1. Supplemental Text to the Manuscript "A Multimethod Latent State-Trait Model for Structurally Different And Interchangeable Methods"

Here we provide the formal definition of the LST-COM model and discuss important implications of the definition. First, we define the LST-COM model based on stochastic measurement theory (Steyer & Eid, 2001) and classical LST theory (Steyer, Schmitt, & Eid, 1999; Steyer, Mayer, Geiser, & Cole, 2015). Second, we discuss important implications of the LST-COM model definition with a special focus on non-permissible covariances and correlations. Third, we provide a description of the implied covariance structure of the LST-COM model using matrix algebra and discuss how the model parameters can be identified. Finally, we provide an annotated Mplus syntax used in the MC simulation study.

1.1. Formal Definition of the LST-COM Model

The LST-COM model will be defined for a longitudinal multimethod (or multirater) design including multiple indicators $i \in I = \{1, ..., c\}$, multiple constructs $j \in J = \{1, ..., d\}$, multiple methods $k \in K = \{1, ..., e\}$, and multiple occasions of measurement $l \in L = \{1, ..., f\}$. The LST-COM model will be defined for one set of interchangeable methods, but an arbitrary number of structurally different methods. It is assumed that multiple interchangeable raters $r \in R = \{1, ..., a\}$ rate a target person $t \in T = \{1, ..., b\}$ on multiple measurement occasion $l \in L = \{1, ..., f\}$. In addition, the self-ratings of the target person as well as ratings from other structurally different methods (e.g., parent ratings, physiological measures) are collected on each measurement occasion. In the remainder, we denote the set of interchangeable methods by $k = 2$. Any additional structurally different methods will be represented by $k > 2$. Without any loss of generality, the first method will serve as the reference methods ($k = 1$).

1.2. Random Experiment and Probability Space

The starting point for defining the LST-COM model is a probability space $(\Omega, \mathscr{A}, \mathscr{P})$ with the following sets:
$\Omega = \Omega_{\mathcal{T}} \times \Omega_{\mathcal{TS}_1} \times \ldots \times \Omega_{\mathcal{TS}_l} \times \ldots \times \Omega_{\mathcal{TS}_f} \times \Omega_{\mathcal{R}} \times \Omega_{\mathcal{R}_2\mathcal{S}_1} \times \ldots \times \Omega_{\mathcal{R}_k\mathcal{S}_l} \times \ldots \times \Omega_{\mathcal{R}_e\mathcal{S}_f} \times \Omega_{\mathcal{O}}$,
where $\Omega_{\mathcal{T}}$ is the set of possible targets, $\Omega_{\mathcal{TS}_l}$ is the set of possible target situations, $\Omega_{\mathcal{R}}$ is the set of possible interchangeable methods, $\Omega_{\mathcal{R}_k\mathcal{S}_l}$ is the set of possible situations referring to method k assessed at time l, and $\Omega_{\mathcal{O}}$ is the set of possible outcomes. Note that the set of possible rater situations contains

the index k, implying that rater situations may differ across methods k, where $k \geq 2$. Note that there is no set of possible raters that refers to a structurally different methods ($k > 2$), as these additional structurally different methods are fixed for the particular target person. For a minimal design including one structurally different method (k=1), and one set of interchangeable methods (k=2), $\Omega_{\mathcal{R}_k \mathcal{S}_l}$ can be replaced by $\Omega_{\mathcal{R}\mathcal{S}_l}$, which will lead to the random experiment described in the article. Furthermore, $\Omega_{\mathcal{O}}$ is itself a set of products representing the set of possible observations with respect to indicator i, construct j, method k, and occasion of measurement l, hence $\Omega_{\mathcal{O}}$ can also be replaced by $\Omega_{\mathcal{O}_{1111}} \times ... \times \Omega_{\mathcal{O}_{ijkl}} \times ... \times \Omega_{\mathcal{O}_{cdef}}$.

1.3. Mappings

Next, we consider four projections in order to define random variables on the probability space. We define the projection $p_{\mathcal{T}} : \Omega \to \Omega_{\mathcal{T}}$ as the mapping of possible outcomes to the set of the possible targets, $p_{\mathcal{T}\mathcal{S}_l} : \Omega \to \Omega_{\mathcal{T}\mathcal{S}_l}$ as the mapping of possible outcomes to the set of possible target situations, the projection $p_{\mathcal{R}} : \Omega \to \Omega_{\mathcal{R}}$ as the mapping of possible outcomes to the set of the possible interchangeable raters, and $p_{\mathcal{R}_k \mathcal{S}_l} : \Omega \to \Omega_{\mathcal{R}_k \mathcal{S}_l}$ as the mapping of possible outcomes to the set of possible rater situations of method k, where k >1. The observed variables Y_{rtij2l} and Y_{tijkl} are random variables on the probability space $(\Omega, \mathscr{A}, \mathscr{P})$ with finite first and second order moments. The observed variables belonging to the reference method Y_{tij1l} (here: target self-reports, Level-2 observed variables) are defined as $Y_{tij1l} : \Omega_{\mathcal{T}} \times \Omega_{\mathcal{T}\mathcal{S}_l} \times \Omega_{\mathcal{O}} \to \mathbb{R}$. The observed variables belonging to the set of interchangeable methods Y_{rtij2l} (Level-1 observed variables) are defined as $Y_{rtij2l} : \Omega_{\mathcal{T}} \times \Omega_{\mathcal{T}\mathcal{S}_l} \times \Omega_{\mathcal{R}} \times \Omega_{\mathcal{R}_2 \mathcal{S}_l} \times \Omega_{\mathcal{O}} \to \mathbb{R}$, and the observed variables pertaining to any additional structurally different method Y_{tijkl} (where k >2) are defined as $Y_{tijkl} : \Omega_{\mathcal{T}} \times \Omega_{\mathcal{T}\mathcal{S}_l} \times \Omega_{\mathcal{R}_k \mathcal{S}_l} \times \Omega_{\mathcal{O}} \to \mathbb{R}$. Note that the values of Y_{rtij2l} are measured at Level-1 (rater-specific level), whereas the values of Y_{tijkl} (e.g., target self-reports, where $k \neq 2$) are measured at Level-2 (target-specific level).

1.4. Formal Definition of the Latent Variables

Without any loss of generality, the first method ($k = 1$) is selected as reference (comparison) method. The second method ($k = 2$) refers to the set of interchangeable methods, which serve as non-reference methods. All other methods ($k > 2$) refer to structurally different methods, which also serve as non-reference methods. Then, the following latent variables are defined as random variables on

$(\Omega, \mathscr{A}, \mathscr{P})$ with finite first- and second-order moments:

Level-1 variables:

$$S_{rtij2l} := \mathbb{E}(Y_{rtij2l} | p_{\mathcal{T}}, p_{\mathcal{TS}_l}, p_{\mathcal{R}}, p_{\mathcal{R}_2\mathcal{S}_l}), \tag{1}$$

$$UM_{rtij2l} := S_{rtij2l} - \mathbb{E}(S_{rtij2l} | p_{\mathcal{T}}, p_{\mathcal{TS}_l}), \tag{2}$$

$$T^{UM}_{rtij2l} := \mathbb{E}(UM_{rtij2l} | p_{\mathcal{T}}, p_{\mathcal{R}}) \tag{3}$$

$$O^{UM}_{rtij2l} := UM_{rtij2l} - T^{UM}_{rtij2l}, \tag{4}$$

$$\epsilon_{rtij2l} := Y_{rtij2l} - \mathbb{E}(Y_{rtij2l} | p_{\mathcal{T}}, p_{\mathcal{TS}_l}, p_{\mathcal{R}}, p_{\mathcal{R}_2\mathcal{S}_l}). \tag{5}$$

Level-2 variables:

$$S_{tij1l} := \mathbb{E}(Y_{tij1l} | p_{\mathcal{T}}, p_{\mathcal{TS}_l}), \tag{6}$$

$$S_{tij2l} := \mathbb{E}(S_{rtij2l} | p_{\mathcal{T}}, p_{\mathcal{TS}_l}), \tag{7}$$

$$S_{tijkl} := \mathbb{E}(Y_{tijkl} | p_{\mathcal{T}}, p_{\mathcal{TS}_l}, p_{\mathcal{R}_k\mathcal{S}_l}), \qquad \forall\ k > 2, \tag{8}$$

$$T_{tijkl} := \mathbb{E}(S_{tijkl} | p_{\mathcal{T}}), \tag{9}$$

$$O_{tijkl} := S_{tijkl} - T_{tijkl}, \tag{10}$$

$$T^{CM}_{tij2l} := T_{tij2l} - \mathbb{E}(T_{tij2l} | T_{tij1l}), \tag{11}$$

$$O^{CM}_{tij2l} := O_{tij2l} - \mathbb{E}(O_{tij2l} | O_{tij1l}), \tag{12}$$

$$T^{M}_{tijkl} := T_{tijkl} - \mathbb{E}(T_{tijkl} | T_{tij1l}), \qquad \forall\ k > 2, \tag{13}$$

$$O^{M}_{tijkl} := O_{tijkl} - \mathbb{E}(O_{tijkl} | O_{tij1l}), \qquad \forall\ k > 2, \tag{14}$$

$$\epsilon_{tij1l} := Y_{tij1l} - \mathbb{E}(Y_{tij1l} | p_{\mathcal{T}}, p_{\mathcal{TS}_l}), \tag{15}$$

$$\epsilon_{tijkl} := Y_{tijkl} - \mathbb{E}(Y_{tijkl} | p_{\mathcal{T}}, p_{\mathcal{TS}_l}, p_{\mathcal{R}_k\mathcal{S}_l}), \qquad \forall\ k > 2. \tag{16}$$

Remark 1. First, the Level-1 latent variables are defined. Note that the Level-1 latent variables belong to the set of interchangeable methods (k=2). The latent state variables S_{rtij2l} are defined as conditional expectations of the Level-1 observed variables Y_{rtij2l} given the target variable $p_{\mathcal{T}}$, the target-situation variable $p_{\mathcal{TS}_l}$, the rater-variable $p_{\mathcal{R}}$, and the rater-situation variable $p_{\mathcal{R}_2\mathcal{S}_l}$ (see Eq. 1). Then, the measurement error variable (at Level-1) is defined as the difference between the observed variable and the latent state variable (see Eq. 5). The unique method variable UM_{rtij2l} is defined as the difference between the Level-1 latent state variable S_{rtij2l} and the Level-2 latent state variable S_{tij2l} (i.e., conditional expectation of the S_{rtij2l}-variable given the target $p_{\mathcal{T}}$ and the target-situation variable $p_{\mathcal{TS}_l}$). That is, the unique method variable UM_{rtij2l} is defined as a latent residual with respect to the target $p_{\mathcal{T}}$ and the target-situation variable $p_{\mathcal{TS}_l}$, and thus has a mean of zero. Next, it is also possible to define latent trait T^{UM}_{rtij2l} as well as occasion-specific O^{UM}_{rtij2l} unique method variables. The latent trait unique method variables T^{UM}_{rtij2l} are defined as conditional expectations of the

UM_{rtij2l}-variables given the target $p_{\mathcal{T}}$ and the rater $p_{\mathcal{R}}$ variable. Thus, the T^{UM}_{rtij2l} variables are free of occasion-specific effects and capture stable (time-invariant) target, rater, and target-rater interaction effects. The occasion-specific unique method variables O^{UM}_{rtij2l} are defined as the difference between the UM_{rtij2l}-variables and the latent trait unique method T^{UM}_{rtij2l} variables. The O^{UM}_{rtij2l}-variables capture momentary (occasion-specific) target, rater, and target-rater interaction effects.

Second, the Level-2 latent variables are defined. Again, the latent state variables (S_{tij1l}, S_{tij2l}, S_{tijkl}) are defined as conditional expectations (see Eq. 6 to 8). The measurement error variables are defined as differences between the observed variables and the corresponding latent state variables. Note that there are no measurement error variables at Level-2 for the set of interchangeable methods (k=2), as measurement error influences are already captured at Level-1 with respect to this method. Next, the latent trait and occasion-specific variables are defined. The latent trait variables T_{tij1l} are defined as conditional expectations of the latent state S_{tij1l} variables given the target variable $p_{\mathcal{T}}$. The latent occasion-specific variables O_{tij1l} (called: state-residuals) are defined as differences between the latent state S_{tij1l} and the latent trait T_{tij1l} variables (see Equation 10). The latent occasion-specific variables O_{tij1l} are defined as residuals with respect to the latent trait variables pertaining to the same indicator i, construct j, method k and occasion of measurement l. Hence, the latent trait and latent occasion-specific variables are uncorrelated.

Third, the latent method variables are defined at Level-2. The latent trait (common) method variables (T^{CM}_{tij2l} and T^{M}_{tijkl}) are defined as residuals with respect to the latent regression of the non-reference trait variables on the reference trait variables (see Equations 11 and 13). Therefore, these latent variables reflect the consistent bias of the other ratings which is not shared with the consistent view of the target's self-perception (see Equations 11 and 13). The latent occasion-specific method variables are also defined as latent residuals with respect to the latent regression of the occasion-specific variables pertaining to the non-reference method on the occasion-specific variables measured by the reference method (see Equations 12 and 14). These latent variables represent the occasion-specific (momentary) method bias which is not shared with the occasion-specific (momentary) view of the target (reference method).

1.5. Linearity Assumptions & Definition of Latent Factors

In order to define unidimensional latent factors, the following linearity assumptions must be made in the LST-COM model. Typically, assumptions like those below are made implicitly in the context of structural equation modeling.

(a) With respect to the same indicator i, same construct j, and same occasion of measurement l, it is assumed that the regression of the trait variable belonging to a non-reference method k on the latent trait variable belonging to the reference method ($k = 1$) is linear. For each indicator i, construct j, measured by a non-reference method k on occasion of measurement l there is a constant $\alpha_{Tijkl} \in \mathbb{R}$ as well as a constant $\lambda_{Tijkl} \in \mathbb{R}$, such that

$$\mathbb{E}(T_{tijkl}|T_{tij1l}) = \alpha_{Tijkl} + \lambda_{Tijkl} T_{tij1l}. \tag{17}$$

(b) Definition of common *trait* variables. For each indicator i, construct j, measured by a reference method k $(k = 1)$ and for each pair $(l, l') \in L \times L'$, $(l \neq l')$ there is a constant $\alpha_{Tij1ll'}$ as well as a constant $\lambda_{Tij1ll'}$, such that

$$T_{tij1l} = \alpha_{Tij1ll'} + \lambda_{Tij1ll'} T_{tij1l'}. \tag{18}$$

(c) For each indicator i, construct j, measured by a non-reference method k $(k \neq 1)$ on occasion of measurement l there is a constant $\lambda_{Oijkl} \in \mathbb{R}$, such that

$$\mathbb{E}(O_{tijkl}|O_{tij1l}) = \lambda_{Oijkl} O_{tij1l}. \tag{19}$$

(d) Definition of common *method trait* variables. For each indicator i, construct j, measured by a non-reference method k $(k \neq 1)$ and for each pair $(l, l') \in L \times L'$, $(l \neq l')$ there are constants $\lambda^{CM}_{Tij2ll'}$, $\lambda^{UM}_{Tij2ll'}$, as well as $\lambda^{M}_{Tijkll'} \in \mathbb{R}_+$, such that

$$T^{CM}_{tij2l} = \lambda^{CM}_{Tij2ll'} T^{CM}_{tij2l'}, \tag{20}$$

$$T^{UM}_{rtij2l} = \lambda^{UM}_{Tij2ll'} T^{UM}_{rtij2l'}, \tag{21}$$

$$T^{M}_{tijkl} = \lambda^{M}_{Tijkll'} T^{M}_{tijkl'}, \qquad \forall\ k > 2. \tag{22}$$

(e) Definition of common *method state residual* variables. For each construct j, measured by the non-reference method k $(k \neq 1)$ and for each pair $(i, i') \in I \times I'$, $(i \neq i')$ there are constants $\lambda^{CM}_{Oii'j2l}$, $\lambda^{UM}_{Oii'j2l}$, as well as $\lambda^{M}_{Oii'jkl} \in \mathbb{R}_+$, such that

$$O^{CM}_{tij2l} = \lambda^{CM}_{Oii'j2l} O^{CM}_{ti'j2l}, \tag{23}$$

$$O^{UM}_{rtij2l} = \lambda^{UM}_{Oii'j2l} O^{UM}_{rti'j2l}, \tag{24}$$

$$O^{M}_{tijkl} = \lambda^{M}_{Oii'jkl} O^{M}_{ti'jkl}, \qquad \forall\ k > 2. \tag{25}$$

Remark 2. Assumption (a) states a latent linear regression of the latent trait variables pertaining to the non-reference method on the latent trait variables pertaining to the reference method.

Assumption (b) implies that the latent trait variables pertaining to different occasions of measurement l and l' are linear functions of each other. With regard to this assumption, it is possible to replace each latent trait variable T_{tij1l} by $\alpha_{Tij1l} + \lambda_{Tij1l} T_{tij1}$. As T_{tij1} is always measured by the same method (namely the reference method, $k = 1$), one may also omit the index k and thus write $\alpha_{Tij1l} + \lambda_{Tij1l} T_{tij}$. Equation (c) states a linear regression at the level of the occasion-specific variables. With regard to the Assumptions stated in (d), it is possible to define latent trait method factors (T^{CM}_{tij2}, T^{UM}_{rtij2}, and T^{M}_{tijk}). Note that these latent factors are defined for each indicator i. Finally, we define latent occasion-specific method factors according to Assumptions (e). The latent occasion-specific method variables are therefore assumed to be homogeneous for all indicators i and i' pertaining to the same construct-method-occasion-unit.

1.6. *Conditional Regressive Independence Assumptions*

In order to derive the variance and covariance structure of the LST-COM model, additional assumptions need to be imposed. In the next theorem, we discuss these important assumptions.

Theorem 1. (Conditional Regressive Independence (CRI) Assumptions.) Let $\mathcal{M} \equiv \langle (\Omega, \mathscr{A}, \mathscr{P}), \mathbf{T_t}, \mathbf{T^{UM}_{rt}}, \mathbf{T^{CM}_{t}}, \mathbf{T^{M}_{t}}, \mathbf{O_t}, \mathbf{O^{UM}_{rt}}, \mathbf{O^{CM}_{t}}, \mathbf{O^{M}_{t}}, \boldsymbol{\epsilon}_{rt}, \boldsymbol{\epsilon}_{t}, \boldsymbol{\alpha}_{T}, \boldsymbol{\lambda}_{T}, \boldsymbol{\lambda}^{UM}_{T}, \boldsymbol{\lambda}^{CM}_{T}, \boldsymbol{\lambda}^{M}_{T}, \boldsymbol{\lambda}_{O}, \boldsymbol{\lambda}^{UM}_{O}, \boldsymbol{\lambda}^{CM}_{O}, \boldsymbol{\lambda}^{M}_{O} \rangle$ be the LST-COM measurement model according to the above definition with $(\mathbf{T_t}, ..., \boldsymbol{\lambda}^{M}_{O})$ being vectors containing the model parameters of the LST-COM model. Additionally, it is assumed that

$$\mathbb{E}\left(Y_{tij1l} | p_{\mathcal{T}}, p_{\mathcal{T}\mathcal{S}_1}, ..., p_{\mathcal{T}\mathcal{S}_f}, (Y_{t(ijkl)'}), (Y_{rt(ij2l)'})\right) = \mathbb{E}(Y_{tij1l} | p_{\mathcal{T}}, p_{\mathcal{T}\mathcal{S}_l}), \tag{26}$$

$$\begin{aligned}&\mathbb{E}\left(Y_{tijkl} | p_{\mathcal{T}}, p_{\mathcal{T}\mathcal{S}_1}, ..., p_{\mathcal{T}\mathcal{S}_f}, p_{\mathcal{R}_k\mathcal{S}_1}, ..., p_{\mathcal{R}_k\mathcal{S}_f}, (Y_{t(ijkl)'}), (Y_{rt(ij2l)'})\right)\\ &= \mathbb{E}(Y_{tijkl} | p_{\mathcal{T}}, p_{\mathcal{T}\mathcal{S}_l}, p_{\mathcal{R}_k\mathcal{S}_l}), \text{ for } k > 2,\end{aligned} \tag{27}$$

$$\begin{aligned}&\mathbb{E}\left(Y_{rtij2l} | p_{\mathcal{T}}, p_{\mathcal{T}\mathcal{S}_1}, ..., p_{\mathcal{T}\mathcal{S}_f}, p_{\mathcal{R}}, p_{\mathcal{R}_k\mathcal{S}_1}, ..., p_{\mathcal{R}_k\mathcal{S}_f}, (Y_{t(ijkl)'}), (Y_{rt(ij2l)'})\right)\\ &= \mathbb{E}(Y_{rtij2l} | p_{\mathcal{T}}, p_{\mathcal{T}\mathcal{S}_l}, p_{\mathcal{R}}, p_{\mathcal{R}_2\mathcal{S}_l}),\end{aligned} \tag{28}$$

$$\mathbb{E}\left(S_{tij2l} | p_{\mathcal{T}}, p_{\mathcal{R}}\right) = \mathbb{E}(S_{tij2l} | p_{\mathcal{T}}), \tag{29}$$

$$\mathbb{E}\left(S_{rtij2l} | p_{\mathcal{T}}, p_{\mathcal{T}\mathcal{S}_1}, ..., p_{\mathcal{T}\mathcal{S}_f}, p_{\mathcal{R}_k\mathcal{S}_1}, \ldots, p_{\mathcal{R}_k\mathcal{S}_f}\right) = \mathbb{E}(S_{rtij2l} | p_{\mathcal{T}}, p_{\mathcal{T}\mathcal{S}_l}), \text{ for } k > 2, \tag{30}$$

$$\begin{aligned}&\mathbb{E}\left(S_{tijkl} | p_{\mathcal{T}}, p_{\mathcal{T}\mathcal{S}_1}, ..., p_{\mathcal{T}\mathcal{S}_{l-1}}, p_{\mathcal{T}\mathcal{S}_{l+1}}, ..., p_{\mathcal{T}\mathcal{S}_f}, p_{\mathcal{R}_{k'}\mathcal{S}_{l-1}}, p_{\mathcal{R}_{k'}\mathcal{S}_{l+1}}, ..., p_{\mathcal{R}_{k'}\mathcal{S}_f}\right)\\ &= \mathbb{E}(S_{tijkl} | p_{\mathcal{T}}), \text{for } k = k' \text{ or } k \neq k',\end{aligned} \tag{31}$$

$$\begin{aligned}&\mathbb{E}\left(S_{rtij2l} | p_{\mathcal{T}}, p_{\mathcal{T}\mathcal{S}_1}, ..., p_{\mathcal{T}\mathcal{S}_{l-1}}, p_{\mathcal{T}\mathcal{S}_{l+1}}, ..., p_{\mathcal{T}\mathcal{S}_f}, p_{\mathcal{R}}, p_{\mathcal{R}_2\mathcal{S}_1}, ..., p_{\mathcal{R}_2\mathcal{S}_{l-1}}, p_{\mathcal{R}_2\mathcal{S}_{l+1}}, ..., p_{\mathcal{R}_2\mathcal{S}_f}\right)\\ &= \mathbb{E}(S_{rtij2l} | p_{\mathcal{T}}, p_{\mathcal{R}}).\end{aligned} \tag{32}$$

where $(i, j, k, l) \neq (i, j, k, l)'$. Then, $\mathcal{M}$ is called LST-COM model with conditional regressive independence (CRI).

Remark 3. Assumption 26 states that the conditional expectations of the Level-2 observed variables Y_{tij1l} pertaining to the reference method ($k = 1$) only depend on the target variable $p_{\mathcal{T}}$ and the particular target-situation variable $p_{\mathcal{TS}_l}$, but not on other target-situations realized on different occasions of measurement nor on the values of other observed variables $Y_{t(ijkl)'}$ and $Y_{rt(ij2l)'}$, where $(ijkl) \neq (ijkl)'$. This assumption implies, for example, that the Level-1 error variables pertaining to the reference method are uncorrelated with any other error variable in the model. Similarly, the Level-2 observed variables pertaining to remaining structurally different methods ($k > 2$) only depend on the target variable $p_{\mathcal{T}}$, the particular target-situation variable $p_{\mathcal{TS}_l}$ and the particular rater situation $p_{\mathcal{R}_k\mathcal{S}_l}$ of this method, but not on other target- or rater-situations realized on different occasions of measurement nor on the values of other observed variables $Y_{t(ijkl)'}$ and $Y_{rt(ij2l)'}$ with $(ijkl) \neq (ijkl)'$ (see Eq. 27). Assumption 28 implies that observed variables belonging to the set of interchangeable raters ($k = 2$) only depend on the target variable $p_{\mathcal{T}}$, the particular target-situation variable $p_{\mathcal{TS}_l}$, the rater variable $p_{\mathcal{R}}$, and the particular rater-situation variable $p_{\mathcal{R}_2\mathcal{S}_l}$ pertaining to the set of the interchangeable method, but do not depend on other target-specific or other rater-specific situations realized on different occasions of measurement nor on the values of other observed variables [$Y_{rt(ijkl)'}$ or $Y_{t(ijkl)'}$, where $(ijkl) \neq (ijkl)'$].

Assumption 29 states that the conditional expectations of the Level-2 latent state variables pertaining to the set of interchangeable methods ($k = 2$) are conditionally regressively independent from the rater variable $p_{\mathcal{R}}$ given the target variable . Thus, the rater variable $p_{\mathcal{R}}$ does not provide any additional information for the expected value of S_{tij2l} given the target variable $p_{\mathcal{T}}$. With respect to this assumption, it is possible to define latent trait unique method variables as follows: $T_{rtij2l} - T_{tij2l}$.

Assumption 30 expresses that the Level-1 latent state variables S_{rtij2l} only depend on the target $p_{\mathcal{T}}$ variable and the particular target situation $p_{\mathcal{TS}_l}$ variable, but, given the target and the target situation, do not depend on other target situations realized on different occasions of measurement or on rater-situations of different methods $k > 2$. Assumption 31 states that the Level-2 latent state variables S_{tijkl} depend only on the target variable $p_{\mathcal{T}}$, but, given the target, neither depend on target situations nor on rater-situations realized on different occasions of measurement. Similarly, Assumption 32 states that the Level-1 latent state variables S_{rtij2l} depend only on the target variable $p_{\mathcal{T}}$ and the rater variable $p_{\mathcal{R}}$, but neither on other target situations $p_{\mathcal{TS}_l}$ or other rater situations $p_{\mathcal{R}_2\mathcal{S}_l}$ realized on different occasions of measurement. The last two suppositions imply, for example, that occasion-specific variables belonging to different occasions of measurement l and l' are uncorrelated with each other.

1.7. Implications of the Model Definition

Next, we discuss important implications of the above model definition and provide proofs for a selection of important implications. Following a similar logic, the remaining proofs can be easily shown.

Corollary 1. (Non-Permissible Correlations) Let $\mathcal{M} \equiv \langle(\Omega, \mathscr{A}, \mathscr{P}), \mathbf{T_t}, \mathbf{T_{rt}^{UM}}, \mathbf{T_t^{CM}}, \mathbf{T_t^{M}}, \mathbf{O_t}, \mathbf{O_{rt}^{UM}}, \mathbf{O_t^{CM}}, \mathbf{O_t^{M}}, \boldsymbol{\epsilon}_{rt}, \boldsymbol{\epsilon}_t, \boldsymbol{\alpha}_T, \boldsymbol{\lambda}_T, \boldsymbol{\lambda}_T^{UM}, \boldsymbol{\lambda}_T^{CM}, \boldsymbol{\lambda}_T^{M}, \boldsymbol{\lambda}_O, \boldsymbol{\lambda}_O^{UM}, \boldsymbol{\lambda}_O^{CM}, \boldsymbol{\lambda}_O^{M}\rangle$ be called an LST-COM model with conditional regressive independence according to the above Definition 1, then for $r \in R$, $t \in T$, $i, i' \in I$, $j, j' \in J$, $k, k' \in K$, $l, l' \in L$ where i can be equal to i', j to j', k to k' and l to l' but $(ijkl) \neq (ijkl)'$:

Uncorrelatedness of latent residual variables:

$$Cov(\epsilon_{rt(ij2l)}, \epsilon_{rt(ij2l)'}) = 0, \tag{33}$$
$$Cov(\epsilon_{t(ijkl)}, \epsilon_{t(ijkl)'}) = 0, \tag{34}$$
$$Cov(\epsilon_{rt(ij2l)}, \epsilon_{t(ijkl)'}) = 0. \tag{35}$$

Uncorrelatedness of latent variables and latent residual variables:

$$Cov(T_{ti'j'1}, \epsilon_{(r)tijkl}) = 0, \tag{36}$$
$$Cov(T_{rti'j'2}^{UM}, \epsilon_{(r)tijkl}) = 0, \tag{37}$$
$$Cov(T_{ti'j'2}^{CM}, \epsilon_{(r)tijkl}) = 0, \tag{38}$$
$$Cov(T_{ti'j'k'}^{M}, \epsilon_{(r)tijkl}) = 0, \tag{39}$$
$$Cov(O_{ti'j'k'l'}, \epsilon_{(r)tijkl}) = 0, \tag{40}$$
$$Cov(O_{rtj'2l'}^{UM}, \epsilon_{(r)tijkl}) = 0, \tag{41}$$
$$Cov(O_{tj'2l'}^{CM}, \epsilon_{(r)tijkl}) = 0, \tag{42}$$
$$Cov(O_{tj'k'l'}^{M}, \epsilon_{(r)tijkl}) = 0. \tag{43}$$

Uncorrelatedness of latent trait variables and latent trait method variables:

$$Cov(T_{tij1}, T_{rti'j'2}^{UM}) = 0, \tag{44}$$
$$Cov(T_{tij1}, T_{tij2}^{CM}) = 0, \tag{45}$$
$$Cov(T_{tij1}, T_{tijk}^{M}) = 0, \qquad \forall\ k > 2. \tag{46}$$

Uncorrelatedness of latent trait variables and latent occasion-specific (method) variables:

$$Cov(T_{tij1}, O_{ti'j'k'l'}) = 0, \tag{47}$$
$$Cov(T_{tij1}, O_{rtj'2l'}^{UM}) = 0, \tag{48}$$
$$Cov(T_{tij1}, O_{tj'2l'}^{CM}) = 0, \tag{49}$$
$$Cov(T_{tij1}, O_{tj'k'l'}^{M}) = 0, \qquad \forall\ k > 2. \tag{50}$$

Uncorrelatedness of latent trait method variables and latent occasion-specific (method) variables:

$$Cov(T^{M}_{tijk}, O_{ti'j'1l'}) = 0, \tag{51}$$

$$Cov(T^{M}_{tijk}, O^{M}_{tj'k'l'}) = 0, \qquad \forall\ k > 2 \tag{52}$$

$$Cov(T^{M}_{tijk}, O^{CM}_{tj'2l'}) = 0, \tag{53}$$

$$Cov(T^{M}_{tijk}, O^{UM}_{rtj'2l'}) = 0, \tag{54}$$

$$Cov(T^{CM}_{tij2}, O_{ti'j'1l'}) = 0, \tag{55}$$

$$Cov(T^{CM}_{tij2}, O^{M}_{tj'k'l'}) = 0, \qquad \forall\ k > 2 \tag{56}$$

$$Cov(T^{CM}_{tij2}, O^{CM}_{tj'2l'}) = 0, \tag{57}$$

$$Cov(T^{CM}_{tij2}, O^{UM}_{rtj'2l'}) = 0, \tag{58}$$

$$Cov(T^{UM}_{rtij2}, O_{ti'j'1l'}) = 0, \tag{59}$$

$$Cov(T^{UM}_{rtij2}, O^{M}_{tj'k'l'}) = 0,, \qquad \forall\ k > 2 \tag{60}$$

$$Cov(T^{UM}_{rtij2}, O^{CM}_{tj'2l'}) = 0, \tag{61}$$

$$Cov(T^{UM}_{rtij2}, O^{UM}_{rtj'2l'}) = 0. \tag{62}$$

Uncorrelatedness of latent trait method variables:

$$Cov(T^{M}_{tijk}, T^{UM}_{rti'j'2}) = 0, \tag{63}$$

$$Cov(T^{CM}_{tij2}, T^{UM}_{rti'j'2}) = 0. \tag{64}$$

Uncorrelatedness of latent state and occasion-specific method variables:

$$Cov(O_{tij1l}, O^{M}_{tjkl'}) = 0, \qquad \forall\ k > 2 \tag{65}$$

$$Cov(O_{tij1l}, O^{CM}_{tj2l'}) = 0, \tag{66}$$

$$Cov(O_{tij1l}, O^{UM}_{rtj'2l'}) = 0. \tag{67}$$

Uncorrelatedness of latent occasion-specific method variables:

$$Cov(O_{tijkl}, O_{tijkl'}) = 0, \quad \forall\ l \neq l', \tag{68}$$

$$Cov(O^{M}_{tjkl}, O^{M}_{tjkl'}) = 0, \quad \forall\ l \neq l', k > 2 \tag{69}$$

$$Cov(O^{CM}_{tj2l}, O^{CM}_{tj2l'}) = 0, \quad \forall\ l \neq l', \tag{70}$$

$$Cov(O^{UM}_{rtj2l}, O^{UM}_{rtj2l'}) = 0, \quad \forall\ l \neq l'. \tag{71}$$

Proof of the Unique Trait Variables. First, we show that $T^{UM}_{rtij2l} := T_{rtij2l} - T_{tij2l}$. According to Equation 3 in Definition 1, the latent trait unique method T^{UM}_{rtij2l} variables are defined as conditional expectations of the latent occasion-specific unique method variables given the target $p_{\mathcal{T}}$ and the rater $p_{\mathcal{R}}$:

$$\begin{aligned} T^{UM}_{rtij2l} :=& \mathbb{E}(UM_{rtij2l}|p_{\mathcal{T}}, p_{\mathcal{R}}) \\ =& \mathbb{E}[(S_{rtij2l} - S_{tij2l})|p_{\mathcal{T}}, p_{\mathcal{R}}] \\ =& \mathbb{E}(S_{rtij2l}|p_{\mathcal{T}}, p_{\mathcal{R}}) - \mathbb{E}(S_{tij2l}|p_{\mathcal{T}}, p_{\mathcal{R}}). \end{aligned}$$

According to Equation 29 in Theorem 1, the expression $\mathbb{E}(S_{rtij2l}|p_{\mathcal{T}}, p_{\mathcal{R}}) - \mathbb{E}(S_{tij2l}|p_{\mathcal{T}}, p_{\mathcal{R}})$ can be simplified as follows:

$$\begin{aligned} T^{UM}_{rtij2l} :=& \mathbb{E}(S_{rtij2l}|p_{\mathcal{T}}, p_{\mathcal{R}}) - \mathbb{E}(S_{tij2l}|p_{\mathcal{T}}) \\ =& T_{rtij2l} - T_{tij2l}, \end{aligned}$$

with $T_{rtij2l} := \mathbb{E}(S_{rtij2l}|p_{\mathcal{T}}, p_{\mathcal{R}})$ and $T_{tij2l} := \mathbb{E}(S_{tij2l}|p_{\mathcal{T}})$. □

Proof of Equation 33. Equation 33 can be rewritten as

$$Cov(Y_{rt(ij2l)} - S_{rt(ij2l)}, Y_{rt(ij2l)'} - S_{rt(ij2l)'}),$$

where $S_{rt(ij2l)}$ is defined as conditional expectation of $Y_{rt(ij2l)}$ given $p_{\mathcal{T}}$, $p_{\mathcal{TS}_l}$, $p_{\mathcal{R}}$, $p_{\mathcal{R}_2\mathcal{S}_l}$, whereas $S_{rt(ij2l)'}$ is defined as conditional expectation of $Y_{rt(ij2l)'}$ given $p_{\mathcal{T}}$, $p_{\mathcal{TS}_{l'}}$, $p_{\mathcal{R}}$, $p_{\mathcal{R}_2\mathcal{S}_{l'}}$. This means that $\epsilon_{rt(ij2l)}$ is defined as residual with respect to any measurable function of $Y_{rt(ij2l)}$ and $p_{\mathcal{T}}$, $p_{\mathcal{TS}_l}$, $p_{\mathcal{R}}$, $p_{\mathcal{R}_2\mathcal{S}_l}$. According to Equation 27 (see Theorem 1), it is possible to replace $E\left(Y_{rtij2l}|p_{\mathcal{T}}, p_{\mathcal{TS}_1}, ..., p_{\mathcal{TS}_f}, p_{\mathcal{R}}, p_{\mathcal{R}_2\mathcal{S}_1}, ..., p_{\mathcal{R}_2\mathcal{S}_f}, (Y_{t(ijkl)'}), (Y_{rt(ij2l)'})\right)$ by $E(Y_{rtij2l}|p_{\mathcal{T}}, p_{\mathcal{TS}_l}, p_{\mathcal{R}}, p_{\mathcal{R}_2\mathcal{S}_l})$. Hence, $\epsilon_{rt(ij2l)'}$ is also a residual with respect to Y_{rtij2l} and $p_{\mathcal{T}}$, $p_{\mathcal{TS}_l}$, $p_{\mathcal{R}}$, as well as $p_{\mathcal{R}_2\mathcal{S}_l}$. Given that residuals are always uncorrelated with their regressors or with functions of their regressors (c.f Steyer, 1988, 1989; Steyer & Eid, 2001), the condition above holds. Following a similar logic, Equations (34) and (35) can be shown. □

Proof of Equation 36. First, $\epsilon_{(r)tijkl}$ may denote either a Level-1 or a Level-2 error variable. Second, the latent trait factors $T_{ti'j'1}$ are functions of the latent trait variables $T_{ti'j'1l'}$, namely $\frac{T_{ti'j'1l'} - \alpha_{Ti'j'1l'}}{\lambda_{Ti'j'1l'}}$. Thus, $Cov(T_{ti'j'1}, \epsilon_{(r)tijkl}) = 0$ if $Cov(T_{ti'j'1l'}, \epsilon_{(r)tijkl}) = 0$. From Equation 26 in Theorem 1, it follows directly that $Cov(T_{ti'j'1l'}, \epsilon_{tij1l}) = 0$. Next, according to Equation 27 in Theorem 1, it follows that $Cov(T_{ti'j'1l'}, \epsilon_{tijkl}) = 0$, where $k > 2$, because ϵ_{tijkl} is also defined as residual with respect to $Y_{ti'j'1l'}$. Since $T_{ti'j'1l'}$ is a ($Y_{ti'j'1l'}$, $p_{\mathcal{T}}$)-measurable function, Equation

$Cov(T_{ti'j'1l'}, \epsilon_{tijkl}) = 0$ holds. Similarly, according to Equation 28 in Theorem 1, it can be shown that $Cov(T_{ti'j'1l'}, \epsilon_{rtij2l}) = 0$, because ϵ_{rtij2l} is defined as a residual with respect to $Y_{ti'j'1l'}$, and $T_{ti'j'1l'}$ is defined as a $(Y_{ti'j'1l'},\, p_{\mathcal{T}})$-measurable function. Thus, $Cov(T_{ti'j'1l'}, \epsilon_{(r)tijkl}) = 0$ holds. □

Proof of Equation 44. T_{tij1} can be written as $\frac{T_{tij1l} - \alpha_{Tij1l}}{\lambda_{Tij1l}}$ and $T^{UM}_{rti'j'2}$ can be written as $\frac{T^{UM}_{rti'j'2l'}}{\lambda^{UM}_{Ti'j'2l'}}$. Thus, $Cov(T_{tij1}, T^{UM}_{rti'j'2}) = 0$ if $Cov(T_{tij1l}, T^{UM}_{rti'j'2l'}) = 0$. $T^{UM}_{rti'j'2l'}$ is defined as $\mathbb{E}(S_{rti'j'2l'}|p_{\mathcal{T}}, p_{\mathcal{R}}) - \mathbb{E}(S_{ti'j'2l'}|p_{\mathcal{T}})$, which could be also rewritten as $\mathbb{E}(S_{rti'j'2l'}|p_{\mathcal{T}}, p_{\mathcal{R}}) - \mathbb{E}(S_{rti'j'2l'}|p_{\mathcal{T}})$. Hence, $T^{UM}_{rti'j'2l'}$ is a residual with respect to any $(p_{\mathcal{T}})$-measurable function and therefore uncorrelated with the $(p_{\mathcal{T}})$-measurable function T_{tij1l}. □

Proof of Equation 47. Again, T_{tij1} can be written as $\frac{T_{tij1l} - \alpha_{Tij1l}}{\lambda_{Tij1l}}$ and thus $Cov(T_{tij1}, O_{ti'j'k'l'}) = 0$ holds if $Cov(T_{tij1l}, O_{ti'j'k'l'}) = 0$ is true. $O_{ti'j'k'l'}$ is defined as $S_{ti'j'k'l'} - \mathbb{E}(S_{ti'j'k'l'}|p_{\mathcal{T}})$. Again, this means that $O_{ti'j'k'l'}$ is defined as residual with respect to any $(p_{\mathcal{T}})$-measurable function, and is thereby uncorrelated with the $(p_{\mathcal{T}})$-measurable function T_{tij1l}. □

Proof of Equation 48. $Cov(T_{tij1}, O^{UM}_{rtj'2l'}) = 0$ holds, if $Cov(T_{tij1l}, O^{UM}_{rti'j'2l'}) = 0$, because T_{tij1} can be written as $\frac{T_{tij1l} - \alpha_{Tij1l}}{\lambda_{Tij1l}}$ and $O^{UM}_{tj'2l'}$ can be written as $\frac{O^{UM}_{ti'j'2l'}}{\lambda^{UM}_{Oi'j'2l'}}$. $O^{UM}_{rti'j'2l'}$ can be written as:

$$O^{UM}_{rti'j'2l'} = UM_{rti'j'2l'} - \mathbb{E}(UM_{rti'j'2l'} \mid p_{\mathcal{T}}, p_{\mathcal{R}})$$

It follows that $O^{UM}_{rti'j'2l'}$ is a residual with respect to $(p_{\mathcal{T}})$-measurable functions, and thereby uncorrelated with the $(p_{\mathcal{T}})$-measurable function T_{tij1l}. □

Proof of Equation 52. For all $k, k' > 2$, $Cov(T^M_{tijk}, O^M_{tj'k'l'}) = 0$ holds, if $Cov(T^M_{tijkl}, O^M_{ti'j'k'l'}) = 0$ holds, given that T^M_{tijk} can be expressed as $\frac{T^M_{tijkl}}{\lambda^M_{Tijkl}}$ and $O^M_{tj'k'l'}$ can be rewritten as $\frac{O^M_{ti'j'k'l'}}{\lambda^M_{Oi'j'k'l'}}$. T^M_{tijkl} is defined as $T_{tijkl} - \mathbb{E}(T_{tijkl} \mid T_{tij1l})$ and thereby a direct function of the $(p_{\mathcal{T}})$-measurable functions T_{tijkl} and T_{tij1l}. $O^M_{ti'j'k'l'}$ is defined as $O_{ti'j'k'l'} - \mathbb{E}(O_{ti'j'k'l'} \mid O_{ti'j'1l'})$ and thereby a direct function of $O_{ti'j'k'l'}$ and $O_{ti'j'1l'}$, which are defined as residuals with respect to any $(p_{\mathcal{T}})$-measurable function. Hence $Cov(T^M_{tijk}, O^M_{tj'k'l'})$=0 holds. □

Proof of Equation 63. $Cov(T^M_{tijk}, T^{UM}_{rti'j'2}) = 0$ is true, if $Cov(T^M_{tijkl}, T^{UM}_{rti'j'2l'}) = 0$ is true, because T^M_{tijk} can be expressed as $\frac{T^M_{tijkl}}{\lambda^M_{Tijkl}}$ and $T^{UM}_{rti'j'2}$ can be written as $\frac{T^{UM}_{rti'j'2l'}}{\lambda^{UM}_{Ti'j'2l'}}$. T^M_{tijkl} is defined as $T_{tijkl} - \mathbb{E}(T_{tijkl} \mid T_{tij1l})$ and thereby a direct function of the $(p_{\mathcal{T}})$-measurable functions T_{tijkl} and T_{tij1l}. Hence $Cov(T^M_{tijkl}, T^{UM}_{rti'j'2l'}) = 0$ follows from $Cov(T_{tijkl}, T^{UM}_{rti'j'2l'}) = 0$, which was proven above (see Proof of Equation 44). □

Proof of Equation 68. $Cov(O_{tijkl}, O_{tijkl'}) = 0$ $(\forall l \neq l')$ holds, because O_{tijkl} is a $(p_{\mathcal{T}}, p_{\mathcal{T}\mathcal{S}_l}, p_{\mathcal{R}_k\mathcal{S}_l})$-measurable function as S_{tijkl} is a $(p_{\mathcal{T}}, p_{\mathcal{T}\mathcal{S}_l}, p_{\mathcal{R}_k\mathcal{S}_l})$-measurable function and

$O_{tijkl} := S_{tijkl} - \mathbb{E}(S_{tijkl} \mid p_{\mathcal{T}})$. According to Assumption 31, it holds that

$$\mathbb{E}\left(S_{tijkl} \mid p_{\mathcal{T}}, p_{\mathcal{T}\mathcal{S}_1}, \ldots, p_{\mathcal{T}\mathcal{S}_{l-1}}, p_{\mathcal{T}\mathcal{S}_{l+1}}, \ldots, p_{\mathcal{T}\mathcal{S}_f}, p_{\mathcal{R}_{k'}\mathcal{S}_1}, \ldots, p_{\mathcal{R}_{k'}\mathcal{S}_{l-1}}, p_{\mathcal{R}_{k'}\mathcal{S}_{l+1}}, \ldots, p_{\mathcal{R}_{k'}\mathcal{S}_f}\right) = \mathbb{E}(S_{tijkl} \mid p_{\mathcal{T}}),$$

where $k = k'$ or $k \neq k'$. It follows that O_{tijkl} is also a residual with respect to a $(p_{\mathcal{T}}, p_{\mathcal{T}\mathcal{S}_{l'}}, p_{\mathcal{R}_k\mathcal{S}_{l'}})$-measurable function, and therefore uncorrelated with the $(p_{\mathcal{T}}, p_{\mathcal{T}\mathcal{S}_{l'}}, p_{\mathcal{R}_k\mathcal{S}_{l'}})$-measurable function $O_{tijkl'}$. Equations 69 and 70 can be shown following a similar logic. □

Proof of Equation 65. First, note that $Cov(O_{tij1l}, O^M_{tijkl}) = 0$ follows directly, because O^M_{tijkl} is defined as $O_{tijkl} - \mathbb{E}(O_{tijkl}|O_{tij1l})$ and thus O^M_{tijkl} is a residual with respect to O_{tij1l}. $Cov(O_{tij1l}, O^M_{tijkl'}) = 0$ holds if $Cov(O_{tij1l}, O_{tijkl'}) = 0$ and $Cov(O_{tij1l}, O_{tij1l'}) = 0$, as $O^M_{tijkl'} = O_{tijkl'} - \mathbb{E}(O_{tijkl'} \mid O_{tij1l'})$. $Cov(O_{tij1l}, O_{tij1l'}) = 0$ follows from Equation (68), which was proven above. By Assumption 31

$$\mathbb{E}(S_{tijkl} \mid p_{\mathcal{T}}, p_{\mathcal{T}\mathcal{S}_1}, \ldots, p_{\mathcal{T}\mathcal{S}_{l-1}}, p_{\mathcal{T}\mathcal{S}_{l+1}}, \ldots, p_{\mathcal{T}\mathcal{S}_f}, p_{\mathcal{R}_{k'}\mathcal{S}_1}, \ldots, p_{\mathcal{R}_{k'}\mathcal{S}_{l-1}}, p_{\mathcal{R}_{k'}\mathcal{S}_{l+1}}, \ldots, p_{\mathcal{R}_{k'}\mathcal{S}_f}) = \mathbb{E}(S_{tijkl} \mid p_{\mathcal{T}})$$

$O_{tij1l} = S_{tij1l} - \mathbb{E}(S_{tij1l} \mid p_{\mathcal{T}})$ is also a residual with respect to a $(p_{\mathcal{T}}, p_{\mathcal{T}\mathcal{S}_{l'}}, p_{\mathcal{R}_k\mathcal{S}_{l'}})$-measurable function, with $k \neq 1$. Therefore O_{tij1l} is uncorrelated with the $(p_{\mathcal{T}}, p_{\mathcal{T}\mathcal{S}_{l'}}, p_{\mathcal{R}_k\mathcal{S}_{l'}})$-measurable function $O_{tijkl'}$ and $Cov(O_{tij1l}, O_{tijkl'}) = 0$. □

1.9. Covariance Structure: LST-COM Model with CRI

In the following section the total variance-covariance matrix of the LST-COM model for three indicators × two constructs × two methods × three occasions of measurements is described. Similar to the previous chapters, the total covariance matrix $\mathbf{\Sigma}$ of size 36×36 (i.e., $ijkl \times ijkl$) can be decomposed into a within $\mathbf{\Sigma}_W$ and a between $\mathbf{\Sigma}_B$ matrix:

$$\mathbf{\Sigma} = \mathbf{\Sigma}_W + \mathbf{\Sigma}_B.$$

As a consequence of the definition of the model, each of these matrices $\mathbf{\Sigma}_W$ and $\mathbf{\Sigma}_B$ can be further decomposed into a trait T, occasion-specific variables O and residual $\boldsymbol{\theta}$ matrix. This decomposition follows directly, given that latent trait variables are uncorrelated with latent occasion-specific variables (see above Theorem 1). Thus, the within $\mathbf{\Sigma}_W$ and the between $\mathbf{\Sigma}_B$ variance-covariance matrices may be represented as

$$\mathbf{\Sigma}_W = \mathbf{\Sigma}_{T_W} + \mathbf{\Sigma}_{O_W} + \mathbf{\Sigma}_{\boldsymbol{\theta}_W}, \qquad \text{and} \qquad \mathbf{\Sigma}_B = \mathbf{\Sigma}_{T_B} + \mathbf{\Sigma}_{O_B} + \mathbf{\Sigma}_{\boldsymbol{\theta}_B}.$$

$\mathbf{\Sigma}_{T_W}$ refers to the within trait matrix, $\mathbf{\Sigma}_{O_W}$ refers to the within occasion-specific matrix, $\mathbf{\Sigma}_{\boldsymbol{\theta}_W}$ refers to the within residual matrix, $\mathbf{\Sigma}_{T_B}$ refers to the between trait matrix, $\mathbf{\Sigma}_{O_B}$ refers to the between occasion-specific matrix, and $\mathbf{\Sigma}_{\boldsymbol{\theta}_B}$ is the between residual matrix. The within and between trait and occasion-specific matrices $\mathbf{\Sigma}_{T_W}$, $\mathbf{\Sigma}_{O_W}$, $\mathbf{\Sigma}_{\boldsymbol{\theta}_W}$, and $\mathbf{\Sigma}_{T_B}$ are then further decomposed into:

$$\mathbf{\Sigma}_{T_W} = \mathbf{\Lambda}_{T_W}\mathbf{\Phi}_{T_W}\mathbf{\Lambda}_{T_W}^{'}, \quad \text{and} \quad \mathbf{\Sigma}_{O_W} = \mathbf{\Lambda}_{O_W}\mathbf{\Phi}_{O_W}\mathbf{\Lambda}_{O_W}^{'},$$

$$\mathbf{\Sigma}_{T_B} = \mathbf{\Lambda}_{T_B}\mathbf{\Phi}_{T_B}\mathbf{\Lambda}_{T_B}^{'}, \quad \text{and} \quad \mathbf{\Sigma}_{O_B} = \mathbf{\Lambda}_{O_B}\mathbf{\Phi}_{O_B}\mathbf{\Lambda}_{O_B}^{'}.$$

$\mathbf{\Lambda}_{T_W}$ refers to the factor loading matrix for the trait-specific latent variables on the within level, with $\mathbf{\Lambda}_{T_W}^{'}$ being its transpose, $\mathbf{\Phi}_{T_W}$ is the variance and covariance matrix of the latent trait-specific variables on the within level, $\mathbf{\Lambda}_{O_W}$ is the factor loading matrix for the latent occasion-specific variables on the within level, with $\mathbf{\Lambda}_{O_W}^{'}$ being the transposed matrix, $\mathbf{\Phi}_{O_W}$ is the variance and covariance matrix of the latent occasion-specific variables on the within level. In a similar way, the target-level matrices are denoted by the subscript $\mathbf{B}$ for between level.

In order to illustrate the complete covariance matrix of the LST-COM model for three indicators, two constructs, two methods, and three occasions of measurement, the index (j, l) which can take the following values in the given ordering, is defined: $(1,1),(1,2),(1,3),(2,1),(2,2),(2,3)$. The index $(1,1)$ indicates that a given parameter (e.g., factor loading) refers to the first construct $j = 1$ measured on the first occasion of measurement $l = 1$. In addition, the function $Pos((j, l))$ is defined. The function maps the index (j, l) on its position $\mathbf{p}$ with respect to the ordering above. The function therefore takes the values given in Table 1. Then, the matrix $\mathbf{\Lambda}_{T_W}$ of size 36×6 (i.e., $ijkl \times ij$) containing the factor

Function	Values					
(j, l)	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)
$\mathbf{p}$=$Pos((j, l))$	1	2	3	4	5	6

Table 1.
Function for the mapping of the index (j, l) to $\mathbf{p}$.

loadings of the latent trait unique method variables T_{rtij2}^{UM} is given by:

$$\mathbf{\Lambda_{T_W}} = \sum_{\mathbf{p=1}}^{\mathbf{6}} \mathbf{I}_{\mathbf{\Lambda_T}}^{\mathbf{p}} \otimes \mathbf{\Lambda}_{\mathbf{T_W}}^{\mathbf{p}},$$

$\sum_{p=1}^{6}$ refers to the sum over all constructs j and measurement occasions l. $\mathbf{I}_{\mathbf{\Lambda_T}}^{\mathbf{p}}$ is a contrast (or dummy coding) matrix for a particular combination of construct and occasion of measurement (e.g., $j = 1$ and $l = 1$). $\otimes$ is the Kronecker product and $\mathbf{\Lambda}_{T_W}^{p}$ is the within trait unique method factor loading matrix of size 6×3 (i.e., $ik \times i$). The contrast matrix $\mathbf{I}_{\mathbf{\Lambda_T}}^{\mathbf{p}}$, where $\mathbf{p} \in \mathbb{N} = \{1, ..., 6\}$ is defined as 6×2 matrix (i.e., $jl \times j$):

$$\mathbf{I^1_{\Lambda_T}} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \mathbf{I^2_{\Lambda_T}} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \mathbf{I^3_{\Lambda_T}} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix},$$

$$\mathbf{I^4_{\Lambda_T}} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \mathbf{I^5_{\Lambda_T}} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \mathbf{I^6_{\Lambda_T}} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

Then, the within trait unique method factor loading matrix $\mathbf{\Lambda}^p_{T_W}$ of size 6×3 (i.e., $ik \times i$), where the elements λ^{UM}_{T1j2l}, λ^{UM}_{T2j2l}, $\lambda^{UM}_{T3j2l} > 0$ and all other elements are zero, is given by:

$$\mathbf{\Lambda}^p_{T_W} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \lambda^{UM}_{T1j2l} & 0 & 0 \\ 0 & \lambda^{UM}_{T2j2l} & 0 \\ 0 & 0 & \lambda^{UM}_{T3j2l} \end{pmatrix}.$$

Similarly, the occasion-specific unique method factor loading matrix at Level-1 $\mathbf{\Lambda}_{O_W}$ of size 36×6 (i.e., $ijkl \times jl$) can be defined:

$$\mathbf{\Lambda}_{O_W} = \sum_{p=1}^{6} \mathbf{I^p_{\Lambda_O}} \otimes \mathbf{\Lambda^p_{O_W}}.$$

$\mathbf{I^p_{\Lambda_O}}$ refers to a contrast matrix of size 6×6 (i.e., $jl \times jl$) where $\mathbf{p} \in \mathbb{N} = \{1, ..., 6\}$ with a one on the pth diagonal element and zeros elsewhere, e.g. for $\mathbf{p}$=2:

$$\mathbf{I^2_{\Lambda_O}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Again, the occasion-specific unique method factor loading vector at Level-1$\mathbf{\Lambda}^p_{O_W}$ of size 6×1 (i.e., $ik \times 1$) is given by

$$\mathbf{\Lambda}^p_{O_W} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \lambda^{UM}_{O1j2l} \\ \lambda^{UM}_{O2j2l} \\ \lambda^{UM}_{O3j2l} \end{pmatrix}.$$

The complete within covariance matrix of the latent trait variables $\mathbf{\Phi}_{T_W}$ of size 6×6 (i.e., $ij \times ij$) can be represented as follows (see Figure 1):

$$\mathbf{\Phi}_{T_W} = \mathbb{E}\left[(\mathbf{V_{\Phi_{T_W}}} - \mathbb{E}[\mathbf{V_{\Phi_{T_W}}}])(\mathbf{V_{\Phi_{T_W}}} - \mathbb{E}[\mathbf{V_{\Phi_{T_W}}}])^{'}\right],$$

where $\mathbf{V_{\Phi_{T_W}}}$ refers to the vector of size 6×1 (i.e., $ij \times 1$) including all latent trait unique method factors on the within level, namely $\left(T^{UM}_{rt112}, T^{UM}_{rt212}, T^{UM}_{rt312}, T^{UM}_{rt122}, T^{UM}_{rt222}, T^{UM}_{rt322}\right)^{'}$. Note that all covariances and correlations between latent trait unique method variables are permissible (see Theorem 1). Consequently, $\mathbf{\Phi}_{T_W}$ does not contain zero-elements. In a similar way, $\mathbf{\Phi}_{O_W}$ is given by:

$$\mathbf{\Phi}_{O_W} = \mathbb{E}\left[(\mathbf{V_{\Phi_{O_W}}} - \mathbb{E}[\mathbf{V_{\Phi_{O_W}}}])(\mathbf{V_{\Phi_{O_W}}} - \mathbb{E}[\mathbf{V_{\Phi_{O_W}}}])^{'}\right],$$

where $\mathbf{V_{\Phi_{O_W}}}$ refers to the vector of size 6×1 (i.e., $jl \times 1$) including all latent occasion-specific unique method factors at Level-1, namely $\left(O^{UM}_{rt121}, O^{UM}_{rt122}, O^{UM}_{rt123}, O^{UM}_{rt221}, O^{UM}_{rt222}, O^{UM}_{rt223}\right)^{'}$. Note that O^{UM}_{rtij2l} are assumed to be homogeneous across items, therefore the index i has been dropped. In contrast to $\mathbf{V_{\Phi_{T_W}}}$, the within variance and covariance matrix $\mathbf{V_{\Phi_{O_W}}}$ of the latent occasion-specific variables O^{UM}_{rtj2l} of size 6×6 (i.e., $jl \times jl$) contains zero-elements. The zero-elements (see Theorem 1) refer to the correlations among the latent occasion-specific unique method variables pertaining to the same construct j, but different occasions of measurement l and l', that is $Cov(O^{UM}_{rtj2l}, O^{UM}_{rtj2l'}) = 0, \ \forall\, l \neq l'$ (see white cells in Figure 2). Furthermore, it is also recommended to fix all of the following correlations referring to associations between latent occasion-specific unique method factors pertaining to different constructs $j \neq j'$ and different occasions of measurement $l \neq l'$ to zero as well: $Cov(O^{UM}_{rtj2l}, O^{UM}_{rtj'2l'}) = 0, \ \forall\, j, l \neq j', l'$ (see light gray cells in Figure 2). In most empirical applications these correlations will be close to zero, and therefore may be fixed to zero for parsimony.

The target-level matrices can be defined following a similar logic. First, the between latent trait factor loadings matrix $\mathbf{\Lambda_{T_B}}$ of size 36×12 (i.e., $ijkl \times jkl$) containing the latent factor loading onto the latent trait variables T_{tij1} and T^{CM}_{tij2} is given by:

$$\mathbf{\Lambda_{T_B}} = \sum_{\mathbf{p=1}}^{\mathbf{6}} \mathbf{I^p_{\Lambda_T}} \otimes \mathbf{\Lambda^p_{T_B}},$$

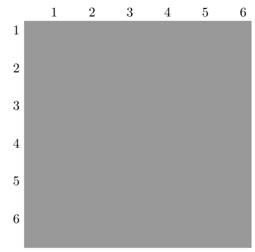

FIGURE 1.

Within variance-covariance matrix $\mathbf{\Phi}_{T_W}$ of the LST-COM, where 1=T^{UM}_{rt112}, 2=T^{UM}_{rt212}, 3=T^{UM}_{rt312}, 4=T^{UM}_{rt122}, 5=T^{UM}_{rt222}, 6=T^{UM}_{rt322}. Cells colored in dark gray indicate permissible and interpretable variances and covariances among the latent variables.

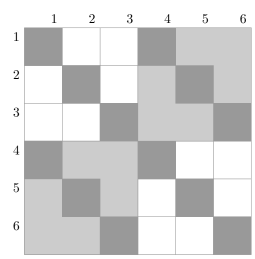

FIGURE 2.

Within variance-covariance matrix $\mathbf{\Phi}_{O_W}$ of the LST-COM, where 1=O^{UM}_{rt121}, 2=O^{UM}_{rt122}, 3=O^{UM}_{rt123}, 4=O^{UM}_{rt221}, 5=O^{UM}_{rt122}, 6=O^{UM}_{rt223}. Cells colored in dark gray indicate permissible and interpretable variances and covariances among the latent variables. Cells colored in light gray refer to covariances that can be fixed to zero for parsimony. White cells refer to non-permissible covariances among the latent variables.

for which the elements $\lambda_{T1j1l}, \lambda_{T2j1l}, \lambda_{T3j1l}, \lambda_{T1j2l}, \lambda_{T2j2l}, \lambda_{T3j2l}, \lambda^{CM}_{T1j2l}, \lambda^{CM}_{T2j2l}, \lambda^{CM}_{T3j2l} > 0$ and all other elements are necessarily zero. $\mathbf{I^p_{\Lambda_T}}$ refers to a contrast matrix of size 6×2 (i.e., $jl \times j$) described above. Then, $\mathbf{\Lambda^p_{T_B}}$ is the matrix of the between factor loadings of size $6{\times}6$ (i.e., $ik \times ik$) which is given by:

$$\mathbf{\Lambda^p_{T_B}} = \begin{pmatrix} \lambda_{T1j1l} & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_{T2j1l} & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_{T3j1l} & 0 & 0 & 0 \\ \lambda_{T1j2l} & 0 & 0 & \lambda^{CM}_{T1j2l} & 0 & 0 \\ 0 & \lambda_{T2j2l} & 0 & 0 & \lambda^{CM}_{T2j2l} & 0 \\ 0 & 0 & \lambda_{T3j2l} & 0 & 0 & \lambda^{CM}_{T3j2l} \end{pmatrix}.$$

In a similar way, the matrix $\mathbf{\Lambda_{O_B}}$ of size $36{\times}12$ (i.e., $ijkl \times jkl$) containing the between latent occasion-specific factor loadings of the common latent occasion-specific variables O_{tj1l} and O^{CM}_{tj2l} is given by:

$$\mathbf{\Lambda_{O_B}} = \sum_{\mathbf{p=1}}^{\mathbf{6}} \mathbf{I^p_{\Lambda_O}} \otimes \mathbf{\Lambda^p_{O_B}}.$$

Again, $\mathbf{I^p_{\Lambda_O}}$ refers to the contrast matrix of size 6×6 (i.e., $jl \times jl$) described above and $\mathbf{\Lambda^p_{O_B}}$ is the between factor loadings matrix[1], represented by:

$$\mathbf{\Lambda^p_{O_B}} = \begin{pmatrix} \lambda_{O1j1l} & 0 \\ \lambda_{O2j1l} & 0 \\ \lambda_{O3j1l} & 0 \\ \lambda_{O1j2l} & \lambda^{CM}_{O1j2l} \\ \lambda_{O2j2l} & \lambda^{CM}_{O2j2l} \\ \lambda_{O3j2l} & \lambda^{CM}_{O3j2l} \end{pmatrix}.$$

The between variance and covariance matrix of the latent trait variables $\mathbf{\Phi_{TB}}$ of size $12{\times}12$ (i.e., $ijk \times ijk$) is given by:

$$\mathbf{\Phi_{T_B}} = \mathbb{E}\left[(\mathbf{V_{\Phi_{T_B}}} - \mathbb{E}[\mathbf{V_{\Phi_{T_B}}}])(\mathbf{V_{\Phi_{T_B}}} - \mathbb{E}[\mathbf{V_{\Phi_{T_B}}}])^{'}\right],$$

where $\mathbf{V_{\Phi_{T_B}}}$ refers to the vector of size $12{\times}1$ including all latent trait unique method factors on the between level, namely $\left(T_{t11}, T_{t21}, T_{t31}, T^{CM}_{t11}, T^{CM}_{t21}, T^{CM}_{t31}, T_{t12}, T_{t22}, T_{t32}, T^{CM}_{t12}, T^{CM}_{t22}, T^{CM}_{t32}\right)^{'}$. As a consequence of the definition of the model, all elements referring to $Cov(T_{tij}, T^{CM}_{tij2}) = 0$ are zero-elements. For parsimony reasons, it is recommended to also fix the elements referring to $Cov(T_{ti'j'}, T^{CM}_{tij2}) = 0,\ \forall\ i, j \neq i', j'$ to zero. In Figure 3 the structure of the variance-covariance matrix $\mathbf{\Phi_{T_B}}$ is depicted.

[1]Note that for the sake of simplicity, it is assumed that the latent occasion-specific variables O_{tij1l} are homogeneous across items. Hence, it is assumed that the latent occasion-specific variables O_{tij1l} are measured by a common latent occasion-specific factor O_{tj1l}. The matrix $\mathbf{\Lambda_{OB_p}}$ refers therefore to the factor loading matrix of common O_{tj1l} and O^{CM}_{tj2l} variables. Note that this model differs slightly from the model in Definition 2.

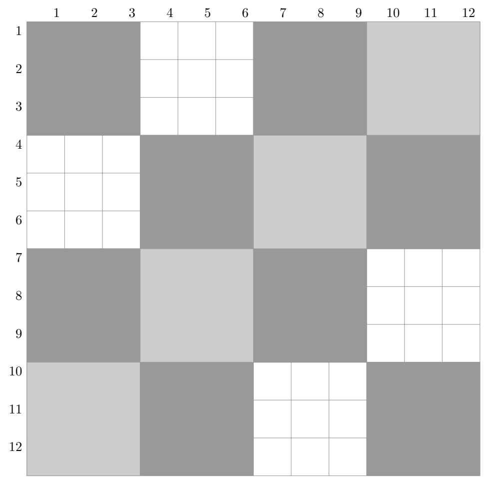

FIGURE 3.

Between variance-covariance matrix $\mathbf{\Phi_{T_B}}$ of the LST-COM model, where 1=T_{t11}, 2=T_{t21}, 3=T_{t31}, 4=T^{CM}_{t11}, 5=T^{CM}_{t21}, 6=T^{CM}_{t31}, 7=T_{t12}, 8=T_{t22}, 9=T_{t32}, 10=T^{CM}_{t12}, 11=T^{CM}_{t22}, 12=T^{CM}_{t32}. Cells colored in white indicate zero-covariances, cells colored in gray indicate permissible and interpretable variances and covariances. Cells in light gray indicate covariances that should be fixed to zero for parsimony.

The between variance and covariance matrix of the latent occasion-specific factors $\mathbf{\Phi_{O_B}}$ of size 12×12 (i.e., $jkl \times jkl$) is given by:

$$\mathbf{\Phi_{O_B}} = \mathbb{E}\left[(\mathbf{V_{\Phi_{O_B}}} - \mathbb{E}[\mathbf{V_{\Phi_{O_B}}}])(\mathbf{V_{\Phi_{O_B}}} - \mathbb{E}[\mathbf{V_{\Phi_{O_B}}}])^{'}\right],$$

where $\mathbf{V_{\Phi_{O_W}}}$ refers to the vector of size 12×1 including all latent occasion-specific factors on the between level, namely $\left(O_{t111}, O^{CM}_{t121}, O_{t121}, O^{CM}_{t122}, O_{t131}, O^{CM}_{t123}, O_{t211}, O^{CM}_{t221}, O_{t221}, O^{CM}_{t222}, O_{t231}, O^{CM}_{t223}\right)^{'}$. By definition, all elements referring to $Cov(O_{tjl}, O_{tjl'}) = Cov(O_{tjl}, O^{CM}_{tj2l'}) = Cov(O^{CM}_{tj2l}, O^{CM}_{tj2l'}) = 0,\ \forall\ l \neq l'$ are zero elements. Again, for parsimony reasons, it is recommended to fix the elements referring to $Cov(O_{tjl}, O^{CM}_{tj'2l'})\ \forall\ j, l \neq j', l'$ to zero as well. As we explained earlier in the manuscript, occasion-specific variables may be correlated with latent occasion-specific method variables measured at Level-2, if they belong to different constructs j and j'. However, theses correlations will often be low and non-significant in empirical applications. Figure 4 illustrates the complete between variance-covariance matrix for the latent occasion-specific variables.

1.10. Mean Structure

Theorem 2. (Mean structure) Iff $\mathcal{M} \equiv \langle(\Omega, \mathscr{A}, \mathscr{P}), \mathbf{T_t}, \mathbf{T^{UM}_{rt}}, \mathbf{T^{CM}_t}, \mathbf{T^M_t}, \mathbf{O_t}, \mathbf{O^{UM}_{rt}}, \mathbf{O^{CM}_t}, \mathbf{O^M_t}, \boldsymbol{\epsilon}_{rt}, \boldsymbol{\epsilon}_t, \boldsymbol{\alpha}_T, \boldsymbol{\lambda}_T, \boldsymbol{\lambda}^{UM}_T, \boldsymbol{\lambda}^{CM}_T, \boldsymbol{\lambda}^M_T, \boldsymbol{\lambda}_O, \boldsymbol{\lambda}^{UM}_O, \boldsymbol{\lambda}^{CM}_O, \boldsymbol{\lambda}^M_O\rangle$ is a LST-COM model with CRI as defined in Definition 2, where the first method

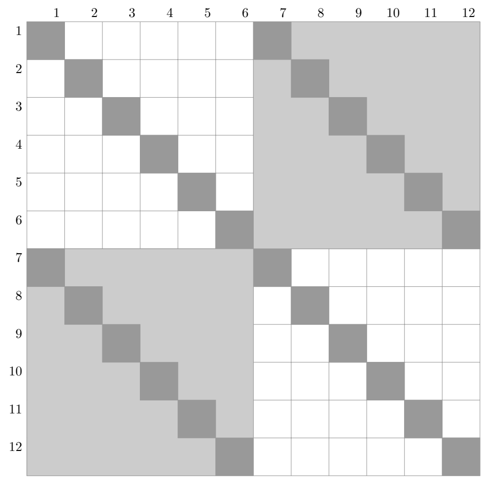

FIGURE 4.

Between variance-covariance matrix $\mathbf{\Phi_{OB}}$ of the LST-COM model, where 1=O_{t111}, 2=O^{CM}_{t121}, 3=O_{t121}, 4=O^{CM}_{t122}, 5=O_{t131}, 6=O^{CM}_{t123}, 7=O_{t211}, 8=O^{CM}_{t221}, 9=O_{t221}, 10=O^{CM}_{t222}, 11=O_{t231}, 12=O^{CM}_{t223}. Cells colored in white indicate zero covariances, cells colored in gray indicate permissible and interpretable variances and covariances among the latent variables. Cells in light gray indicate covariances among the latent variables that should be fixed to zero for parsimony.

$(k = 1)$ is chosen as reference method (without loss of generality), then the following mean structure holds for all $r \in R \equiv \{1, \ldots, a\}$, $t \in T \equiv \{1, \ldots, b\}$, $i \in I \equiv \{1, \ldots, c\}$, $j \in J \equiv \{1, \ldots, d\}$, $k \in K \equiv \{1, \ldots, e\}$, $l \in L \equiv \{1, \ldots, f\}$:

$$\mathbb{E}(Y_{tijkl}) = \alpha_{Tijkl} + \lambda_{Tijkl}\mathbb{E}(T_{tij1}), \tag{72}$$

$$\mathbb{E}(Y_{rtij2l}) = \alpha_{Tij2l} + \lambda_{Tij2l}\mathbb{E}(T_{tij1}). \tag{73}$$

$$\mathbb{E}(O_{tij1l}) = 0, \tag{74}$$

$$\mathbb{E}(T^{UM}_{rtij2}) = 0, \tag{75}$$

$$\mathbb{E}(T^{CM}_{tij2}) = 0, \tag{76}$$

$$\mathbb{E}(T^{M}_{tijk}) = 0, \qquad \forall\ k > 2, \tag{77}$$

$$\mathbb{E}(O^{UM}_{rtj2l}) = 0, \tag{78}$$

$$\mathbb{E}(O^{CM}_{tj2l}) = 0, \tag{79}$$

$$\mathbb{E}(O^{M}_{tjkl}) = 0, \qquad \forall\ k > 2, \tag{80}$$

$$\mathbb{E}(\epsilon_{tijkl}) = 0, \qquad \forall k\ \neq 2, \tag{81}$$

$$\mathbb{E}(\epsilon_{rtij2l}) = 0, \tag{82}$$

where $\mathbb{E}(.)$ denotes expected value.

Proof. Mean structure

The observed variables Y_{tij1l} pertaining to the reference method are given by:

$$Y_{tij1l} = \alpha_{Tij1l} + \lambda_{Tij1l} T_{tij1} + O_{tij1l} + \epsilon_{tij1l}.$$

The expectation (mean) of Y_{tij1l} is

$$\mathbb{E}(Y_{tij1l}) = \mathbb{E}(\alpha_{Tij1l}) + \mathbb{E}(\lambda_{Tij1l} T_{tij1}) + \mathbb{E}(O_{tij1l}) + \mathbb{E}(\epsilon_{tij1l}).$$

According to the Equations 74 and 81 in the above Theorem 2, it follows:

$$\mathbb{E}(Y_{tij1l}) = \alpha_{Tij1l} + \lambda_{Tij1l} \mathbb{E}(T_{tij1}).$$

Equations 74 and 81 in the above Theorem 2 state that the latent occasion-specific variables O_{tij1l} as well as the measurement error variables ϵ_{tij1l} are defined as residuals. If one sets α_{Tij1l} to zero and λ_{Tij1l} to one, it follows that $\mathbb{E}(Y_{tij1l}) = \mathbb{E}(T_{tij1})$. In a similar way, the observed variable Y_{rtij2l} of the interchangeable non reference method is given by

$$\begin{aligned} Y_{rtij2l} =& \alpha_{Tij2l} + \lambda_{Tij2l} T_{tij1} + \lambda^{CM}_{Tij2l} T^{CM}_{tij2} + \lambda^{UM}_{Tij2l} T^{UM}_{rtij2} + \\ & \lambda_{Oij2l} O_{tij1l} + \lambda^{CM}_{Oij2l} O^{CM}_{tj2l} + \lambda^{UM}_{Oij2l} O^{UM}_{rtj2l} + \epsilon_{rtij2l}. \end{aligned}$$

Hence, the expectation (mean) of Y_{rtij2l} is given by

$$\begin{aligned} \mathbb{E}(Y_{rtij2l}) =& \mathbb{E}(\alpha_{Tij2l}) + \mathbb{E}(\lambda_{Tij2l} T_{tij1}) + \mathbb{E}(\lambda^{CM}_{Tij2l} T^{CM}_{tij2}) + \mathbb{E}(\lambda^{UM}_{Tij2l} T^{UM}_{rtij2}) + \\ & \mathbb{E}(\lambda_{Oij2l} O_{tij1l}) + \mathbb{E}(\lambda^{CM}_{Oij2l} O^{CM}_{tj2l}) + \mathbb{E}(\lambda^{UM}_{Oij2l} O^{UM}_{rtj2l}) + \mathbb{E}(\epsilon_{rtij2l}). \end{aligned}$$

Again, the expected values of the T^{CM}_{tij2}, T^{UM}_{rtij2}, O_{tij1l}, O^{CM}_{tj2l}, O^{UM}_{rtj2l}, and ϵ_{rtij2l}-variables are zero with respect to the above theorem. Thus, the equation simplifies to (see Equation 73):

$$\mathbb{E}(Y_{rtij2l}) = \alpha_{Tij2l} + \lambda_{Tij2l} \mathbb{E}(T_{tij1}).$$

By definition of the LST-COM model the latent variables T^{CM}_{tij2}, T^{UM}_{rtij2}, O_{tij1l}, O^{CM}_{tj2l}, O^{UM}_{rtj2l}, and ϵ_{rtij2l} are defined as zero-mean residual variables. The observed variable Y_{tij3l} of the structurally different non reference method is decomposed into:

$$Y_{tij3l} = \alpha_{Tij3l} + \lambda_{Tij3l} T_{tij1} + \lambda^{M}_{Tij3l} T^{M}_{tij3} + \lambda_{Oij3l} O_{tij1l} + \lambda^{M}_{Oij3l} O^{M}_{tj3l} + \epsilon_{tij3l}.$$

The expectation (mean) of Y_{tij3l} is then given by

$$\begin{aligned} \mathbb{E}(Y_{tij3l}) =& E(\alpha_{Tij3l}) + \mathbb{E}(\lambda_{Tij3l} T_{tij1}) + \mathbb{E}(\lambda^{M}_{Tij3l} T^{M}_{tij3}) \\ & + \mathbb{E}(\lambda_{Oij3l} O_{tij1l}) + \mathbb{E}(\lambda^{M}_{Oij3l} O^{M}_{tj3l}) + \mathbb{E}(\epsilon_{tij3l}). \end{aligned}$$

According to the Equations 74, 77, 80, and 81 of the above Theorem 2, it follows that the expectations (means) of the latent variables O_{tij1l}, T^M_{tij3}, O^M_{tj3l} and ϵ_{tij3l} are zero. Thus, the above equation simplifies to (see Equation 72):

$$\mathbb{E}(Y_{tij3l}) = \alpha_{Tij3l} + \lambda_{Tij3l}\mathbb{E}(T_{tij1}).$$

Equations 74, 77, 80 and 81 follow by definition of the LST-COM model, given that O_{tij1l}, T^M_{tij3}, O^M_{tj3l} and ϵ_{tij3l} are defined as zero-mean residual variables (Steyer, 1989; Steyer & Eid, 2001). The intercepts (α_{Tijkl}) of the non reference method indicators are identified, if the factor loading parameters (λ_{Tijkl}) are set to a value greater than 0 (usually 1) or if the $\mathbb{E}(T_{tij1})$ is set to zero.

Remark 4. Equations 72 and 73 clarify that the means of the observed variables are equal to $\alpha_{Tijkl} + \lambda_{Tijkl}\mathbb{E}(T_{tij1})$ and $\alpha_{Tij2l} + \lambda_{Tij2l}\mathbb{E}(T_{tij1})$, respectively. Equations 74 to 80 reveal that the latent occasion-specific variables as well as the trait-specific and occasion-specific method factors are defined as residuals and therefore have an expected value of zero. The same holds for the measurement error variables (see Equation 81 and 82).

1.11. Identification

Theorem 3. (Identification of the LST-COM covariance structure) Let $\mathcal{M} \equiv \langle(\Omega, \mathscr{A}, \mathscr{P}), \mathbf{T_t}, \mathbf{T^{UM}_{rt}}, \mathbf{T^{CM}_{t}}, \mathbf{T^{M}_{t}}, \mathbf{O_t}, \mathbf{O^{UM}_{rt}}, \mathbf{O^{CM}_{t}}, \mathbf{O^{M}_{t}}, \boldsymbol{\epsilon}_{rt}, \boldsymbol{\epsilon}_{t}, \boldsymbol{\alpha}_T, \boldsymbol{\lambda}_T, \boldsymbol{\lambda}^{UM}_T, \boldsymbol{\lambda}^{CM}_T, \boldsymbol{\lambda}^{M}_T,$ $\boldsymbol{\lambda}_O, \boldsymbol{\lambda}^{UM}_O, \boldsymbol{\lambda}^{CM}_O, \boldsymbol{\lambda}^{M}_O\rangle$ be a LST-COM model with CRI, then the parameters of the vector $\boldsymbol{\mu}_{T_B}$ and the matrices $\boldsymbol{\Lambda}_{T_W}$, $\boldsymbol{\Phi}_{T_W}$, $\boldsymbol{\Lambda}_{O_W}$, $\boldsymbol{\Phi}_{O_W}$, $\boldsymbol{\Sigma}_{\boldsymbol{\theta}_W}$, $\boldsymbol{\Lambda}_{\mathbf{T_B}}$, $\boldsymbol{\Phi}_{\mathbf{T_B}}$, $\boldsymbol{\Lambda}_{\mathbf{O_B}}$, $\boldsymbol{\Phi}_{\mathbf{O_B}}$, $\sum_{\boldsymbol{\theta}_\mathbf{W}}$ are identified, if either one factor loading λ_{Tijkl}, λ^M_{Tijkl}, λ^{CM}_{Tij2l}, λ^{UM}_{Tij2l}, λ_{Oijkl}, λ^M_{Oijkl}, λ^{CM}_{Oij2l}, λ^{UM}_{Oij2l} for each factor T_{tij}, T^{CM}_{tij2}, T^{UM}_{rtij2}, O_{tijl}, O^{CM}_{tj2l}, O^{UM}_{rtj2l} or the variance of these factors are set to any real value larger than 0 (typically to 1), and

(a) iff $i = 2$, $j \geq 2$, $k \geq 2$, $l \geq 3$ and $\boldsymbol{\Phi}_{T_W}$, $\boldsymbol{\Phi}_{O_W}$, $\boldsymbol{\Phi}_{T_B}$, $\boldsymbol{\Phi}_{O_B}$ contain permissible correlations among the latent variables (i.e., nonzero elements in the off-diagonal), otherwise

(b) iff $i \geq 3$, $j \geq 1$, $k \geq 3$, $l \geq 3$.

Remark 5. According to the above Theorem 3 the LST-COM model parameter are uniquely identified for the minimal condition of two indicators, two constructs, two sets of methods (one structurally different and one set of interchangeable methods) and three occasions of measurement. Given that the between covariance matrix $\boldsymbol{\Sigma}_B$ of the LST-COM can be seen as restrictive variant of the total covariance matrix of the MM-LST model by Courvoisier (2006), the model identification for the parameter with respect to the between covariance matrix $\boldsymbol{\Sigma}_B$ is shown by Courvoisier (2006, chapter 5.4.11). The identification for the parameters of the within covariance matrix $\boldsymbol{\Sigma}_W$ is demonstrated for the case of a $2 \times 2 \times 2 \times 3$ measurement design (Koch, 2013).

```
Title:
        LST-COM Model Simulation
         1 Constructs
         2 Methods
         2 Occasions
Montecarlo:
        names =Y1111 Y2111 Y3111
          Y1112 Y2112 Y3112
          Y1121 Y2121 Y3121
          Y1122 Y2122 Y3122;
         nreps = 500;
         seed = 55719;
         repsave = ALL;
         save = Data*.dat;
         ncsizes = 1;
         csizes = 350 ( 2 );
         nobservations = 700 ;
         between =Y1111 Y2111 Y3111
                            Y1112 Y2112 Y3112;
        Model Population:
        %Within%
!UM State Loadings
UMS121 BY Y1121@1
          Y2121*1 (gZum212)
          Y3121*1 (gZum312);
UMS122 BY Y1122@1
          Y2122*1 (gZum212)
          Y3122*1 (gZum312);
!UM Trait Loadings
UMT112 BY Y1121@1
          Y1122@1;
UMT212 BY Y2121@1
          Y2122@1;
UMT312 BY Y3121@1
          Y3122@1;
!Latent Variances
UMS121*0.16;
UMS122*0.16;
UMT112*0.16;
UMT212*0.16;
UMT312*0.16;
!Latent Covariances
UMS121 with UMS122@0 UMT112@0 UMT212@0 UMT312@0 ;
```

```
UMS122 with UMT112@0 UMT212@0 UMT312@0 ;
UMT112 with UMT212*0.096 UMT312*0.096;
UMT212 with UMT312*0.096;
!Residual Variances
Y1121*0.19;
Y2121*0.19;
Y3121*0.19;
Y1122*0.19;
Y2122*0.19;
Y3122*0.19;
%Between%
!CM State Loadings
CMS121 BY Y1121@1
          Y2121*1 (gZcm212)
          Y3121*1 (gZcm312);
CMS122 BY Y1122@1
          Y2122*1 (gZcm212)
          Y3122*1 (gZcm312);
 !State Loadings
S111 BY Y1111@1
          Y2111*1 (lZ211)
          Y3111*1 (lZ311)
          Y1121*0.5833333 (lZ112)
          Y2121*0.5833333 (lZ212)
          Y3121*0.5833333 (lZ312);
S112 BY Y1112@1
          Y2112*1 (lZ211)
          Y3112*1 (lZ311)
          Y1122*0.5833333 (lZ112)
          Y2122*0.5833333 (lZ212)
          Y3122*0.5833333 (lZ312);
!CM Trait Loadings
CMT112 BY Y1121@1
          Y1122@1;
CMT212 BY Y2121@1
          Y2122@1;
CMT312 BY Y3121@1
          Y3122@1;
!Trait Loadings (Reference)
T111 BY Y1111@1
          Y1112@1
          Y1121*0.5(lX112)
          Y1122*0.5(lX112);
T211 BY Y2111@1
```

```
          Y2112@1
          Y2121*0.5(lX212)
          Y2122*0.5(lX212);
T311 BY Y3111@1
          Y3112@1
          Y3121*0.5(lX312)
          Y3122*0.5(lX312);
 !Latent Variances
S111*0.36;
 CMS121*0.1225;
 S112*0.36;
 CMS122*0.1225;
 T111*0.49;
 T211*0.49;
 T311*0.49;
 CMT112*0.1225;
 CMT212*0.1225;
 CMT312*0.1225;
 !Latent Covariances
S111 with CMS121@0 S112@0 CMS122@0 T111@0 T211@0 T311@0 CMT112@0 CMT212@0 CMT312@0;
CMS121 with S112@0 CMS122@0 T111@0 T211@0 T311@0 CMT112@0 CMT212@0 CMT312@0;
S112 with CMS122@0 T111@0 T211@0 T311@0 CMT112@0 CMT212@0 CMT312@0;
CMS122 with T111@0 T211@0 T311@0 CMT112@0 CMT212@0 CMT312@0;
T111 with T211*0.392 T311*0.392 CMT112@0 CMT212@0 CMT312@0;
T211 with T311*0.392 CMT112@0 CMT212@0 CMT312@0 ;
T311 with CMT112@0 CMT212@0 CMT312@0;
CMT112 with CMT212*0.0735 CMT312*0.0735;
CMT212 with CMT312*0.0735;
!Residual Variances
Y1111*0.15;
Y2111*0.15;
Y3111*0.15;
Y1112*0.15;
Y2112*0.15;
Y3112*0.15;
Y1121@0;
Y2121@0;
Y3121@0;
Y1122@0;
Y2122@0;
Y3122@0;
!Intercepts
 [Y1111@0];
 [Y2111@0];
```

```
  [Y3111@0];
  [Y1112@0];
  [Y2112@0];
  [Y3112@0];
  [Y1121@0];
  [Y2121@0];
  [Y3121@0];
  [Y1122@0];
  [Y2122@0];
  [Y3122@0];
!Latent Means
  [T111*0];
  [T211*0];
  [T311*0];
Analysis:
        Type=Twolevel;
        H1Iterations=15000;
        Processors=4;
Model:
! repeat model from above!
 Output: Tech9;
```

References

Courvoisier, D. S. (2006). *Unfolding the constituents of psychological scores: Development and application of mixture and multitrait-multimethod lst models.* (Doctoral dissertation). University of Geneva, Switzerland. Retrieved from `https://archive-ouverte.unige.ch/unige:447`

Koch, T. (2013). *Multilevel structural equation modeling of multitrait-multimethod-multioccasion data.* (Doctoral dissertation). Freie Universität Berlin, Germany.

Steyer, R. (1988). *Experiment, Regression und Kausalität. Die logische Struktur kausaler Regressionsmodelle [Experiment, regression, and causality. The logical structure of causal regression models].* (Habilitation thesis). Universität Trier, Germany.

Steyer, R. (1989). Models of classical psychometric test theory as stochastic measurement models: Representation, uniqueness, meaningfulness, identifiability, and testability. *Methodika*, *3*, 25-60.

Steyer, R., & Eid, M. (2001). *Messen und Testen. [Measurement and testing.]* (2nd ed.). Heidelberg: Springer.

Steyer, R., Mayer, A., Geiser, C., & Cole, D. A. (2015). A theory of states and traits—revised. *Annual Review of Clinical Psychology*, *11*(1), 71-98. (PMID: 25062476) doi: 10.1146/annurev-clinpsy-032813-153719

Steyer, R., Schmitt, M., & Eid, M. (1999). Latent state-trait theory and research in personality and individual differences. *European Journal of Personality*, *13*(5), 389-408. doi:

10.1002/(SICI)1099-0984(199909/10)13:53.0.CO;2-A