Supplementary Materials for "Latent Class Dynamic Mediation Model with Application to Smoking Cessation Data"

1. Conditional posterior distributions for the MCMC algorithm

We first define some notations.

$$\boldsymbol{Y_i} = (Y_{i1}, Y_{i2}, ..., Y_{iT})',\ \boldsymbol{X_i} = (X_{i1}, X_{i2}, ..., X_{iT})',\ \boldsymbol{M_i} = (M_{i1}, M_{i2}, ..., M_{iT})'$$

$$\boldsymbol{\xi} = (\xi_1, ...\xi_i, ..., \xi_N)',\ \boldsymbol{\zeta} = (\zeta_1, ...\zeta_i, ..., \zeta_N)'$$

$$\boldsymbol{a}_l = (a_{0l}, a_{1l}, a_{2l})',\ \boldsymbol{a} = (\boldsymbol{a}'_1, \boldsymbol{a}'_2, ..., \boldsymbol{a}'_L)'$$

$$\boldsymbol{b}_l = (b_{0l}, b_{1l}, b_{2l})',\ \boldsymbol{b} = (\boldsymbol{b}'_1, \boldsymbol{b}'_2, ..., \boldsymbol{b}'_L)'$$

$$\boldsymbol{c}_l = (c_{0l}, c_{1l}, c_{2l})',\ \boldsymbol{c} = (\boldsymbol{c}'_1, \boldsymbol{c}'_2, ..., \boldsymbol{c}'_L)'$$

$$\boldsymbol{d}_l = (d_{0l}, d_{1l}, d_{2l})',\ \boldsymbol{d} = (\boldsymbol{d}'_1, \boldsymbol{d}'_2, ..., \boldsymbol{d}'_L)'$$

$$\boldsymbol{e}_l = (e_{0l}, e_{1l}, e_{2l})',\ \boldsymbol{e} = (\boldsymbol{e}'_1, \boldsymbol{e}'_2, ..., \boldsymbol{e}'_L)'$$

$$\boldsymbol{\phi}_l = (\phi_{1l}, \phi_{2l}, ..., \phi_{Kl})',\ \boldsymbol{\phi} = (\boldsymbol{\phi}'_1, \boldsymbol{\phi}'_2, ..., \boldsymbol{\phi}'_L)'$$

$$\boldsymbol{\varphi}_l = (\varphi_{1l}, \varphi_{2l}, ..., \varphi_{Kl})',\ \boldsymbol{\varphi} = (\boldsymbol{\varphi}'_1, \boldsymbol{\varphi}'_2, ..., \boldsymbol{\varphi}'_L)'$$

$$\boldsymbol{\chi}_l = (\chi_{1l}, \chi_{2l}, ..., \chi_{Kl})',\ \boldsymbol{\chi} = (\boldsymbol{\chi}'_1, \boldsymbol{\chi}'_2, ..., \boldsymbol{\chi}'_L)'$$

$$\boldsymbol{\psi}_l = (\psi_{1l}, \psi_{2l}, ..., \psi_{Kl})',\ \boldsymbol{\psi} = (\boldsymbol{\psi}'_1, \boldsymbol{\psi}'_2, ..., \boldsymbol{\psi}'_L)'$$

$$\boldsymbol{\varpi}_l = (\varpi_{1l}, \varpi_{2l}, ..., \varpi_{Kl})',\ \boldsymbol{\varpi} = (\boldsymbol{\varpi}'_1, \boldsymbol{\varpi}'_2, ..., \boldsymbol{\varpi}'_L)'$$

$$\boldsymbol{\Sigma} = (\sigma^2_{\xi 1}, \sigma^2_{\xi 2}, ..., \sigma^2_{\xi L}, \sigma^2_{\zeta 1}, \sigma^2_{\zeta 2}, ..., \sigma^2_{\zeta L}, \sigma^2_{11}, \sigma^2_{12}, ..., \sigma^2_{1L}, \sigma^2_{21}, \sigma^2_{22}, ..., \sigma^2_{2L}, \sigma^2_{\phi}, \sigma^2_{\varphi}, \sigma^2_{\chi}, \sigma^2_{\psi}, \sigma^2_{\varpi})'$$

$$\boldsymbol{\omega} = (\omega_{01}, ..., \omega_{0(L-1)}, \omega_1, ..., \omega_p)'$$

$$
\boldsymbol{W}_{1i} = \begin{bmatrix}
1 & t_{i1} & t_{i1}^2 & (t_{i1}-\kappa_1)_+^2 & \cdots & (t_{i1}-\kappa_K)_+^2 & X_{i1} & X_{i1}t_{i1} & X_{i1}t_{i1}^2 & X_{i1}(t_{i1}-\kappa_1)_+^2 & \cdots & X_{i1}(t_{i1}-\kappa_K)_+^2 \\
1 & t_{i2} & t_{i2}^2 & (t_{i2}-\kappa_1)_+^2 & \cdots & (t_{i2}-\kappa_K)_+^2 & X_{i2} & X_{i2}t_{i2} & X_{i2}t_{i2}^2 & X_{i2}(t_{i2}-\kappa_1)_+^2 & \cdots & X_{i2}(t_2-\kappa_K)_+^2 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
1 & t_{im_i} & t_{im_i}^2 & (t_{im_i}-\kappa_1)_+^2 & \cdots & (t_{im_i}-\kappa_K)_+^2 & X_{im_i} & X_{im_i}t_{im_i} & X_{im_i}t_{im_i}^2 & X_{im_i}(t_{im_i}-\kappa_1)_+^2 & \cdots & X_{im_i}(t_{im_i}-\kappa_K)_+^2
\end{bmatrix},
$$

$$
\boldsymbol{W}_{2i} = \begin{bmatrix}
1 & t_{i1} & t_{i1}^2 & (t_{1i}-\kappa_1)_+^2 & \cdots & (t_{i1}-\kappa_K)_+^2 & M_{i1} & M_{i1}t_{i1} & M_{i1}t_1^2 & M_{i1}(t_{i1}-\kappa_1)_+^2 & \cdots & M_{i1}(t_{i1}-\kappa_K)_+^2 & X_{i1} & X_{i1}t_{i1} & X_{i1}t_{i1}^2 & X_{i1}(t_{i1}-\kappa_1)_+^2 & \cdots & X_{i1}(t_{i1}-\kappa_K)_+^2 \\
1 & t_{i2} & t_{i2}^2 & (t_{i2}-\kappa_1)_+^2 & \cdots & (t_{i2}-\kappa_K)_+^2 & M_{i2} & M_{i2}t_{i2} & M_{i2}t_{i2}^2 & M_{i2}(t_{i2}-\kappa_1)_+^2 & \cdots & M_{i2}(t_{i2}-\kappa_K)_+^2 & X_{i2} & X_{i2}t_{i2} & X_{i2}t_{i2}^2 & X_{i2}(t_{i2}-\kappa_1)_+^2 & \cdots & X_{i2}(t_{i2}-\kappa_K)_+^2 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
1 & t_{im_i} & t_{im_i}^2 & (t_{im_i}-\kappa_1)_+^2 & \cdots & (t_{im_i}-\kappa_K)_+^2 & M_{im_i} & M_{im_i}t_{im_i} & M_{im_i}t_{im_i}^2 & M_{im_i}(t_{im_i}-\kappa_1)_+^2 & \cdots & M_{im_i}(t_{im_i}-\kappa_K)_+^2 & X_{im_i} & X_{im_i}t_{im_i} & X_{im_i}t_{im_i}^2 & X_{im_i}(t_{im_i}-\kappa_1)_+^2 & \cdots & X_{im_i}(t_{im_i}-\kappa_K)_+^2
\end{bmatrix}
$$

The Metropolis-within-Gibbs algorithm is carried out by sequentially sampling the following conditional posterior distributions:

$$C_i|. \sim \mathrm{MULTI}\left(1, \boldsymbol{\pi}_i^*\right), \quad \text{where} \quad \boldsymbol{\pi}_i^* = (\tilde{\pi}_{i1}/\sum_{l=1}^{L}\tilde{\pi}_{il}, ..., \tilde{\pi}_{iL}/\sum_{l=1}^{L}\tilde{\pi}_{il},), \quad \text{and}$$

$$\tilde{\pi}_{il} = (\sigma_{1l}^2)^{-\frac{m_i}{2}} \exp\left[-\frac{1}{2\sigma_{1l}^2}\left\{\boldsymbol{M}_i - \xi_i\mathbf{1}_{m_i} - \boldsymbol{W}_{1i}\cdot(\boldsymbol{c}_l', \boldsymbol{\chi}_l', \boldsymbol{a}_l', \boldsymbol{\phi}_l')'\right\}'\left\{\boldsymbol{M}_i - \xi_i\mathbf{1}_{m_i} - \boldsymbol{W}_{1i}\cdot(\boldsymbol{c}_l', \boldsymbol{\chi}_l', \boldsymbol{a}_l', \boldsymbol{\phi}_l')'\right\}\right]$$
$$\cdot (\sigma_{2l}^2)^{-\frac{m_i}{2}} \exp\left[-\frac{1}{2\sigma_{2l}^2}\left\{\boldsymbol{Y}_i - \zeta_i\mathbf{1}_{m_i} - \boldsymbol{W}_{2i}\cdot(\boldsymbol{d}_l', \boldsymbol{\psi}_l', \boldsymbol{b}_l', \boldsymbol{\varphi}_l', \boldsymbol{e}_l', \boldsymbol{\varpi}_l')'\right\}'\left\{\boldsymbol{Y}_i - \zeta_i\mathbf{1}_{m_i} - \boldsymbol{W}_{2i}\cdot(\boldsymbol{d}_l', \boldsymbol{\psi}_l', \boldsymbol{b}_l', \boldsymbol{\varphi}_l', \boldsymbol{e}_l', \boldsymbol{\varpi}_l')'\right\}\right]$$
$$\cdot\, \pi_l(\boldsymbol{Z}_i)$$

$$\boldsymbol{\omega}|. \propto \prod_{i=1}^{n}\prod_{l=1}^{L}\pi_l(\boldsymbol{Z}_i)^{\boldsymbol{I}(C_i=l)} \cdot |10^4\boldsymbol{I}_{\boldsymbol{(p+L-1)\times(p+L-1)}}|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}10^{-4}\boldsymbol{\omega}'\boldsymbol{I}_{\boldsymbol{(p+L-1)\times(p+L-1)}}\boldsymbol{\omega}\right)$$

$$(\boldsymbol{c}_l', \boldsymbol{\chi}_l', \boldsymbol{a}_l', \boldsymbol{\phi}_l')'|. \sim N\left(\boldsymbol{\mu}_{1l}, V_{1l}\right)$$

where,

$$V_{1l} = \left\{\Psi_l^{-1} + \sigma_{1l}^{-2}\sum_{i=1}^{n}\boldsymbol{I}(C_i = l)\boldsymbol{W}_{1i}'\boldsymbol{W}_{1i}\right\}^{-1}$$
$$\boldsymbol{\mu}_{1l} = V_1 \cdot \left\{\sigma_{1l}^{-2}\sum_{i=1}^{n}\boldsymbol{I}(C_i = l)\boldsymbol{W}_{1i}'\left\{\boldsymbol{M}_i - \xi_i\mathbf{1}_{m_i}\right\}\right\}$$
$$\Psi_l = \mathrm{diag}(10^4\mathbf{1}_3', \sigma_{\chi l}^2\mathbf{1}_K', 10^4\mathbf{1}_3', \sigma_{\phi l}^2\mathbf{1}_K')$$

$$(\boldsymbol{d}_l', \boldsymbol{\psi}_l', \boldsymbol{b}_l', \boldsymbol{\varphi}_l', \boldsymbol{e}_l', \boldsymbol{\varpi}_l')'|. \sim N(\boldsymbol{\mu}_{2l}, V_{2l})$$

where,

$$V_{2l} = \left\{ \Omega_l^{-1} + \sigma_{2l}^{-2} \sum_{i=1}^{n} \boldsymbol{I}(C_i = l) \boldsymbol{W}_{2i}' \boldsymbol{W}_{2i} \right\}^{-1}$$

$$\boldsymbol{\mu}_{2l} = V_2 \cdot \left\{ \sigma_{2l}^{-2} \sum_{i=1}^{n} \boldsymbol{I}(C_i = l) \boldsymbol{W}_{2i}' \{ \boldsymbol{Y}_i - \zeta_i \boldsymbol{1}_{m_i} \} \right\}$$

$$\Omega_l = \mathrm{diag}(10^4 \boldsymbol{1}_3', \sigma_{\psi l}^2 \boldsymbol{1}_K', 10^4 \boldsymbol{1}_3', \sigma_{\varphi l}^2 \boldsymbol{1}_K', 10^4 \boldsymbol{1}_3', \sigma_{\varpi l}^2 \boldsymbol{1}_K')$$

$$\xi_i|. \sim N\left(\left(m_i \sigma_{1l}^{-2} + \sigma_{\xi l}^{-2}\right)^{-1} \sigma_{1l}^{-2} \cdot (\boldsymbol{c}_l', \boldsymbol{\chi}_l', \boldsymbol{a}_l', \boldsymbol{\phi}_l') \boldsymbol{W}_{1i} \boldsymbol{1}_{m_i}, \quad \left(m_i \sigma_{1l}^{-2} + \sigma_{\xi l}^{-2}\right)^{-1} \right)$$

$$\zeta_i|. \sim N\left(\left(m_i \sigma_{2l}^{-2} + \sigma_{\zeta l}^{-2}\right)^{-1} \sigma_{2l}^{-2} \cdot (\boldsymbol{d}_l', \boldsymbol{\psi}_l', \boldsymbol{b}_l', \boldsymbol{\varphi}_l', \boldsymbol{e}_l', \boldsymbol{\varpi}_l') \boldsymbol{W}_{2i} \boldsymbol{1}_{m_i}, \quad \left(m_i \sigma_{2l}^{-2} + \sigma_{\zeta l}^{-2}\right)^{-1} \right)$$

$$\sigma_{\xi l}^2|. \sim IG(10^{-3} + \frac{\sum_{i=1}^{n} \boldsymbol{I}(C_i = l)}{2}, 10^{-3} + \frac{1}{2} \sum_{i=1}^{n} \boldsymbol{I}(C_i = l) \xi_i^2)$$

$$\sigma_{\zeta l}^2|. \sim IG(10^{-3} + \frac{\sum_{i=1}^{n} \boldsymbol{I}(C_i = l)}{2}, 10^{-3} + \frac{1}{2} \sum_{i=1}^{n} \boldsymbol{I}(C_i = l) \zeta_i^2)$$

$$\sigma_{\phi l}^2|. \sim IG(10^{-3} + \frac{K}{2}, 10^{-3} + \frac{1}{2} \sum_{k=1}^{K} \phi_{kl}^2)$$

$$\sigma_{\varphi l}^2|. \sim IG(10^{-3} + \frac{K}{2}, 10^{-3} + \frac{1}{2} \sum_{k=1}^{K} \varphi_{kl}^2)$$

$$\sigma^2_{\chi l}|. \sim IG(10^{-3} + \frac{K}{2}, 10^{-3} + \frac{1}{2}\sum_{k=1}^{K}\chi^2_{kl})$$

$$\sigma^2_{\psi l}|. \sim IG(10^{-3} + \frac{K}{2}, 10^{-3} + \frac{1}{2}\sum_{k=1}^{K}\psi^2_{kl})$$

$$\sigma^2_{\varpi l}|. \sim IG(10^{-3} + \frac{K}{2}, 10^{-3} + \frac{1}{2}\sum_{k=1}^{K}\varpi^2_{kl})$$

$$\sigma^2_{\epsilon l}|. \sim IG\Bigg(10^{-3} + \frac{\sum_{i=1}^{n}\boldsymbol{I}(C_i = l)}{2}, \\ 10^{-3} + \frac{1}{2}\sum_{i=1}^{n}\boldsymbol{I}(C_i = l)\left\{\boldsymbol{M}_i - \xi_i\mathbf{1}_{m_i} - \boldsymbol{W}_{1i}\cdot(\boldsymbol{c}'_l, \boldsymbol{\chi}'_l, \boldsymbol{a}'_l, \boldsymbol{\phi}'_l)'\right\}'\left\{\boldsymbol{M}_i - \xi_i\mathbf{1}_{m_i} - \boldsymbol{W}_{1i}\cdot(\boldsymbol{c}'_l, \boldsymbol{\chi}'_l, \boldsymbol{a}'_l, \boldsymbol{\phi}'_l)'\right\}\Bigg)$$

$$\sigma^2_{\eta l}|. \sim IG\Bigg(10^{-3} + \frac{\sum_{i=1}^{n}\boldsymbol{I}(C_i = l)}{2}, \\ 10^{-3} + \frac{1}{2}\sum_{i=1}^{n}\boldsymbol{I}(C_i = l)\left\{\boldsymbol{Y}_i - \zeta_i\mathbf{1}_{m_i} - \boldsymbol{W}_{2i}\cdot(\boldsymbol{d}'_l, \boldsymbol{\psi}'_l, \boldsymbol{b}'_l, \boldsymbol{\varphi}'_l, \boldsymbol{e}'_l, \boldsymbol{\varpi}'_l)'\right\}'\left\{\boldsymbol{Y}_i - \zeta_i\mathbf{1}_{m_i} - \boldsymbol{W}_{2i}\cdot(\boldsymbol{d}'_l, \boldsymbol{\psi}'_l, \boldsymbol{b}'_l, \boldsymbol{\varphi}'_l, \boldsymbol{e}'_l, \boldsymbol{\varpi}'_l)'\right\}\Bigg)$$