Supplementary Materials for

"A Doubly Latent Space Joint Model for Local Item and Person Dependence in the Analysis of Item Response Data"

June 1, 2018

A DRV Data Analysis Results

A.1 Jumping Rule

- **Jumping Rule**: (1.0, 0.30, 0.30, 0.25, 0.25, 0.25, 0.25, 0.20, 0.20, 0.20, 0.20, 0.15, 0.15, 0.15, 0.15, 0.10, 0.10, 0.10, 0.10, 0.05, 0.05, 0.05, 0.025, 0.025, 0.025)

A.2 The Item Intercept Parameter Estimates

Item	$\hat{\beta}$	95% HPD Interval ($\hat{\beta}$)	Item	$\hat{\beta}$	95% HPD Interval ($\hat{\beta}$)
MP, CO, NE	1.0508	(1.0348, 1.0667)	NA, CO, NE	1.0217	(1.0057, 1.0367)
AC, CO, NE	0.6654	(0.6495, 0.6809)	MT, CO, NE	-0.2187	(-0.2356, -0.2002)
MP, CO, NN	0.7634	(0.7464, 0.7779)	NA, CO, NN	0.7916	(0.7760, 0.8074)
AC, CO, NN	0.2115	(0.1950, 0.2294)	MT, CO, NN	0.0602	(0.0444, 0.0776)
MP, AB, NE	2.3509	(2.3333, 2.3681)	NA, AB, NE	-0.7510	(-0.7712, -0.7311)
AC, AB, NE	-1.5159	(-1.5419, -1.4888)	MT, AB, NE	1.5013	(1.4858, 1.5175)
MP, AB, NN	1.9278	(1.9109, 1.9450)	NA, AB, NN	-1.2888	(-1.3131, -1.2651)
AC, AB, NN	-2.2635	(-2.2986, -2.2267)	MT, AB, NN	0.2252	(0.2088, 0.2415)
MP, CF, NE	1.6535	(1.6372, 1.6689)	NA, CF, NE	-1.0601	(-1.0812, -1.0374)
AC, CF, NE	-1.4959	(-1.5232, -1.4711)	MT, CF, NE	1.4846	(1.4680, 1.5001)
MP, CF, NN	1.6534	(1.6372, 1.6701)	NA, CF, NN	-1.7200	(-1.7479, -1.6914)
AC, CF, NN	-1.9421	(-1.9722, -1.9078)	MT, CF, NN	0.4855	(0.4695, 0.5019)

Table 1: The item intercept parameter estimates (β) and their 95% HPD intervals for the DRV data.

A.3 Five Number Summary of the Person Intercept Parameter Estimates

Item Score	min	25%	median	75%	max
4	-3.4608	-3.4608	-3.4608	-3.4608	-3.4608
5	-2.9073	-2.9051	-2.9030	-2.9002	-2.8975
6	-2.4674	-2.4621	-2.4603	-2.4598	-2.4586
7	-2.0943	-2.0927	-2.0909	-2.0893	-2.0863
8	-1.7759	-1.7697	-1.7672	-1.7645	-1.7617
9	-1.4842	-1.4794	-1.4773	-1.4760	-1.4705
10	-1.2165	-1.2124	-1.2112	-1.2086	-1.2066
11	-0.9656	-0.9625	-0.9608	-0.9588	-0.9564
12	-0.7284	-0.7241	-0.7228	-0.7212	-0.7169
13	-0.4985	-0.4932	-0.4921	-0.4907	-0.4879
14	-0.2698	-0.2669	-0.2656	-0.2632	-0.2590
15	-0.0430	-0.0384	-0.0375	-0.0369	-0.0344
16	0.1903	0.1915	0.1922	0.1936	0.1959
17	0.4275	0.4302	0.4320	0.4331	0.4358
18	0.6797	0.6829	0.6842	0.6863	0.6876
19	0.9600	0.9608	0.9623	0.9635	0.9637
20	1.2697	1.2722	1.2728	1.2735	1.2755
21	1.6396	1.6437	1.6444	1.6452	1.6488
22	2.1318	2.1338	2.1340	2.1371	2.1396
23	2.9105	2.9115	2.9122	2.9148	2.9162
24	12.6810	12.7162	12.7514	12.7789	12.8064

Table 2: Five number summary of the person intercept parameter estimates (θ) categorized by the total scores for the DRV data (minimum, first quartile, median, third quartile, maximum).

B Trace Plots for Checking Convergence

B.1 Item Intercept Parameters

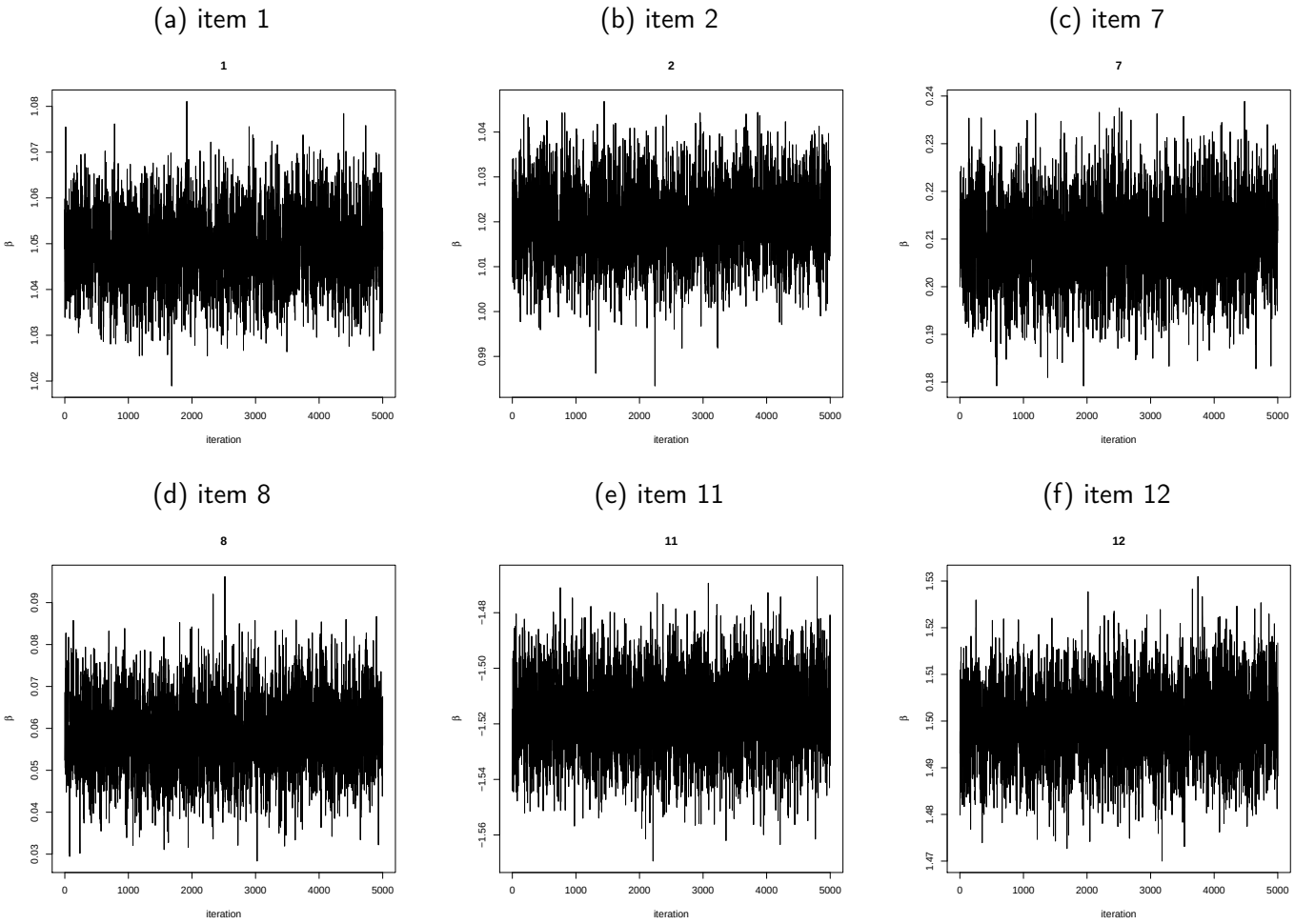

Figure 1: Trace plots of the item intercept parameters for the DRV data..

B.2 Person Intercept Parameters

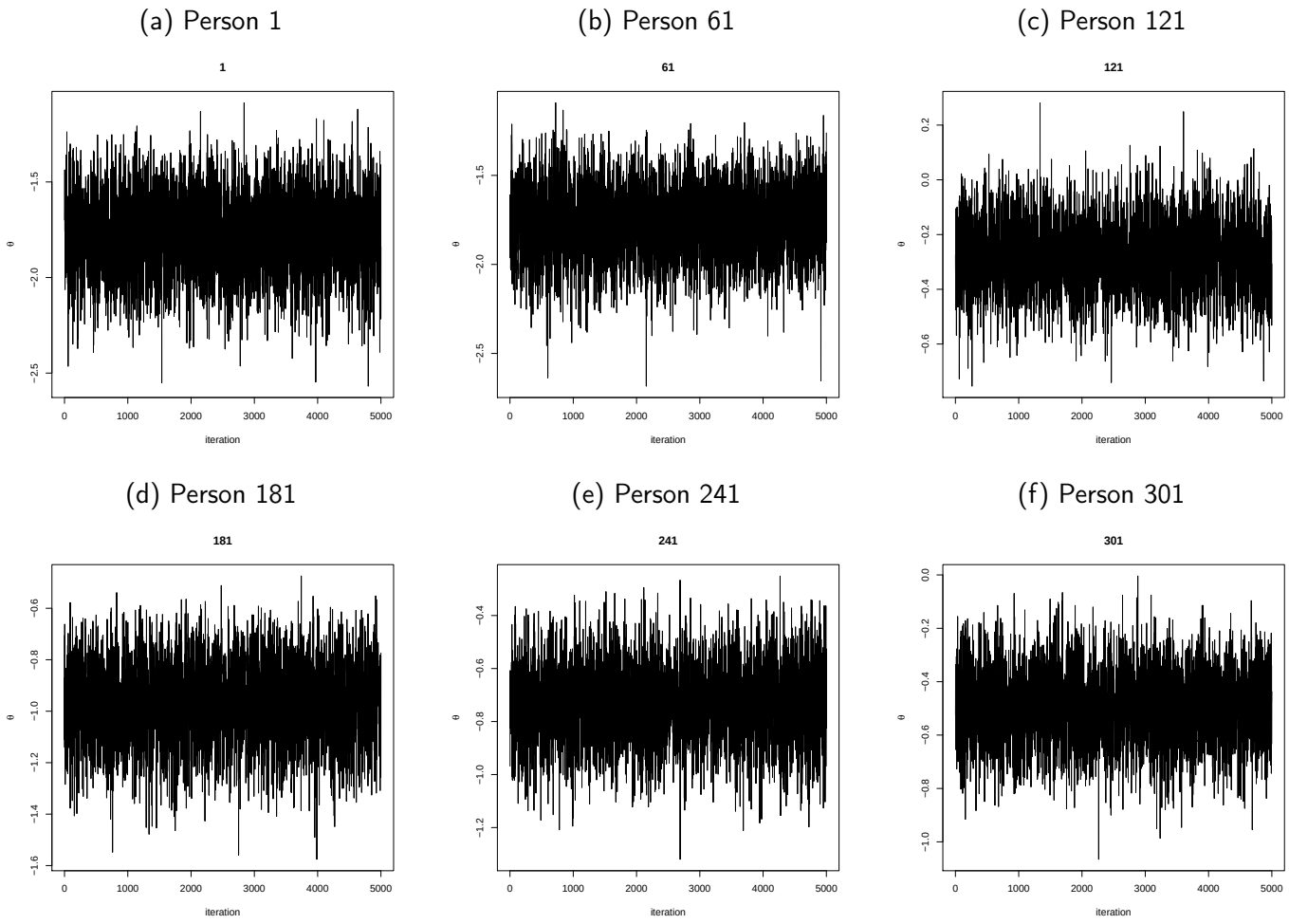

Figure 2: Trace plots of the person intercept parameters for the DRV data..

B.3 Item Distances

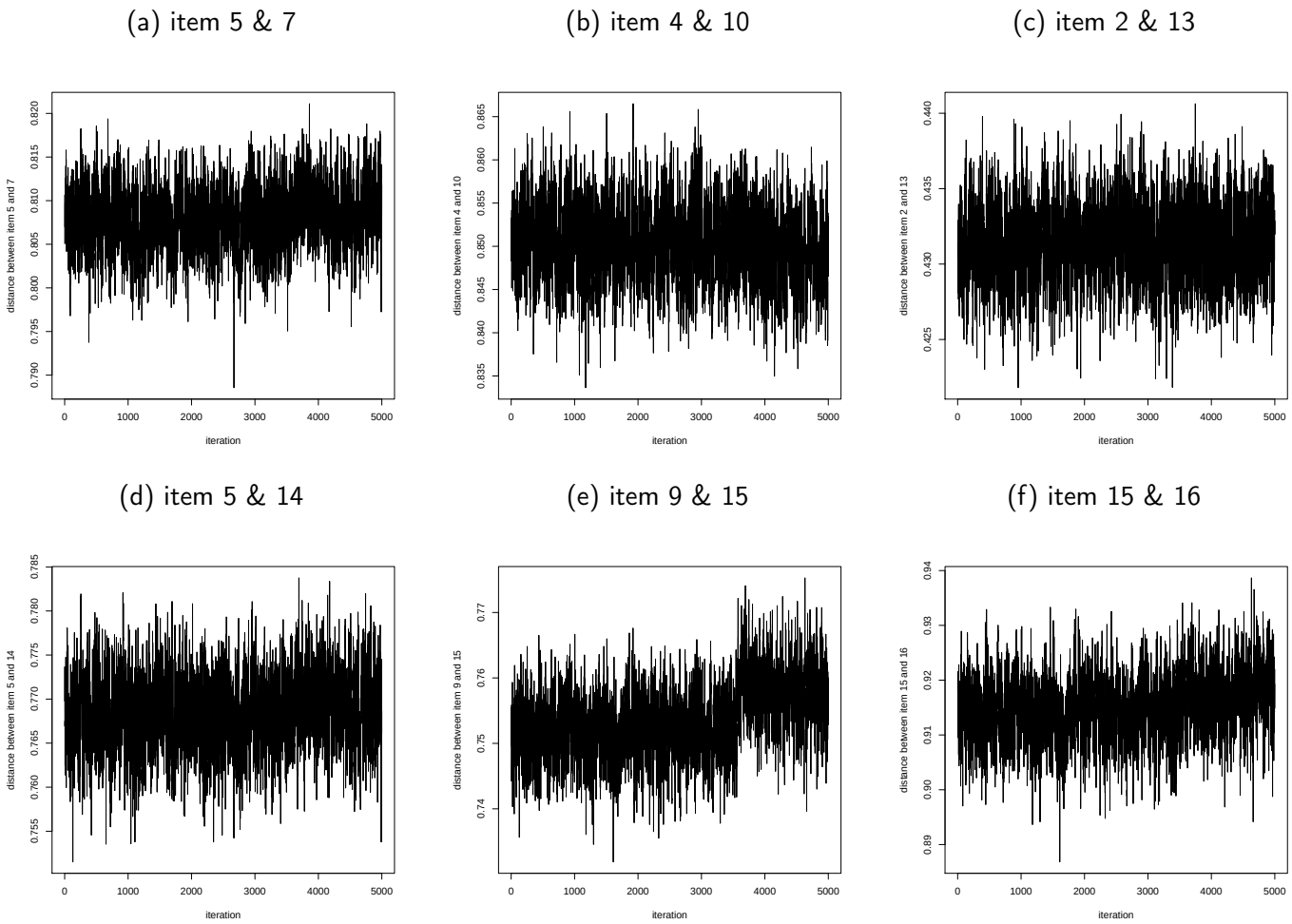

Figure 3: Trace plots of the item distances for the DRV data (Part 1).

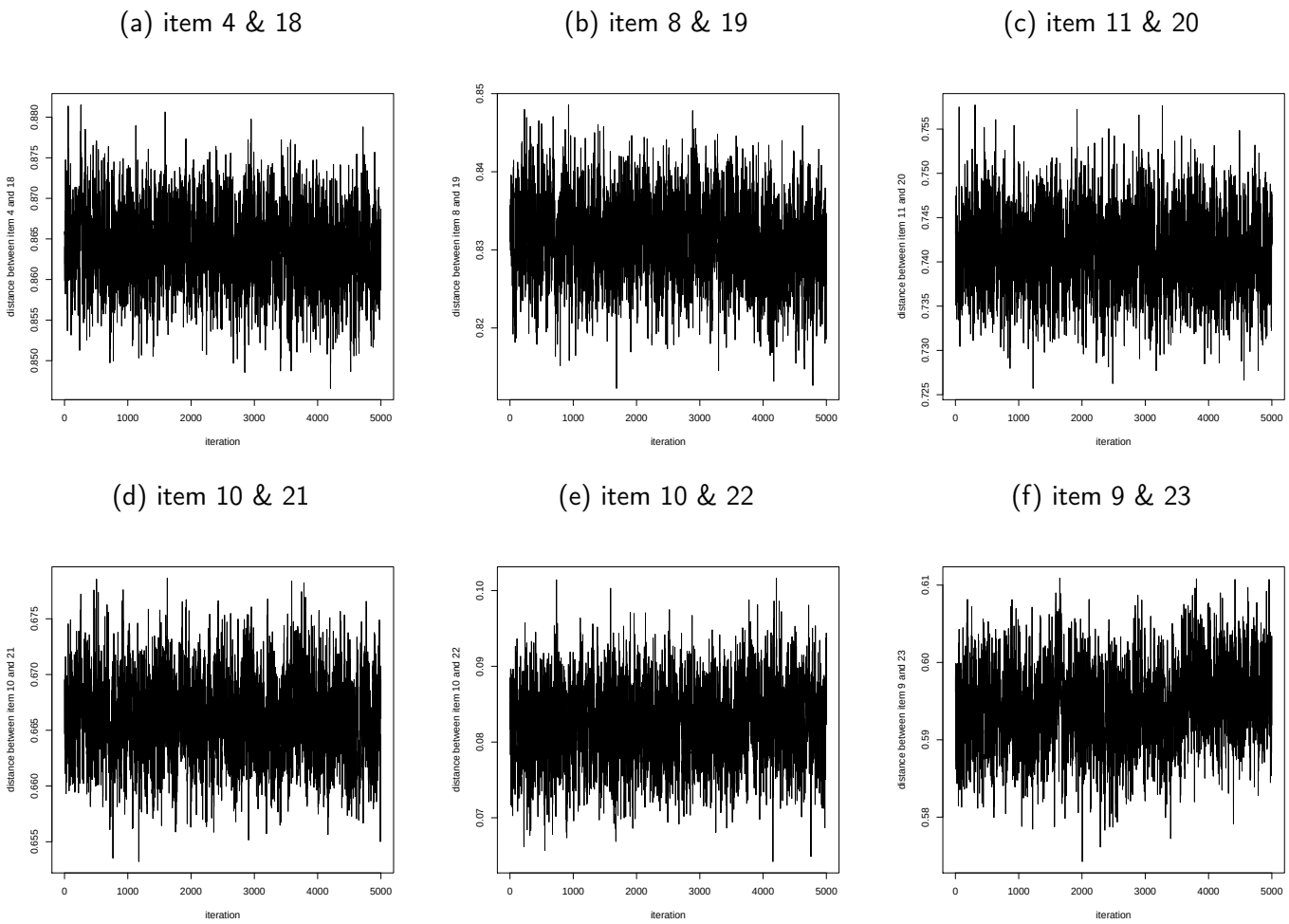

Figure 4: Trace plots of the item distances for the DRV data (Part 2).

C Item Index Number

Index No.	Type of Inference	Content of the Conditional	Presentation of the Antecedent
1	MP	CO	NN
2	NA	CO	NN
3	AC	CO	NN
4	MT	CO	NN
5	MP	CO	NE
6	NA	CO	NE
7	AC	CO	NE
8	MT	CO	NE
9	MP	AB	NN
10	NA	AB	NN
11	AC	AB	NN
12	MT	AB	NN
13	MP	AB	NE
14	NA	AB	NE
15	AC	AB	NE
16	MT	AB	NE
17	MP	CF	NN
18	NA	CF	NN
19	AC	CF	NN
20	MT	CF	NN
21	MP	CF	NE
22	NA	CF	NE
23	AC	CF	NE
24	MT	CF	NE

Table 3: Item index numbers and their corresponding design factors

D Comparison DLSJM's Spectral Clustering with the Mixture Rasch Analysis

Total Scores	DLJSM	Mixture	Response Patterns of the Students
11	1	2	0 1 0 0 0 1 0 1 1 0 0 1 1 0 1 0 1 0 0 1 1 0 0 1
10	1	2	1 1 1 1 1 0 0 1 1 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1
10	1	2	1 1 1 1 1 0 0 0 1 0 0 1 1 0 0 0 1 0 0 1 0 0 0 0
11	1	2	1 1 1 0 1 0 0 0 1 0 0 1 0 0 0 1 1 0 0 1 1 0 0 1
12	1	2	0 1 1 1 1 0 0 0 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
8	1	2	0 1 0 0 0 0 0 1 1 0 0 1 1 0 0 0 1 0 0 0 0 1 0 1
10	1	2	1 1 1 1 0 0 0 1 1 0 0 1 1 0 0 0 0 0 0 0 1 0 0 1
11	1	2	0 0 1 0 0 1 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
10	1	2	0 1 1 0 1 0 0 0 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 0
11	1	2	0 1 0 1 0 0 1 0 1 0 0 1 1 0 0 1 1 1 0 1 1 0 0 0
9	1	2	0 1 0 1 1 0 1 0 1 0 0 1 1 0 0 1 0 0 0 1 0 0 0 0
10	1	2	0 1 0 0 1 0 1 0 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 0
9	1	2	0 1 0 1 0 1 0 0 1 0 0 1 1 0 0 1 1 0 0 1 0 0 0 0
10	1	2	0 0 1 0 1 1 0 1 1 0 0 1 1 0 0 0 1 0 0 1 1 0 0 0
10	1	3	0 0 1 1 1 0 0 1 0 1 0 0 1 0 0 0 0 1 1 0 1 0 0 1
10	2	3	0 1 1 0 0 1 1 0 1 1 1 0 1 0 0 0 0 0 0 1 0 1 0 0
10	2	3	0 1 1 0 0 0 0 1 0 0 1 0 0 0 1 1 0 1 0 1 1 0 0 1
9	2	3	1 0 1 0 0 0 1 0 0 1 0 0 0 0 1 0 1 0 1 1 1 0 0 0
11	2	3	1 0 1 0 0 1 1 0 1 0 0 1 1 0 0 1 0 1 1 0 0 0 1 0
13	2	3	0 1 1 0 0 1 1 0 1 0 0 1 1 0 0 1 1 1 1 0 0 1 1 0
13	2	3	1 1 1 0 0 1 1 0 1 1 1 1 0 0 0 0 0 1 1 0 0 1 1 0
9	2	3	0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 0 0 0 0 0 0
12	2	3	0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 1 0 0 1 1 0 0 1
13	2	3	1 1 1 1 0 1 1 0 1 1 1 0 1 1 1 0 0 1 0 0 0 0 0 0
9	2	3	1 1 1 0 0 1 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0 1 1 0
16	2	3	0 1 1 0 0 1 1 0 1 1 1 1 1 1 1 1 0 1 1 0 1 0 1 0
12	2	3	0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 1 0 0 1 1 0 0 1
13	2	3	0 1 1 0 0 1 1 0 1 1 1 1 0 1 1 0 1 0 0 1 1 0 0 0
8	2	3	0 0 0 0 0 0 0 1 1 0 0 1 1 0 0 1 0 1 1 0 0 0 1 0

Table 4: The response patterns of all mismatch cases between spectral clustering of students' latent spaces, which were estimated from DLSJM (DLSJM) and clustering from the mixture IRT approach (Mixture). (Part 1)

Total Scores	DLJSM	Mixture	Response Patterns of the Students
8	2	3	0 0 0 0 0 0 1 0 0 1 0 0 0 0 1 0 1 1 1 0 0 1 1 0
12	2	3	0 1 1 0 0 1 1 0 1 1 0 0 1 1 0 1 0 0 1 0 0 1 1 0
9	2	3	0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 0 1 0 0 0 0 0
13	2	3	1 0 1 0 1 1 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0
8	2	3	0 1 0 0 0 1 1 1 0 1 0 1 1 0 0 0 0 1 0 0 0 0 0 0
12	2	3	0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0
10	2	3	0 1 1 0 0 1 1 0 0 0 0 1 1 0 0 0 0 1 1 1 0 1 0 0
11	2	3	0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 0 1 0 0 1 1 0
11	2	3	0 1 1 0 0 1 1 0 0 1 1 0 1 1 1 0 0 1 0 0 0 0 0 1
13	2	3	1 1 0 0 1 1 1 0 0 1 0 0 1 1 1 0 0 1 1 0 0 1 1 0
13	2	3	1 1 1 0 0 1 1 0 0 1 1 0 0 1 1 1 0 1 1 0 0 0 1 0
12	2	3	0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0
12	2	3	0 1 1 0 0 1 1 0 1 1 1 0 0 0 1 0 1 0 0 1 1 0 0 1
8	2	3	0 0 0 0 1 0 1 0 1 0 0 0 1 0 1 0 0 0 0 1 1 0 1 0
13	2	3	0 1 1 0 0 1 1 0 0 1 1 0 0 1 0 0 1 1 1 1 0 1 1 0
12	2	3	1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0
12	2	3	0 1 1 0 0 1 1 0 1 1 1 1 1 1 0 0 0 1 0 0 0 1 0 0
11	2	3	0 1 1 0 0 1 1 0 1 0 0 1 1 0 0 0 0 1 1 0 0 1 1 0
5	2	3	0 1 1 0 0 1 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
15	2	3	0 1 1 0 1 1 1 0 1 0 1 0 1 1 0 0 0 1 1 1 1 1 1 0
14	2	3	0 1 1 0 0 1 1 0 1 1 0 0 1 0 1 0 1 1 1 1 1 1 0 0
11	2	3	0 1 1 0 0 1 1 0 1 1 0 1 1 0 1 1 0 1 0 0 0 0 0 0
11	2	3	0 1 1 0 0 1 1 0 0 1 0 0 0 1 1 0 1 0 0 1 1 0 0 1
7	2	3	1 0 1 0 0 0 0 0 0 1 1 0 1 1 0 0 0 1 0 0 0 0 0 0
14	2	3	0 1 1 0 0 1 1 0 1 1 1 0 1 0 1 1 1 1 1 0 1 0 0 0
16	2	3	1 1 1 1 1 1 1 1 0 1 1 0 0 1 1 0 1 1 1 0 0 1 0 0
11	2	3	0 1 1 0 0 1 1 0 1 0 0 1 1 0 0 1 0 1 1 0 0 1 0 0
10	2	3	0 1 1 0 0 1 1 0 1 1 0 0 0 0 0 0 0 1 1 0 0 1 1 0
12	2	3	0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0
13	2	3	1 1 1 0 0 1 0 0 1 1 0 0 0 1 1 0 1 0 1 1 1 0 1 0
10	2	3	0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 0 0 0 1 0 0
11	2	3	1 1 1 0 0 1 1 0 1 0 0 0 0 1 0 0 0 1 1 0 1 0 1 0
7	2	3	0 1 0 0 0 1 0 0 0 1 1 0 0 1 0 0 0 0 0 1 0 0 1 0
11	2	3	1 1 0 0 0 0 0 1 1 1 1 0 1 1 0 1 0 0 0 1 1 0 0 0
12	2	3	0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0
15	2	3	1 1 1 0 1 1 0 0 0 1 1 1 1 1 0 0 0 1 1 0 1 1 1 0
13	2	3	0 1 1 0 0 1 1 0 1 1 0 0 1 1 1 0 0 1 1 0 1 1 0 0
6	2	3	0 1 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 1 0 1 0 0 0

Table 5: The response patterns of all mismatch cases between spectral clustering of students' latent spaces, which were estimated from DLSJM (DLSJM) and clustering from the mixture IRT approach (Mixture). (Part 2)

Total Scores	DLJSM	Mixture	Response Patterns of the Students
8	2	3	0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0
14	2	3	0 1 1 1 0 1 1 0 1 1 1 0 1 1 0 0 0 1 1 0 0 1 1 0
9	2	3	0 1 1 0 0 0 0 0 0 1 1 0 1 1 0 0 0 1 0 0 0 0 0 1 1
11	2	3	0 1 1 0 1 0 0 1 1 1 0 0 0 0 0 1 0 1 1 0 0 1 1 0
8	2	3	0 1 1 0 0 1 0 0 1 0 0 0 1 1 0 0 0 1 0 0 0 0 0 0 1
11	2	3	1 0 1 0 1 1 1 1 0 0 1 0 1 1 0 1 0 0 0 1 0 0 0 0
17	3	1	1 1 0 1 1 0 0 1 1 0 0 1 1 1 1 1 1 0 1 1 1 0 1 1
16	3	1	1 1 1 1 1 1 0 1 1 0 0 1 1 0 0 0 1 1 0 1 1 0 1 1
13	3	1	1 0 1 0 1 1 0 1 1 0 0 1 0 0 1 0 1 0 0 1 1 0 1 1
15	3	1	1 1 0 1 0 1 0 1 0 0 0 1 1 1 0 1 1 1 0 1 1 1 0 1
17	3	1	1 1 1 1 1 1 0 1 1 1 0 1 1 0 0 1 1 0 1 1 1 0 0 1
15	3	1	1 0 0 1 1 1 0 1 1 0 0 1 1 1 0 0 1 0 0 1 1 1 1 1
16	3	1	1 1 1 1 1 1 0 1 1 1 0 1 1 1 0 0 1 0 0 1 1 0 0 1
16	3	1	1 1 0 1 1 1 0 1 1 1 0 1 1 1 0 0 1 1 0 0 1 1 0 1
13	3	1	1 0 1 0 1 1 0 1 1 1 0 1 1 0 0 1 1 0 0 1 1 0 0 0
12	3	1	1 1 0 0 1 1 0 1 1 1 0 0 1 0 0 0 1 0 0 1 1 0 0 1
18	3	1	1 1 1 1 1 1 0 1 1 1 0 1 1 1 0 1 1 1 0 1 1 0 0 1
14	3	1	1 1 0 0 1 1 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 1 0 1
12	3	2	1 1 0 0 1 1 0 0 1 1 0 1 1 1 0 0 1 0 0 1 1 0 0 0
12	3	2	1 1 1 0 1 0 0 0 1 0 0 1 0 1 1 1 1 0 0 1 1 0 0 0
15	3	2	1 1 1 1 1 1 0 1 1 0 0 1 0 0 0 0 1 1 0 1 1 1 0 1
11	1	2	1 1 0 0 1 1 0 0 1 0 0 1 1 0 0 0 1 1 0 0 1 0 0 1
14	3	2	1 1 1 0 1 1 0 1 0 1 0 1 1 1 0 0 1 0 0 1 1 0 0 1
13	3	2	0 1 1 1 0 1 1 0 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
14	3	2	0 1 1 1 0 1 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 1 0 1
13	3	2	1 1 1 0 1 1 1 1 1 0 1 1 0 0 0 1 0 0 0 1 1 0 0 0
16	3	2	1 1 1 0 0 1 1 0 1 1 0 1 1 1 1 1 1 1 0 0 1 1 0 0 1
22	3	2	1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 1 1 1
12	3	2	0 1 1 0 1 0 1 0 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
19	3	2	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 0 1 1 0 0 1
16	3	2	1 1 1 1 1 1 1 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
16	3	2	0 1 1 1 1 1 1 0 1 0 1 1 1 0 0 1 1 0 0 1 1 1 0 1
12	3	2	1 0 1 0 1 1 1 0 1 0 1 1 1 0 0 1 1 0 0 0 1 0 0 0
19	3	2	1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 0 0 1 1 1 0 0 1
18	3	2	1 1 1 1 0 1 1 1 1 1 1 1 1 0 0 0 1 1 0 1 1 1 0 1
14	3	2	1 1 1 1 0 1 1 0 1 0 0 1 1 0 0 1 0 0 1 1 1 0 0 1
18	3	2	1 1 1 1 1 1 1 1 1 0 0 1 1 1 0 1 1 1 1 0 1 0 0 1
15	3	2	1 1 1 1 0 1 1 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1

Table 6: The response patterns of all mismatch cases between spectral clustering of students' latent spaces, which were estimated from DLSJM (DLSJM) and clustering from the mixture IRT approach (Mixture). (Part 3)

Total Scores	DLJSM	Mixture	Response Patterns of the Students
16	3	2	1 1 1 0 0 1 1 1 1 1 1 1 1 0 0 1 0 0 1 1 0 0 1
13	3	2	1 1 1 0 1 1 1 0 1 0 0 0 1 1 0 1 1 0 0 0 1 0 0 1
13	3	2	1 1 1 0 1 1 0 0 1 0 0 0 1 1 0 1 1 1 0 0 1 0 0 1
14	3	2	1 1 1 0 1 1 0 1 1 0 1 1 1 0 0 1 1 1 0 0 1 0 0 0
18	3	2	1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 0 0 0 0
14	3	2	0 1 1 1 0 1 1 0 1 1 0 1 1 0 0 1 0 1 0 1 0 1 0 1
15	3	2	1 1 1 0 1 1 1 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
12	3	2	0 1 1 0 0 1 1 0 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
21	3	2	1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 0 1 1
21	3	2	1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 0 1 1
17	3	2	1 1 1 1 1 1 1 0 1 0 0 1 1 0 0 1 1 1 1 1 1 0 1 0
21	3	2	1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 0 1 1 1 1 1 1
12	3	2	0 1 1 0 0 1 1 0 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
12	3	2	0 1 1 0 0 1 1 0 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
18	3	2	1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 0 1 1 0 0 0
13	3	2	0 1 1 0 1 1 1 1 1 0 0 1 0 0 0 1 1 0 0 1 1 0 0 1
11	3	2	0 1 0 0 1 1 1 0 1 0 0 1 1 0 0 0 1 0 0 1 1 0 0 1
18	3	2	1 1 1 1 0 1 1 1 1 1 1 1 1 0 0 1 1 1 0 1 1 0 0 1
19	3	2	1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 0 0 1 1 0 0 1
14	3	2	1 1 1 1 1 1 1 0 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 0
17	3	2	1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 0 0 1 1 0 0 0
12	3	2	0 1 1 0 0 1 1 0 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
12	3	2	1 1 1 0 1 1 1 0 1 0 0 1 0 0 0 0 1 0 0 1 1 0 0 1
16	3	2	1 1 1 1 1 1 1 1 1 0 0 1 0 0 0 1 0 1 0 1 1 1 0 1
17	3	2	1 1 1 1 1 1 1 1 1 1 0 1 1 0 0 1 1 0 0 1 1 0 0 1
12	1	2	0 1 1 1 0 0 1 1 1 0 0 1 1 0 0 1 1 0 0 0 1 0 0 1
13	3	2	1 1 1 0 0 1 1 0 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
16	3	2	1 1 1 1 1 1 1 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
17	3	2	1 1 1 1 0 1 1 1 1 1 1 1 1 0 0 1 1 0 0 1 1 0 0 1
13	3	2	1 1 1 0 1 1 1 0 1 0 0 1 1 0 0 0 1 0 0 1 1 0 0 1
13	3	2	1 1 1 1 0 1 0 1 1 0 0 1 0 0 1 0 0 0 0 1 1 1 0 1
12	3	2	0 1 1 0 0 1 1 1 1 0 0 1 1 0 0 0 1 0 0 1 1 0 0 1
18	3	2	1 1 1 1 1 1 1 1 1 1 0 1 1 1 0 1 1 0 0 1 1 0 0 1
12	3	2	1 1 1 0 0 1 1 0 1 1 0 1 0 0 0 0 1 0 0 1 1 0 0 1
17	3	2	1 1 1 1 1 1 1 0 1 1 1 1 1 1 0 0 1 0 0 1 1 0 0 1
18	3	2	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 0 0 1 1 0 0 1
14	3	2	1 1 1 1 1 1 0 0 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
12	3	2	0 0 1 0 1 1 1 0 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1

Table 7: The response patterns of all mismatch cases between spectral clustering of students' latent spaces, which were estimated from DLSJM (DLSJM) and clustering from the mixture IRT approach (Mixture). (Part 4)

Total Scores	DLJSM	Mixture	Response Patterns of the Students
14	3	2	1 1 1 1 0 1 0 1 1 1 1 1 1 0 0 0 1 0 0 1 0 0 0 1
13	3	2	1 1 0 0 1 0 1 0 1 1 0 1 1 1 0 0 1 0 0 1 1 0 0 1
13	3	2	0 1 1 1 0 1 1 1 1 0 0 1 1 0 0 0 1 0 0 1 1 0 0 1
14	3	2	1 1 1 0 1 1 1 0 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
15	3	2	0 1 1 1 1 1 1 1 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0
12	3	2	1 1 0 0 1 1 1 0 1 0 0 1 1 0 0 0 1 0 0 1 1 0 0 1
14	3	2	1 1 1 0 0 1 1 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
12	3	2	0 1 1 0 0 1 1 0 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
12	3	2	0 1 1 0 0 1 1 0 1 1 0 0 1 1 0 0 1 0 0 1 1 0 0 1
20	3	2	1 1 1 0 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 0 1 1
21	3	2	1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 0 1 1
12	3	2	0 1 1 0 0 1 1 0 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
12	3	2	0 1 1 0 0 1 1 0 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
13	3	2	1 1 0 1 0 0 1 0 1 1 0 1 1 0 0 1 1 0 0 1 1 0 0 1
12	3	2	0 1 1 0 0 1 1 0 1 1 0 1 1 0 0 0 1 0 0 1 1 0 0 1
13	3	2	1 1 1 0 0 1 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
15	3	2	1 1 1 0 1 1 1 0 1 0 0 1 1 0 0 1 1 1 0 1 1 1 0 0
12	3	2	0 1 1 0 0 1 1 0 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
13	3	2	1 1 1 0 1 1 1 0 1 0 1 1 1 0 0 0 1 0 0 1 1 0 0 0
11	3	2	0 1 1 0 0 1 1 0 1 0 0 1 1 0 0 0 1 0 0 1 1 0 0 1
13	3	2	1 1 1 0 0 1 1 0 0 1 0 1 1 0 0 1 1 0 0 1 1 0 0 1
12	3	2	0 1 1 0 1 1 0 0 1 1 1 0 1 0 0 0 1 0 0 1 1 0 0 1
12	3	2	1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 1 0 0 1 1 0 0 1
15	3	2	1 1 1 0 1 1 1 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
14	3	2	1 0 1 1 1 1 1 0 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1
18	3	2	1 1 1 1 1 1 1 0 1 1 0 1 1 0 0 0 1 1 1 1 1 1 0 1
16	3	2	1 0 1 0 1 1 1 1 1 0 0 1 1 0 0 1 1 1 1 1 1 0 0 1
22	3	2	1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 0 1 1 1 1 1
17	3	2	1 1 1 1 1 1 1 0 1 1 1 1 1 0 0 0 1 0 1 0 1 0 1 1
14	3	2	1 1 1 0 0 1 1 0 1 0 0 1 1 0 0 1 0 1 0 1 1 1 1 0
13	3	2	1 1 1 0 1 1 1 0 1 0 0 1 1 0 0 0 1 1 0 0 1 1 0 0
15	3	2	0 1 1 0 0 1 1 0 1 1 1 1 1 1 1 0 1 0 0 1 1 0 0 1
15	3	2	1 1 1 0 1 1 1 0 1 0 0 1 1 0 0 1 1 1 1 0 1 0 1 0
17	3	2	1 1 1 0 1 1 1 0 1 1 0 0 1 1 0 1 1 1 0 1 1 0 1 1
13	3	2	1 1 1 1 1 0 1 0 0 0 0 1 1 1 0 0 1 0 0 1 1 0 1 0
15	3	2	1 1 1 1 0 1 1 1 1 0 0 1 0 0 0 0 0 0 0 1 1 1 1 1

Table 8: The response patterns of all mismatch cases between spectral clustering of students' latent spaces, which were estimated from DLSJM (DLSJM) and clustering from the mixture IRT approach (Mixture). (Part 5)