Model-Based Measures for Detecting and Quantifying Response Bias

Abstract

On-line supplement for the article “Model-Based Measures for Detecting and Quantifying Response Bias”. Contains tables of Type I error rates, power rates, bias, RMSE, and RE estimates from the Monte Carlo simulation study using the 2PL model. Also contains a brief simulation study for the SIBTEST family of statistics using an “all non-focal items as anchors” approach to demonstrate their behavior in the select IRT models which were under investigation.

Model-Based Measures for Detecting and Quantifying Response Bias

Detection Behavior when using All Non-studied Item as Anchors

According to Shealy and Stout (1993) and Li and Stout (1996), the SIBTEST family of statistics should consider all non-studied items as anchor items when investigating response bias. The authors suggest this approach because they claim the SIBTEST statistics remain robust when including items which contain DIF in the anchor item set, although Shealy and Stout did report that Type I error rates become slightly inflated and power to detect response bias slightly decreases. This section briefly re-investigates the robustness of these statistics using the above simulation conditions and DIF definitions.

Table 1 presents the Type I error rates when using all non-studied items as anchors for SIBTEST and CSIBTEST. This simulation included the five DIF items defined above to demonstrate the effect of including items with known DIF as anchors, as well as the behavior of these statistics when all anchor items contain no DIF. The results indicate that, when items with no DIF items are included in the anchor set, both SIBTEST and CSIBTEST are often able to achieve detection rates reasonably close to the nominal α level. With respect to testing for DIF, these results were consistent with those found by Chalmers (2018). CSIBTEST tended to have slightly higher Type I error control than SIBTEST, particularly for larger studied item sets, as well as in tests with fewer items, while both statistics were influenced by the latent trait distributions and the test length.

Finally, Table 1, as well as the associated power table in the on-line appendix, highlight the negative consequences when including contaminated anchor items. Specifically, the Type I error rates for both SIBTEST and CSIBTEST are often unacceptably liberal when biased items are included as anchor items. Detection rates tended to decrease at a rate proportional to the test length (or, analogously, as the number of non-DIF anchor items increased), increased more in larger studied item sets, were negatively influenced by the latent trait distribution, and increased in larger sample size conditions. Finally, power to detect response bias tended to decrease when including contaminated anchor items, which is consistent with the results reported by Shealy and

Stout (1993). Based on these results alone, the recommendation to use all non-studied items as anchor items should be considered with caution, particularly if unpredictably inflated or conservative Type I error control is costly to the test analyst.

References

Chalmers, R. P. (2018). Improving the Crossing-SIBTEST statistic for detecting non-uniform DIF. *Psychometrika*, *83*(2), 376–386. doi: 10.1007/s11336-017-9583-8

Li, H.-H., & Stout, W. (1996). A new procedure for detection of crossing DIF. *Psychometrika*, *61*(4), 647–677.

Shealy, R., & Stout, W. (1993). A model-based standardization approach that separates true bias/DIF from group ability differences and detect test bias/DTF as well as item bias/DIF. *Psychometrika*, *58*(2), 159–194.

Contaminated and Non-Contaminated SIBTEST Type I Error Rates

			$n_s = 1$				$n_s = 3$				$n_s = 5$			
			SIBTEST		CSIBTEST		SIBTEST		CSIBTEST		SIBTEST		CSIBTEST	
N_R/N_F	J	δ_θ	No-DIF	DIF	No-DIF	DIF	No-DIF	DIF	No-DIF	DIF	No-DIF	DIF	No-DIF	DIF
250/250	20	0	.040	.050	.052	.063	.051	.087	.067	.128	.059	.151	.081	.198
		0.5	.034	.039	.050	.053	.048	.078	.072	.110	.056	.103	.074	.167
	30	0	.029	.034	.038	.037	.035	.057	.055	.072	.047	.077	.077	.103
		0.5	.024	.024	.031	.029	.035	.027	.050	.049	.042	.034	.060	.063
	40	0	.019	.017	.021	.023	.047	.059	.058	.073	.052	.055	.070	.080
		0.5	.017	.015	.023	.017	.029	.011	.040	.024	.020	.015	.039	.036
500/250	20	0	.043	.062	.057	.083	.058	.118	.072	.161	.057	.168	.070	.226
		0.5	.044	.043	.063	.055	.059	.089	.083	.131	.077	.142	.109	.202
	30	0	.039	.040	.052	.047	.049	.079	.069	.087	.054	.094	.071	.115
		0.5	.032	.020	.042	.036	.026	.024	.040	.041	.027	.027	.035	.051
	40	0	.026	.019	.045	.027	.048	.056	.063	.071	.055	.062	.077	.089
		0.5	.019	.012	.026	.018	.010	.016	.026	.023	.007	.005	.023	.021
500/500	20	0	.055	.075	.072	.110	.059	.147	.086	.233	.043	.226	.072	.297
		0.5	.046	.050	.075	.082	.062	.111	.095	.171	.060	.182	.086	.249
	30	0	.049	.063	.063	.072	.050	.099	.056	.122	.057	.140	.082	.167
		0.5	.040	.034	.050	.047	.034	.033	.057	.063	.041	.067	.067	.103
	40	0	.033	.036	.040	.045	.045	.055	.060	.078	.048	.080	.067	.103
		0.5	.031	.027	.035	.034	.028	.026	.043	.042	.023	.024	.043	.043
1000/500	20	0	.055	.077	.070	.098	.054	.176	.080	.262	.050	.288	.067	.393
		0.5	.048	.053	.072	.085	.049	.129	.079	.199	.057	.223	.087	.288
	30	0	.049	.079	.061	.096	.057	.119	.081	.147	.056	.161	.082	.201
		0.5	.037	.026	.047	.040	.038	.048	.057	.070	.045	.066	.068	.099
	40	0	.036	.035	.058	.050	.042	.066	.061	.082	.042	.087	.066	.125
		0.5	.031	.029	.047	.043	.017	.012	.026	.034	.015	.016	.029	.047
1000/1000	20	0	.050	.095	.067	.133	.040	.229	.066	.318	.056	.393	.085	.490
		0.5	.048	.071	.067	.106	.047	.178	.070	.243	.046	.289	.064	.394
	30	0	.049	.074	.060	.092	.051	.132	.071	.174	.048	.200	.079	.248
		0.5	.036	.053	.052	.076	.039	.089	.070	.113	.056	.127	.078	.142
	40	0	.041	.060	.049	.065	.049	.092	.064	.123	.038	.132	.070	.170
		0.5	.035	.048	.053	.066	.036	.041	.063	.074	.030	.055	.043	.087
2000/1000	20	0	.052	.136	.070	.187	.058	.314	.080	.428	.053	.537	.082	.653
		0.5	.060	.084	.082	.118	.065	.223	.088	.300	.073	.380	.107	.457
	30	0	.042	.068	.065	.085	.051	.151	.058	.204	.058	.225	.065	.291
		0.5	.047	.039	.076	.064	.050	.078	.070	.108	.058	.144	.092	.159
	40	0	.052	.067	.067	.085	.046	.111	.065	.149	.053	.139	.063	.212
		0.5	.039	.049	.052	.063	.042	.048	.055	.066	.035	.059	.059	.113

Table 1

Type I error rate estimates for SIBTEST and CSIBTEST when using all non-studied items as anchors. The No-DIF/DIF columns indicate whether or not the five defined DIF items were included in the anchor set.

Contaminated SIBTEST Power Rates

N_R/N_F	J	δ_θ	DIF1	DIF2	DIF3	DIF4	DIF5	DBF3a	DBF3b	DBF5
250/250	20	0	.546	.312	.112	.045	.055	.849	.068	.696
		0.5	.477	.239	.075	.046	.088	.638	.067	.375
	30	0	.499	.260	.113	.052	.037	.821	.106	.721
		0.5	.367	.219	.082	.021	.052	.400	.033	.221
	40	0	.357	.173	.079	.038	.034	.781	.109	.651
		0.5	.296	.149	.057	.021	.031	.302	.035	.165
500/250	20	0	.697	.407	.164	.060	.062	.925	.108	.807
		0.5	.574	.316	.089	.043	.108	.727	.052	.550
	30	0	.675	.382	.160	.060	.037	.897	.113	.829
		0.5	.343	.214	.075	.033	.070	.290	.040	.223
	40	0	.594	.316	.135	.040	.039	.837	.109	.744
		0.5	.223	.133	.046	.025	.043	.148	.019	.096
500/500	20	0	.889	.613	.252	.059	.058	.989	.121	.929
		0.5	.857	.502	.135	.068	.203	.951	.075	.714
	30	0	.888	.625	.285	.082	.072	.987	.173	.945
		0.5	.653	.451	.141	.043	.134	.649	.040	.567
	40	0	.885	.630	.263	.069	.040	.971	.200	.934
		0.5	.552	.398	.148	.036	.097	.427	.035	.349
1000/500	20	0	.963	.732	.310	.065	.090	.997	.142	.977
		0.5	.893	.590	.218	.074	.196	.957	.080	.833
	30	0	.962	.750	.328	.093	.052	.995	.206	.987
		0.5	.577	.467	.173	.039	.149	.570	.047	.626
	40	0	.958	.749	.375	.099	.066	.963	.270	.969
		0.5	.429	.330	.148	.037	.087	.265	.025	.253
1000/1000	20	0	.994	.900	.452	.086	.118	1.000	.201	.999
		0.5	.988	.827	.274	.077	.378	.997	.084	.950
	30	0	.996	.905	.510	.113	.071	.999	.285	1.000
		0.5	.843	.749	.278	.049	.265	.897	.051	.915
	40	0	.996	.909	.542	.135	.071	.999	.381	1.000
		0.5	.662	.644	.261	.046	.206	.568	.042	.622
2000/1000	20	0	1.000	.961	.553	.101	.126	1.000	.218	1.000
		0.5	.993	.891	.352	.067	.386	1.000	.061	.989
	30	0	1.000	.969	.641	.147	.085	1.000	.378	1.000
		0.5	.793	.763	.347	.041	.268	.866	.048	.933
	40	0	.993	.976	.685	.180	.068	.995	.464	.996
		0.5	.527	.534	.302	.054	.194	.463	.046	.564

Table 2

Power rates for the compensatory SIBTEST statistic when using all non-studied items as anchors.

N_R/N_F	J	δ_θ	DIF1	DIF2	DIF3	DIF4	DIF5	DBF3a	DBF3b	DBF5
250/250	20	0	.554	.336	.190	.183	.248	.862	.660	.845
		0.5	.488	.259	.130	.140	.251	.662	.515	.624
	30	0	.504	.262	.129	.114	.113	.832	.568	.799
		0.5	.372	.227	.095	.070	.108	.431	.345	.430
	40	0	.351	.176	.092	.059	.101	.778	.611	.833
		0.5	.292	.151	.066	.045	.074	.335	.355	.464
500/250	20	0	.698	.438	.320	.326	.471	.936	.818	.918
		0.5	.580	.339	.184	.228	.416	.747	.620	.764
	30	0	.677	.416	.285	.258	.349	.899	.760	.905
		0.5	.344	.239	.138	.148	.254	.337	.378	.540
	40	0	.586	.329	.228	.280	.391	.846	.845	.936
		0.5	.229	.158	.088	.123	.253	.199	.403	.532
500/500	20	0	.887	.634	.421	.448	.646	.987	.945	.985
		0.5	.853	.533	.303	.418	.652	.954	.902	.932
	30	0	.879	.633	.381	.353	.532	.985	.923	.974
		0.5	.654	.477	.257	.260	.481	.696	.753	.852
	40	0	.879	.608	.379	.356	.524	.979	.976	.998
		0.5	.554	.415	.240	.286	.507	.524	.802	.909
1000/500	20	0	.962	.757	.541	.629	.848	.996	.982	.998
		0.5	.899	.610	.395	.537	.805	.958	.941	.954
	30	0	.962	.766	.542	.598	.796	.996	.988	.996
		0.5	.577	.509	.377	.394	.665	.609	.805	.931
	40	0	.952	.748	.611	.695	.849	.973	.998	1.000
		0.5	.430	.386	.343	.494	.669	.365	.845	.959
1000/1000	20	0	.993	.912	.706	.825	.961	1.000	.999	1.000
		0.5	.987	.853	.547	.761	.949	.997	.996	.998
	30	0	.996	.904	.712	.760	.938	.999	.998	1.000
		0.5	.842	.767	.544	.635	.882	.906	.968	.995
	40	0	.995	.911	.761	.864	.979	.999	1.000	1.000
		0.5	.664	.701	.565	.755	.914	.657	.979	.996
2000/1000	20	0	1.000	.965	.836	.926	.997	1.000	1.000	1.000
		0.5	.992	.886	.657	.858	.987	1.000	1.000	1.000
	30	0	1.000	.971	.860	.918	.984	1.000	.999	1.000
		0.5	.793	.781	.676	.757	.925	.871	.993	1.000
	40	0	.993	.977	.915	.975	.997	.995	1.000	1.000
		0.5	.527	.633	.760	.845	.905	.535	.992	.999

Table 3

Power rates for the non-compensatory Crossing-SIBTEST statistic when using all non-studied items as anchors.

Type I Error Rates with Unequal Sample Sizes

				$n_s = 1$		$n_s = 3$		$n_s = 5$		$n_s = J - n_a$	
N_R/N_F	δ_θ	n_a	J	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB
500/250	0	5	20	.057	.055	.049	.076	.055	.075	.048	.095
			30	.048	.061	.034	.059	.039	.064	.043	.080
			40	.045	.046	.051	.068	.052	.072	.060	.105
		10	20	.050	.054	.040	.057	.046	.066	.040	.064
			30	.049	.048	.047	.060	.054	.063	.057	.079
			40	.053	.061	.052	.069	.053	.061	.042	.065
	0.5	5	20	.047	.078	.044	.084	.045	.102	.050	.140
			30	.047	.088	.054	.101	.051	.108	.058	.163
			40	.055	.073	.054	.095	.080	.125	.049	.169
		10	20	.048	.056	.057	.082	.058	.097	.057	.105
			30	.061	.066	.045	.068	.051	.077	.053	.112
			40	.048	.055	.052	.068	.051	.078	.045	.115
1000/500	0	5	20	.058	.055	.061	.064	.054	.076	.049	.089
			30	.050	.051	.045	.064	.049	.069	.052	.100
			40	.061	.052	.051	.073	.055	.080	.057	.098
		10	20	.047	.048	.048	.051	.049	.058	.052	.068
			30	.056	.059	.064	.065	.054	.060	.050	.079
			40	.042	.048	.045	.051	.045	.055	.050	.075
	0.5	5	20	.048	.072	.048	.100	.055	.112	.055	.145
			30	.059	.072	.060	.114	.054	.121	.056	.164
			40	.047	.073	.051	.086	.044	.104	.045	.152
		10	20	.054	.055	.043	.066	.047	.086	.054	.098
			30	.042	.056	.056	.075	.058	.079	.045	.094
			40	.048	.060	.046	.077	.054	.082	.054	.106
2000/1000	0	5	20	.052	.071	.053	.080	.053	.082	.045	.081
			30	.056	.073	.059	.073	.054	.084	.058	.115
			40	.052	.058	.036	.057	.054	.075	.034	.092
		10	20	.055	.056	.059	.056	.050	.059	.051	.067
			30	.039	.045	.058	.064	.060	.076	.052	.066
			40	.046	.048	.045	.051	.048	.057	.049	.073
	0.5	5	20	.054	.077	.049	.095	.037	.102	.038	.131
			30	.048	.073	.037	.086	.055	.121	.054	.166
			40	.039	.065	.047	.088	.048	.105	.052	.161
		10	20	.053	.059	.049	.070	.055	.076	.050	.083
			30	.038	.060	.043	.071	.059	.079	.052	.105
			40	.048	.056	.065	.079	.053	.073	.059	.118

Table 4

Type I error rate estimates for detecting compensatory response bias with the CDRF and SIBTEST (SIB) statistics.

				$n_s = 1$				$n_s = 3$		$n_s = 5$		$n_s = J - n_a$	
N_R/N_F	δ_θ	n_a	J	LR	Wald	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB
500/250	0	5	20	.054	.046	.047	.071	.044	.090	.044	.089	.047	.126
			30	.049	.041	.034	.081	.028	.086	.029	.084	.039	.108
			40	.053	.043	.042	.069	.056	.092	.043	.098	.044	.142
		10	20	.049	.042	.041	.066	.036	.079	.044	.081	.047	.083
			30	.056	.047	.036	.065	.049	.083	.042	.091	.051	.118
			40	.050	.050	.029	.084	.043	.093	.040	.085	.035	.099
	0.5	5	20	.041	.041	.034	.098	.049	.103	.037	.119	.038	.169
			30	.047	.043	.045	.107	.044	.121	.036	.136	.032	.206
			40	.050	.046	.046	.087	.045	.120	.051	.146	.040	.209
		10	20	.045	.045	.040	.076	.049	.109	.051	.134	.054	.147
			30	.063	.059	.053	.090	.039	.100	.046	.112	.053	.160
			40	.048	.044	.043	.074	.037	.095	.036	.112	.041	.161
1000/500	0	5	20	.057	.060	.052	.066	.044	.083	.047	.093	.044	.110
			30	.058	.050	.045	.065	.038	.086	.044	.093	.042	.125
			40	.061	.057	.052	.067	.031	.093	.033	.109	.043	.126
		10	20	.053	.048	.045	.070	.037	.065	.043	.072	.040	.097
			30	.049	.047	.041	.075	.054	.091	.044	.089	.044	.116
			40	.049	.046	.032	.069	.033	.071	.042	.073	.037	.104
	0.5	5	20	.050	.043	.047	.085	.044	.122	.047	.135	.046	.171
			30	.060	.054	.053	.093	.045	.136	.050	.150	.054	.192
			40	.052	.050	.044	.098	.051	.116	.040	.125	.040	.188
		10	20	.050	.047	.044	.075	.051	.102	.052	.109	.054	.129
			30	.045	.040	.039	.083	.055	.097	.046	.107	.054	.129
			40	.039	.041	.035	.093	.043	.106	.047	.114	.038	.151
2000/1000	0	5	20	.062	.056	.055	.085	.050	.106	.051	.100	.038	.106
			30	.058	.055	.055	.088	.048	.096	.044	.107	.048	.142
			40	.045	.046	.040	.076	.023	.075	.038	.095	.029	.117
		10	20	.058	.056	.046	.074	.041	.076	.051	.089	.046	.096
			30	.039	.036	.030	.058	.040	.089	.049	.098	.042	.095
			40	.041	.039	.035	.067	.022	.072	.033	.082	.038	.103
	0.5	5	20	.048	.047	.048	.088	.047	.116	.036	.125	.036	.159
			30	.046	.051	.036	.091	.031	.110	.038	.143	.039	.204
			40	.054	.054	.049	.086	.032	.110	.037	.136	.043	.187
		10	20	.059	.056	.058	.080	.052	.093	.052	.102	.047	.133
			30	.040	.035	.032	.075	.038	.104	.049	.111	.047	.143
			40	.053	.049	.048	.078	.050	.109	.052	.098	.044	.158

Table 5

Type I error rate estimates for detecting non-compensatory response bias with the likelihood ratio (LR), Wald, NCDRF, and CSIBTEST (CSIB) methods.

Power Rates with Unequal Sample Sizes

				DBF3a		DBF3b		DBF5		DTF	
N_R/N_F	δ_θ	n_a	J	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB
500/250	0	5	20	.847	.825	.924	.352	.896	.688	.247	.232
			30	.849	.840	.916	.313	.897	.667	.113	.164
			40	.868	.845	.939	.350	.923	.683	.081	.168
		10	20	.915	.917	.953	.699	.963	.858	.597	.496
			30	.906	.909	.965	.693	.964	.855	.229	.250
			40	.896	.896	.962	.692	.956	.834	.141	.192
	0.5	5	20	.706	.719	.802	.295	.710	.493	.164	.218
			30	.731	.760	.846	.282	.732	.497	.089	.224
			40	.740	.735	.850	.291	.738	.487	.068	.219
		10	20	.824	.836	.896	.587	.840	.693	.428	.382
			30	.812	.838	.914	.593	.846	.668	.189	.237
			40	.804	.814	.927	.590	.851	.631	.106	.216
1000/500	0	5	20	.993	.985	.996	.544	.993	.902	.425	.354
			30	.986	.984	1.000	.563	.998	.903	.197	.233
			40	.994	.988	.998	.588	1.000	.909	.123	.193
		10	20	.996	.995	1.000	.949	.999	.986	.896	.763
			30	.998	.995	1.000	.947	1.000	.989	.456	.401
			40	.998	.997	1.000	.945	1.000	.993	.227	.272
	0.5	5	20	.967	.949	.984	.452	.949	.755	.332	.314
			30	.970	.954	.993	.459	.966	.759	.149	.211
			40	.964	.947	.992	.454	.967	.741	.107	.249
		10	20	.975	.978	.999	.892	.985	.903	.727	.592
			30	.988	.990	1.000	.888	.991	.926	.303	.320
			40	.987	.986	1.000	.894	.994	.922	.172	.253
2000/1000	0	5	20	1.000	1.000	1.000	.860	1.000	.995	.758	.580
			30	1.000	.999	1.000	.853	1.000	.994	.388	.350
			40	1.000	1.000	1.000	.843	1.000	.992	.204	.229
		10	20	1.000	1.000	1.000	1.000	1.000	1.000	.999	.965
			30	1.000	1.000	1.000	.999	1.000	1.000	.753	.649
			40	1.000	1.000	1.000	.999	1.000	1.000	.432	.431
	0.5	5	20	.998	.995	1.000	.704	.997	.951	.556	.424
			30	.998	.997	1.000	.704	1.000	.934	.260	.311
			40	.999	.999	1.000	.691	1.000	.943	.148	.279
		10	20	1.000	1.000	1.000	.996	1.000	.995	.954	.851
			30	1.000	1.000	1.000	.997	1.000	.997	.577	.501
			40	1.000	1.000	1.000	.994	1.000	.997	.328	.351

Table 6

Power rate estimates for detecting non-compensatory response bias (DBF and DTF) with the NCDRF and CSIBTEST (CSIB) statistics.

				DIF1				DIF2				DIF3				DIF4				DIF5			
N_R/N_F	δ_θ	n_a	J	LR	Wald	NCDRF	CSIB	LR	Wald	NCDRF	CSIB	LR	Wald	NCDRF	CSIB	LR	Wald	NCDRF	CSIB	LR	Wald	NCDRF	CSIB
500/250	0	5	20	.708	.693	.691	.702	.509	.518	.490	.464	.523	.561	.453	.259	.638	.697	.576	.177	.822	.856	.762	.206
			30	.696	.677	.685	.718	.505	.516	.474	.477	.510	.542	.429	.244	.679	.705	.589	.160	.830	.863	.764	.204
			40	.720	.706	.711	.709	.542	.557	.503	.483	.539	.566	.442	.264	.664	.705	.559	.181	.862	.890	.776	.201
		10	20	.710	.696	.695	.767	.549	.558	.522	.546	.535	.558	.472	.357	.655	.674	.574	.331	.827	.848	.771	.425
			30	.741	.738	.746	.795	.537	.549	.487	.520	.566	.582	.466	.350	.721	.747	.625	.320	.880	.890	.817	.450
			40	.730	.720	.733	.787	.534	.543	.499	.538	.570	.584	.472	.331	.717	.739	.637	.320	.895	.906	.841	.476
	0.5	5	20	.680	.667	.655	.690	.362	.386	.361	.401	.309	.367	.304	.194	.505	.558	.462	.159	.737	.774	.694	.249
			30	.668	.670	.672	.694	.344	.375	.334	.395	.354	.413	.315	.202	.538	.592	.480	.154	.783	.813	.721	.240
			40	.684	.681	.653	.700	.362	.399	.351	.410	.371	.417	.309	.195	.549	.613	.479	.172	.822	.846	.764	.249
		10	20	.708	.706	.695	.778	.399	.418	.390	.463	.340	.378	.312	.240	.545	.577	.492	.278	.801	.821	.739	.426
			30	.694	.687	.673	.751	.416	.438	.394	.478	.369	.407	.321	.249	.590	.620	.520	.274	.845	.863	.791	.469
			40	.693	.693	.661	.739	.409	.422	.393	.487	.387	.418	.316	.243	.613	.654	.511	.260	.856	.876	.791	.458
1000/500	0	5	20	.952	.948	.946	.947	.793	.803	.767	.726	.807	.827	.757	.420	.910	.926	.874	.253	.988	.991	.980	.343
			30	.949	.946	.941	.939	.799	.808	.763	.721	.846	.861	.790	.442	.942	.949	.905	.291	.988	.991	.978	.356
			40	.949	.947	.951	.942	.829	.839	.783	.742	.826	.849	.770	.428	.946	.958	.914	.278	.987	.991	.977	.364
		10	20	.959	.959	.957	.976	.833	.840	.801	.824	.832	.851	.785	.595	.934	.942	.913	.576	.994	.995	.990	.760
			30	.961	.959	.961	.974	.834	.847	.803	.811	.863	.873	.802	.561	.943	.950	.913	.570	.995	.995	.989	.774
			40	.967	.967	.967	.984	.842	.846	.810	.821	.885	.894	.820	.620	.951	.955	.923	.565	.994	.994	.990	.748
	0.5	5	20	.936	.938	.925	.932	.660	.685	.660	.679	.551	.597	.533	.311	.783	.824	.759	.208	.969	.979	.953	.391
			30	.932	.933	.923	.930	.651	.678	.636	.663	.590	.642	.544	.314	.868	.896	.827	.237	.976	.981	.967	.378
			40	.935	.934	.922	.927	.666	.681	.655	.660	.647	.687	.593	.319	.855	.887	.814	.212	.984	.988	.979	.409
		10	20	.945	.944	.946	.966	.648	.664	.635	.673	.611	.643	.577	.438	.842	.862	.809	.506	.984	.986	.979	.764
			30	.955	.955	.949	.970	.724	.741	.708	.741	.661	.688	.617	.422	.866	.883	.829	.494	.987	.990	.980	.781
			40	.943	.942	.942	.967	.708	.721	.690	.740	.678	.705	.623	.453	.902	.915	.854	.485	.990	.992	.985	.749
2000/1000	0	5	20	1.000	.999	.999	.998	.978	.978	.975	.946	.976	.981	.971	.691	.999	.999	.994	.488	1.000	1.000	1.000	.612
			30	.999	.999	.999	.997	.975	.978	.971	.951	.984	.986	.979	.696	.999	.999	.997	.505	1.000	1.000	1.000	.608
			40	.999	.999	1.000	.997	.980	.983	.977	.944	.987	.991	.976	.671	.999	.999	.996	.484	1.000	1.000	1.000	.640
		10	20	.999	.999	1.000	1.000	.989	.992	.985	.974	.996	.997	.989	.850	1.000	1.000	.998	.890	1.000	1.000	1.000	.976
			30	1.000	1.000	1.000	1.000	.989	.989	.986	.980	.995	.996	.987	.854	1.000	1.000	1.000	.891	1.000	1.000	1.000	.970
			40	.999	.999	.999	.999	.988	.989	.989	.984	.989	.989	.983	.880	1.000	1.000	1.000	.883	1.000	1.000	1.000	.973
	0.5	5	20	.998	.998	.997	.999	.931	.935	.931	.910	.860	.879	.850	.503	.983	.988	.976	.362	1.000	1.000	1.000	.703
			30	1.000	1.000	1.000	1.000	.912	.919	.910	.908	.890	.910	.874	.467	.994	.995	.992	.386	1.000	1.000	1.000	.673
			40	.999	1.000	1.000	.998	.923	.927	.915	.910	.913	.926	.893	.520	.994	.996	.990	.387	1.000	1.000	1.000	.681
		10	20	1.000	1.000	1.000	1.000	.930	.936	.927	.940	.897	.906	.885	.711	.989	.992	.987	.803	1.000	1.000	1.000	.974
			30	.999	.999	.998	.998	.946	.950	.944	.945	.910	.916	.893	.672	.995	.997	.991	.801	1.000	1.000	1.000	.964
			40	.999	.999	.999	1.000	.956	.963	.946	.954	.942	.951	.924	.692	.993	.993	.991	.797	1.000	1.000	1.000	.965

Table 7

Power rate estimates for detecting non-compensatory response bias with the LR, Wald, NCDRF, and CSIBTEST (CSIB) methods.

				DIF1		DIF2		DIF3		DIF4		DIF5		DBF3a		DBF3b		DBF5		DTF	
N_R/N_F	δ_θ	n_a	J	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB
500/250	0	5	20	.784	.698	.524	.458	.263	.231	.108	.100	.065	.074	.867	.822	.174	.179	.687	.644	.164	.199
			30	.788	.717	.526	.469	.241	.224	.097	.091	.056	.068	.865	.836	.165	.166	.665	.634	.092	.130
			40	.800	.706	.558	.476	.280	.238	.104	.100	.042	.058	.873	.843	.179	.196	.693	.655	.089	.138
		10	20	.777	.756	.556	.527	.274	.263	.110	.105	.053	.056	.921	.911	.198	.207	.771	.755	.386	.414
			30	.815	.794	.538	.508	.269	.252	.106	.096	.054	.057	.928	.907	.187	.180	.788	.767	.152	.193
			40	.811	.789	.546	.520	.257	.253	.103	.098	.061	.063	.908	.900	.176	.184	.750	.739	.122	.150
	0.5	5	20	.753	.678	.419	.395	.163	.175	.060	.093	.064	.101	.764	.715	.077	.105	.458	.461	.105	.185
			30	.747	.684	.400	.380	.170	.179	.069	.089	.074	.100	.789	.753	.075	.119	.463	.462	.074	.180
			40	.747	.680	.418	.400	.154	.166	.068	.096	.071	.104	.790	.727	.064	.114	.465	.445	.060	.178
		10	20	.776	.763	.449	.445	.170	.176	.067	.062	.071	.090	.860	.828	.072	.083	.601	.555	.269	.282
			30	.760	.737	.463	.480	.179	.175	.063	.061	.092	.094	.861	.834	.085	.107	.561	.541	.107	.157
			40	.766	.727	.471	.462	.173	.174	.063	.064	.077	.084	.849	.812	.078	.090	.587	.523	.093	.151
1000/500	0	5	20	.970	.947	.804	.724	.440	.384	.140	.142	.046	.057	.994	.985	.274	.268	.921	.887	.286	.324
			30	.974	.939	.788	.719	.453	.416	.145	.154	.057	.069	.993	.983	.273	.276	.925	.876	.156	.199
			40	.969	.942	.803	.737	.443	.396	.147	.146	.051	.060	.993	.988	.280	.286	.924	.894	.092	.153
		10	20	.981	.976	.834	.817	.469	.452	.140	.149	.052	.058	.997	.995	.325	.311	.965	.962	.668	.665
			30	.984	.976	.819	.795	.458	.433	.151	.145	.063	.060	.997	.995	.325	.308	.969	.963	.269	.307
			40	.987	.984	.834	.819	.481	.478	.151	.136	.067	.066	.997	.997	.326	.308	.968	.961	.157	.192
	0.5	5	20	.962	.929	.716	.672	.271	.279	.062	.087	.095	.121	.975	.949	.087	.137	.743	.726	.198	.285
			30	.953	.927	.686	.658	.288	.287	.064	.090	.097	.126	.980	.954	.092	.128	.760	.731	.104	.173
			40	.958	.923	.704	.651	.282	.292	.060	.085	.092	.122	.967	.947	.115	.145	.759	.710	.097	.203
		10	20	.969	.967	.708	.677	.302	.307	.076	.093	.091	.125	.986	.981	.104	.121	.836	.817	.439	.469
			30	.980	.963	.770	.735	.303	.294	.058	.081	.109	.135	.991	.988	.091	.108	.863	.833	.199	.229
			40	.969	.958	.744	.722	.308	.311	.065	.070	.101	.125	.987	.983	.103	.112	.848	.815	.119	.168
2000/1000	0	5	20	1.000	.998	.975	.946	.726	.662	.253	.238	.047	.088	1.000	1.000	.492	.486	.996	.993	.498	.553
			30	1.000	.997	.974	.950	.716	.651	.243	.246	.052	.066	1.000	.999	.479	.469	.998	.989	.240	.307
			40	1.000	.997	.982	.944	.734	.639	.241	.230	.048	.075	1.000	1.000	.482	.435	.997	.986	.149	.195
		10	20	1.000	1.000	.987	.977	.755	.744	.242	.244	.053	.050	1.000	1.000	.549	.545	1.000	1.000	.922	.920
			30	1.000	1.000	.978	.982	.765	.752	.272	.265	.056	.054	1.000	1.000	.577	.567	1.000	1.000	.507	.538
			40	.999	.999	.989	.985	.763	.751	.282	.276	.064	.052	1.000	1.000	.582	.580	1.000	1.000	.292	.334
	0.5	5	20	.999	.999	.954	.910	.516	.471	.064	.078	.149	.195	.998	.995	.116	.139	.971	.941	.324	.397
			30	.999	.998	.932	.905	.465	.429	.081	.105	.151	.178	1.000	.997	.132	.152	.952	.922	.181	.273
			40	.999	.997	.950	.909	.514	.494	.097	.115	.139	.188	1.000	.999	.128	.179	.959	.928	.125	.240
		10	20	1.000	1.000	.953	.938	.541	.524	.081	.089	.162	.184	1.000	1.000	.156	.140	.989	.985	.772	.751
			30	.999	.998	.962	.939	.495	.483	.073	.085	.161	.187	1.000	1.000	.133	.136	.993	.978	.346	.360
			40	1.000	1.000	.964	.948	.526	.500	.095	.108	.146	.174	1.000	1.000	.156	.158	.992	.980	.197	.235

Table 8

Power rate estimates for detecting compensatory response bias with the CDRF and SIBTEST (SIB) statistics.

Type I Error Study Bias Estimates

				$n_s = 1$		$n_s = 3$		$n_s = 5$		$n_s = J - n_a$	
N_R/N_F	δ_θ	n_a	J	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB
250/250	0	5	20	.001	.001	.002	.002	.002	.001	.004	.000
			30	.002	.003	.000	.001	.001	.004	.007	.009
			40	.003	.004	.006	.008	.007	.011	.046	.056
		10	20	-.001	-.001	.001	.002	.000	.002	.001	.003
			30	-.001	.000	-.001	.000	-.001	.000	-.001	.003
			40	.003	.004	.003	.004	.002	.003	.015	.016
	0.5	5	20	-.001	.001	-.002	.004	.002	.012	.003	.035
			30	.000	.001	.001	.006	.003	.011	.009	.051
			40	.001	.003	.001	.007	-.001	.009	-.031	.033
		10	20	-.002	-.003	-.003	.000	-.005	.002	-.003	.011
			30	.000	-.002	.002	.005	.001	.006	-.003	.022
			40	-.001	-.002	.003	.006	.005	.011	.019	.056
500/500	0	5	20	.001	.001	.002	.002	.003	.005	.009	.013
			30	-.001	.000	-.003	-.004	-.004	-.004	.003	-.002
			40	.000	.001	.001	.002	.004	.004	.021	.019
		10	20	-.001	-.001	-.001	.000	.000	.000	.000	.001
			30	.001	.001	.002	.002	.002	.002	.003	.006
			40	.001	.001	.001	.000	.000	-.002	.007	.004
	0.5	5	20	.003	.004	.003	.008	.002	.012	.004	.033
			30	.000	.001	-.001	.003	-.001	.005	-.008	.026
			40	.000	.002	.001	.005	.002	.009	.036	.085
		10	20	-.001	-.001	-.002	.000	-.002	.002	-.005	.003
			30	.001	.002	-.001	.001	.001	.007	-.005	.012
			40	.000	.000	-.001	.001	-.001	.003	.003	.027
1000/1000	0	5	20	.000	-.001	.000	.001	.000	.001	.003	.006
			30	.000	.001	.001	.000	.004	.002	.012	.006
			40	-.001	.000	.000	.001	-.002	.001	-.021	-.006
		10	20	.001	.001	.002	.002	.004	.004	.005	.005
			30	.000	.000	.001	.001	.002	.003	.001	.003
			40	.001	.001	.000	-.001	.000	.000	-.001	-.003
	0.5	5	20	-.002	-.001	.001	.004	.003	.008	.009	.019
			30	-.001	.000	-.001	.003	-.001	.004	-.002	.022
			40	.001	.002	.002	.005	.003	.007	.011	.040
		10	20	.001	.001	.001	.002	.002	.005	.002	.007
			30	.000	.001	.001	.003	.000	.004	.000	.011
			40	.001	.002	.002	.003	.002	.005	.003	.021

Table 9

Bias estimates given $\beta_C = 0$ for the CDRF and SIBTEST (SIB) methods.

				$n_s = 1$		$n_s = 3$		$n_s = 5$		$n_s = J - n_a$	
N_R/N_F	δ_θ	n_a	J	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB
500/250	0	5	20	.000	.000	.000	.000	.000	.001	-.003	-.007
			30	.000	-.001	.000	.000	-.002	-.003	-.009	-.021
			40	.001	.001	.003	.002	.003	.002	.030	.024
		10	20	.000	.000	.000	.000	.001	.000	.002	.001
			30	.001	.001	.002	.003	.002	.003	.003	.006
			40	.000	.000	-.001	-.001	.001	.002	.007	.014
	0.5	5	20	-.002	.000	-.002	.004	-.002	.011	.001	.044
			30	-.001	.001	-.003	.006	-.003	.010	-.003	.055
			40	.000	.002	.000	.006	-.001	.011	-.014	.058
		10	20	.000	-.001	-.001	.002	-.003	.002	.002	.015
			30	-.001	-.003	-.002	.001	-.001	.006	.003	.029
			40	.000	-.001	.000	.004	-.005	.002	-.012	.033
1000/500	0	5	20	-.002	-.002	-.001	-.001	-.003	-.003	-.003	-.005
			30	.000	.000	-.001	-.001	.000	.001	-.002	.003
			40	.001	.001	-.001	-.001	.000	-.001	.000	-.007
		10	20	.000	-.001	-.002	-.002	-.003	-.003	-.004	-.005
			30	-.001	-.001	-.002	-.003	-.003	-.003	-.010	-.012
			40	.000	.000	-.001	-.001	-.001	-.002	-.008	-.014
	0.5	5	20	-.001	.000	-.001	.003	-.002	.004	-.007	.008
			30	.001	.003	.001	.007	.003	.012	.011	.054
			40	.001	.002	.004	.007	.003	.007	.005	.040
		10	20	.000	-.001	-.001	.001	.000	.004	-.001	.007
			30	.001	.001	.001	.004	.000	.004	.000	.017
			40	.000	.001	.000	.003	.003	.007	.012	.038
2000/1000	0	5	20	-.001	-.001	-.003	-.003	-.003	-.003	-.006	-.005
			30	.000	.000	-.002	-.003	-.002	-.003	-.013	-.014
			40	-.001	-.001	-.001	-.001	.000	-.001	.007	.000
		10	20	.000	.000	.000	.000	.000	.001	.000	.001
			30	-.001	-.001	-.001	-.001	-.002	-.002	-.001	-.002
			40	.000	.000	.001	.000	.001	.001	.005	.004
	0.5	5	20	-.001	.000	-.002	.002	-.002	.004	-.006	.011
			30	.000	.001	.002	.004	.003	.006	.008	.027
			40	-.001	.000	-.003	.001	-.005	.001	-.023	.017
		10	20	.000	.001	-.001	.001	-.004	-.001	-.004	.003
			30	.000	.001	-.001	.002	.000	.004	.003	.016
			40	.001	.001	.000	.002	.001	.004	.002	.021

Table 10

Bias estimates given $\beta_C = 0$ for the CDRF and SIBTEST (SIB) methods.

				$n_s = 1$		$n_s = 3$		$n_s = 5$		$n_s = J - n_a$	
N_R/N_F	δ_θ	n_a	J	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB
250/250	0	5	20	.047	.050	.094	.100	.136	.144	.333	.360
			30	.047	.052	.099	.103	.141	.150	.538	.569
			40	.046	.050	.093	.103	.131	.144	.682	.755
		10	20	.047	.047	.088	.090	.121	.124	.196	.199
			30	.045	.046	.083	.089	.117	.122	.335	.357
			40	.045	.049	.086	.091	.121	.127	.467	.493
	0.5	5	20	.051	.053	.102	.104	.146	.151	.356	.375
			30	.050	.053	.099	.108	.138	.153	.523	.579
			40	.048	.054	.097	.107	.137	.153	.721	.809
		10	20	.048	.049	.092	.094	.129	.129	.203	.212
			30	.046	.048	.090	.095	.124	.134	.348	.383
			40	.047	.050	.087	.094	.120	.131	.476	.532
500/500	0	5	20	.034	.036	.067	.070	.098	.102	.244	.251
			30	.033	.036	.067	.072	.095	.101	.362	.382
			40	.033	.037	.067	.074	.095	.104	.501	.540
		10	20	.032	.034	.061	.064	.084	.088	.136	.145
			30	.032	.034	.060	.062	.084	.087	.235	.249
			40	.031	.034	.061	.065	.083	.089	.332	.353
	0.5	5	20	.035	.038	.069	.073	.102	.106	.246	.264
			30	.035	.037	.070	.076	.101	.110	.389	.432
			40	.034	.038	.069	.076	.100	.111	.523	.583
		10	20	.035	.035	.063	.067	.089	.094	.145	.153
			30	.034	.036	.064	.065	.088	.093	.243	.259
			40	.033	.035	.062	.066	.088	.094	.348	.370
1000/1000	0	5	20	.024	.026	.049	.051	.069	.073	.167	.178
			30	.024	.025	.048	.050	.069	.073	.271	.280
			40	.023	.025	.047	.051	.068	.074	.346	.383
		10	20	.023	.024	.044	.044	.061	.063	.099	.102
			30	.022	.023	.042	.043	.058	.061	.159	.170
			40	.022	.023	.041	.045	.057	.061	.227	.242
	0.5	5	20	.026	.027	.053	.055	.075	.079	.179	.188
			30	.024	.026	.050	.051	.072	.074	.273	.282
			40	.024	.026	.048	.053	.070	.075	.375	.412
		10	20	.024	.024	.044	.044	.061	.063	.101	.106
			30	.023	.024	.044	.047	.061	.064	.171	.186
			40	.023	.025	.044	.047	.060	.064	.234	.265

Table 11

Bias estimates given $\beta_{NC} = 0$ for the NCDRF and Crossing-SIBTEST (CSIB) methods.

				$n_s = 1$		$n_s = 3$		$n_s = 5$		$n_s = J - n_a$	
N_R/N_F	δ_θ	n_a	J	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB
500/250	0	5	20	.043	.044	.083	.090	.119	.127	.300	.317
			30	.041	.044	.083	.088	.115	.123	.456	.471
			40	.040	.044	.082	.090	.117	.127	.617	.674
		10	20	.040	.041	.076	.076	.106	.106	.170	.169
			30	.038	.041	.075	.077	.104	.108	.287	.306
			40	.039	.042	.074	.079	.101	.108	.390	.426
	0.5	5	20	.043	.046	.087	.091	.126	.134	.314	.334
			30	.044	.046	.088	.095	.126	.137	.477	.539
			40	.043	.047	.085	.092	.126	.138	.646	.716
		10	20	.042	.043	.083	.085	.115	.120	.185	.195
			30	.042	.044	.079	.083	.111	.116	.312	.344
			40	.041	.043	.077	.081	.106	.116	.429	.490
1000/500	0	5	20	.029	.031	.059	.062	.086	.091	.207	.217
			30	.029	.031	.057	.061	.082	.089	.317	.337
			40	.029	.032	.057	.063	.083	.092	.435	.468
		10	20	.028	.029	.052	.055	.073	.077	.122	.127
			30	.027	.029	.053	.055	.074	.077	.203	.212
			40	.027	.028	.051	.052	.072	.073	.278	.302
	0.5	5	20	.031	.032	.063	.067	.089	.095	.224	.235
			30	.031	.033	.062	.068	.087	.099	.335	.379
			40	.030	.034	.062	.067	.087	.093	.450	.500
		10	20	.030	.030	.057	.057	.081	.081	.132	.134
			30	.029	.031	.056	.059	.079	.082	.220	.233
			40	.028	.030	.054	.058	.075	.082	.294	.329
2000/1000	0	5	20	.021	.022	.043	.045	.061	.064	.146	.154
			30	.020	.022	.042	.044	.060	.065	.231	.242
			40	.020	.022	.039	.043	.057	.062	.297	.320
		10	20	.020	.021	.038	.039	.052	.053	.083	.085
			30	.019	.020	.037	.039	.052	.054	.143	.152
			40	.019	.020	.036	.037	.050	.052	.196	.206
	0.5	5	20	.022	.022	.045	.046	.065	.066	.154	.162
			30	.022	.023	.043	.047	.063	.068	.237	.267
			40	.021	.023	.043	.047	.062	.068	.317	.357
		10	20	.021	.021	.040	.041	.057	.057	.092	.095
			30	.020	.021	.039	.041	.055	.057	.154	.168
			40	.020	.021	.040	.043	.054	.057	.215	.233

Table 12

Bias estimates given $\beta_{NC} = 0$ for the NCDRF and Crossing-SIBTEST (CSIB) methods.

Type I Error Study RMSE and RE Estimates

				CDRF (RMSE)				SIBTEST (RE)			
N_R/N_F	δ_θ	n_a	J	$n_s = 1$	3	5	$J - n_a$	$n_s = 1$	3	5	$J - n_a$
250/250	0	5	20	.002	.007	.014	.084	1.95	2.03	1.91	1.99
			30	.002	.008	.016	.231	1.97	1.93	1.87	1.87
			40	.002	.007	.014	.386	1.98	2.02	2.06	2.06
		10	20	.002	.006	.011	.030	1.33	1.36	1.35	1.34
			30	.002	.005	.010	.083	1.43	1.42	1.45	1.42
			40	.002	.006	.012	.176	1.47	1.37	1.40	1.52
	0.5	5	20	.002	.008	.015	.094	2.11	2.09	2.23	2.28
			30	.002	.008	.015	.223	2.18	2.26	2.42	2.53
			40	.002	.008	.016	.429	2.37	2.43	2.41	2.60
		10	20	.002	.006	.012	.031	1.45	1.35	1.34	1.49
			30	.002	.007	.013	.098	1.44	1.48	1.57	1.73
			40	.002	.006	.011	.186	1.46	1.55	1.62	1.72
500/500	0	5	20	.001	.003	.007	.046	1.91	1.85	1.79	1.72
			30	.001	.004	.007	.101	1.98	1.82	1.80	1.64
			40	.001	.004	.008	.194	1.98	1.75	1.68	1.86
		10	20	.001	.003	.005	.014	1.33	1.31	1.33	1.39
			30	.001	.003	.005	.042	1.38	1.42	1.40	1.38
			40	.001	.003	.005	.089	1.32	1.32	1.38	1.36
	0.5	5	20	.001	.004	.008	.046	2.11	2.04	2.07	2.25
			30	.001	.004	.008	.124	2.22	2.23	2.28	2.43
			40	.001	.004	.008	.219	2.11	2.45	2.41	2.59
		10	20	.001	.003	.006	.015	1.37	1.48	1.48	1.50
			30	.001	.003	.006	.046	1.46	1.42	1.53	1.59
			40	.001	.003	.006	.097	1.37	1.42	1.45	1.54
1000/1000	0	5	20	.000	.002	.003	.020	1.78	1.86	1.96	1.95
			30	.000	.002	.004	.055	1.98	1.88	1.78	1.91
			40	.000	.002	.004	.103	1.87	2.05	1.89	1.93
		10	20	.000	.001	.003	.007	1.35	1.32	1.34	1.32
			30	.000	.001	.003	.020	1.33	1.35	1.39	1.35
			40	.000	.001	.003	.040	1.28	1.43	1.47	1.44
	0.5	5	20	.001	.002	.004	.025	2.02	2.14	2.14	2.31
			30	.000	.002	.004	.054	2.13	2.24	2.18	2.14
			40	.000	.002	.004	.108	2.24	2.42	2.30	2.53
		10	20	.000	.001	.003	.008	1.42	1.45	1.47	1.52
			30	.000	.001	.003	.022	1.39	1.61	1.58	1.61
			40	.000	.002	.003	.044	1.46	1.45	1.52	1.71

Table 13

RMSE and relative efficiency (RE) estimates given $\beta_C = 0$ for the CDRF and SIBTEST methods.

				CDRF (RMSE)				SIBTEST (RE)			
N_R/N_F	δ_θ	n_a	J	$n_s = 1$	3	5	$J - n_a$	$n_s = 1$	3	5	$J - n_a$
500/250	0	5	20	.001	.005	.011	.070	1.99	1.99	2.12	2.03
			30	.001	.005	.010	.151	1.97	1.94	1.95	1.95
			40	.001	.005	.011	.299	1.89	1.97	1.90	1.91
		10	20	.001	.004	.008	.021	1.44	1.41	1.37	1.40
			30	.001	.004	.008	.063	1.37	1.29	1.33	1.44
			40	.001	.004	.008	.116	1.52	1.49	1.43	1.47
	0.5	5	20	.001	.005	.011	.071	2.29	2.53	2.64	2.87
			30	.001	.006	.012	.174	2.50	2.80	2.69	3.07
			40	.001	.006	.013	.323	2.33	2.29	2.38	2.98
		10	20	.001	.005	.010	.024	1.48	1.57	1.66	1.78
			30	.001	.004	.009	.073	1.51	1.71	1.82	1.83
			40	.001	.004	.009	.136	1.62	1.74	1.88	2.28
1000/500	0	5	20	.001	.003	.005	.033	1.81	1.88	1.94	1.92
			30	.001	.003	.005	.082	1.92	1.87	1.89	1.86
			40	.001	.003	.006	.159	2.00	1.91	1.77	1.84
		10	20	.001	.002	.004	.011	1.36	1.41	1.39	1.41
			30	.001	.002	.004	.032	1.32	1.31	1.30	1.34
			40	.001	.002	.004	.064	1.41	1.35	1.36	1.40
	0.5	5	20	.001	.003	.006	.039	2.17	2.34	2.33	2.51
			30	.001	.003	.006	.092	2.19	2.43	2.95	3.02
			40	.001	.003	.006	.160	2.37	2.41	2.59	2.80
		10	20	.001	.002	.005	.012	1.38	1.55	1.65	1.64
			30	.001	.002	.005	.035	1.48	1.52	1.49	1.71
			40	.001	.002	.004	.068	1.68	1.78	1.77	1.85
2000/1000	0	5	20	.000	.001	.003	.015	2.01	1.95	1.89	1.97
			30	.000	.001	.003	.043	1.94	1.90	2.01	1.83
			40	.000	.001	.003	.070	2.03	1.98	1.90	1.88
		10	20	.000	.001	.002	.005	1.34	1.33	1.30	1.32
			30	.000	.001	.002	.016	1.38	1.41	1.33	1.38
			40	.000	.001	.002	.031	1.31	1.31	1.33	1.33
	0.5	5	20	.000	.001	.003	.016	2.19	2.32	2.53	2.65
			30	.000	.001	.003	.043	2.32	2.56	2.47	2.91
			40	.000	.001	.003	.078	2.23	2.26	2.43	3.00
		10	20	.000	.001	.002	.006	1.45	1.48	1.53	1.64
			30	.000	.001	.002	.018	1.55	1.62	1.64	1.82
			40	.000	.001	.002	.035	1.45	1.66	1.72	1.97

Table 14

RMSE and relative efficiency (RE) estimates given $\beta_C = 0$ for the CDRF and SIBTEST statistics.

				NCDRF (RMSE)				CSIBTEST (RE)			
N_R/N_F	δ_θ	n_a	J	$n_s = 1$	3	5	$J - n_a$	$n_s = 1$	3	5	$J - n_a$
250/250	0	5	20	.003	.011	.023	.139	1.44	1.50	1.45	1.62
			30	.003	.012	.025	.374	1.64	1.33	1.43	1.36
			40	.003	.011	.022	.601	1.58	1.71	1.73	1.70
		10	20	.003	.010	.019	.049	1.20	1.26	1.24	1.18
			30	.003	.009	.017	.140	1.30	1.55	1.44	1.51
			40	.003	.009	.019	.277	1.50	1.38	1.41	1.41
	0.5	5	20	.003	.013	.027	.162	1.30	1.27	1.32	1.42
			30	.003	.012	.025	.353	1.52	1.68	1.80	1.76
			40	.003	.012	.024	.665	1.73	1.71	1.83	1.91
		10	20	.003	.011	.021	.052	1.26	1.24	1.18	1.36
			30	.003	.010	.019	.155	1.30	1.50	1.57	1.68
			40	.003	.010	.018	.292	1.51	1.58	1.66	1.71
500/500	0	5	20	.001	.006	.012	.075	1.47	1.38	1.34	1.33
			30	.001	.006	.012	.166	1.56	1.44	1.40	1.34
			40	.001	.006	.012	.318	1.62	1.66	1.62	1.53
		10	20	.001	.005	.009	.024	1.30	1.46	1.40	1.44
			30	.001	.004	.009	.070	1.38	1.39	1.31	1.39
			40	.001	.005	.009	.144	1.45	1.50	1.50	1.40
	0.5	5	20	.002	.006	.013	.077	1.54	1.52	1.41	1.50
			30	.002	.006	.013	.192	1.54	1.54	1.54	1.75
			40	.002	.006	.013	.348	1.77	1.67	1.72	1.78
		10	20	.002	.005	.010	.026	1.13	1.50	1.47	1.45
			30	.001	.005	.010	.076	1.36	1.32	1.44	1.49
			40	.001	.005	.010	.153	1.37	1.46	1.52	1.46
1000/1000	0	5	20	.001	.003	.006	.035	1.49	1.36	1.46	1.46
			30	.001	.003	.006	.093	1.44	1.35	1.37	1.33
			40	.001	.003	.006	.158	1.56	1.60	1.60	1.65
		10	20	.001	.002	.005	.013	1.20	1.18	1.27	1.25
			30	.001	.002	.004	.032	1.29	1.34	1.41	1.43
			40	.001	.002	.004	.067	1.36	1.51	1.45	1.44
	0.5	5	20	.001	.003	.007	.041	1.48	1.37	1.43	1.39
			30	.001	.003	.007	.095	1.44	1.27	1.29	1.32
			40	.001	.003	.006	.178	1.60	1.63	1.55	1.65
		10	20	.001	.002	.005	.013	1.31	1.15	1.30	1.40
			30	.001	.002	.005	.037	1.26	1.50	1.39	1.57
			40	.001	.002	.005	.069	1.46	1.53	1.55	1.87

Table 15

RMSE and relative efficiency (RE) estimates given $\beta_{NC} = 0$ for the NCDRF and Crossing-SIBTEST (CSIBTEST) statistics.

				NCDRF				CSIBTEST (RE)			
N_R/N_F	δ_θ	n_a	J	$n_s = 1$	3	5	$J - n_a$	$n_s = 1$	3	5	$J - n_a$
500/250	0	5	20	.002	.009	.018	.115	1.35	1.47	1.45	1.42
			30	.002	.008	.017	.263	1.62	1.52	1.48	1.32
			40	.002	.009	.018	.490	1.59	1.61	1.55	1.58
		10	20	.002	.007	.014	.037	1.16	1.14	1.17	1.09
			30	.002	.007	.014	.106	1.40	1.29	1.32	1.47
			40	.002	.007	.013	.193	1.50	1.50	1.47	1.56
	0.5	5	20	.002	.010	.020	.124	1.61	1.41	1.51	1.52
			30	.002	.010	.020	.290	1.50	1.63	1.67	1.91
			40	.002	.009	.020	.534	1.63	1.58	1.66	1.81
		10	20	.002	.009	.017	.043	1.24	1.28	1.35	1.43
			30	.002	.008	.015	.125	1.33	1.46	1.48	1.69
			40	.002	.007	.014	.236	1.37	1.46	1.66	1.89
1000/500	0	5	20	.001	.004	.009	.055	1.33	1.43	1.38	1.36
			30	.001	.004	.009	.129	1.53	1.57	1.54	1.38
			40	.001	.004	.009	.245	1.50	1.78	1.73	1.50
		10	20	.001	.003	.007	.019	1.35	1.38	1.32	1.33
			30	.001	.004	.007	.053	1.42	1.33	1.33	1.37
			40	.001	.003	.007	.100	1.39	1.30	1.20	1.49
	0.5	5	20	.001	.005	.010	.064	1.25	1.43	1.50	1.46
			30	.001	.005	.010	.145	1.53	1.60	1.86	1.93
			40	.001	.005	.009	.262	1.77	1.68	1.57	1.77
		10	20	.001	.004	.008	.022	1.18	1.19	1.18	1.24
			30	.001	.004	.008	.061	1.34	1.30	1.32	1.45
			40	.001	.004	.007	.111	1.53	1.59	1.64	1.72
2000/1000	0	5	20	.001	.002	.005	.027	1.46	1.43	1.38	1.37
			30	.001	.002	.005	.068	1.55	1.40	1.56	1.41
			40	.001	.002	.004	.112	1.67	1.61	1.64	1.51
		10	20	.001	.002	.003	.009	1.23	1.17	1.17	1.24
			30	.000	.002	.003	.026	1.33	1.46	1.33	1.43
			40	.000	.002	.003	.049	1.41	1.35	1.38	1.41
	0.5	5	20	.001	.003	.005	.030	1.18	1.19	1.23	1.35
			30	.001	.002	.005	.073	1.48	1.63	1.62	1.77
			40	.001	.002	.005	.130	1.62	1.62	1.61	1.75
		10	20	.001	.002	.004	.011	1.11	1.22	1.25	1.32
			30	.001	.002	.004	.030	1.33	1.46	1.40	1.59
			40	.001	.002	.004	.058	1.30	1.52	1.43	1.61

Table 16

RMSE and relative efficiency (RE) estimates given $\beta_{NC} = 0$ for the NCDRF and Crossing-SIBTEST (CSIBTEST) statistics.

Power Study Bias Estimates

				DIF1		DIF2		DIF3		DIF4		DIF5		DBF3a		DBF3b		DBF5		DTF	
N_R/N_F	δ_θ	n_a	J	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB
250/250	0	5	20	.000	-.004	.001	-.004	.000	-.005	-.001	-.004	.000	.000	.002	-.013	-.001	-.009	.000	-.018	.001	-.016
			30	.001	-.006	.000	-.006	.000	-.004	.002	-.002	.001	.002	.001	-.015	.002	-.004	.004	-.016	.028	.003
			40	.002	-.003	.000	-.005	-.001	-.005	-.001	-.002	.000	-.001	.001	-.012	-.003	-.008	.000	-.015	.000	.001
		10	20	-.001	-.002	-.002	-.004	.000	-.001	.000	-.002	.001	.002	-.003	-.010	.000	-.003	-.003	-.010	-.002	-.009
			30	.000	-.003	.001	-.002	.000	-.001	-.001	-.003	-.001	-.001	.000	-.008	-.002	-.006	-.002	-.012	-.001	-.014
			40	.002	.000	.000	-.003	-.002	-.004	.000	.000	.002	.001	.000	-.008	.000	-.003	.002	-.006	.002	-.002
	0.5	5	20	.001	-.003	-.001	-.005	-.001	-.005	.001	-.001	-.001	-.003	-.002	-.014	-.002	-.009	-.002	-.018	-.011	-.008
			30	.000	-.002	.001	-.004	-.001	-.005	.000	-.003	.000	-.002	.001	-.010	-.001	-.010	.000	-.015	.007	.024
			40	-.001	-.004	-.003	-.007	.000	-.004	.000	-.004	.002	.000	-.003	-.015	.001	-.008	-.002	-.019	-.024	.025
		10	20	.001	-.002	.001	-.002	-.001	-.003	-.001	-.003	.001	-.001	.001	-.004	.000	-.002	.002	-.003	.000	.002
			30	.001	-.002	.000	-.003	-.001	-.005	.000	-.002	.001	-.001	-.001	-.007	.000	-.004	.001	-.007	.010	.021
			40	.000	-.003	.000	-.003	.000	-.002	.000	-.003	.000	-.002	.000	-.004	.000	-.001	-.001	-.005	-.017	.010
500/500	0	5	20	.000	-.006	.001	-.004	.000	-.004	.000	-.002	.001	.001	.000	-.014	.000	-.005	.001	-.015	.005	-.013
			30	-.002	-.009	.001	-.003	-.001	-.004	.000	-.002	-.002	-.001	-.002	-.017	-.002	-.008	-.004	-.020	-.014	-.032
			40	.000	-.005	.000	-.005	-.001	-.006	.001	-.001	.001	.000	-.002	-.016	.001	-.007	.000	-.018	.002	-.020
		10	20	.001	-.002	-.001	-.004	.000	-.002	.000	-.001	.000	.000	.000	-.007	-.001	-.003	.000	-.009	.002	-.006
			30	.001	-.002	.000	-.003	-.001	-.002	-.001	-.001	.001	.001	.000	-.007	.000	-.003	.001	-.008	-.001	-.010
			40	-.001	-.004	-.002	-.005	-.001	-.003	.000	-.001	-.001	-.001	-.004	-.012	-.001	-.004	-.004	-.014	-.013	-.027
	0.5	5	20	-.001	-.005	.000	-.005	.000	-.003	.001	-.003	.002	-.002	.000	-.013	.003	-.008	.002	-.018	.006	-.004
			30	.001	-.003	.000	-.004	.000	-.004	-.001	-.004	.001	-.001	.000	-.011	-.001	-.010	.000	-.017	.003	.019
			40	-.001	-.005	.001	-.004	-.001	-.005	.000	-.002	.001	-.001	.000	-.014	.001	-.009	.001	-.018	-.001	.015
		10	20	.001	-.002	.001	-.002	.002	.000	-.001	-.003	-.001	-.002	.004	-.004	.001	-.005	.002	-.008	.006	.000
			30	-.001	-.004	-.001	-.004	.001	-.002	-.002	-.003	.000	-.001	-.001	-.008	-.001	-.005	-.003	-.012	-.017	-.014
			40	.000	-.002	-.001	-.003	.000	-.003	.001	-.001	.001	-.001	-.001	-.008	.001	-.005	.001	-.010	.006	.022
1000/1000	0	5	20	.000	-.005	-.001	-.006	.001	-.002	.001	.000	-.001	-.001	.000	-.014	.001	-.004	.000	-.015	-.003	-.015
			30	.000	-.006	.001	-.004	.000	-.003	.000	-.001	.001	.001	.001	-.013	.001	-.003	.001	-.013	.009	-.007
			40	-.001	-.006	.000	-.005	-.001	-.005	.001	-.002	-.001	-.002	-.001	-.017	-.001	-.010	-.002	-.021	-.002	-.031
		10	20	.001	-.003	.000	-.003	.000	-.002	-.001	-.002	.000	.000	.001	-.008	-.001	-.004	.000	-.009	.000	-.010
			30	.000	-.003	.001	-.002	.000	-.002	.001	-.001	-.001	-.001	.000	-.008	-.001	-.004	.000	-.010	-.006	-.016
			40	.000	-.003	.000	-.003	.001	-.001	.000	-.001	.000	.001	.001	-.008	.001	-.002	.001	-.008	.003	-.003
	0.5	5	20	.001	-.004	.002	-.003	.001	-.004	.000	-.004	.000	-.003	.003	-.011	.000	-.011	.003	-.018	.001	-.014
			30	.000	-.004	.000	-.005	-.001	-.006	.001	-.003	.001	-.002	-.001	-.014	.000	-.011	.000	-.019	.002	.004
			40	.001	-.003	.000	-.005	.000	-.006	.001	-.003	.000	-.002	.001	-.014	.001	-.012	.002	-.020	.022	.023
		10	20	.001	-.002	.000	-.002	-.001	-.003	.001	-.001	-.001	-.002	.000	-.007	-.001	-.007	.000	-.011	-.002	-.010
			30	.000	-.002	.001	-.001	-.001	-.003	-.001	-.002	-.001	-.002	.000	-.006	-.003	-.007	-.001	-.010	-.005	-.002
			40	.001	-.002	.000	-.002	.000	-.002	.000	-.002	-.001	-.002	.000	-.007	-.002	-.006	-.001	-.011	-.006	.004

Table 17

Bias estimates given $\beta_C = 0$ for the CDRF and SIBTEST (SIB) statistics.

				DIF1		DIF2		DIF3		DIF4		DIF5		DBF3a		DBF3b		DBF5		DTF	
N_R/N_F	δ_θ	n_a	J	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB	CDRF	SIB
500/250	0	5	20	.000	-.005	.000	-.005	.000	-.003	-.001	-.002	.000	.000	.001	-.013	-.001	-.005	.000	-.014	.004	-.004
			30	.000	-.007	-.001	-.006	.000	-.004	.000	-.002	.000	.000	-.001	-.017	.000	-.005	-.001	-.018	.006	-.009
			40	-.001	-.007	-.002	-.006	-.001	-.005	.000	-.002	-.001	-.001	-.004	-.018	-.003	-.009	-.006	-.021	-.022	-.039
		10	20	.001	-.001	.000	-.003	-.002	-.004	.000	-.001	-.001	-.001	-.001	-.009	-.003	-.007	-.002	-.012	.000	-.011
			30	-.001	-.004	.001	-.002	.001	.000	-.001	-.002	.000	-.001	.001	-.006	.000	-.003	-.001	-.009	.005	-.004
			40	.000	-.003	.000	-.003	.001	-.002	.001	.000	.001	.001	.001	-.008	.004	-.001	.004	-.007	.005	-.012
	0.5	5	20	.000	-.003	.002	-.001	.001	.001	.002	.004	.003	.006	.003	-.003	.006	.011	.008	.007	.020	.039
			30	.001	-.003	.003	.000	-.001	-.002	.004	.004	.001	.005	.003	-.006	.004	.006	.007	.003	.019	.048
			40	.000	-.004	.002	-.001	.004	.003	.002	.003	.001	.005	.006	-.002	.007	.011	.009	.006	.018	.066
		10	20	.000	-.004	.000	-.003	.002	.001	.000	.000	.003	.006	.002	-.002	.006	.011	.005	.007	.007	.017
			30	.002	-.003	-.002	-.004	.002	.002	.003	.004	.001	.004	.003	-.001	.006	.013	.007	.009	.020	.041
			40	.003	-.001	-.001	-.004	.002	.001	.001	.002	.001	.003	.004	.001	.004	.011	.006	.009	-.009	.041
1000/500	0	5	20	-.002	-.007	.001	-.004	.000	-.003	.001	.000	.001	.001	-.001	-.014	.002	-.002	.001	-.013	.000	-.014
			30	-.001	-.006	.001	-.003	.000	-.004	.000	-.002	.001	.001	-.001	-.014	.000	-.005	.000	-.015	-.003	-.019
			40	.000	-.006	.000	-.004	.000	-.003	.000	-.003	.001	.001	.001	-.013	.001	-.004	.001	-.014	-.007	-.022
		10	20	.000	-.003	.000	-.002	.000	-.002	.001	.000	-.001	.000	.001	-.007	.001	-.002	.002	-.007	.003	-.005
			30	-.001	-.004	.002	-.001	.001	.000	.001	.000	-.001	-.002	.002	-.005	.001	-.002	.002	-.007	.001	-.008
			40	-.001	-.005	.000	-.002	.000	-.002	.000	.000	.000	.000	-.001	-.009	.000	-.002	.000	-.009	.021	.016
	0.5	5	20	.001	-.004	.000	-.004	.003	.001	.001	.001	.001	.004	.003	-.007	.005	.006	.005	-.002	.006	.010
			30	.001	-.004	.001	-.003	.002	.000	.003	.004	.003	.006	.004	-.007	.007	.009	.009	.002	.013	.029
			40	.000	-.004	.001	-.004	.001	-.001	.001	.001	.001	.004	.002	-.009	.003	.005	.004	-.004	-.013	.010
		10	20	.001	-.002	.003	.001	.001	.000	.001	.001	.002	.004	.006	-.001	.005	.006	.009	.005	.011	.012
			30	-.001	-.004	-.001	-.003	.000	-.001	.003	.003	.003	.005	-.001	-.006	.006	.007	.005	.001	.007	.016
			40	.000	-.003	.001	-.002	.001	.000	.002	.003	.003	.005	.001	-.004	.005	.008	.006	.003	-.002	.015
2000/1000	0	5	20	-.001	-.007	.001	-.004	.000	-.004	.000	-.002	-.001	-.001	-.001	-.015	-.001	-.007	-.002	-.019	.002	-.016
			30	.001	-.005	.001	-.004	.000	-.004	.000	-.002	.001	.000	.002	-.012	.001	-.005	.003	-.014	.004	-.015
			40	.001	-.004	.000	-.005	.000	-.003	.000	-.001	.002	.002	.000	-.012	.001	-.003	.002	-.012	.003	-.008
		10	20	.001	-.002	.000	-.003	.001	-.001	.000	-.001	.001	.001	.002	-.006	.002	-.001	.003	-.006	.004	-.006
			30	.000	-.004	.000	-.003	.001	-.002	.000	-.002	.000	.000	.000	-.009	.000	-.003	.000	-.010	.001	-.010
			40	-.001	-.005	.000	-.004	.000	-.002	-.001	-.002	.001	.000	-.002	-.011	-.001	-.004	-.002	-.012	-.006	-.018
	0.5	5	20	-.001	-.006	.001	-.002	.001	.000	.002	.003	.003	.006	.001	-.009	.006	.008	.006	.000	.008	.015
			30	.001	-.004	.002	-.002	.003	.001	.002	.002	.003	.005	.005	-.005	.007	.008	.010	.002	.014	.028
			40	.000	-.006	.001	-.002	.001	-.002	.002	.001	.002	.004	.002	-.010	.004	.003	.005	-.005	-.002	.020
		10	20	.000	-.003	.000	-.002	.001	-.001	.002	.003	.001	.003	.001	-.006	.004	.006	.005	.001	.003	.003
			30	-.001	-.004	.001	-.001	.002	.001	.002	.003	.002	.004	.002	-.004	.006	.008	.006	.002	.009	.014
			40	.000	-.003	.000	-.002	.002	.001	.002	.002	.002	.003	.003	-.004	.006	.007	.006	.002	.012	.025

Table 18

Bias estimates given $\beta_C = 0$ for the CDRF and SIBTEST (SIB) statistics.

				DIF1		DIF2		DIF3		DIF4		DIF5		DBF3a		DBF3b		DBF5		DTF	
N_R/N_F	δ_θ	n_a	J	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB
250/250	0	5	20	.004	.007	.012	.011	.014	.001	.009	-.019	.007	-.033	.014	.016	.012	-.092	.025	-.047	.133	.086
			30	.002	.008	.011	.011	.013	-.001	.009	-.019	.007	-.033	.015	.017	.012	-.091	.023	-.053	.269	.251
			40	.002	.006	.012	.011	.015	.000	.010	-.019	.009	-.034	.014	.014	.018	-.089	.027	-.054	.437	.437
		10	20	.005	.005	.013	.011	.011	.003	.007	-.011	.005	-.021	.017	.014	.007	-.049	.021	-.029	.048	.010
			30	.004	.006	.011	.009	.013	.002	.007	-.010	.005	-.018	.013	.013	.009	-.044	.019	-.026	.131	.120
			40	.001	.003	.012	.010	.014	.005	.007	-.012	.007	-.016	.015	.013	.010	-.045	.019	-.031	.248	.245
	0.5	5	20	.004	.007	.016	.015	.015	.000	.010	-.017	.008	-.031	.019	.018	.011	-.093	.026	-.051	.154	.118
			30	.004	.006	.014	.014	.014	.002	.011	-.018	.008	-.030	.019	.018	.012	-.093	.029	-.050	.312	.340
			40	.005	.008	.016	.016	.015	.001	.011	-.017	.009	-.029	.022	.021	.015	-.093	.029	-.051	.488	.520
		10	20	.004	.006	.012	.011	.014	.005	.009	-.010	.007	-.017	.014	.012	.010	-.048	.017	-.035	.052	.013
			30	.003	.006	.013	.014	.013	.005	.009	-.009	.007	-.017	.016	.012	.009	-.049	.018	-.034	.149	.132
			40	.003	.006	.013	.012	.013	.005	.009	-.006	.008	-.015	.013	.010	.011	-.042	.020	-.026	.261	.288
500/500	0	5	20	.001	.006	.006	.005	.006	-.010	.003	-.031	.003	-.041	.008	.013	.004	-.110	.010	-.066	.066	.006
			30	.003	.009	.005	.004	.006	-.010	.003	-.030	.002	-.043	.010	.016	.004	-.111	.013	-.062	.152	.120
			40	.001	.005	.006	.006	.008	-.008	.005	-.030	.003	-.042	.010	.016	.006	-.109	.013	-.062	.268	.256
		10	20	-.001	.002	.006	.005	.005	-.005	.004	-.014	.002	-.022	.007	.007	.004	-.054	.008	-.045	.021	-.023
			30	.000	.003	.005	.004	.005	-.005	.003	-.014	.003	-.021	.006	.008	.003	-.054	.007	-.045	.060	.027
			40	.002	.005	.007	.006	.007	-.004	.003	-.016	.003	-.021	.011	.012	.004	-.054	.012	-.040	.133	.120
	0.5	5	20	.002	.006	.008	.007	.007	-.010	.004	-.030	.003	-.040	.010	.013	.005	-.113	.010	-.076	.077	.028
			30	.000	.003	.007	.006	.007	-.010	.005	-.030	.003	-.041	.009	.011	.006	-.107	.013	-.074	.180	.154
			40	.002	.006	.006	.005	.008	-.009	.005	-.030	.003	-.041	.010	.014	.007	-.110	.014	-.075	.308	.316
		10	20	.000	.002	.006	.005	.007	-.003	.005	-.013	.003	-.021	.005	.005	.007	-.052	.008	-.043	.026	-.018
			30	.002	.005	.007	.006	.007	-.005	.004	-.015	.003	-.021	.009	.009	.004	-.054	.010	-.045	.079	.061
			40	.001	.004	.007	.007	.007	-.003	.004	-.015	.004	-.020	.008	.009	.006	-.054	.008	-.044	.139	.133
1000/1000	0	5	20	.000	.005	.004	.005	.003	-.017	.001	-.038	.002	-.046	.005	.013	.002	-.122	.006	-.073	.036	-.041
			30	.000	.006	.002	.003	.003	-.016	.001	-.039	.000	-.047	.003	.012	.000	-.126	.003	-.074	.074	.025
			40	.001	.006	.003	.004	.004	-.014	.002	-.036	.003	-.045	.005	.016	.005	-.118	.009	-.066	.143	.123
		10	20	.000	.003	.003	.002	.003	-.009	.002	-.017	.001	-.020	.003	.007	.002	-.054	.004	-.050	.012	-.036
			30	.000	.004	.003	.002	.002	-.010	.002	-.017	.002	-.021	.003	.007	.003	-.055	.004	-.051	.037	.002
			40	.000	.003	.003	.003	.003	-.009	.002	-.018	.001	-.022	.003	.007	.002	-.057	.004	-.053	.058	.023
	0.5	5	20	.000	.004	.003	.002	.003	-.018	.003	-.037	.002	-.045	.003	.010	.003	-.122	.005	-.088	.042	-.032
			30	.000	.004	.004	.004	.003	-.017	.003	-.036	.002	-.044	.006	.013	.003	-.120	.005	-.085	.090	.039
			40	-.001	.004	.004	.004	.003	-.016	.002	-.037	.002	-.044	.004	.013	.002	-.121	.004	-.084	.159	.139
		10	20	-.001	.002	.003	.003	.003	-.009	.002	-.018	.000	-.023	.005	.007	.000	-.060	.002	-.054	.010	-.040
			30	.000	.002	.003	.001	.003	-.009	.002	-.019	.002	-.023	.004	.005	.002	-.058	.004	-.055	.036	-.006
			40	.000	.002	.003	.003	.003	-.009	.002	-.017	.001	-.022	.004	.006	.001	-.056	.004	-.051	.070	.046

Table 19

Bias estimates given $\beta_{NC} = 0$ for the NCDRF and Crossing-SIBTEST (CSIB) statistics.

				DIF1		DIF2		DIF3		DIF4		DIF5		DBF3a		DBF3b		DBF5		DTF	
N_R/N_F	δ_θ	n_a	J	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB	NCDRF	CSIB
500/250	0	5	20	.002	.006	.010	.009	.011	-.005	.007	-.024	.005	-.037	.013	.014	.010	-.098	.020	-.057	.103	.047
			30	.003	.008	.009	.009	.008	-.005	.006	-.025	.004	-.038	.013	.017	.006	-.105	.018	-.057	.219	.183
			40	.003	.008	.010	.009	.010	-.005	.005	-.024	.004	-.038	.015	.018	.007	-.100	.019	-.052	.376	.381
		10	20	.001	.003	.009	.008	.009	.001	.004	-.010	.003	-.018	.012	.012	.003	-.049	.013	-.032	.032	-.007
			30	.003	.005	.008	.006	.008	.000	.006	-.010	.004	-.018	.010	.009	.005	-.050	.014	-.035	.095	.067
			40	.002	.004	.008	.007	.007	-.001	.005	-.011	.005	-.017	.008	.010	.005	-.051	.009	-.038	.173	.170
	0.5	5	20	.004	.007	.008	.006	.010	-.006	.007	-.023	.005	-.034	.010	.005	.008	-.103	.012	-.081	.105	.057
			30	.002	.006	.008	.007	.011	-.007	.008	-.024	.005	-.035	.011	.008	.010	-.108	.015	-.081	.241	.236
			40	.003	.007	.008	.008	.009	-.007	.007	-.024	.006	-.035	.007	.005	.008	-.106	.014	-.080	.394	.434
		10	20	.003	.007	.010	.009	.009	-.002	.007	-.012	.004	-.018	.011	.005	.006	-.057	.014	-.049	.035	-.016
			30	.001	.004	.011	.009	.008	-.002	.007	-.011	.006	-.016	.009	.004	.008	-.057	.012	-.052	.106	.083
			40	.000	.003	.009	.009	.008	-.002	.005	-.012	.004	-.017	.006	.002	.004	-.057	.006	-.056	.186	.207
1000/500	0	5	20	.002	.007	.004	.003	.004	-.013	.001	-.035	.002	-.045	.007	.013	.002	-.121	.007	-.074	.049	-.021
			30	.002	.007	.003	.003	.005	-.013	.002	-.033	.002	-.044	.007	.013	.003	-.116	.008	-.069	.112	.076
			40	.001	.006	.005	.004	.004	-.014	.003	-.033	.003	-.044	.006	.012	.004	-.115	.009	-.072	.209	.197
		10	20	.001	.003	.004	.003	.005	-.006	.002	-.016	.002	-.021	.005	.007	.003	-.054	.005	-.047	.013	-.034
			30	.001	.004	.003	.002	.004	-.008	.002	-.017	.002	-.020	.003	.005	.003	-.056	.005	-.049	.051	.015
			40	.001	.005	.004	.002	.004	-.006	.002	-.017	.001	-.022	.006	.008	.002	-.055	.005	-.047	.085	.062
	0.5	5	20	.000	.004	.006	.004	.004	-.017	.002	-.036	.001	-.042	.004	.006	.000	-.122	.003	-.091	.051	-.020
			30	.000	.004	.004	.003	.004	-.017	.004	-.035	.003	-.043	.003	.006	.004	-.122	.005	-.096	.122	.076
			40	.000	.005	.004	.004	.004	-.016	.002	-.036	.003	-.041	.005	.008	.003	-.122	.005	-.090	.231	.225
		10	20	-.001	.003	.003	.001	.004	-.008	.002	-.017	.003	-.021	.001	.001	.003	-.058	.002	-.063	.013	-.046
			30	.001	.005	.005	.004	.004	-.009	.002	-.018	.003	-.021	.007	.006	.003	-.060	.003	-.060	.044	.004
			40	.001	.004	.004	.003	.005	-.008	.003	-.018	.002	-.022	.004	.005	.004	-.060	.003	-.060	.109	.088
2000/1000	0	5	20	.001	.007	.002	.003	.003	-.017	.002	-.039	.002	-.049	.004	.014	.004	-.126	.007	-.070	.028	-.048
			30	-.001	.005	.002	.003	.003	-.016	.001	-.039	.000	-.048	.002	.011	.001	-.125	.003	-.075	.066	.002
			40	-.001	.004	.003	.004	.002	-.018	.001	-.039	.000	-.047	.003	.011	.000	-.126	.002	-.076	.103	.061
		10	20	-.001	.002	.003	.002	.002	-.010	.002	-.018	.001	-.022	.001	.005	.003	-.056	.004	-.056	.010	-.045
			30	.000	.004	.002	.003	.002	-.011	.001	-.018	.001	-.022	.003	.008	.001	-.057	.003	-.053	.021	-.018
			40	.001	.005	.002	.003	.002	-.010	.001	-.017	.000	-.022	.004	.010	.000	-.057	.002	-.053	.048	.015
	0.5	5	20	.001	.006	.002	.002	.002	-.022	.001	-.041	.001	-.045	.003	.008	.000	-.129	.000	-.101	.022	-.072
			30	.000	.004	.001	.001	.000	-.023	.001	-.040	.002	-.045	-.001	.004	.000	-.129	-.002	-.102	.059	.001
			40	.001	.006	.001	.002	.001	-.021	.001	-.040	.001	-.046	.002	.009	.001	-.130	.000	-.096	.121	.098
		10	20	.000	.004	.002	.001	.002	-.011	.001	-.019	.002	-.023	.002	.005	.002	-.059	.001	-.065	.005	-.057
			30	.001	.004	.002	.001	.001	-.013	.001	-.020	.001	-.022	.001	.003	.000	-.061	.000	-.065	.020	-.027
			40	.000	.003	.002	.001	.002	-.012	.000	-.020	.002	-.022	.001	.003	.001	-.061	.001	-.066	.049	.012

Table 20

Bias estimates given $\beta_{NC} = 0$ for the NCDRF and SIBTEST (SIB) methods.

Power Study RMSE and RE Estimates

N_R/N_F	δ_θ	n_a	J	DIF1	DIF2	DIF3	DIF4	DIF5	DBF3a	DBF3b	DBF5	DTF
250/250	0	5	20	.002	.002	.002	.002	.002	.007	.008	.015	.082
			30	.002	.002	.002	.002	.002	.007	.007	.014	.201
			40	.002	.002	.002	.002	.002	.007	.007	.014	.382
		10	20	.002	.002	.002	.002	.002	.006	.006	.012	.030
			30	.002	.002	.002	.002	.001	.006	.006	.011	.086
			40	.002	.002	.002	.002	.002	.006	.006	.012	.178
	0.5	5	20	.002	.002	.002	.002	.002	.007	.008	.016	.095
			30	.002	.002	.002	.002	.002	.008	.008	.016	.230
			40	.002	.002	.002	.002	.002	.007	.008	.015	.416
		10	20	.002	.002	.002	.002	.002	.007	.007	.013	.032
			30	.002	.002	.002	.002	.002	.006	.007	.011	.094
			40	.002	.002	.002	.002	.002	.006	.007	.013	.183
500/500	0	5	20	.001	.001	.001	.001	.001	.004	.004	.008	.045
			30	.001	.001	.001	.001	.001	.004	.004	.008	.103
			40	.001	.001	.001	.001	.001	.004	.004	.008	.208
		10	20	.001	.001	.001	.001	.001	.003	.003	.006	.015
			30	.001	.001	.001	.001	.001	.003	.003	.005	.036
			40	.001	.001	.001	.001	.001	.003	.003	.006	.089
	0.5	5	20	.001	.001	.001	.001	.001	.004	.004	.008	.047
			30	.001	.001	.001	.001	.001	.004	.004	.008	.114
			40	.001	.001	.001	.001	.001	.004	.004	.008	.236
		10	20	.001	.001	.001	.001	.001	.003	.003	.006	.016
			30	.001	.001	.001	.001	.001	.003	.003	.006	.047
			40	.001	.001	.001	.001	.001	.003	.003	.006	.089
1000/1000	0	5	20	.000	.000	.000	.000	.000	.002	.002	.003	.021
			30	.000	.000	.000	.000	.000	.002	.002	.004	.056
			40	.001	.000	.000	.000	.000	.002	.002	.004	.100
		10	20	.000	.000	.000	.000	.000	.001	.001	.003	.007
			30	.000	.000	.000	.000	.000	.001	.002	.003	.022
			40	.000	.000	.000	.000	.000	.002	.002	.003	.041
	0.5	5	20	.001	.001	.001	.001	.001	.002	.002	.004	.025
			30	.000	.000	.000	.001	.000	.002	.002	.004	.055
			40	.001	.001	.001	.001	.001	.002	.002	.004	.112
		10	20	.000	.000	.000	.000	.000	.002	.002	.003	.008
			30	.000	.000	.000	.000	.000	.002	.002	.003	.022
			40	.000	.000	.000	.000	.000	.002	.002	.003	.050

Table 21

RMSE estimates for the CDRF statistic.

N_R/N_F	δ_θ	n_a	J	DIF1	DIF2	DIF3	DIF4	DIF5	DBF3a	DBF3b	DBF5	DTF
250/250	0	5	20	1.80	1.95	1.90	2.01	2.01	1.90	2.03	1.95	1.93
			30	2.18	1.92	2.08	2.13	2.11	2.04	2.13	2.04	1.97
			40	1.92	2.08	1.93	2.16	2.16	1.86	2.19	2.02	1.87
		10	20	1.33	1.38	1.44	1.39	1.40	1.37	1.35	1.36	1.36
			30	1.35	1.36	1.31	1.42	1.49	1.37	1.41	1.44	1.43
			40	1.30	1.37	1.33	1.41	1.36	1.40	1.37	1.40	1.33
	0.5	5	20	2.08	2.06	1.98	1.93	2.10	2.13	2.09	2.21	2.28
			30	2.06	2.22	2.42	2.14	2.29	2.33	2.57	2.45	2.52
			40	2.09	2.21	2.03	2.30	2.20	2.36	2.26	2.38	2.45
		10	20	1.33	1.39	1.40	1.52	1.55	1.44	1.52	1.55	1.63
			30	1.38	1.43	1.48	1.51	1.54	1.54	1.57	1.59	1.60
			40	1.42	1.34	1.48	1.49	1.63	1.53	1.50	1.51	1.75
500/500	0	5	20	1.88	1.89	2.15	2.33	2.51	1.84	2.48	2.18	1.73
			30	1.96	1.98	1.99	2.16	2.29	1.97	2.21	2.09	1.85
			40	2.04	1.98	2.35	1.89	1.97	2.24	2.07	1.97	1.68
		10	20	1.33	1.37	1.37	1.37	1.41	1.38	1.39	1.42	1.38
			30	1.36	1.38	1.39	1.44	1.34	1.36	1.41	1.39	1.31
			40	1.25	1.37	1.37	1.32	1.32	1.36	1.33	1.32	1.33
	0.5	5	20	1.93	2.04	2.12	2.11	2.32	2.22	2.35	2.37	2.29
			30	2.06	2.42	2.16	2.06	2.02	2.39	2.19	2.37	2.54
			40	1.95	2.05	2.25	2.29	2.22	2.16	2.47	2.34	2.44
		10	20	1.23	1.45	1.26	1.39	1.37	1.32	1.38	1.32	1.35
			30	1.46	1.51	1.38	1.40	1.37	1.56	1.46	1.60	1.69
			40	1.47	1.49	1.54	1.39	1.46	1.65	1.49	1.60	1.72
1000/1000	0	5	20	2.07	1.98	2.12	2.16	2.25	2.20	2.48	2.36	1.91
			30	2.02	2.05	2.29	2.26	2.46	2.31	2.39	2.38	1.78
			40	1.95	2.08	2.12	2.28	2.39	2.18	2.50	2.33	1.80
		10	20	1.34	1.42	1.39	1.35	1.35	1.46	1.41	1.44	1.42
			30	1.39	1.42	1.38	1.41	1.40	1.52	1.49	1.54	1.43
			40	1.39	1.42	1.38	1.30	1.39	1.40	1.39	1.36	1.26
	0.5	5	20	1.78	1.93	1.95	2.16	2.60	2.13	2.56	2.46	2.28
			30	2.16	2.13	2.50	2.17	2.25	2.69	2.68	2.81	2.34
			40	2.05	2.14	1.97	2.02	2.30	2.24	2.28	2.42	2.31
		10	20	1.37	1.42	1.43	1.38	1.52	1.46	1.55	1.54	1.43
			30	1.43	1.51	1.39	1.58	1.50	1.52	1.65	1.66	1.62
			40	1.39	1.48	1.40	1.49	1.55	1.46	1.61	1.61	1.48

Table 22

RE estimates for the compensatory SIBTEST statistic relative to CDRF.

N_R/N_F	δ_θ	n_a	J	DIF1	DIF2	DIF3	DIF4	DIF5	DBF3a	DBF3b	DBF5	DTF
500/250	0	5	20	.001	.001	.001	.001	.001	.005	.006	.012	.065
			30	.001	.001	.001	.001	.001	.005	.005	.011	.148
			40	.001	.001	.001	.001	.001	.005	.006	.011	.321
		10	20	.001	.001	.001	.001	.001	.004	.004	.009	.022
			30	.001	.001	.001	.001	.001	.004	.004	.008	.066
			40	.001	.001	.001	.001	.001	.005	.004	.009	.139
	0.5	5	20	.002	.002	.002	.002	.001	.006	.006	.012	.070
			30	.001	.002	.001	.002	.001	.006	.006	.012	.163
			40	.002	.002	.002	.002	.001	.006	.006	.012	.313
		10	20	.001	.002	.001	.001	.001	.005	.005	.009	.024
			30	.001	.001	.001	.001	.002	.005	.005	.009	.068
			40	.001	.001	.001	.001	.001	.005	.005	.009	.132
1000/500	0	5	20	.001	.001	.001	.001	.001	.003	.003	.005	.029
			30	.001	.001	.001	.001	.001	.003	.003	.005	.079
			40	.001	.001	.001	.001	.001	.003	.003	.005	.145
		10	20	.001	.001	.001	.001	.001	.002	.002	.004	.011
			30	.001	.001	.001	.001	.001	.002	.002	.004	.029
			40	.001	.001	.001	.001	.001	.002	.002	.004	.068
	0.5	5	20	.001	.001	.001	.001	.001	.003	.003	.006	.034
			30	.001	.001	.001	.001	.001	.003	.003	.006	.079
			40	.001	.001	.001	.001	.001	.003	.003	.007	.168
		10	20	.001	.001	.001	.001	.001	.002	.003	.005	.012
			30	.001	.001	.001	.001	.001	.002	.002	.005	.034
			40	.001	.001	.001	.001	.001	.003	.002	.005	.070
2000/1000	0	5	20	.000	.000	.000	.000	.000	.001	.001	.003	.017
			30	.000	.000	.000	.000	.000	.001	.001	.003	.039
			40	.000	.000	.000	.000	.000	.001	.001	.003	.076
		10	20	.000	.000	.000	.000	.000	.001	.001	.002	.005
			30	.000	.000	.000	.000	.000	.001	.001	.002	.014
			40	.000	.000	.000	.000	.000	.001	.001	.002	.032
	0.5	5	20	.000	.000	.000	.000	.000	.001	.001	.003	.017
			30	.000	.000	.000	.000	.000	.002	.002	.003	.046
			40	.000	.000	.000	.000	.000	.001	.002	.003	.087
		10	20	.000	.000	.000	.000	.000	.001	.001	.002	.006
			30	.000	.000	.000	.000	.000	.001	.001	.002	.017
			40	.000	.000	.000	.000	.000	.001	.001	.002	.034

Table 23

RMSE estimates for the CDRF statistic.

N_R/N_F	δ_θ	n_a	J	DIF1	DIF2	DIF3	DIF4	DIF5	DBF3a	DBF3b	DBF5	DTF
500/250	0	5	20	2.08	1.88	1.84	1.99	2.04	1.90	2.00	1.97	1.77
			30	2.14	1.89	2.24	2.00	2.31	2.03	2.20	2.17	1.85
			40	2.19	1.90	2.10	1.94	2.01	2.26	2.13	2.15	1.82
		10	20	1.34	1.40	1.33	1.32	1.43	1.45	1.37	1.47	1.44
			30	1.34	1.33	1.35	1.34	1.35	1.35	1.39	1.40	1.32
			40	1.24	1.37	1.41	1.39	1.39	1.33	1.37	1.36	1.38
	0.5	5	20	2.29	1.94	2.27	2.18	2.39	2.43	2.48	2.59	2.79
			30	2.23	2.11	2.39	2.28	2.47	2.54	2.56	2.63	3.42
			40	2.18	2.09	2.46	2.42	2.46	2.23	2.67	2.43	2.99
		10	20	1.47	1.33	1.39	1.39	1.64	1.56	1.71	1.65	1.78
			30	1.29	1.34	1.49	1.51	1.56	1.62	1.80	1.84	2.00
			40	1.40	1.45	1.59	1.54	1.56	1.75	1.89	1.88	2.32
1000/500	0	5	20	2.06	1.92	1.92	2.32	2.18	1.95	2.33	2.11	1.91
			30	2.25	1.98	2.22	2.20	2.26	2.23	2.49	2.38	1.97
			40	2.01	2.00	1.98	2.24	2.11	1.97	2.18	2.03	1.80
		10	20	1.31	1.31	1.34	1.34	1.40	1.30	1.39	1.36	1.40
			30	1.50	1.28	1.32	1.31	1.35	1.45	1.39	1.45	1.41
			40	1.38	1.35	1.37	1.36	1.42	1.42	1.44	1.48	1.36
	0.5	5	20	1.87	1.99	2.00	2.06	2.48	2.04	2.57	2.31	2.63
			30	2.09	1.96	2.02	2.35	2.30	2.19	2.57	2.39	2.68
			40	1.84	2.20	2.11	2.25	2.38	2.17	2.39	2.28	2.47
		10	20	1.37	1.54	1.53	1.59	1.64	1.55	1.69	1.68	1.81
			30	1.40	1.51	1.50	1.58	1.68	1.45	1.69	1.56	1.86
			40	1.48	1.49	1.56	1.56	1.73	1.63	1.68	1.64	1.71
2000/1000	0	5	20	2.51	2.15	2.14	2.36	2.35	2.68	2.80	2.83	2.12
			30	2.10	1.97	2.53	2.56	2.41	2.56	3.04	2.91	1.96
			40	2.04	2.09	2.23	2.41	2.46	2.36	2.65	2.59	1.80
		10	20	1.32	1.34	1.32	1.51	1.33	1.39	1.50	1.49	1.48
			30	1.39	1.41	1.28	1.48	1.32	1.54	1.46	1.57	1.40
			40	1.38	1.39	1.41	1.32	1.37	1.56	1.41	1.48	1.32
	0.5	5	20	2.35	2.18	1.81	2.15	2.60	2.42	2.46	2.51	3.08
			30	2.09	1.84	2.05	2.30	2.28	2.08	2.36	2.15	2.71
			40	2.10	2.15	2.21	2.35	2.86	2.37	2.89	2.71	3.10
		10	20	1.50	1.40	1.37	1.47	1.57	1.50	1.53	1.44	1.53
			30	1.54	1.48	1.47	1.62	1.59	1.60	1.66	1.69	1.85
			40	1.53	1.62	1.49	1.54	1.63	1.58	1.63	1.57	1.93

Table 24

RE estimates for the compensatory SIBTEST statistic relative to CDRF.

N_R/N_F	δ_θ	n_a	J	DIF1	DIF2	DIF3	DIF4	DIF5	DBF3a	DBF3b	DBF5	DTF
250/250	0	5	20	.002	.001	.001	.001	.001	.006	.006	.011	.061
			30	.001	.001	.001	.001	.001	.006	.005	.010	.156
			40	.002	.001	.001	.001	.001	.006	.005	.011	.360
		10	20	.002	.001	.001	.001	.001	.005	.005	.009	.023
			30	.001	.001	.001	.001	.001	.005	.004	.008	.064
			40	.002	.001	.001	.001	.001	.005	.004	.008	.138
	0.5	5	20	.002	.002	.001	.001	.001	.007	.006	.013	.073
			30	.002	.002	.001	.001	.001	.006	.006	.012	.199
			40	.002	.002	.001	.001	.001	.006	.005	.012	.396
		10	20	.002	.001	.001	.001	.001	.006	.005	.010	.024
			30	.001	.001	.001	.001	.001	.005	.005	.009	.071
			40	.002	.001	.001	.001	.001	.006	.004	.009	.143
500/500	0	5	20	.001	.001	.001	.001	.001	.003	.003	.006	.031
			30	.001	.001	.001	.001	.001	.003	.002	.005	.078
			40	.001	.001	.001	.001	.001	.003	.003	.005	.166
		10	20	.001	.001	.001	.001	.001	.003	.002	.004	.011
			30	.001	.001	.001	.001	.001	.003	.002	.004	.029
			40	.001	.001	.001	.001	.001	.003	.002	.004	.067
	0.5	5	20	.001	.001	.001	.001	.001	.004	.004	.007	.034
			30	.001	.001	.001	.001	.001	.003	.003	.006	.085
			40	.001	.001	.001	.001	.001	.003	.003	.006	.189
		10	20	.001	.001	.001	.001	.001	.003	.003	.005	.012
			30	.001	.001	.001	.001	.001	.003	.002	.005	.035
			40	.001	.001	.001	.001	.001	.003	.002	.004	.063
1000/1000	0	5	20	.000	.000	.000	.000	.000	.002	.001	.003	.016
			30	.000	.000	.000	.000	.000	.002	.001	.003	.039
			40	.001	.000	.000	.000	.000	.002	.001	.003	.073
		10	20	.000	.000	.000	.000	.000	.001	.001	.002	.006
			30	.000	.000	.000	.000	.000	.001	.001	.002	.017
			40	.000	.000	.000	.000	.000	.001	.001	.002	.028
	0.5	5	20	.001	.000	.000	.000	.000	.002	.002	.004	.019
			30	.000	.000	.000	.000	.000	.002	.002	.003	.040
			40	.000	.000	.000	.000	.000	.002	.002	.003	.080
		10	20	.000	.000	.000	.000	.000	.002	.001	.003	.007
			30	.000	.000	.000	.000	.000	.001	.001	.002	.017
			40	.000	.000	.000	.000	.000	.002	.001	.003	.032

Table 25

RMSE estimates for the NCDRF statistic.

N_R/N_F	δ_θ	n_a	J	DIF1	DIF2	DIF3	DIF4	DIF5	DBF3a	DBF3b	DBF5	DTF
250/250	0	5	20	1.78	1.87	1.11	1.37	3.67	2.12	6.94	2.37	1.10
			30	2.20	2.03	1.10	1.73	3.75	2.42	7.01	3.12	1.29
			40	2.00	1.94	1.21	1.87	4.74	1.97	7.92	2.72	1.24
		10	20	1.42	1.33	1.10	1.33	2.26	1.44	3.15	1.36	1.13
			30	1.45	1.43	1.17	1.54	2.27	1.56	2.90	1.55	1.20
			40	1.33	1.38	1.24	1.49	2.39	1.58	3.71	1.68	1.47
	0.5	5	20	1.96	1.81	0.89	1.19	2.66	1.88	5.91	1.82	1.01
			30	1.82	1.71	0.96	1.36	3.42	2.29	6.87	2.15	1.56
			40	1.90	1.83	0.86	1.55	2.82	2.31	7.40	2.18	1.72
		10	20	1.42	1.33	0.90	1.16	1.77	1.47	2.41	1.68	1.12
			30	1.40	1.34	1.23	1.40	2.28	1.73	3.42	1.76	1.20
			40	1.44	1.39	1.16	1.52	2.09	1.49	3.78	1.77	1.92
500/500	0	5	20	2.03	2.13	1.83	6.42	13.44	2.28	33.44	5.89	1.17
			30	2.10	2.02	2.06	7.54	18.38	2.35	38.85	5.68	1.14
			40	2.24	2.21	2.25	7.66	19.83	2.84	38.14	5.76	1.33
		10	20	1.32	1.45	1.32	2.41	4.02	1.58	7.25	2.76	1.49
			30	1.37	1.47	1.44	2.69	5.08	1.57	7.62	2.86	1.16
			40	1.31	1.37	1.35	3.05	5.58	1.52	8.19	2.21	1.20
	0.5	5	20	2.06	1.76	1.12	3.45	10.23	2.48	22.42	4.74	0.83
			30	2.09	2.34	1.42	4.70	14.93	2.83	24.98	5.69	1.26
			40	2.00	2.14	1.49	5.66	15.72	2.57	26.35	5.47	1.61
		10	20	1.26	1.49	1.17	1.86	3.57	1.51	5.07	2.05	1.42
			30	1.45	1.58	1.28	2.61	4.46	1.70	7.53	2.82	1.12
			40	1.45	1.61	1.57	3.12	5.03	1.73	7.46	3.03	1.40
1000/1000	0	5	20	2.14	2.35	5.31	27.11	58.34	2.59	129.79	13.40	1.83
			30	2.07	2.56	5.80	34.39	66.17	2.76	180.15	14.54	0.99
			40	2.00	2.79	4.91	28.92	70.78	2.64	166.34	12.35	1.09
		10	20	1.37	1.54	1.84	4.77	7.08	1.73	15.02	5.76	1.99
			30	1.40	1.60	2.26	6.21	8.51	1.79	16.27	5.41	1.11
			40	1.41	1.63	2.36	5.88	10.95	1.66	21.48	7.02	1.27
	0.5	5	20	1.80	1.95	3.89	17.64	40.09	2.41	97.90	12.52	1.33
			30	2.23	2.62	4.26	20.53	45.85	3.26	119.03	16.28	1.00
			40	2.06	2.60	3.63	22.90	55.33	2.65	131.51	16.26	1.18
		10	20	1.38	1.46	1.82	4.51	8.71	1.64	14.88	4.88	1.83
			30	1.44	1.76	2.00	5.69	8.67	1.69	16.49	6.93	1.42
			40	1.46	1.52	2.10	6.03	9.46	1.65	18.12	5.53	1.37

Table 26

RE estimates for the Crossing-SIBTEST statistic relative to NCDRF.

N_R/N_F	δ_θ	n_a	J	DIF1	DIF2	DIF3	DIF4	DIF5	DBF3a	DBF3b	DBF5	DTF
500/250	0	5	20	.001	.001	.001	.001	.001	.005	.004	.009	.047
			30	.001	.001	.001	.001	.001	.005	.004	.008	.125
			40	.001	.001	.001	.001	.001	.004	.004	.008	.271
		10	20	.001	.001	.001	.001	.001	.004	.004	.007	.017
			30	.001	.001	.001	.001	.001	.004	.003	.006	.046
			40	.001	.001	.001	.001	.001	.004	.003	.006	.099
	0.5	5	20	.001	.001	.001	.001	.001	.006	.006	.012	.052
			30	.001	.001	.001	.001	.001	.005	.005	.011	.133
			40	.001	.001	.001	.001	.001	.005	.005	.010	.295
		10	20	.001	.001	.001	.001	.001	.005	.004	.009	.020
			30	.001	.001	.001	.001	.001	.004	.004	.008	.052
			40	.001	.001	.001	.001	.001	.004	.004	.008	.102
1000/500	0	5	20	.001	.001	.001	.000	.001	.003	.002	.004	.023
			30	.001	.001	.000	.000	.000	.002	.002	.004	.056
			40	.001	.001	.000	.000	.001	.002	.002	.004	.115
		10	20	.001	.001	.001	.000	.000	.002	.002	.003	.009
			30	.001	.001	.000	.000	.000	.002	.002	.003	.023
			40	.001	.001	.000	.000	.000	.002	.002	.003	.044
	0.5	5	20	.001	.001	.001	.001	.001	.003	.003	.005	.026
			30	.001	.001	.001	.001	.001	.003	.003	.005	.061
			40	.001	.001	.001	.001	.001	.003	.002	.005	.137
		10	20	.001	.001	.001	.001	.001	.002	.002	.005	.011
			30	.001	.001	.001	.001	.001	.002	.002	.004	.026
			40	.001	.001	.001	.001	.001	.002	.002	.004	.050
2000/1000	0	5	20	.000	.000	.000	.000	.000	.001	.001	.002	.013
			30	.000	.000	.000	.000	.000	.001	.001	.002	.029
			40	.000	.000	.000	.000	.000	.001	.001	.002	.053
		10	20	.000	.000	.000	.000	.000	.001	.001	.002	.004
			30	.000	.000	.000	.000	.000	.001	.001	.001	.011
			40	.000	.000	.000	.000	.000	.001	.001	.002	.024
	0.5	5	20	.000	.000	.000	.000	.000	.001	.001	.003	.015
			30	.000	.000	.000	.000	.000	.001	.001	.003	.034
			40	.000	.000	.000	.000	.000	.001	.001	.003	.063
		10	20	.000	.000	.000	.000	.000	.001	.001	.002	.006
			30	.000	.000	.000	.000	.000	.001	.001	.002	.014
			40	.000	.000	.000	.000	.000	.001	.001	.002	.026

Table 27

RMSE estimates for the NCDRF statistic.

N_R/N_F	δ_θ	n_a	J	DIF1	DIF2	DIF3	DIF4	DIF5	DBF3a	DBF3b	DBF5	DTF
500/250	0	5	20	2.14	1.97	1.25	2.47	7.38	2.36	13.51	3.43	0.99
			30	2.29	2.02	1.47	3.22	7.20	2.43	16.74	3.53	1.14
			40	2.50	1.82	1.67	3.04	8.88	2.93	16.47	3.77	1.47
		10	20	1.39	1.46	1.07	1.79	2.30	1.59	4.28	1.79	1.35
			30	1.44	1.50	1.37	1.90	3.21	1.53	4.92	2.11	1.17
			40	1.36	1.48	1.50	2.15	2.91	1.53	5.27	2.21	1.28
	0.5	5	20	2.38	1.40	0.66	1.57	3.53	2.20	8.22	2.54	0.85
			30	2.17	1.56	0.98	2.01	4.69	2.43	10.70	3.24	1.60
			40	2.16	1.75	1.05	2.05	5.86	2.22	11.38	3.37	1.60
		10	20	1.48	1.26	0.83	1.33	1.66	1.44	3.87	1.79	1.06
			30	1.49	1.14	1.06	1.32	2.30	1.59	4.18	2.11	1.27
			40	1.50	1.37	0.99	1.72	2.57	1.73	5.79	3.05	1.70
1000/500	0	5	20	2.15	2.30	2.87	13.46	26.18	2.28	71.34	8.35	1.07
			30	2.33	2.39	3.41	12.61	33.95	2.59	72.69	8.95	1.12
			40	2.16	2.41	3.54	14.05	24.40	2.48	62.32	8.61	1.21
		10	20	1.38	1.45	1.49	3.29	6.08	1.49	9.77	3.32	1.66
			30	1.55	1.36	1.76	3.89	5.68	1.77	12.39	4.32	1.13
			40	1.41	1.51	1.63	4.40	6.94	1.62	13.28	4.22	1.33
	0.5	5	20	1.97	1.94	1.56	6.93	18.39	2.30	44.43	8.32	0.94
			30	2.13	1.83	1.95	8.51	19.22	2.25	47.06	9.53	0.93
			40	1.91	2.20	2.18	11.29	20.35	2.47	57.70	9.26	1.32
		10	20	1.41	1.31	1.29	2.61	3.55	1.68	6.78	4.11	1.52
			30	1.41	1.48	1.34	3.12	4.40	1.49	10.22	4.11	1.32
			40	1.50	1.50	1.39	3.53	5.92	1.64	11.04	4.60	1.36
2000/1000	0	5	20	2.55	2.69	6.93	42.40	114.25	3.10	250.02	17.02	2.15
			30	2.13	2.39	7.52	51.13	129.68	3.15	310.00	23.13	1.31
			40	2.09	2.53	9.09	63.27	145.65	2.71	351.42	26.17	1.15
		10	20	1.33	1.54	2.78	5.71	11.84	1.59	28.30	10.30	2.90
			30	1.40	1.63	3.03	9.08	11.44	1.74	28.90	10.05	1.39
			40	1.39	1.73	2.56	8.61	14.86	1.73	30.18	8.87	1.20
	0.5	5	20	2.43	2.19	5.56	36.10	61.12	2.52	180.23	23.54	2.02
			30	2.11	2.09	8.29	50.71	62.83	2.36	237.28	27.83	0.82
			40	2.11	2.24	7.70	38.53	96.80	2.55	255.83	27.06	1.18
		10	20	1.51	1.44	1.90	4.94	8.17	1.49	16.38	8.36	2.50
			30	1.53	1.45	2.73	7.22	11.27	1.67	24.57	9.72	1.57
			40	1.56	1.63	2.86	7.60	9.06	1.76	24.03	10.28	1.21

Table 28

RE estimates for the Crossing-SIBTEST statistic relative to NCDRF.