AN EXTENDED GFFIT STATISTIC DEFINED ON ORTHOGONAL COMPONENTS OF PEARSON'S CHI-SQUARE

ON-LINE SUPPLEMENT

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1. Appendix B

1.1. Type I Error and Power Study for Omnibus Statistics

Although the topic of this paper is the $GFfit$ statistic, a small Monte Carlo simulation study for the $X^2_{[2]}$ statistic is presented here because $X^2_{[2]}$ applied to ordinal variables has not been previously studied.

A simulation study was conducted using GLLVM to assess the accuracy of the Type I error rates for X^2_{PF}, M_2 (Maydue-Olivares & Joe, 2006), and $X^2_{[2]}$, where $X^2_{[2]}$ is calculated by using orthogonal components obtained from sequential sum of squares. Degrees of freedom for $X^2_{[2]}$ applied to the 2PL GLLVM are $\frac{1}{2}q(q-1)(c-1)^2$. The simulation had several conditions, with $q = 4$, $q = 5$, and $q = 6$ variables, and $c = 3$ and $c = 4$ categories. Each condition used 500 pseudo samples of size $n = 500$ each. Simulations were conducted under the assumption $\boldsymbol{X} \sim N_p(0, \boldsymbol{\Sigma_X})$. Pseudo samples were generated from a model with one latent factor, and the GLLVM was fit with the specification of one latent factor. Calculation of X^2_{PF} and components involves estimation of model parameters. If the parameter estimator has large bias or variance, then the performance of $X^2_{[2]}$ and components in terms of Type I error and power may be affected through the fitted probabilities for the cells of the multinomial distribution. In the GLLVM model, the maximum likelihood estimator is consistent and efficient, but the estimator has large mean square error in finite samples when model parameters have large magnitudes, i.e, large intercepts, either positive or negative, and large slopes. Since this study is concerned with the performance of the test statistics, only modest values for GLLVM parameters were used in order to avoid confounding of the performance of the test statistic with bias in parameter estimation. To generate the pseudo data, intercept values were specified in as -1.5 and 1.5 for three categories and -1.5, 0, and 1.5 with four categories. Slope parameters were specified as follows: For $q = 4$, $\boldsymbol{\beta}_1 = (0.2,\ 0.5,\ 1.0,\ 2.0)$; for $q = 5$, $\boldsymbol{\beta}_1 = (0.2,\ 0.5,\ 1.0,\ 1.5,\ 2.0)$; for $q = 6$, $\boldsymbol{\beta}_1 = (0.2,\ 0.5,\ 0.75,\ 1.0,\ 1.5,\ 2.0)$. All intercept and slope parameters were estimated using the *grm* function from the *ltm* package in R. Test statistics were calculated using a custom ***R*** script.

Simulation results for Type I error are shown in Tables 8 and 9. The tables show empirical Type I error for nominal $\alpha = 0.05$. The results demonstrate the well-known outcome that the chi-square approximation for the Pearson statistic may be invalid when the cross-classified table becomes very sparse. $X^2_{[2]}$ and M_2 results show empirical Type I error very close to the nominal level. $X^2_{[2]}$ calculated from sequential sum of squares gives reliable results due to numeric stability.

The power of $X^2_{[2]}$ was examined by calculating asymptotic power and then approximating empirical power by using a simulation study. Asymptotic power was calculated by the method given in Reiser (2008). For the simulation study, pseudo data for 1000 samples each of size 500 were generated from a confirmatory two-factor model with all parameters fixed and then fit with a model specifying one factor. Four variables and six variables each with four categories were studied. Intercept parameter values for the data generating model were -1.5, 0, and 1.5 for all

Table 8: Monte Carlo Simulation Results for $X^2_{[2]}$
Type I Error with n=300

q	4	4	5	6	8
c	3	4	4	4	3
X^2_{PF}	0.068	0.057	0.083	0.148	0.324
$X^2_{[2]}$	0.063	0.045	0.038	0.060	0.067
M_2	0.060	0.050	0.044	0.059	0.053

Table 9: Monte Carlo Simulation Results for $X^2_{[2]}$
Type I Error with n=500

q	4	4	5	6	8
c	3	4	4	4	3
X^2_{PF}	0.058	0.050	0.068	0.11	0.369
$X^2_{[2]}$	0.042	0.046	0.042	0.052	0.063
M_2	0.042	0.044	0.040	0.044	0.063

variables. Slope parameters for the data generating model with four variables included slopes for factor 1, $\boldsymbol{\beta}_1 = (0.2,\ 0.5,\ 1.0,\ 2.0)'$, and slopes for factor 2, $\boldsymbol{\beta}_2 = (0.8,\ 0.8,\ 0.0,\ 0.0)'$. For six variables, slope parameter values were $\boldsymbol{\beta}_1 = (0.2, 0.5, 0.75, 1.0, 1.5, 2.0)'$, and $\boldsymbol{\beta}_2 = (0.8, 0.8, 0.8, 0.0, 0.0, 0.0)'$. The two latent variables were specified as uncorrelated, each with variance equal to 1.0. Estimation of the one-factor GLLVM converged for all 1000 samples for both four and six variables.

Tables 10 and 11 show empirical power for $X^2_{[2]}$ and M_2. $X^2_{[2]}$ and M_2 show similar empirical power, somewhat below the asymptotic power. $X^2_{[2]}$, again calculated using sequential sum of squares, gives numerically stable results.

Table 10: Power Simulation Results for $X^2_{[2]}$
Four Variables Four Categories

	Mean	SD	Power
X^2_{PF}	251.55	23.37	0.149
$X^2_{[2]}$	66.64	13.13	0.314
M_2	62.40	13.02	0.319
A.Power	.	.	0.328

Table 11: Power Simulation Results for $X^2_{[2]}$
Six Variables Four Categories

	Mean	SD	Power
X^2_{PF}	4113.92	114.42	0.173
$X^2_{[2]}$	166.78	20.76	0.549
M_2	160.43	20.32	0.557
A.Power	.	.	0.580

Based on these simulation results, $X^2_{[2]}$ and M_2 appear to have similar performance, a result that is consistent with other studies that have compared these statistics for binary variables (Mavridis, D., and Moustaki, I. and Knott, M., 2007). For $X^2_{[2]}$ we recommend the calculation by the sequential sum of squares method because it is very stable numerically. The generalized inverse and ordinary Cholesky methods may produce results with lower reliability due to collinearity among the rows of the $\mathbf{H}$ matrix. However, for limited-information statistics calculated on second-order marginals for ordinal variables there is an issue that two-way marginal tables may have cells with low expected frequencies when the number of categories is four or larger. This issue for $GFfit_{\perp}^{(ij)}$ will be addressed in Section 1.2 of this paper.

Although a test may be intended for the global null hypothesis $H_o\colon \boldsymbol{\pi} = \boldsymbol{\pi}(\boldsymbol{\beta})$, there may be advantages in terms of both sparseness and power to instead test $H_o\colon \mathbf{H}\boldsymbol{\pi} = \mathbf{H}\boldsymbol{\pi}(\boldsymbol{\beta})$ with a statistic on lower-order components since "Reject $H_o\colon \mathbf{H}\boldsymbol{\pi} = \mathbf{H}\boldsymbol{\pi}(\boldsymbol{\beta})$" is a sufficient although not necessary condition for "Reject $H_o\colon \boldsymbol{\pi} = \boldsymbol{\pi}(\boldsymbol{\beta})$." On the other hand,"Do not reject $H_o\colon \mathbf{H}\boldsymbol{\pi} = \mathbf{H}\boldsymbol{\pi}(\boldsymbol{\beta})$" is a necessary but not sufficient condition for "Do not reject $H_o\colon \boldsymbol{\pi} = \boldsymbol{\pi}(\boldsymbol{\beta})$" because it is possible that lack of fit may be manifest only in a direction not represented in $\mathbf{H}_{[t:u]}$.

1.2. Sparse Marginal Tables

When two-way marginal tables are formed from variables with a larger number of categories, such as five or six, the cell frequencies in the two-way marginal tables may become sparse, especially if the univariate marginal distributions across the variables are skewed. Cai and Hansen (2013) found that cells in corner of larger two-way tables tended to have low expected frequencies, although other cells may have low expected values depending on the model parameters for the manifest variables. If a two-way marginal tables is sparse, then the asymptotic approximation using the chi-square distribution for $GFfit_{\perp}^{ij}$ may not be valid, just as if the full table is sparse, the asymptotic approximation for the Pearson statistic may not be valid. Sparseness in a two-way marginal table needs to be fairly severe before the effects can be seen in simulation results obtained by using the chi-square distribution to calculate the achieved significance level. The $GFfit_{\perp}^{ij}$ statistic itself is composed of orthogonal components, designed on cells of the two-way table. It is possible to partition these individual components, by a method which should be chosen before the test, to create a test statistic that is focused on a location in the two-way table. For example, $GFfit_{\perp}^{ij}$ can be partitioned into a set of individual components for the main diagonal, subdiagonal and superdiagonal and another set of components for the other off-diagonals. When interpreting the result for a $GFfit_{\perp}^{ij}$ statistic, the results for the two partitions can be compared. They may show similar results. If the two-way table is sparse, the partition with larger cell counts will be more reliable in terms of Type I error and power based on the asymptotic chi-square approximation. Another possibility would be two partitions, one for the center of the table and a second partition for the non-center cells.

Many different partitions of $GFfit_{\perp}^{ij}$ are possible, and components that are summed to produce $GFfit_{\perp}^{ij}$ must be chosen in a way that identifies the two-way table. While the order of rows in $\mathbf{H}_{[2]}^{(ij)}$ is arbitrary, the standard $\mathbf{H}_{[2]}^{(ij)}$ matrix generates components for the lower right portion of the two-way table. A partition for the main diagonal, subdiagonal and superdiagonal of the table would include $(c-1)+2(c-2)=3c-5$ components, and a partition for the other off-diagonals would include $(c-1)^2-(3c-5)$ components. Then,

$$GFfit_{\perp}^{ij} = GFfit_{\perp}^{ij(d)} + GFfit_{\perp}^{ij(od)}, \tag{1.1}$$

where superscript (d) indicates diagonal and (od) indicates off-diagonal. The distinct tallies for partitions can be accomplished by simply reordering the rows of the $\mathbf{H}_{[2]}^{(ij)}$ matrix by using a permutation matrix. For example, if $q = 3$ and $c = 4$, $\mathbf{H}_{[2]}^{(1,2)}$ is a nine by 64 partition of the matrix in expression (2.14), and $\mathbf{H}_{[2]}^{(1,2)}$ has the following pattern:

$$\mathbf{H}_{[2]}^{(1,2)} = \begin{pmatrix} \mathbf{0}_{16}' & \mathbf{0}_4' & \boldsymbol{j}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' \\ \mathbf{0}_{16}' & \mathbf{0}_4' & \mathbf{0}_4' & \boldsymbol{j}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' \\ \mathbf{0}_{16}' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \boldsymbol{j}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' \\ \mathbf{0}_{16}' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \boldsymbol{j}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' \\ \mathbf{0}_{16}' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \boldsymbol{j}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' \\ \mathbf{0}_{16}' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \boldsymbol{j}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' \\ \mathbf{0}_{16}' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \boldsymbol{j}_4' & \mathbf{0}_4' & \mathbf{0}_4' \\ \mathbf{0}_{16}' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \boldsymbol{j}_4' & \mathbf{0}_4' \\ \mathbf{0}_{16}' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \mathbf{0}_4' & \boldsymbol{j}_4' \end{pmatrix} \tag{1.2}$$

where $\boldsymbol{j}_4 = (1\ 1\ 1\ 1)'$,$\mathbf{0}_4 = (0\ 0\ 0\ 0)'$, and $\mathbf{0}_{16}$ is a vector of zeros with dimension 16. Keeping in mind that components are order dependent, premultiplying $\mathbf{H}_{[2]}^{(1,2)}$ by the permutation matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \tag{1.3}$$

will position the seven rows of $\mathbf{H}$ for the diagonals of the table in the top partition of $\mathbf{H}_{[2]}^{(1,2)}$ and the two rows from $\mathbf{H}$ for off-diagonal of the table in the other partition:

$$\mathbf{H}_{[2]}^{(1,2)} = \begin{pmatrix} \mathbf{H}_{[2]}^{1,2(d)} \\ \dots \\ \mathbf{H}_{[2]}^{1,2(od)} \end{pmatrix} \tag{1.4}$$

Then using $\hat{\boldsymbol{\gamma}}$ obtained from this reordered $\mathbf{H}_{[2]}^{ij}$ matrix,

$$GFfit_{\perp}^{1,2(d)} = \sum_{\ell=1}^{\ell=7} \hat{\gamma}_{\ell}^{2} \text{ and } GFfit_{\perp}^{1,2(od)} = \sum_{\ell=8}^{\ell=9} \hat{\gamma}_{\ell}^{2} \tag{1.5}$$

$GFfit_{\perp}^{(ij)}$ is invariant with respect to ordering of the rows within $\mathbf{H}_{[2]}^{(ij)}$, and the original ordering is arbitrary, so the rows could have been created in the diagonal vs. off-diagonal order initially.

The omnibus lower-order $X_{[2]}^{2}$ statistic can also be partitioned into diagonal and off-diagonal components. Expression (4.7) established that $X_{[2]}^{2}$ is a sum of $GFfit_{\perp}^{ij}$ statistics. Because each $GFfit_{\perp}^{ij}$ statistic can be partitioned into diagonal and off-diagonal components, $X_{[2]}^{2}$ can be partitioned as

$$X_{[2]}^{2} = \sum_{i=1}^{i=q-1} \sum_{j=i+1}^{j=q} GFfit_{\perp}^{ij(d)} + \sum_{i=1}^{i=q-1} \sum_{j=i+1}^{j=q} GFfit_{\perp}^{ij(od)}, \tag{1.6}$$

When interpreting the result of the $X_{[2]}^{2}$ statistic, the result for the diagonal partition can be compared to the result for the off-diagonal partition.

1.3. Monte Carlo Simulation for Sparse Marginal Tables

A Monte Carlo simulation was conducted to examine Type I error and power for $GFfit_{\perp(d)}^{(ij)}$ using four variables, each with six categories. Tables **??** to 14 show results related to the simulation. The parameters for the data generating model are the following: $\boldsymbol{\alpha}_{0(1)} = (-3.5, -3.5, -3.5, -3.5)'$, $\boldsymbol{\alpha}_{0(2)} = (-2.5, -2.5, -2.5, -2.5)'$, $\boldsymbol{\alpha}_{0(3)} = (0, 0, 0, 0)'$, $\boldsymbol{\alpha}_{0(4)} = (2.5, 2.5, 2.5, 2.5)'$, $\boldsymbol{\alpha}_{0(5)} = (3.5, 3.5, 3.5, 3.5)'$, $\boldsymbol{\alpha}_{1} = (2.0, 2.0, 2.0, 2.0)'$. Average cell frequencies under these parameters for the six by six marginal table with $n = 300$ are shown in Table **??**. It can be seen from the table that expected counts in the upper right and lower left corners are quite small, but even the diagonal partition has a few cells with low expected counts. Type I error rates were calculated for the diagonal partition using 13 cells from the six by six marginal table as marked with the * symbol shown in Table 12.

Table 12: Average Cell Frequencies Six by Six Marginal Table

Category of Variable *j*	Category of Variable *i*					
	1	*2*	*3*	*4*	*5*	*6*
1	3.32	1.62	3.28	1.08	0.11	0.07
2	1.62	1.21	3.31	1.40	0.15	0.13
3	3.28	3.31	13.90	9.74	1.40	1.08
4	1.08	1.40	9.74	13.90	3.31	3.28
5	0.11	0.15	1.40	3.31	1.21	1.62
6	0.07	0.11	1.08	3.28	1.62	3.32

This diagonal partition excludes 12 cells with low expected counts from two corners of the table. So for the partitioning of $GFfit_{\perp}^{(ij)}$, the diagonal partition has 13 degrees of freedom, and the off-diagonal partition has 12 degrees of freedom. Two sample sizes were used for the simulation, $n = 300$ and $n = 1000$. 1000 pseudo samples were generated for each sample size. For omnibus statistics, at $n = 1000$, $X^2_{[2]}$ has type I error rate 0.0704, $X^2_{[2,od]}$ has type I error rate 0.0690, and $X^2_{[2,d]}$ has Type I error rate 0.0660. All three of these values are outside a 95% interval for Monte Carlo error although the value for $X^2_{[2,d]}$ is just outside the interval. At $n = 300$, $X^2_{[2]}$ has type I error rate 0.0815, $X^2_{[2,od]}$ has type I error rate 0.0841, and $X^2_{[2,d]}$ has type I error rate 0.0586. The Type I error rate for $X^2_{[2,d]}$ is within a 95% interval for Monte Carlo error, but the other two values are outside the interval. Table 13 shows empirical Type I error rates for $GFfit_{\perp}^{(ij)}$ and $GFfit_{\perp(d)}^{(ij)}$

Table 13: Cells for Six by Six Marginal Table

Category of Variable *j*	Category of Variable *i*					
	1	*2*	*3*	*4*	*5*	*6*
1	1	7	13	19	25	31
2	2	8*	14*	20	26	32
3	3	9*	15*	21*	27	33
4	4	10	16*	22*	28*	34
5	5	11	17	23*	29*	35*
6	6	12	19	24	30*	36*

Table 14: Type I Error Rates for $GFfit^{(ij)}_{\perp(d)}$, M_{ij}, X^2_{ij}, and $\bar{\bar{X}}^2_{ij}$
Four Variables, Six Categories

(ij)	$GFfit^{(ij)}_{\perp}$	$GFfit^{(ij)}_{\perp(d)}$	$GFfit^{(ij)}_{\perp(od)}$	$M_2^{(ij)}$	X^2_{ij}	$\bar{\bar{X}}^2_{ij}$
(12)	**0.087**	0.054	**0.091**	**0.105**	**0.092**	**0.106**
(13)	**0.095**	0.058	**0.093**	**0.110**	**0.103**	**0.110**
(14)	**0.095**	0.055	**0.080**	**0.092**	**0.091**	**0.094**
(23)	**0.100**	0.055	**0.011**	**0.106**	**0.098**	**0.107**
(24)	**0.089**	0.055	**0.082**	**0.097**	**0.090**	**0.096**
(34)	**0.100**	0.053	**0.088**	**0.109**	**0.103**	**0.106**

n=100, 1000 samples; 98.9% convergence

from simulations with $n = 300$ and $n = 1000$. These simulation results show that the Type I error rates for $GFfit_{\perp}^{(ij)}$ are inflated, but the magnitude of the inflation is modest. Type I error rates are also inflated for $GFfit_{\perp(od)}^{(ij)}$, but Type I error rates for $GFfit_{\perp(d)}^{(ij)}$ are essentially at the nominal level.

A power study for $X^2_{[2,d]}$ was conducted with sample size $n = 500$ and 1000 replications for the Monte Carlo simulation. The intercepts parameters were the same as given above for the Type I error simulation study. Values for slope parameters were $\boldsymbol{\alpha}_1 = (1.0, 1.0, 2.0, 2.0)'$, and $\boldsymbol{\alpha}_2 = (1.0, 1.0, 0.0, 0.0)'$. From the results, $X^2_{[2]}$ has asymptotic power 0.312 and empirical power 0.334. M_2 also has empirical power 0.334. Power was calculated for the diagonal partition using 13 cells as described above. Asymptotic power for $X^2_{[2,d]}$ is 0.233, and empirical power 0.246, which indicates a reduction in power could occur if only the components in the diagonal partition are used to form the test statistic. However, if the marginal tables are sparse, $X^2_{[2]}$ may not be reliable, so $X^2_{[2,d]}$ may be preferable even if there is a possibility of reduced power. Table 14 shows asymptotic power and empirical power for $GFfit_{\perp}^{(ij)}$ and $GFfit_{\perp(d)}^{(ij)}$.

Table 15: Power Results for $GFfit_{\perp(d)}^{(ij)}$
Four Variables, Six Categories

	$GFfit_{\perp}^{(ij)}$		$GFfit_{\perp(d)}^{(ij)}$	
(ij)	A.Power	E.Power	A.Power	E.Power
(12)	0.137	0.122	0.100	0.140
(13)	0.052	0.125	0.051	0.089
(14)	0.051	0.098	0.051	0.065
(23)	0.052	0.114	0.052	0.075
(24)	0.051	0.130	0.051	0.077
(34)	0.050	0.124	0.050	0.100

n=100, 1000 samples, 99.3% convergence

The results appear to show a reduction in power for $GFfit_{\perp(d)}^{(12)}$ compared to $GFfit_{\perp}^{(12)}$, although $GFfit_{\perp(d)}^{(12)}$ may be more reliable if the marginal tables are sparse. Empirical power for the other $GFfit_{\perp(d)}^{(ij)}$ is inflated compared to the asymptotic power. The inflation is due to the extreme magnitudes of some of the intercept parameters. If intercept parameter values are instead close to 0.0, then empirical power does not exceed asymptotic power.

2. Appendix C

Table 16: Type I Error Results for $GFfit_{\perp}^{(ij)}$, M_{ij}, X_{ij}^2, and $\bar{\bar{X}}_{ij}^2$
Four variables, Four categories

	$n=300$				$n=500$			
(ij)	$GFfit_{\perp}^{(ij)}$	M_{ij}	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$	$GFfit_{\perp}^{(ij)}$	M_{ij}	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$
(12)	0.046	0.054	0.039	0.057	0.054	0.038	**0.036**	0.040
(13)	0.065	**0.068**	0.047	0.060	0.062	0.060	0.052	0.060
(14)	0.056	0.048	0.041	0.049	0.058	**0.036**	**0.030**	0.040
(23)	0.051	0.061	0.042	0.061	0.058	0.064	0.038	0.052
(24)	0.041	0.053	0.037	0.055	0.044	0.052	**0.036**	**0.064**
(34)	0.048	0.042	**0.031**	0.044	**0.064**	0.042	**0.020**	0.042

1000 samples, 100% convergence

Table 17: Type I Error Results for $GFfit_{\perp}^{(ij)}$, M_{ij}, X_{ij}^2, and $\bar{\bar{X}}_{ij}^2$
Five variables, Four categories

	$n = 300$				$n = 500$			
(ij)	$GFfit_{\perp}^{(ij)}$	M_{ij}	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$	$GFfit_{\perp}^{(ij)}$	M_{ij}	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$
(12)	0.049	0.055	0.042	0.048	0.047	0.058	0.044	0.046
(13)	0.051	0.051	0.043	0.051	0.039	0.045	**0.028**	**0.035**
(14)	0.049	0.045	0.038	0.046	0.058	**0.067**	0.038	0.056
(15)	0.055	0.062	0.041	0.063	0.060	0.059	0.054	**0.070**
(23)	0.037	0.044	**0.032**	0.045	0.052	0.039	0.038	0.045
(24)	0.060	0.062	0.047	0.054	0.056	0.051	**0.030**	0.049
(25)	0.050	0.043	0.037	0.045	0.051	0.050	**0.036**	0.052
(34)	0.052	0.053	0.040	0.055	0.044	0.044	**0.023**	0.040
(35)	0.051	0.041	0.031	0.046	0.045	0.045	**0.027**	0.047
(45)	0.043	0.043	**0.024**	0.037	0.044	0.044	**0.025**	0.043

1000 samples, 100% convergence

Table 18: Type I Error Results for $GFfit_{\perp}^{(ij)}$, M_{ij}, X_{ij}^2, and $\bar{\bar{X}}_{ij}^2$
Six variables, Four categories

	$n = 1000$				$n = 5000$			
(ij)	$GFfit_{\perp}^{(ij)}$	M_{ij}	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$	$GFfit_{\perp}^{(ij)}$	M_{ij}	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$
(12)	0.051	0.058	0.044	0.046	0.046	0.050	0.045	0.046
(13)	0.056	0.043	0.048	0.053	0.061	0.052	0.047	0.058
(14)	0.059	0.055	0.049	0.058	0.039	0.037	0.037	0.044
(15)	0.047	0.052	0.036	0.045	0.046	0.046	0.038	0.047
(16)	0.053	0.055	0.035	0.049	0.045	0.046	**0.031**	0.042
(23)	0.055	0.050	0.047	0.051	0.051	0.051	0.050	0.051
(24)	0.047	0.049	0.037	0.042	0.059	0.058	0.041	0.053
(25)	0.062	0.049	0.036	0.047	0.049	0.048	**0.035**	0.048
(26)	0.050	0.040	**0.026**	0.038	0.049	0.037	**0.027**	0.040
(34)	0.043	0.049	**0.029**	0.037	**0.035**	0.041	**0.030**	0.040
(35)	0.053	0.058	0.038	0.049	0.044	0.039	0.030	0.035
(36)	0.055	0.055	**0.036**	0.051	0.046	0.044	**0.029**	0.045
(45)	0.040	0.039	**0.024**	0.037	0.057	0.054	**0.028**	0.048
(46)	0.038	**0.035**	**0.020**	**0.030**	0.051	0.056	0.037	0.059
(56)	0.040	0.049	**0.036**	0.055	0.042	0.054	**0.033**	0.047

1000 samples, 100% convergence

Table 19: Simulation Results for Type I Error
Variance of $GFfit_{\perp}^{(ij)}$, $M_2^{(ij)}$, $\bar{\bar{X}}_{ij}^2$, $\bar{\bar{X}}_{ij}^2$
Eight variables, Four categories

	$n = 300$				$n = 500$			
(ij)	$GFfit_{\perp}^{(ij)}$	$M_2^{(ij)}$	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$	$GFfit_{\perp}^{(ij)}$	$M_2^{(ij)}$	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$
(12)	17.722	13.926	17.313	13.778	16.515	12.882	16.053	12.799
(13)	19.453	15.562	18.979	15.322	19.083	14.722	18.575	14.963
(14)	18.846	13.860	18.232	14.823	19.051	14.731	18.327	14.853
(15)	16.948	13.464	15.587	12.850	18.405	13.770	16.959	14.005
(16)	18.587	13.343	16.340	13.708	17.510	13.660	15.968	13.407
(17)	19.990	14.854	18.176	15.406	19.367	16.367	18.589	15.773
(18)	16.415	12.665	14.853	12.806	17.125	14.052	16.034	13.833
(23)	17.878	14.322	17.199	13.975	17.211	12.937	16.195	13.187
(24)	16.957	13.746	16.022	13.113	18.093	14.167	17.146	14.048
(25)	17.413	13.895	16.133	13.435	17.523	13.683	16.096	13.388
(26)	17.361	13.370	15.891	13.412	15.953	12.205	14.963	12.640
(27)	16.670	12.506	15.034	12.789	16.333	12.677	14.897	12.691
(28)	17.022	13.553	15.218	13.137	16.985	14.260	15.628	13.483
(34)	18.017	14.094	17.594	14.484	17.701	14.060	17.138	14.108
(35)	17.555	14.330	16.720	13.969	18.548	14.375	17.388	14.544
(36)	18.822	14.614	16.913	14.289	17.379	13.427	15.833	13.384
(37)	18.913	14.668	16.697	14.274	19.434	15.785	18.226	15.544
(38)	16.791	13.180	14.600	12.635	18.257	14.178	16.459	14.218
(45)	18.447	15.135	17.727	14.880	18.107	14.647	17.561	14.729
(46)	18.044	13.645	16.773	14.226	15.981	12.817	14.889	12.615
(47)	17.871	14.055	16.746	14.325	17.285	13.688	15.669	13.401
(48)	17.887	12.999	15.119	13.095	17.617	13.655	16.101	13.940
(56)	18.632	14.011	16.151	13.829	20.980	14.988	17.924	15.352
(57)	17.465	13.032	15.519	13.342	20.567	15.281	17.776	15.305
(58)	17.794	13.571	16.151	14.024	17.211	13.709	15.953	13.835
(67)	18.235	13.288	16.749	14.484	18.996	14.442	17.272	14.962
(68)	18.091	14.269	15.871	13.842	17.759	13.660	15.707	13.678
(78)	18.655	14.550	16.770	14.644	17.613	14.656	16.662	14.550

1000 samples. Degrees of freedom are larger for $GFfit_{\perp}^{(ij)}$

Table 20: Power Results for $GFfit_{\perp}^{(ij)}$, M_{ij}, X^2_{ij}, and $\bar{\bar{X}}^2_{ij}$
Six variables, Four categories

	$n = 300$					$n = 500$				
(ij)	A.Power	$GFfit_{\perp}^{(ij)}$	M_{ij}	X^2_{ij}	$\bar{\bar{X}}^2_{ij}$	A.Power	$GFfit_{\perp}^{(ij)}$	M_{ij}	X^2_{ij}	$\bar{\bar{X}}^2_{ij}$
(12)	**0.120**	**0.115**	**0.056**	**0.071**	**0.078**	**0.180**	**0.167**	**0.039**	**0.103**	**0.113**
(13)	**0.087**	**0.088**	**0.049**	**0.048**	**0.057**	**0.116**	**0.120**	**0.064**	**0.063**	**0.082**
(14)	0.069	0.057	0.048	0.033	0.039	0.084	0.062	0.047	0.034	0.041
(15)	0.053	0.067	0.052	0.054	0.063	0.055	0.048	0.045	0.042	0.055
(16)	0.060	0.057	0.049	0.038	0.057	0.067	0.066	0.056	0.056	0.077
(23)	0.071	0.068	0.045	0.038	0.049	0.087	0.092	0.037	0.038	0.055
(24)	0.061	0.060	0.055	0.030	0.041	0.069	0.053	0.039	0.024	0.036
(25)	0.052	0.047	0.044	0.032	0.049	0.054	0.055	0.049	0.041	0.052
(26)	0.055	0.056	0.056	0.049	0.057	0.058	0.071	0.067	0.059	0.074
(34)	0.057	0.049	0.055	0.041	0.051	0.063	0.062	0.040	0.043	0.050
(35)	0.052	0.052	0.057	0.035	0.049	0.053	0.056	0.052	0.036	0.053
(36)	0.053	0.049	0.054	0.031	0.048	0.055	0.057	0.045	0.036	0.049
(45)	0.053	0.052	0.051	0.032	0.050	0.055	0.054	0.050	0.033	0.044
(46)	0.051	0.055	0.046	0.035	0.053	0.052	0.047	0.052	0.030	0.049
(56)	0.057	0.054	0.041	0.035	0.051	0.062	0.055	0.035	0.037	0.056

1000 samples, 100% convergence

Table 21: Type I Error Results for $GFfit_{\perp}^{(ij)}$, M_{ij}, X_{ij}^2, and $\bar{\bar{X}}_{ij}^2$
Eight variables, Four categories

	$n = 1000$				$n = 5000$			
(ij)	$GFfit_{\perp}^{(ij)}$	M_{ij}	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$	$GFfit_{\perp}^{(ij)}$	M_{ij}	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$
(12)	0.055	0.050	0.051	0.052	0.046	0.043	0.044	0.046
(13)	0.049	0.053	0.049	0.050	0.049	0.052	0.049	0.050
(14)	0.061	0.062	0.055	0.061	0.061	0.051	0.056	0.060
(15)	0.055	0.055	0.041	0.047	0.056	0.048	0.040	0.046
(16)	0.040	0.044	**0.033**	0.038	0.048	0.049	0.040	0.052
(17)	0.051	0.051	0.038	0.048	0.066	0.062	0.048	0.060
(18)	0.038	0.037	**0.028**	0.040	0.044	0.049	**0.033**	0.046
(23)	0.058	0.044	0.053	0.056	0.048	0.041	0.039	0.044
(24)	0.052	0.051	0.043	0.047	0.052	0.054	0.048	0.050
(25)	0.030	0.046	**0.027**	**0.034**	0.047	0.043	0.039	0.044
(26)	0.047	0.056	0.040	0.050	0.041	0.041	**0.029**	0.039
(27)	0.043	0.046	**0.033**	0.043	0.044	0.044	**0.031**	0.039
(28)	0.046	0.040	**0.035**	0.049	0.044	0.050	**0.034**	0.045
(34)	0.046	0.047	0.042	0.045	0.049	0.042	0.043	0.050
(35)	0.051	0.052	0.044	0.051	0.063	0.062	0.046	0.055
(36)	0.052	0.056	0.038	0.048	0.046	0.048	**0.034**	0.043
(37)	0.063	0.055	0.046	0.057	0.050	0.058	0.046	0.058
(38)	0.059	0.055	0.043	0.058	0.047	0.049	0.039	0.050
(45)	0.045	0.048	0.039	0.052	0.058	0.054	0.049	0.055
(46)	0.043	0.042	0.037	0.044	**0.036**	0.048	**0.033**	0.040
(47)	0.048	0.046	**0.032**	0.048	0.044	0.055	**0.036**	0.046
(48)	0.052	0.049	**0.030**	0.045	0.048	0.046	**0.030**	0.046
(56)	0.047	0.052	**0.035**	0.047	0.060	0.045	0.043	0.051
(57)	0.060	**0.072**	0.058	**0.076**	**0.068**	**0.066**	0.049	**0.065**
(58)	0.045	0.049	0.037	0.057	0.040	0.056	**0.035**	0.055
(67)	0.051	0.051	0.038	0.043	**0.064**	0.051	**0.033**	0.052
(68)	**0.069**	0.062	0.046	0.066	0.045	0.051	**0.039**	0.047
(78)	0.056	0.045	0.037	0.056	0.053	0.048	**0.034**	0.055

1000 samples, 100% convergence

Table 22: Type I Error Results for $GFfit_{\perp}^{(ij)}$, M_{ij}, X_{ij}^2, and $\bar{\bar{X}}_{ij}^2$
Ten variables, Four categories

	$n = 300$				$n = 500$			
(ij)	$GFfit_{\perp}^{(ij)}$	M_{ij}	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$	$GFfit_{\perp}^{(ij)}$	M_{ij}	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$
(12)	0.053	0.051	0.048	0.048	0.052	0.052	0.048	0.050
(13)	**0.036**	**0.028**	0.030	**0.032**	0.058	0.061	0.058	0.059
(14)	0.054	0.060	0.050	0.053	0.057	0.050	0.051	0.053
(15)	0.058	0.055	0.044	0.051	0.060	0.061	0.055	0.060
(16)	0.050	0.052	0.040	0.053	0.042	0.048	0.041	0.048
(17)	0.045	0.055	0.049	0.054	0.056	0.049	0.040	0.048
(18)	0.046	0.047	0.037	0.044	0.049	0.041	0.041	0.045
(19)	0.045	0.045	**0.033**	0.042	0.052	0.043	0.038	0.047
(1,10)	0.042	0.038	**0.029**	0.040	0.048	0.055	**0.035**	0.049
(23)	0.050	0.058	0.041	0.046	0.043	0.043	0.039	0.043
(24)	0.060	0.061	0.054	0.063	0.047	0.052	0.046	0.052
(25)	0.043	**0.036**	0.036	0.040	0.054	0.053	0.038	0.051
(26)	0.065	0.050	0.054	0.061	0.050	0.057	0.046	0.055
(27)	0.057	0.056	0.052	0.059	0.057	0.046	0.048	0.057
(28)	0.056	0.052	0.043	0.053	0.058	0.049	0.050	0.060
(29)	0.050	0.046	0.041	0.051	0.050	0.055	0.037	0.051
(2,10)	0.043	0.044	**0.028**	0.038	0.044	0.046	**0.026**	0.039
(34)	0.057	0.057	0.052	0.054	0.040	0.045	0.036	0.042
(35)	0.047	0.053	0.045	0.052	0.055	0.046	0.038	0.047
(36)	0.056	0.057	0.053	0.057	0.045	0.054	0.039	0.057
(37)	0.047	0.051	0.038	0.052	0.048	0.057	0.041	0.050
(38)	0.048	0.049	0.044	0.051	0.057	0.057	0.050	0.061
(39)	0.057	0.054	0.039	0.050	0.053	0.050	0.041	0.048
(3,10)	0.051	0.053	0.039	0.050	0.051	0.051	0.040	0.053
(45)	0.053	0.048	0.046	0.051	0.052	0.046	0.041	0.051
(46)	0.052	0.052	0.040	0.052	0.047	0.047	0.040	0.046
(47)	0.045	0.053	**0.033**	0.040	0.054	0.063	0.041	0.055
(48)	0.054	0.056	0.050	0.060	0.061	0.041	0.038	0.046
(49)	0.046	0.053	**0.034**	0.047	**0.071**	0.052	0.046	0.057
(4,10)	0.040	0.038	**0.028**	0.036	0.049	0.035	**0.025**	0.043
(56)	0.047	0.050	**0.033**	0.043	0.040	0.042	**0.035**	0.045
(57)	0.046	0.052	0.040	0.050	0.045	0.048	0.039	0.050

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Table 22 – continued from previous page

	$n = 300$				$n = 500$			
(ij)	$GFfit_{\perp}^{(ij)}$	M_{ij}	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$	$GFfit_{\perp}^{(ij)}$	M_{ij}	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$
(58)	0.048	**0.034**	**0.029**	0.040	0.053	0.045	0.037	0.047
(59)	0.057	0.054	0.044	0.060	0.056	0.051	0.038	0.053
(5,10)	0.041	0.051	0.042	0.050	0.056	0.051	0.045	0.058
(67)	0.060	0.061	0.053	0.060	0.049	0.052	**0.032**	0.045
(68)	0.050	0.054	0.038	0.049	0.046	0.059	0.043	0.053
(69)	0.048	0.044	**0.028**	0.042	0.054	0.051	0.039	0.056
(6,10)	0.056	0.048	0.039	0.050	0.048	0.048	**0.033**	0.044
(78)	0.050	0.045	0.038	0.049	0.050	0.037	0.029	0.039
(79)	0.055	0.048	**0.028**	0.041	0.059	0.061	0.045	0.061
(7,10)	0.048	0.054	**0.024**	0.040	0.050	0.053	0.041	0.056
(89)	0.054	0.055	0.037	0.051	0.048	0.052	0.037	0.056
(8,10)	**0.069**	0.059	0.044	0.066	0.056	0.045	**0.032**	0.046
(9,10)	0.061	0.050	0.043	0.056	0.056	0.051	**0.034**	0.052

1000 samples, 100% convergence

Table 23: Power Results for $GFfit_{\perp}^{(ij)}$, M_{ij}, X_{ij}^2, and $\bar{\bar{X}}_{ij}^2$
Ten Variables, Four Categories

	A.Power	Empirical power			
(ij)	$GFfit_{\perp}^{(ij)}$	$GFfit_{\perp}^{(ij)}$	M_{ij}	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$
(12)	0.055	0.087	0.050	0.066	0.083
(13)	0.056	0.110	0.060	0.094	0.100
(14)	0.080	0.075	0.052	0.048	0.062
(15)	0.081	0.073	0.050	0.052	0.063
(16)	0.081	0.102	0.070	0.070	0.086
(17)	0.051	0.080	0.052	0.068	0.073
(18)	0.052	0.076	0.044	0.061	0.073
(19)	0.082	0.099	0.065	0.064	0.074
(1,10)	0.051	0.078	0.053	0.070	0.082
(23)	0.059	0.096	0.047	0.077	0.092
(24)	0.096	0.097	0.045	0.064	0.070
(25)	0.096	0.096	0.059	0.070	0.081
(26)	0.098	0.095	0.041	0.047	0.058
(27)	0.052	0.084	0.048	0.074	0.086
(28)	0.053	0.102	0.052	0.081	0.090
(29)	0.098	0.095	0.042	0.047	0.058
(2,10)	0.052	0.079	0.055	0.061	0.082
(34)	0.126	0.120	0.055	0.058	0.067
(35)	0.128	0.127	0.056	0.064	0.075
(36)	0.130	0.141	0.057	0.062	0.075
(37)	0.054	0.079	0.044	0.066	0.075
(38)	0.055	0.094	0.057	0.067	0.079
(39)	0.130	0.140	0.042	0.054	0.066
(3,10)	0.054	0.090	0.046	0.070	0.082
(45)	0.901	0.791	0.062	0.831	0.833
(46)	0.916	0.814	0.063	0.833	0.836
(47)	0.059	0.062	0.053	0.046	0.056
(48)	0.073	0.061	0.044	0.045	0.055
(49)	0.918	0.823	0.073	0.832	0.836
(4,10)	0.052	0.063	0.060	0.062	0.068
(56)	0.980	0.840	0.057	0.838	0.840
(57)	0.060	0.072	0.043	0.057	0.066

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Table 23 – continued from previous page

	A.Power	Empirical power			
(ij)	$GFfit_{\perp}^{(ij)}$	$GFfit_{\perp}^{(ij)}$	M_{ij}	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$
(58)	0.074	0.084	0.057	0.052	0.061
(59)	0.932	0.846	0.048	0.828	0.832
(5,10)	0.052	0.061	0.065	0.061	0.075
(67)	0.060	0.065	0.045	0.031	0.046
(68)	0.075	0.077	0.050	0.048	0.053
(69)	0.945	0.878	0.076	0.826	0.825
(6,10)	0.052	0.067	0.063	0.059	0.067
(78)	0.050	0.052	0.045	0.060	0.067
(79)	0.053	0.057	0.052	0.048	0.057
(7,10)	0.050	0.048	0.032	0.051	0.060
(89)	0.052	0.067	0.053	0.046	0.053
(8,10)	0.051	0.068	0.056	0.078	0.087
(9,10)	0.052	0.054	0.054	0.049	0.061

n=300, 1000 samples, 993 converged

Table 24: Power Results for $GFfit_{\perp}^{(ij)}$, M_{ij}, X_{ij}^2, and $\bar{\bar{X}}_{ij}^2$
Ten Variables, Four Categories

	A.Power	Empirical power			
(ij)	$GFfit_{\perp}^{(ij)}$	$GFfit_{\perp}^{(ij)}$	M_{ij}	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$
(12)	0.057	0.080	0.051	0.060	0.071
(13)	0.060	0.099	0.047	0.061	0.078
(14)	0.104	0.099	0.061	0.080	0.089
(15)	0.105	0.105	0.045	0.059	0.073
(16)	0.106	0.138	0.064	0.080	0.096
(17)	0.052	0.063	0.052	0.054	0.066
(18)	0.053	0.080	0.041	0.055	0.066
(19)	0.107	0.137	0.057	0.082	0.094
(1,10)	0.052	0.063	0.056	0.061	0.075
(23)	0.066	0.114	0.052	0.077	0.087
(24)	0.132	0.116	0.040	0.063	0.076

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Table 24 – continued from previous page

	A.Power	Empirical power			
(ij)	$GFfit_{\perp}^{(ij)}$	$GFfit_{\perp}^{(ij)}$	M_{ij}	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$
(25)	0.134	0.149	0.066	0.083	0.099
(26)	0.136	0.140	0.047	0.061	0.072
(27)	0.053	0.078	0.056	0.067	0.078
(28)	0.055	0.083	0.047	0.071	0.079
(29)	0.137	0.148	0.043	0.070	0.081
(2,10)	0.053	0.071	0.045	0.058	0.070
(34)	0.192	0.197	0.053	0.090	0.098
(35)	0.195	0.211	0.057	0.089	0.109
(36)	0.200	0.210	0.042	0.063	0.076
(37)	0.057	0.088	0.049	0.073	0.085
(38)	0.059	0.083	0.061	0.062	0.073
(39)	0.199	0.236	0.052	0.081	0.092
(3,10)	0.057	0.089	0.053	0.066	0.079
(45)	0.993	0.947	0.073	0.962	0.962
(46)	0.995	0.958	0.086	0.960	0.962
(47)	0.066	0.066	0.051	0.054	0.062
(48)	0.090	0.094	0.053	0.060	0.069
(49)	0.996	0.957	0.089	0.964	0.965
(4,10)	0.053	0.064	0.063	0.063	0.072
(56)	0.997	0.963	0.080	0.961	0.963
(57)	0.066	0.075	0.035	0.046	0.053
(58)	0.092	0.105	0.064	0.064	0.077
(59)	0.997	0.967	0.068	0.965	0.966
(5,10)	0.053	0.057	0.054	0.058	0.064
(67)	0.067	0.078	0.053	0.050	0.062
(68)	0.095	0.102	0.047	0.045	0.059
(69)	0.998	0.986	0.085	0.962	0.962
(6,10)	0.053	0.061	0.049	0.046	0.055
(78)	0.050	0.054	0.048	0.058	0.067
(79)	0.055	0.039	0.038	0.037	0.047
(7,10)	0.050	0.069	0.049	0.058	0.066
(89)	0.054	0.062	0.061	0.060	0.067
(8,10)	0.050	0.072	0.041	0.053	0.068
(9,10)	0.053	0.058	0.053	0.055	0.069

Continued on next page

Table 24 – continued from previous page

	A.Power	Empirical power			
(ij)	$GFfit_{\perp}^{(ij)}$	$GFfit_{\perp}^{(ij)}$	M_{ij}	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$
n=500, 1000 samples, 997 converged					

Table 25: Power Results for $GFfit_{\perp}^{(ij)}$
Eight variables, Four categories
Variable Order Reversed

	$n = 300$					$n = 500$				
(ij)	A.Power	$GFfit_{\perp}^{(ij)}$	$M_2^{(ij)}$	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$	A.Power	$GFfit_{\perp}^{(ij)}$	$M_2^{(ij)}$	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$
(78)	0.050	0.048	0.033	0.034	0.045	0.050	0.057	0.037	0.033	0.040
(68)	0.051	0.057	0.052	0.046	0.055	0.052	0.057	0.050	0.052	0.055
(58)	0.051	0.061	0.048	0.032	0.040	0.052	0.052	0.049	0.038	0.044
(48)	0.051	0.072	0.043	0.040	0.049	0.052	0.070	0.050	0.037	0.049
(38)	0.050	0.060	0.051	0.048	0.062	0.050	0.045	0.059	0.045	0.059
(28)	0.050	0.051	0.046	0.049	0.061	0.050	0.052	0.045	0.035	0.043
(18)	0.050	0.049	0.041	0.042	0.052	0.050	0.050	0.042	0.035	0.052
(67)	0.051	0.058	0.040	0.043	0.054	0.053	0.064	0.045	0.034	0.044
(57)	0.052	0.060	0.046	0.045	0.051	0.053	0.062	0.057	0.049	0.063
(47)	0.052	0.080	0.041	0.036	0.045	0.053	0.073	0.045	0.035	0.046
(37)	0.050	0.060	0.047	0.048	0.063	0.050	0.060	0.056	0.042	0.051
(27)	0.050	0.063	0.052	0.053	0.062	0.050	0.057	0.058	0.042	0.058
(17)	0.050	0.066	0.053	0.038	0.052	0.050	0.053	0.050	0.035	0.051
(56)	**0.956**	**0.906**	0.068	**0.891**	**0.893**	**0.999**	**0.989**	**0.076**	**0.990**	**0.990**
(46)	**0.962**	**0.927**	**0.074**	**0.911**	**0.912**	**0.999**	**0.991**	**0.090**	**0.995**	**0.995**
(36)	0.051	0.052	0.040	0.042	0.052	0.052	0.058	0.045	0.052	0.059
(26)	0.051	0.045	0.043	0.046	0.052	0.052	0.052	0.052	0.045	0.060
(16)	0.050	0.051	0.056	0.056	0.058	0.051	0.054	0.043	0.062	**0.075**
(45)	**0.967**	**0.943**	**0.083**	**0.895**	**0.897**	**0.999**	**0.998**	**0.093**	**0.990**	**0.991**
(35)	0.052	0.043	0.048	0.050	0.057	0.052	0.051	0.046	0.061	0.068
(25)	0.051	0.054	0.058	0.055	0.063	0.052	0.042	0.063	0.067	0.081
(15)	0.051	0.052	0.058	0.052	0.063	0.051	0.050	0.041	0.059	0.064
(34)	0.051	0.053	0.049	0.054	0.061	0.052	0.061	0.047	0.053	0.062
(24)	0.051	0.053	0.055	0.050	0.058	0.052	0.054	0.042	0.044	0.051
(14)	0.051	0.052	0.052	0.045	0.055	0.051	0.048	0.040	0.042	0.052
(23)	0.050	0.055	0.058	0.060	0.076	0.050	0.044	0.053	0.045	0.057
(13)	0.050	0.063	0.040	0.041	0.058	0.050	0.049	0.041	0.031	0.042
(12)	0.050	0.059	0.048	0.034	0.053	0.050	0.052	0.061	0.041	0.065

n=300, 1000 samples, 996 converged; n=500, 1000 samples,994 converged.

Table 26: Power Results for $GFfit_{\perp}^{(ij)}$
Eight variables, Four categories
Orthonormalization Order Reversed

	$n = 300$					$n = 500$				
(ij)	A.Power	$GFfit_{\perp}^{(ij)}$	$M_2^{(ij)}$	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$	A.Power	$GFfit_{\perp}^{(ij)}$	$M_2^{(ij)}$	X_{ij}^2	$\bar{\bar{X}}_{ij}^2$
(78)	0.050	0.047	0.033	0.034	0.045	0.050	0.043	0.037	0.033	0.040
(68)	0.051	0.055	0.052	0.046	0.055	0.052	0.071	0.050	0.052	0.055
(67)	0.052	0.068	0.040	0.043	0.054	0.053	0.055	0.045	0.034	0.044
(58)	0.051	0.044	0.048	0.032	0.040	0.052	0.047	0.049	0.038	0.044
(57)	0.052	0.058	0.046	0.045	0.051	0.053	0.077	0.057	0.049	0.063
(56)	**0.956**	**0.898**	0.068	**0.891**	**0.893**	**0.999**	**0.990**	**0.076**	**0.990**	**0.990**
(48)	0.051	0.061	0.043	0.040	0.049	0.052	0.057	0.050	0.037	0.049
(47)	0.052	0.070	0.041	0.036	0.045	0.053	0.071	0.045	0.035	0.046
(46)	**0.962**	**0.945**	**0.074**	**0.911**	**0.912**	**0.999**	**0.999**	**0.090**	**0.995**	**0.995**
(45)	**0.967**	**0.939**	**0.083**	**0.895**	**0.897**	**0.999**	**0.998**	**0.093**	**0.990**	**0.991**
(38)	0.050	0.058	0.051	0.048	0.062	0.050	0.053	0.059	0.045	0.059
(37)	0.050	0.055	0.047	0.048	0.063	0.050	0.051	0.056	0.042	0.051
(36)	0.051	0.046	0.040	0.042	0.052	0.052	0.052	0.045	0.052	0.059
(35)	0.051	0.049	0.048	0.050	0.057	0.052	0.046	0.046	0.061	0.068
(34)	0.051	0.061	0.049	0.054	0.061	0.052	0.047	0.047	0.053	0.062
(28)	0.050	0.057	0.046	0.049	0.061	0.050	0.054	0.045	0.035	0.043
(27)	0.050	0.056	0.052	0.053	0.062	0.050	0.055	0.058	0.042	0.058
(26)	0.051	0.045	0.043	0.046	0.057	0.052	0.049	0.052	0.045	0.060
(25)	0.051	0.060	0.058	0.055	0.063	0.052	0.066	0.063	0.067	0.081
(24)	0.051	0.065	0.055	0.050	0.058	0.052	0.055	0.042	0.044	0.051
(23)	0.050	0.068	0.058	0.060	0.076	0.050	0.049	0.053	0.045	0.057
(18)	0.050	0.055	0.041	0.042	0.052	0.050	0.066	0.042	0.035	0.052
(17)	0.050	0.056	0.053	0.038	0.052	0.050	0.049	0.050	0.035	0.051
(16)	0.051	0.058	0.056	0.056	0.058	0.051	0.053	0.043	0.062	0.075
(15)	0.051	0.049	0.058	0.052	0.063	0.051	0.047	0.041	0.059	0.064
(14)	0.051	0.050	0.052	0.045	0.055	**0.051**	0.047	0.040	0.042	0.052
(13)	0.050	0.040	0.040	0.041	0.058	0.050	0.050	0.041	0.031	0.042
(12)	0.050	0.058	0.048	0.034	0.053	0.050	0.062	0.061	0.041	0.065

n=300, 1000 samples, 996 converged; n=500, 1000 samples,994 converged.