APPROXIMATED PENALIZED MAXIMUM LIKELIHOOD FOR EXPLORATORY FACTOR ANALYSIS: AN ORTHOGONAL CASE

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Abstract

The problem of penalized maximum likelihood (PML) for an exploratory factor analysis (EFA) model is studied in this paper. An EFA model is typically estimated using maximum likelihood and then the estimated loading matrix is rotated to obtain a sparse representation. Penalized maximum likelihood simultaneously fits the EFA model and produces a sparse loading matrix. To overcome some of the computational drawbacks of PML, an approximation to PML is proposed in this paper. It is further applied to an empirical dataset for illustration. A simulation study shows that the approximation naturally produces a sparse loading matrix and more accurately estimates the factor loadings and the covariance matrix, in the sense of having a lower mean squared error than factor rotations, under various conditions.

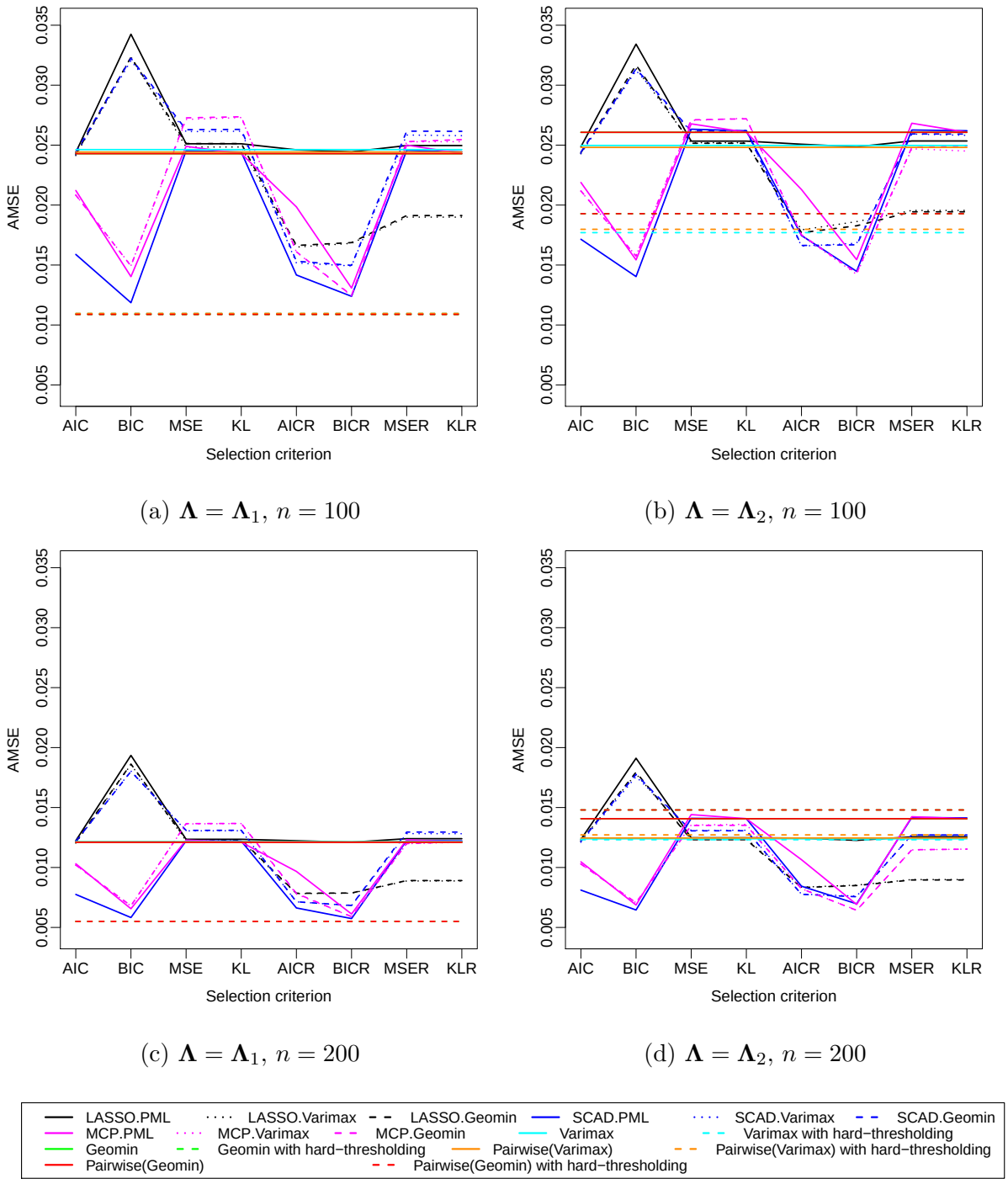
Key words: factor rotation, LASSO, SCAD, MCP, sparsity, shrinkage

1. Supplementary Material

Table 1: Percentage of recovering the correct loading structure when $\mathbf{\Lambda} = \mathbf{\Lambda}_1$, if the covariance matrix is factorized.

				Selection criterion							
n	Penalty	Method	Rotation	AIC	BIC	MSE	KL	AICR	BICR	MSER	KLR
100	LASSO	PML	-	0.10	2.40	0.00	0.00	0.00	0.00	0.00	0.00
		APML	Varimax	0.00	0.70	0.00	0.00	9.20	46.70	0.00	0.00
			Geomin	0.00	0.80	0.00	0.00	8.70	45.70	0.00	0.00
	SCAD	PML	-	26.80	77.50	0.00	0.00	20.40	32.40	0.00	0.00
		APML	Varimax	0.00	0.50	0.00	0.00	33.60	69.00	0.00	0.00
			Geomin	0.00	0.50	0.00	0.00	32.50	67.90	0.00	0.00
	MCP	PML	-	6.20	56.90	0.00	0.00	16.50	62.60	0.00	0.00
		APML	Varimax	5.00	50.20	0.00	0.00	37.10	80.60	0.00	0.00
			Geomin	5.00	50.20	0.00	0.00	36.40	79.70	0.00	0.00
	-	-	Varimax					99.90			
	-	-	Geomin					99.50			
	-	Pairwise	Varimax					99.90			
	-		Geomin					99.40			
200	LASSO	PML	-	0.10	3.60	0.00	0.00	0.00	0.00	0.00	0.00
		APML	Varimax	0.10	1.50	0.00	0.00	10.00	58.10	0.00	0.00
			Geomin	0.00	1.40	0.00	0.00	9.90	57.20	0.00	0.00
	SCAD	PML	-	28.30	86.90	0.00	0.00	50.80	78.30	0.00	0.00
		APML	Varimax	0.10	1.40	0.00	0.00	40.20	87.10	0.00	0.00
			Geomin	0.10	1.30	0.00	0.00	39.90	86.40	0.00	0.00
	MCP	PML	-	9.00	70.90	0.00	0.00	17.60	82.50	0.00	0.00
		APML	Varimax	7.40	66.30	0.00	0.00	41.10	91.70	0.00	0.00
			Geomin	7.30	66.30	0.00	0.00	40.90	91.60	0.00	0.00
	-		Varimax					100.00			
	-		Geomin					100.00			
	-	Pairwise	Varimax					100.00			
	-		Geomin					100.00			

Note: The rotations do not rely on the selection criterion. Thus, the percentages remain the same across different selection criteria. For the pairwise rotation, the analytical criterion can be either varimax or geomin.



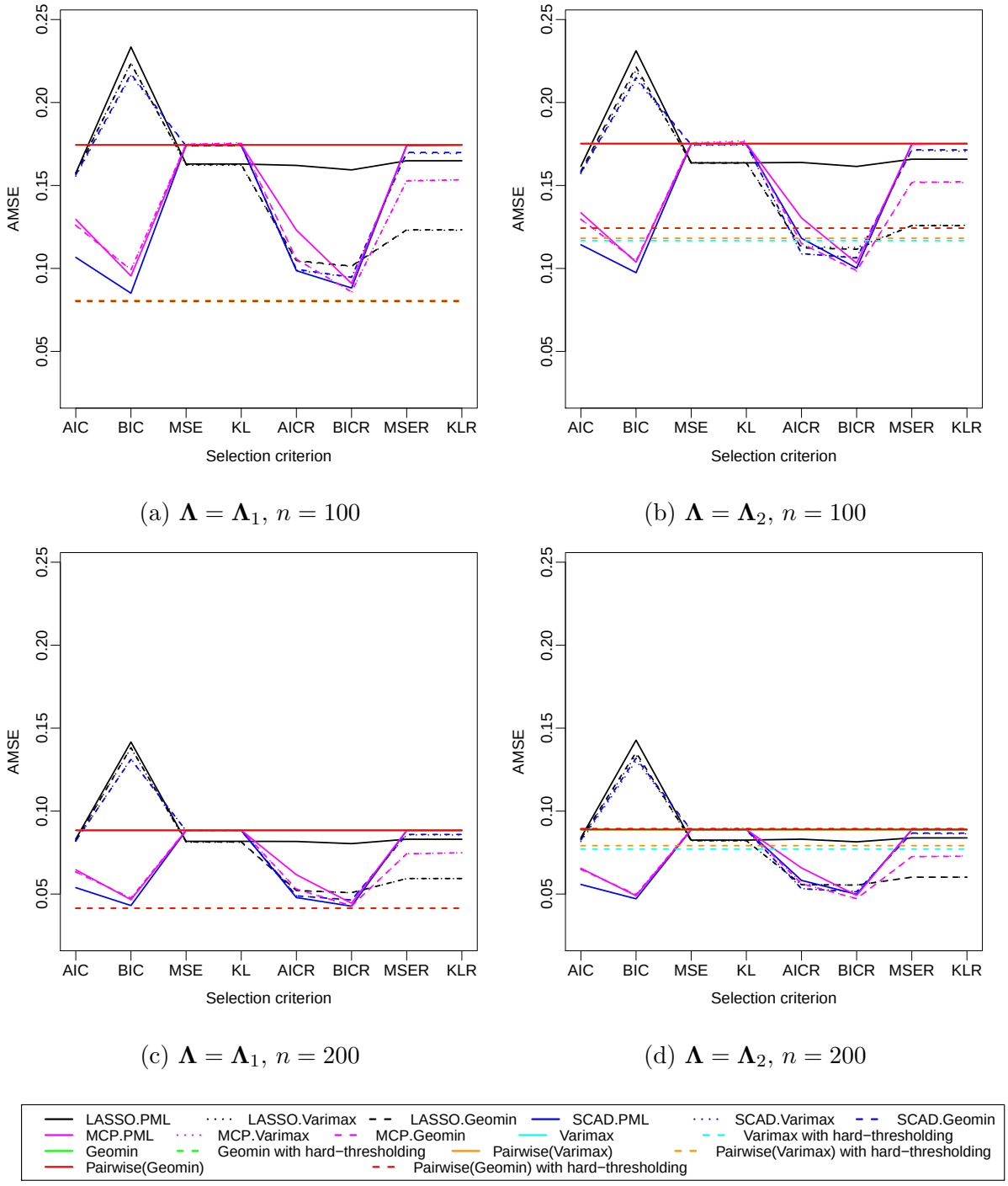
(a) $\mathbf{\Lambda} = \mathbf{\Lambda}_1$, $n = 100$

(b) $\mathbf{\Lambda} = \mathbf{\Lambda}_2$, $n = 100$

(c) $\mathbf{\Lambda} = \mathbf{\Lambda}_1$, $n = 200$

(d) $\mathbf{\Lambda} = \mathbf{\Lambda}_2$, $n = 200$

Figure 1: Average mean squared error (AMSE) of factor loadings if a covariance matrix is factorized. The starting value of APML depends on the factor rotations. The label LASSO.Varimax standards for APML with LASSO penalty and varimax as starting value. The other labels are similarly defined.



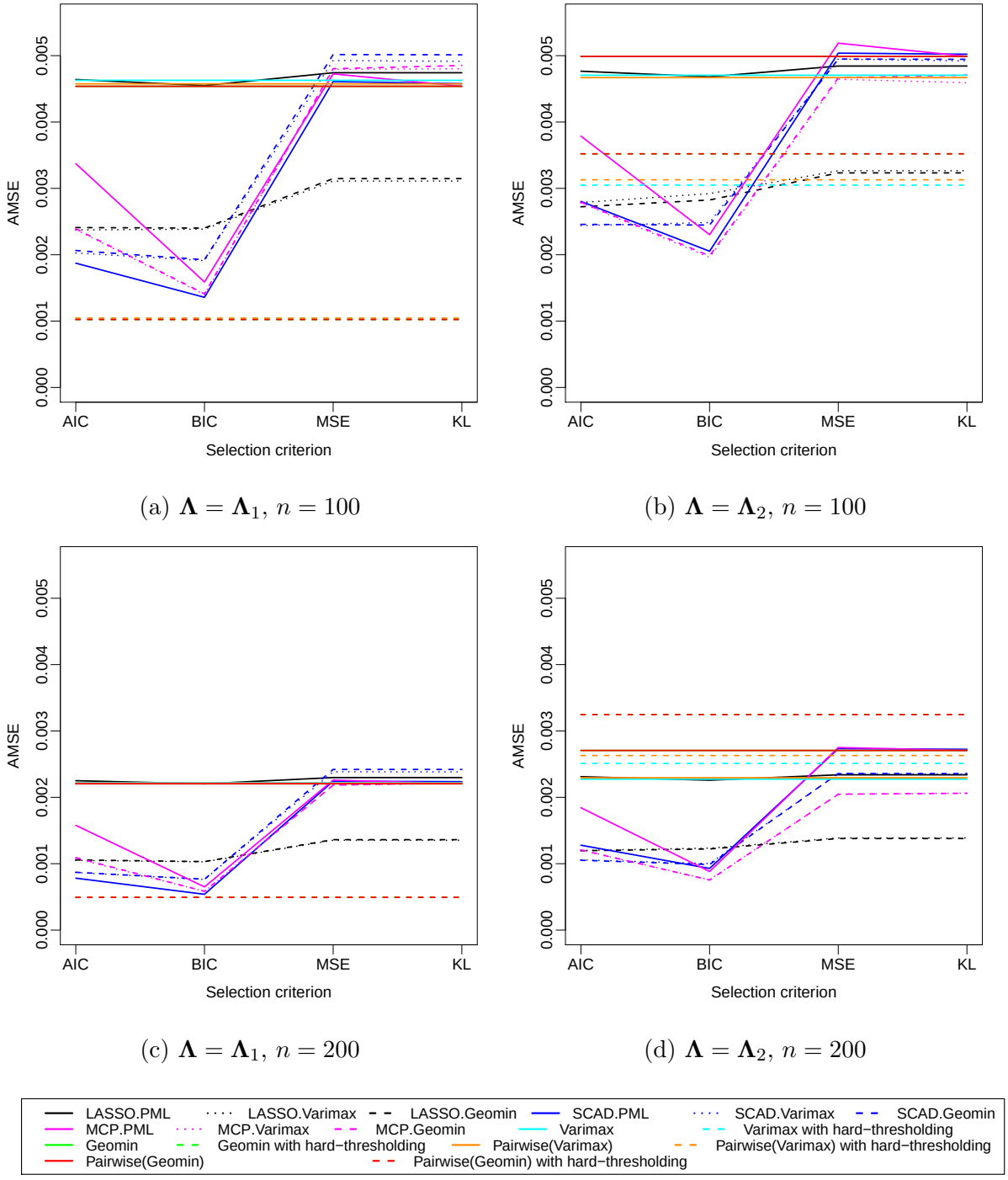
(a) $\mathbf{\Lambda} = \mathbf{\Lambda}_1$, $n = 100$

(b) $\mathbf{\Lambda} = \mathbf{\Lambda}_2$, $n = 100$

(c) $\mathbf{\Lambda} = \mathbf{\Lambda}_1$, $n = 200$

(d) $\mathbf{\Lambda} = \mathbf{\Lambda}_2$, $n = 200$

Figure 2: Average mean squared error (AMSE) of the covariance matrix if a covariance matrix is factorized. The starting value of APML depends on the factor rotations. The label LASSO.Varimax standards for APML with LASSO penalty and varimax as starting value. The other labels are similarly defined.



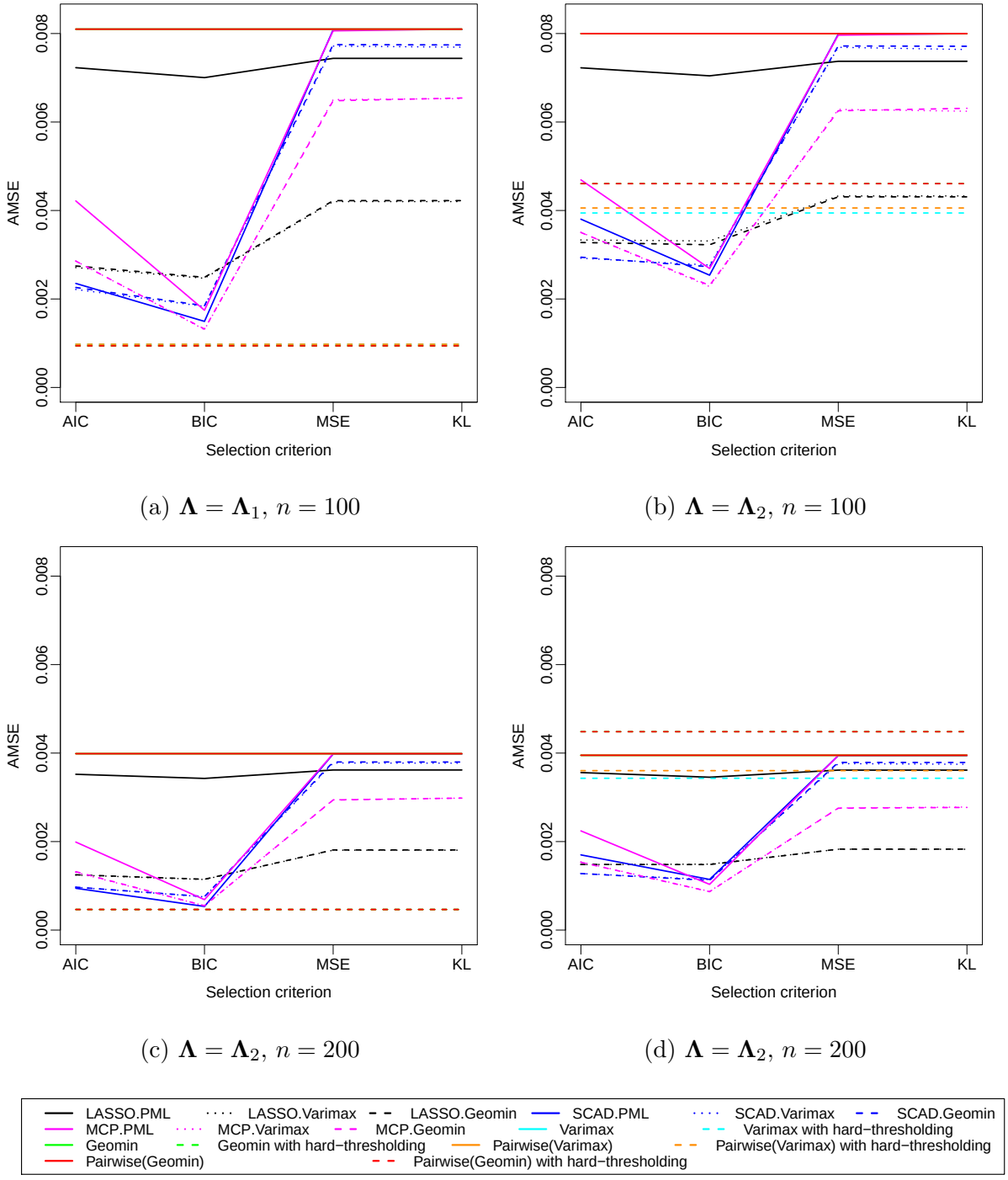
(a) $\mathbf{\Lambda} = \mathbf{\Lambda}_1$, $n = 100$

(b) $\mathbf{\Lambda} = \mathbf{\Lambda}_2$, $n = 100$

(c) $\mathbf{\Lambda} = \mathbf{\Lambda}_1$, $n = 200$

(d) $\mathbf{\Lambda} = \mathbf{\Lambda}_2$, $n = 200$

Figure 3: Average mean squared error (AMSE) of factor loadings if a correlation matrix is factorized. The starting value of APML depends on the factor rotations. The label LASSO.Varimax standards for APML with LASSO penalty and varimax as starting value. The other labels are similarly defined.



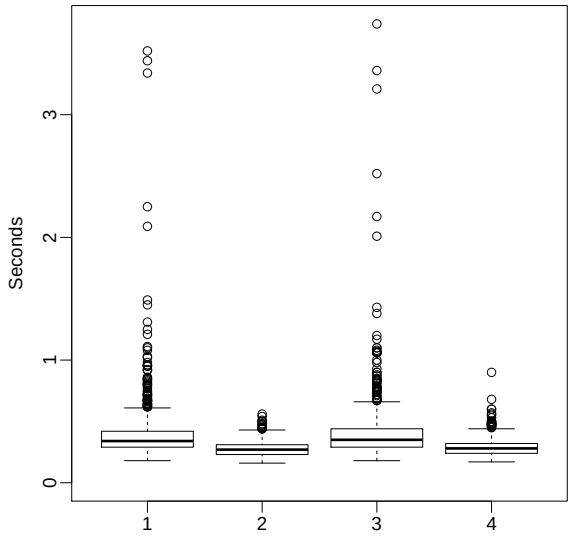
(a) $\mathbf{\Lambda} = \mathbf{\Lambda}_1$, $n = 100$

(b) $\mathbf{\Lambda} = \mathbf{\Lambda}_2$, $n = 100$

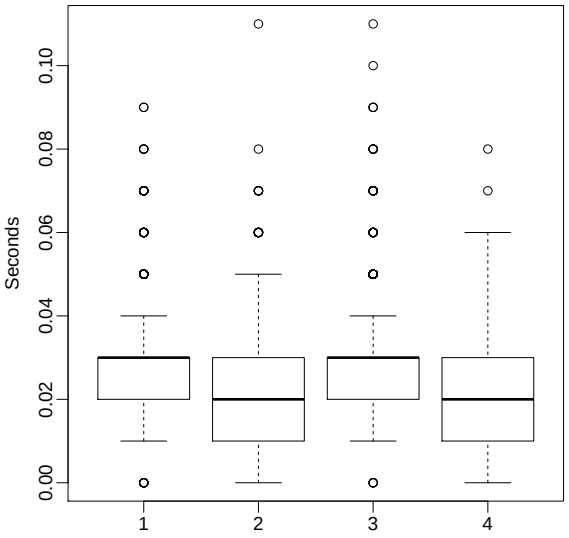
(c) $\mathbf{\Lambda} = \mathbf{\Lambda}_2$, $n = 200$

(d) $\mathbf{\Lambda} = \mathbf{\Lambda}_2$, $n = 200$

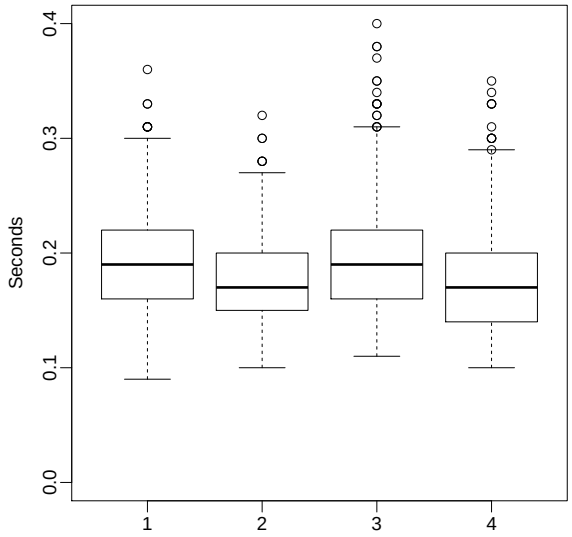
Figure 4: Average mean squared error (AMSE) of the correlation matrix if a correlation matrix is factorized. The starting value of APML depends on the factor rotations. The label LASSO.Varimax standards for APML with LASSO penalty and varimax as starting value. The other labels are similarly defined.



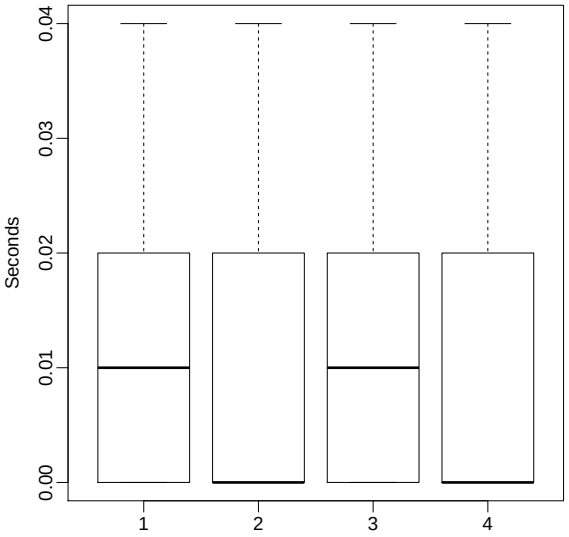
(a) PML when a covariance is factorized



(b) APML when a covariance is factorized



(c) PML when a correlation is factorized



(d) APML when a correlation is factorized

Figure 5: Box plots of elapsed time when the penalty term is the LASSO. The indices 1, 2, 3, and 4 correspond to the settings (1) $\mathbf{\Lambda} = \mathbf{\Lambda}_1$ and $n = 100$, (2) $\mathbf{\Lambda} = \mathbf{\Lambda}_1$ and $n = 200$, (3) $\mathbf{\Lambda} = \mathbf{\Lambda}_2$ and $n = 100$, and (4) $\mathbf{\Lambda} = \mathbf{\Lambda}_2$ and $n = 200$, respectively.

Table 2: Percentage of recovering the correct loading structure when $\mathbf{\Lambda} = \mathbf{\Lambda}_2$, if the covariance matrix is factorized.

				Selection criterion							
n	Penalty	Method	Rotation	AIC	BIC	MSE	KL	AICR	BICR	MSER	KLR
100	LASSO	PML	-	0.00	1.40	0.00	0.00	0.00	0.00	0.00	0.00
		APML	Varimax	0.00	0.30	0.00	0.00	7.40	36.40	0.00	0.00
			Geomin	0.00	0.50	0.00	0.00	7.10	37.80	0.00	0.00
	SCAD	PML	-	23.10	60.30	0.00	0.00	9.80	24.40	0.00	0.00
		APML	Varimax	0.00	0.20	0.00	0.00	21.90	52.60	0.00	0.00
			Geomin	0.00	0.40	0.00	0.00	23.10	53.40	0.00	0.00
	MCP	PML	-	8.40	55.50	0.00	0.00	16.30	50.80	0.00	0.00
		APML	Varimax	6.20	49.50	0.00	0.00	22.30	56.00	0.00	0.00
			Geomin	6.20	49.60	0.00	0.00	23.00	56.80	0.00	0.00
	-	-	Varimax					25.00			
	-	-	Geomin					41.60			
	-	Pairwise	Varimax					25.00			
	-		Geomin					38.20			
200	LASSO	PML	-	0.10	3.30	0.00	0.00	0.00	0.00	0.00	0.00
		APML	Varimax	0.00	0.70	0.00	0.00	8.50	47.30	0.00	0.00
			Geomin	0.00	1.00	0.00	0.00	8.80	48.90	0.00	0.00
	SCAD	PML	-	31.10	81.10	0.00	0.00	14.00	54.00	0.00	0.00
		APML	Varimax	0.00	1.00	0.00	0.00	30.70	75.10	0.00	0.00
			Geomin	0.00	1.00	0.00	0.00	31.00	76.90	0.00	0.00
	MCP	PML	-	12.10	72.50	0.00	0.00	19.30	73.20	0.00	0.00
		APML	Varimax	8.70	68.10	0.00	0.00	33.10	79.50	0.00	0.00
			Geomin	8.80	68.20	0.00	0.00	32.90	79.50	0.00	0.00
	-		Varimax					17.40			
	-		Geomin					40.20			
	-	Pairwise	Varimax					17.40			
	-		Geomin					36.40			

Note: The rotations do not rely on the selection criterion. Thus, the percentages remain the same across different selection criteria. For the pairwise rotation, the analytical criterion can be either varimax or geomin.

Table 3: Percentage of correctly setting a truly zero factor loading as zero in the estimated loading matrix if a covariance matrix is factorized when $\mathbf{\Lambda} = \mathbf{\Lambda}_1$.

				Selection criterion							
n	Penalty	Method	Rotation	AIC	BIC	MSE	KL	AICR	BICR	MSER	KLR
100	LASSO	PML	-	48.67	71.06	18.91	18.91	22.29	29.39	18.17	18.17
		APML	Varimax	44.79	63.35	19.11	19.11	78.58	94.73	42.32	42.32
			Geomin	44.96	63.37	19.07	19.07	78.67	94.53	42.36	42.36
	SCAD	PML	-	86.23	98.18	12.11	9.38	84.31	93.01	12.45	9.72
		APML	Varimax	44.89	63.07	16.49	16.39	87.86	97.44	18.39	18.39
			Geomin	45.29	63.33	16.45	16.33	87.69	97.35	18.32	18.29
	MCP	PML	-	85.12	96.81	11.83	1.79	84.18	96.64	12.84	2.25
		APML	Varimax	84.34	95.90	22.11	21.39	85.61	98.20	49.29	49.63
			Geomin	84.33	95.93	22.11	21.42	85.59	98.11	49.32	49.71
	-	-	Varimax					99.99			
	-	-	Geomin					99.97			
	-	Pairwise	Varimax					99.99			
	-		Geomin					99.97			
200	LASSO	PML	-	48.69	74.52	19.42	19.42	22.79	30.28	18.34	18.34
		APML	Varimax	45.33	68.88	19.69	19.69	79.61	96.29	46.36	46.36
			Geomin	45.30	68.99	19.62	19.62	79.52	96.27	46.43	46.43
	SCAD	PML	-	87.72	99.07	13.91	12.63	90.75	98.59	13.22	12.13
		APML	Varimax	46.69	68.56	16.11	16.05	89.72	99.13	17.96	17.96
			Geomin	46.68	68.57	16.04	15.93	89.79	99.08	17.77	17.77
	MCP	PML	-	86.21	98.11	9.86	2.14	85.67	98.68	10.30	3.13
		APML	Varimax	85.50	97.72	23.00	22.91	87.21	99.41	52.46	52.79
			Geomin	85.47	97.71	22.87	22.80	87.11	99.41	52.47	52.82
	-	-	Varimax					100.00			
	-	-	Geomin					100.00			
	-	Pairwise	Varimax					100.00			
	-		Geomin					100.00			

Note: The rotations do not rely on the selection criterion. Thus, the percentages remain the same across different selection criteria. For the pairwise rotation, the analytical criterion can be either varimax or geomin.

Table 4: Percentage of correctly setting a truly zero factor loading as zero in the estimated loading matrix if a covariance matrix is factorized when $\mathbf{\Lambda} = \mathbf{\Lambda}_2$.

				Selection criterion							
n	Penalty	Method	Rotation	AIC	BIC	MSE	KL	AICR	BICR	MSER	KLR
100	LASSO	PML	-	47.56	68.42	19.85	19.85	22.44	27.70	19.05	19.05
		APML	Varimax	43.11	60.51	19.76	19.76	76.45	92.88	45.06	45.06
			Geomin	43.46	61.32	19.79	19.79	76.66	93.29	45.19	45.19
	SCAD	PML	-	85.58	97.00	11.92	8.66	74.77	90.92	12.37	9.21
		APML	Varimax	44.49	61.55	17.63	17.56	84.04	96.14	19.17	19.32
			Geomin	44.55	61.74	17.43	17.42	84.51	96.19	18.94	18.95
	MCP	PML	-	85.52	96.56	12.87	1.58	81.79	94.82	13.95	1.84
		APML	Varimax	84.72	95.75	23.24	22.53	82.56	96.29	51.84	52.23
			Geomin	84.61	95.79	23.12	22.38	82.59	96.36	51.81	52.34
	-	-	Varimax				99.99				
	-	-	Geomin				99.94				
	-	Pairwise	Varimax				99.99				
	-		Geomin				99.96				
200	LASSO	PML	-	46.86	72.47	20.46	20.46	22.86	29.07	19.59	19.59
		APML	Varimax	44.05	65.79	20.70	20.70	77.21	94.52	49.95	49.95
			Geomin	44.11	66.28	20.69	20.69	77.31	94.76	50.07	50.07
	SCAD	PML	-	88.15	98.54	13.45	11.36	77.62	95.79	13.06	11.42
		APML	Varimax	45.42	66.94	17.68	17.54	86.50	98.12	19.18	19.28
			Geomin	45.86	67.41	17.08	17.01	86.66	98.25	18.87	18.87
	MCP	PML	-	86.69	98.07	10.44	1.99	82.87	97.27	11.09	2.74
		APML	Varimax	85.79	97.72	24.39	24.05	85.71	98.36	55.96	56.48
			Geomin	85.81	97.72	24.23	23.99	85.72	98.38	55.95	56.45
	-	-	Varimax				100.00				
	-	-	Geomin				100.00				
	-	Pairwise	Varimax				100.00				
	-		Geomin				100.00				

Note: The rotations do not rely on the selection criterion. Thus, the percentages remain the same across different selection criteria. For the pairwise rotation, the analytical criterion can be either varimax or geomin.

Table 5: Percentage of falsely setting a nonzero factor loading as zero if a covariance matrix is factorized and $\mathbf{\Lambda} = \mathbf{\Lambda}_2$.

				Selection criterion							
n	Penalty	Method	Rotation	AIC	BIC	MSE	KL	AICR	BICR	MSER	KLR
100	LASSO	PML	-	0.03	0.04	0.02	0.02	0.03	0.03	0.03	0.03
		APML	Varimax	0.03	0.04	0.02	0.02	0.11	0.42	0.06	0.06
			Geomin	0.02	0.03	0.01	0.01	0.11	0.35	0.05	0.05
	SCAD	PML	-	0.22	0.71	0.00	0.00	0.12	0.19	0.00	0.00
		APML	Varimax	0.01	0.02	0.00	0.00	0.17	0.70	0.02	0.02
			Geomin	0.01	0.02	0.00	0.00	0.18	0.70	0.03	0.03
	MCP	PML	-	0.39	0.60	0.00	0.00	0.31	0.49	0.01	0.00
		APML	Varimax	0.25	0.41	0.02	0.04	0.24	0.74	0.11	0.11
			Geomin	0.25	0.40	0.03	0.04	0.24	0.77	0.12	0.12
	-	-	Varimax					7.50			
	-	-	Geomin					5.83			
	-	Pairwise	Varimax					7.50			
	-		Geomin					6.16			
200	LASSO	PML	-	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		APML	Varimax	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
			Geomin	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	SCAD	PML	-	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00
		APML	Varimax	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00
			Geomin	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
	MCP	PML	-	0.06	0.05	0.01	0.00	0.05	0.04	0.01	0.00
		APML	Varimax	0.03	0.04	0.00	0.00	0.01	0.01	0.02	0.02
			Geomin	0.03	0.04	0.00	0.00	0.01	0.01	0.02	0.02
	-	-	Varimax					8.26			
	-	-	Geomin					5.98			
	-	Pairwise	Varimax					8.26			
	-		Geomin					6.36			

Note: The rotations do not rely on the selection criterion. Thus, the percentages remain the same across different selection criteria. For the pairwise rotation, the analytical criterion can be either varimax or geomin. When $\mathbf{\Lambda} = \mathbf{\Lambda}_1$, no falsely zero recovery is encountered.

Table 6: Percentage of containing the correct loading structure in the solution path.

			$\mathbf{\Lambda} = \mathbf{\Lambda}_1$			$\mathbf{\Lambda} = \mathbf{\Lambda}_2$		
n	Method	Rotation	LASSO	SCAD	MCP	LASSO	SCAD	MCP
Factorizing a covariance matrix								
100	PML	-	90.60	100.00	100.00	86.30	93.30	96.40
	APML	Varimax	100.00	98.70	100.00	93.30	89.80	95.80
		Geomin	100.00	98.40	100.00	93.90	91.90	95.40
200	PML	-	99.90	100.00	100.00	99.50	99.50	99.90
	APML	Varimax	100.00	100.00	100.00	99.60	99.10	99.90
		Geomin	100.00	100.00	100.00	99.70	99.50	99.90
Factorizing a correlation matrix								
100	PML	-	95.40	100.00	99.20	85.60	92.30	93.60
	APML	Varimax	100.00	100.00	100.00	91.40	90.20	92.40
		Geomin	100.00	100.00	100.00	91.00	90.70	92.50
200	PML	-	100.00	100.00	100.00	99.10	99.60	99.60
	APML	Varimax	100.00	100.00	100.00	99.40	99.50	99.50
		Geomin	100.00	100.00	100.00	99.40	99.60	99.50