Supplemental Materials

Bayesian Dynamic Borrowing of Historical Information

With Applications to the Analysis of Large-Scale Assessments

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Case Studies

For the single-level case study (Case Study 1) with the full United States sample, in addition to the selected inverse gamma prior values, the estimated coefficients based on all the evaluated inverse gamma priors can be found in Table 1 and the performance of different priors is illustrated in Figure 1. For the single-level case study with a sample size of 500, the estimated coefficients can be found in Table 2 and the performance of different priors is presented in Figure 2. For the multilevel case study (Case Study 2) with a sample size of 500, the estimated coefficients can be found in Tables 3, 4, and 5. The performance of different priors for the multilevel case study is presented in Figure 3 in this supplemental material.

Simulation Studies

Results for the single-level simulation study (Simulation Study 1) with sample sizes of 100, 500 and 2000 with all the evaluated inverse gamma priors can be found in Figures 4, 5 and 6, respectively. Results for the multilevel simulation study (Simulation Study 2) with all the evaluated inverse gamma priors for the sample size conditions (1) 10 schools with 20 students per school, (2) 10 schools with 40 students per school, and (3) 30 schools with 20 students per school can be found in Figures 7, 8, and 9, respectively.

Table 1

Posterior Means and Standard Deviations (SD) of Coefficients for Case Study 1 (Single-Level Model)

Cycle	Method	Intercept		FEMALE		PARED		HOMEPOS		IMMIG	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
2003	BLR non inf	416.89	6.74	-9.49	2.24	3.62	0.49	33.54	1.23	12.08	3.52
2006	BLR non inf	387.85	6.88	-12.86	2.20	6.61	0.49	25.91	1.22	8.04	3.15
2009	BLR non inf	417.92	6.88	-17.22	2.29	5.17	0.51	27.71	1.27	3.85	3.00
2012	BLR non inf	408.43	6.90	-8.58	2.34	5.62	0.52	21.17	1.17	-5.65	3.12
2015	BLR non inf	405.80	5.93	-8.87	2.19	4.72	0.46	19.61	1.07	-0.23	2.78
	BLR non inf	426.29	7.64	-7.21	2.42	4.68	0.54	25.02	1.14	-18.29	3.15
	BLR inf	419.55	5.13	-9.51	1.68	4.66	0.35	25.14	0.79	-7.75	2.12
	BLR pooling	411.46	2.78	-10.57	0.96	5.00	0.21	25.00	0.46	0.34	1.25
	BDB IG(.001,.001)	425.57	6.80	-7.79	2.20	4.69	0.49	25.06	1.07	-17.14	3.06
	BDB IG(.01,.01)	426.12	7.32	-7.34	2.39	4.67	0.52	25.03	1.13	-17.74	3.04
	BDB IG(.1,.1)	426.37	7.59	-7.20	2.39	4.67	0.54	25.07	1.14	-18.18	3.05
2018	BDB IG(1,1)	426.52	7.40	-7.24	2.36	4.66	0.53	25.03	1.11	-18.22	3.07
	BDB IG(1,.1)	426.23	7.24	-7.11	2.39	4.67	0.51	25.01	1.11	-18.09	3.05
	BDB IG(1,.01)	426.20	7.22	-7.41	2.36	4.66	0.51	25.08	1.10	-17.62	3.04
	BDB IG(1,.001)	425.41	6.58	-8.05	2.15	4.68	0.46	25.03	1.08	-16.62	3.04
	PP (.25)	416.67	4.55	-9.51	1.56	4.87	0.33	25.15	0.77	-5.74	2.04
	PP (.50)	413.72	3.71	-10.10	1.25	4.94	0.27	25.07	0.63	-2.20	1.63
	PP (.75)	412.12	3.14	-10.37	1.08	4.99	0.23	25.02	0.54	-0.57	1.45

Table 2

Posterior Means and Standard Deviations (SD) of Coefficients for Small Case Study (Single-Level Model)

Cycle	Method	Intercept		FEMALE		PARED		HOMEPOS		IMMIG	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
2003	BLR non inf	463.43	24.57	-17.64	7.70	1.01	1.72	27.62	4.06	10.54	13.66
2006	BLR non inf	399.97	24.09	-25.47	7.62	7.74	1.71	27.39	4.73	-8.73	9.53
2009	BLR non inf	424.67	19.29	-12.01	7.57	3.84	1.49	27.72	4.33	4.10	9.00
2012	BLR non inf	434.90	21.33	-10.90	7.16	5.79	1.64	25.79	3.59	-20.98	8.83
2015	BLR non inf	417.71	19.49	-14.27	7.85	3.85	1.54	17.07	3.51	4.84	10.27
2018	BLR non inf	418.04	26.72	-8.71	7.46	6.20	1.79	31.82	3.53	-27.91	10.62
	BLR inf	419.56	18.50	-12.86	5.31	5.41	1.19	29.55	2.63	-14.11	7.38
	BLR pooling	426.60	8.80	-15.05	3.10	4.80	0.66	26.15	1.53	-7.26	4.01
	BDB IG(.001,.001)	420.31	21.76	-12.40	5.68	5.65	1.44	30.26	3.10	-18.64	9.60
	BDB IG(.01,.01)	419.43	24.43	-10.27	6.85	5.89	1.63	31.06	3.28	-23.36	10.05
	BDB IG(.1,.1)	418.09	26.90	-9.10	7.37	6.15	1.80	31.55	3.47	-26.77	10.65
	BDB IG(1,1)	417.87	27.18	-9.01	7.46	6.19	1.84	31.70	3.54	-27.39	10.87
	BDB IG(1,.1)	418.72	26.47	-9.34	7.25	6.10	1.76	31.51	3.46	-26.61	10.61
	BDB IG(1,.01)	419.41	23.34	-10.87	6.54	5.84	1.56	30.80	3.21	-21.81	9.58
	BDB IG(1,.001)	421.98	19.06	-13.12	5.17	5.35	1.25	29.50	2.93	-14.88	8.33
	PP (.25)	424.65	15.58	-13.34	5.05	5.20	1.11	28.38	2.48	-14.01	6.93
	PP (.50)	425.71	11.93	-14.37	4.06	4.98	0.87	27.16	1.99	-10.19	5.33
	PP (.75)	426.13	10.19	-14.75	3.45	4.86	0.75	26.51	1.75	-8.11	4.45

Table 3

Posterior Means and Standard Deviations (SD) of Individual-Level Coefficients for Small Case Study (Multilevel Model)

Cycle	Method	Intercept		FEMALE		PARED		HOMEPOS		IMMIG	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
2003	BLR non inf	500.10	42.51	-28.49	8.57	3.40	3.76	32.48	3.76	24.53	12.74
2006	BLR non inf	440.07	33.99	-12.13	7.10	3.15	3.83	16.75	3.83	-3.52	10.69
2009	BLR non inf	466.96	25.68	-24.57	7.72	3.05	4.06	23.46	4.06	8.30	9.85
2012	BLR non inf	440.66	50.15	-12.89	7.54	2.33	3.68	16.52	3.68	-4.92	10.31
2015	BLR non inf	437.70	50.80	-15.90	7.94	2.02	3.38	19.30	3.38	2.78	9.50
	BLR noninf	439.78	29.88	-13.89	8.13	3.25	1.77	21.16	3.61	-19.84	7.85
	BLR inf	448.30	12.77	-17.79	2.75	2.83	0.58	21.54	1.40	1.69	3.24
	BLR pooling	446.97	13.40	-17.51	2.95	2.85	0.62	21.47	1.48	1.17	3.48
	BDB IG(1,1) W2	441.10	28.4	-13.92	7.33	3.14	1.67	21.11	3.48	-19.48	7.51
	BDB IG(1,.1) W2	440.87	26.9	-14.31	7.00	3.10	1.61	21.22	3.38	-18.17	7.07
2018	BDB IG(1,.001) W2	448.33	19.98	-16.71	4.76	2.76	1.00	21.64	2.58	-7.44	4.63
	BDB IG(1,1) W20	441.21	28.77	-14.02	7.28	3.11	1.68	20.85	3.44	-19.27	8.10
	BDB IG(1,.1) W20	442.13	27.98	-14.45	7.08	3.05	1.64	21.02	3.38	-18.38	7.38
	BDB IG(1,.001) W20	447.35	20.16	-16.77	4.88	2.78	1.03	21.45	2.62	-7.01	4.83
	PP (.25)	431.04	17.55	-16.66	4.94	3.78	1.08	24.11	2.46	-3.86	3.86
	PP (.5)	440.48	15.25	-17.28	3.85	3.28	0.84	22.66	1.96	-0.54	3.58
	PP (.75)	444.89	14.20	-17.37	3.26	3.01	0.71	21.89	1.73	0.38	3.47

Table 4

Posterior Means and Standard Deviations (SD) of School-Level Coefficients for Small Case Study (Multilevel Model)

Cycle	Method	TCSHORT		STRATIO		FEMALE:TCSHORT	
		Mean	SD	Mean	SD	Mean	SD
2003	BLR non inf	1.73	10.50	-5.45	2.13	5.21	9.52
2006	BLR non inf	-11.67	8.43	-0.12	1.52	6.05	6.74
2009	BLR non inf	-13.73	7.90	-1.59	0.96	-4.98	8.40
2012	BLR non inf	-8.85	8.79	0.87	2.50	-1.07	6.27
2015	BLR non inf	4.27	11.12	-0.11	2.44	-6.97	7.78
	BLR noninf	-19.77	7.85	0.07	0.94	0.59	7.15
	BLR inf	-10.38	3.24	-0.62	0.55	0.23	2.66
	BLR pooling	-10.67	3.48	-0.53	0.57	0.29	2.81
	BDB IG(1,1) W2	-19.24	7.51	0.09	0.92	0.52	6.50
	BDB IG(1,.1) W2	-18.16	7.07	0.11	0.88	0.21	6.30
2018	BDB IG(1,.001) W2	-13.48	4.63	-0.25	0.72	-0.53	4.13
	BDB IG(1,1) W20	-19.54	8.10	0.09	0.96	0.75	6.59
	BDB IG(1,.1) W20	-17.80	7.38	0.11	0.92	0.10	6.22
	BDB IG(1,.001) W20	-12.88	4.83	-0.22	0.73	-0.76	4.12
	PP (.25)	-12.19	3.86	-0.21	0.53	0.43	4.71
	PP (.5)	-11.02	3.58	-0.44	0.56	0.45	3.65
	PP (.75)	-10.82	3.47	-0.51	0.58	0.35	3.11

Table 5

Posterior Means of Variation Parameters for Small Case Study (Multilevel Model)

Cycle	Method	Level-1 SD	Level-2 Var.-Intercept	Level-2 Covar.	Level-2 Var.-FEMALE
2003	BLR non inf	73.80	908.60	-119.68	125.30
2006	BLR non inf	72.21	1453.94	14.54	43.45
2009	BLR non inf	75.68	540.18	13.13	43.56
2012	BLR non inf	75.70	1830.94	-32.30	33.75
2015	BLR non inf	71.78	1922.74	-148.92	87.51
	BLR noninf	77.79	852.73	-4.03	44.46
	BLR inf	74.56	1127.82	-52.47	17.39
	BLR pooling	74.58	1134.35	-51.92	17.31
	BDB IG(1,1) W2	74.51	830.65	-23.77	13.37
	BDB IG(1,.1) W2	74.47	815.14	-20.18	12.22
2018	BDB IG(1,.001) W2	74.41	863.90	-22.44	11.64
	BDB IG(1,1) W20	74.50	943.18	-32.78	12.10
	BDB IG(1,.1) W20	74.48	926.73	-32.32	11.78
	BDB IG(1,.001) W20	74.44	957.71	-29.19	9.44
	PP (.25)	79.10	438.50	-5.45	11.02
	PP (.5)	76.67	840.22	-18.43	9.17
	PP (.75)	75.35	1035.97	-35.79	12.15

Figure Captions

Figure 1. Total Effective Sample Size (upper panel), and LOOIC (lower panel) for Single-Level Case Study (Full United States Sample).

Figure 2. Total Effective Sample Size (upper panel), and LOOIC (lower panel) for Case Study 1 ($N = 500$)

Figure 3. Total Effective Sample Size (upper panel), and LOOIC (lower panel) for Case Study 2 ($N=500$).

Figure 4. Log MSE (FIG. 3a, upper left) Percent bias (FIG. 3b, upper right), Total Effective sample size (FIG. 3c, lower left), and LOOIC (FIG. 3d, lower right) for Simulation Study 1 ($N=100$).

Figure 5. Log MSE (FIG. 4a, upper left) Percent bias (FIG. 4b, upper right), Total Effective sample size (FIG. 4c, lower left), and LOOIC (FIG. 4d, lower right) for Simulation Study 1 ($N=500$).

Figure 6. Log MSE (FIG. 5a, upper left) Percent bias (FIG. 5b, upper right), Total Effective sample size (FIG. 5c, lower left), and LOOIC (FIG. 5d, lower right) for Simulation Study 1 ($N=2000$).

Figure 7. Log MSE (Figure. 6a, upper left) Percent bias (Figure. 6b, upper right), Total Effective sample size (Figure. 6c, lower left), and LOOIC (Figure. 6d, lower right) for Simulation Study 2 ($N=200$; 10 Schools, 20 Students Each).

Figure 8. Log MSE (Figure. 7a, upper left) Percent bias (Figure. 7b, upper right), Total Effective sample size (Figure. 7c, lower left), and LOOIC (Figure. 7d, lower right) for Simulation Study 2 ($N=400$; 10 Schools, 40 Students Each).

Figure 9. Log MSE (Figure. 8a, upper left) Percent bias (Figure. 8b, upper right), Total Effective sample size (Figure. 8c, lower left), and LOOIC (Figure. 8d, lower right) for Simulation Study 2 (N=600; 30 Schools, 20 Students Each).

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