S6: Comparing error variances of contextual effects estimates among models

From the derived standard errors of contextual effects estimates provided in Supplemental Materials S1, the following relations are obtained:

$$se^2(\hat{\gamma}_2) - se^2(\hat{\gamma}_2^{12}) = \frac{\eta_3^2[I(\rho_1-\rho_2)+(1-\rho_1)]^2}{\eta_2^2(1-\eta_3^2)IJK\sigma_x^2[\eta_2^2(1-\eta_3^2)\rho_2 IJ + (\eta_2^2+\eta_3^2-\eta_2^2\eta_3^2)[I(\rho_1-\rho_2)+(1-\rho_1)]]} \geq 0,$$

$$se^2(\hat{\gamma}_3) - se^2(\hat{\gamma}_3^{13}) = \frac{(1-\eta_2^2)[I(\rho_1-\rho_2)+(1-\rho_1)]^2}{\eta_2^2(1-\eta_3^2)IJK\sigma_x^2[(1-\eta_2^2)(\rho_1-\rho_2)I+(1-\rho_1)]} \geq 0,$$

$$se^2(\hat{\gamma}_3) - se^2(\hat{\gamma}_3^{23}) = 0.$$

So, the error variances of contextual effect estimates based on the model (1) in manuscript are always equal or larger than those for models shown in (2)–(4). These results imply that statistical power becomes lower and confidence interval becomes wider due to larger error variances when the model (1) is used instead of one of the model in (2)–(4). So, in three-level data, if a researcher can specify the specific context effect of interest (i.e., between level-1 and level-2, between level-1 and level-3, or between level-2 and level-3), then using a model from (2)–(4) will be statistically more effective than using (1), condition that true omitted regression coefficient is zero.

Additionally, from the results obtained in Supplemental Materials S4, the following results can also be derived:

$$se^2(\hat{\gamma}_2^{12}) - se^2(\hat{\gamma}_2^{2}) = \frac{1-\rho_1}{(1-\eta_2^2)(1-\eta_3^2)IJK\sigma_x^2} \geq 0,$$

$$se^2(\hat{\gamma}_3^{13}) - se^2(\hat{\gamma}_3^{3}) = \frac{(1-\rho_1)[I(\rho_1-\rho_2)+(1-\rho_1)]}{(1-\eta_3^2)IJK\sigma_x^2[(1-\eta_2^2)(\rho_1-\rho_2)I+(1-\rho_1)]} \geq 0,$$

$$se^2(\hat{\gamma}_3^{23}) - se^2(\hat{\gamma}_3^{3}) = \frac{I(\rho_1-\rho_2)+(1-\rho_1)}{\eta_2^2(1-\eta_3^2)IJK\sigma_x^2} \geq 0.$$

Thus, from the above results, the relations

$$se^2(\hat{\gamma}_2) \geq se^2(\hat{\gamma}_2^{12}) \geq se^2(\hat{\gamma}_2^{2}),$$

$$se^2(\hat{\gamma}_3) \geq se^2(\hat{\gamma}_3^{13}) \geq se^2(\hat{\gamma}_3^{3}),$$

$$se^2(\hat{\gamma}_3) = se^2(\hat{\gamma}_3^{23}) \geq se^2(\hat{\gamma}_3^{3}),$$

can be derived. So, in a model that includes a higher number of predictors, error variances of the corresponding regression coefficient estimates will be larger, condition that true omitted regression coefficient is zero.