIgamma313J<-function(I,K,eta2sq,eta3sq,rho1,rho2,L,alpha,varx){
J<-(4*qnorm(1-alpha/2)^2*(I*(rho1-rho2)+(1-rho1))*((1-eta2sq)*(1-eta3sq)*(rho1-rho2)*I+(1-rho1)))/(
(1-eta3sq)*I*((1-eta2sq)*(rho1-rho2)*I+(1-rho1))*(eta3sq*K*varx*L^2-4*rho2*qnorm(1-alpha/2)^2))
return(J)
}
```

#Supplemental Material S3: For estimating K based on desired statistical power for γ_3^{13}

```
SampleCIgamma313K<-function(I,J,eta2sq,eta3sq,rho1,rho2,L,alpha,varx){
K<-(4*qnorm(1-alpha/2)^2*((1-eta3sq)*rho2*I*J*((1-eta2sq)*(rho1-rho2)*I+(1-rho1))+(I*(rho1-rho2)+(1-rho
1))*((1-eta2sq)*(1-eta3sq)*(rho1-rho2)*I+(1-rho1))))/((1-eta3sq)*eta3sq*I*J*varx*L^2*((1-eta2sq)*(rho1-rho
2)*I+(1-rho1)))
return(K)
}
```

If required sample size of K is needed to be estimated based on desired confidence interval width L=0.40, we can use **SampleCIgamma212K** and **SampleCIgamma313K** functions. Under the settings of parameters/indices described in the example, in this case, each function returns:

```
SampleCIgamma212K(I=28.311,J=3.482,eta2sq=0.031,eta3sq=0.010,rho1=0.082,rho2=0,L=0.40,alpha=0.05,varx
=4.1543)
[1] 18.89443
SampleCIgamma313K(I=28.311,J=3.482,eta2sq=0.031,eta3sq=0.010,rho1=0.082,rho2=0,L=0.40,alpha=0.05,varx
=4.1543)
[1] 76.19073
```

Thus K=19 and K=77 are required in estimating γ_2^{12} and γ_3^{13} in the specified conditions, respectively.

#Supplemental Material S3: For estimating I based on desired statistical power for γ_3^{23}

SampleCIgamma323I<-function(J,K,eta2sq,eta3sq,rho1,rho2,L,alpha,varx){
I<-(4*(eta2sq+eta3sq-eta2sq*eta3sq)*(1-rho1)*qnorm(1-alpha/2)^2)/(eta2sq*(1-eta3sq)*J*(eta3sq*K*varx*L^2-4*rho2*qnorm(1-alpha/2)^2)-4*(eta2sq+eta3sq-eta2sq*eta3sq)*(rho1-rho2)*qnorm(1-alpha/2)^2)
return(I)
}

#Supplemental Material S3: For estimating J based on desired statistical power for γ_3^{23}

SampleCIgamma323J<-function(I,K,eta2sq,eta3sq,rho1,rho2,L,alpha,varx){
J<-(4*(eta2sq+eta3sq-eta2sq*eta3sq)*qnorm(1-alpha/2)^2*(I*(rho1-rho2)+(1-rho1)))/(eta2sq*(1-eta3sq)*I*(eta3sq*K*varx*L^2-4*rho2*qnorm(1-alpha/2)^2))
return(J)
}

#Supplemental Material S3: For estimating K based on desired statistical power for γ_3^{23}

SampleCIgamma323K<-function(I,J,eta2sq,eta3sq,rho1,rho2,L,alpha,varx){
K<-(4*qnorm(1-alpha/2)^2*(eta2sq*(1-eta3sq)*rho2*I*J+(eta2sq+eta3sq-eta2sq*eta3sq)*(I*(rho1-rho2)+(1-rho1))))/(eta2sq*(1-eta3sq)*eta3sq*I*J*varx*L^2)
return(K)
}

#Supplemental Material S3: For estimating I based on desired statistical power for $\gamma_{2|2}$

SampleCIgamma22I<-function(J,eta2sq,rho1,L,alpha,varx){
I<-(4*(1-rho1)*qnorm(1-alpha/2)^2)/((1-eta2sq)*(J*varx*L^2*eta2sq-4*rho1*qnorm(1-alpha/2)^2))
return(I)
}

#Supplemental Material S3: For estimating J based on desired statistical power for $\gamma_{2|2}$

SampleCIgamma22J<-function(I,eta2sq,rho1,L,alpha,varx){
J<-(4*qnorm(1-alpha/2)^2*(rho1*I*(1-eta2sq)+(1-rho1)))/((1-eta2sq)*eta2sq*I*varx*L^2)
return(J)
}

If required sample size of K is needed to be estimated based on desired confidence interval width L=0.40 for estimating γ_3^{23}, we can use **SampleCIgamma323K** function. Under the settings of parameters/indices described in the example, in this case, the function returns:

SampleCIgamma323K(I=28.311,J=3.482,eta2sq=0.031,eta3sq=0.010,rho1=0.076,rho2=0,L=0.40,alpha=0.05,varx=4.1543)
[1] 95.6269

Thus K=96 in estimating γ_3^{23} in the specified condition.

If required sample size of J is needed to be estimated based on desired confidence interval width L=0.40 for estimating $\gamma_{2|2}$, we can use **SampleCIgamma22J** function. Under the settings of parameters/indices described in the example, in this case, the function returns:

SampleCIgamma22J (I=28.311,eta2sq=0.041,rho1=0.082,L=0.40,alpha=0.05,varx=4.1543)
[1] 65.29914

Thus J=66 in estimating $\gamma_{2|2}$ in the specified condition.