Supplementary Materials for

Modeling Intensive Polytomous Time Series Eye-Tracking Data: A Dynamic Tree-Based Item Response Model

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$AR(1)$ **Effects in the Dynamic IRTree Model**

The $AR(1)$ parameters in Equation 1 can be presented as

$$\lambda_{jir} = \lambda_r + \lambda_{1jr} + \lambda_{2ir}. \tag{1}$$

The λ_{ji1} is the model-based conditional log odds ratio between y^*_{tlji1} and $y^*_{(t-1)lji1}$ at Node 1:

$$\lambda_{ji1} = \frac{\frac{P(y^*_{tlji1}=1|y^*_{(t-1)lji1}=1, time_{tlji}, \mathbf{X}, \delta_{lji1}, \lambda_{1j1}, \theta_{j1}, \lambda_{2i1}, \beta_{i1})}{P(y^*_{tlji1}=1|y^*_{(t-1)lji1}=-1, time_{tlji}, \mathbf{X}, \delta_{lji1}, \lambda_{1j1}, \theta_{j1}, \lambda_{2i1}, \beta_{i1})}}{\frac{P(y^*_{tlji1}=0|y^*_{(t-1)lji1}=1, time_{tlji}, \mathbf{X}, \delta_{lji1}, \lambda_{1j1}, \theta_{j1}, \lambda_{2i1}, \beta_{i1})}{P(y^*_{tlji1}=0|y^*_{(t-1)lji1}=-1, time_{tlji}, \mathbf{X}, \delta_{lji1}, \lambda_{1j1}, \theta_{j1}, \lambda_{2i1}, \beta_{i1})}}. \tag{2}$$

The λ_{ji2} is the model-based conditional log odds ratio between y^*_{tlji2} and $y^*_{(t-1)lji2}$ at Node 2:

$$\lambda_{ji2} = \frac{\frac{P(y^*_{tlji2}=1|y^*_{(t-1)lji2}=1, time_{tlji}, \mathbf{X}, \delta_{lji2}, \lambda_{1j2}, \theta_{j2}, \lambda_{2i2}, \beta_{i2})}{P(y^*_{tlji2}=1|y^*_{(t-1)lji2}=-1, time_{tlji}, \mathbf{X}, \delta_{lji2}, \lambda_{1j2}, \theta_{j2}, \lambda_{2i2}, \beta_{i2})}}{\frac{P(y^*_{tlji2}=0|y^*_{(t-1)lji2}=1, time_{tlji}, \mathbf{X}, \delta_{lji2}, \lambda_{1j2}, \theta_{j2}, \lambda_{2i2}, \beta_{i2})}{P(y^*_{tlji2}=0|y^*_{(t-1)lji2}=-1, time_{tlji}, \mathbf{X}, \delta_{lji2}, \lambda_{1j2}, \theta_{j2}, \lambda_{2i2}, \beta_{i2})}}. \tag{3}$$

In λ_{jir} $(r = 1, 2)$, own-lag (TC $\rightarrow$ T&C; T $\rightarrow$ T) and cross-lag (T&C $\rightarrow$ O; T $\rightarrow$ C) effects were considered, presented in the following diagram. Paths from time point $t-1$ to time point t indicate the comparison structure in the log odds ratio.

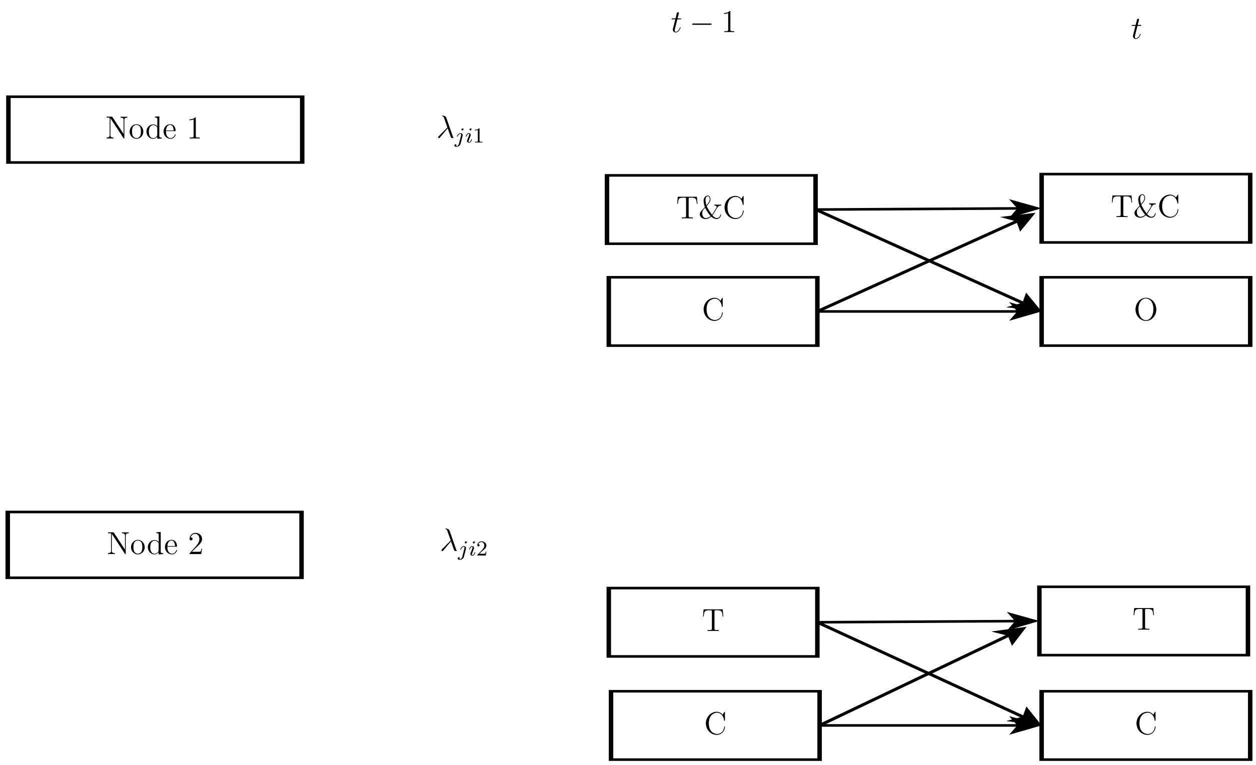


Figure A.1-1 Graphical representation for λ_{jir}.
Note. T, C, and O indicates “Target”, “Competitor”, and “Unrelated Objects” respectively.

The $AR(1)$ parameters in Equation 2 can be presented as

$$\lambda_{Tjir} = \lambda_{Tr} + \lambda_{T1jr} + \lambda_{T2ir} \tag{4}$$

and

$$\lambda_{Cjir} = \lambda_{Cr} + \lambda_{C1jr} + \lambda_{C2ir}. \tag{5}$$

The λ_{Tji1} is the model-based conditional log odds ratio between y^*_{tlji1} and $x_{T(t-1)lji}$ at Node 1:

$$\lambda_{Tji1} = \frac{\frac{P(y^*_{tlji1}=1|x_{T(t-1)lji}=1,x_{C(t-1)lji}=0,time_{tlji},\mathbf{X},\delta_{lji1},\lambda_{1j1},\theta_{j1},\lambda_{2i1},\beta_{i1})}{P(y^*_{tlji1}=1|x_{T(t-1)lji}=-1,x_{C(t-1)lji}=0,time_{tlji},\mathbf{X},\delta_{lji1},\lambda_{1j1},\theta_{j1},\lambda_{2i1},\beta_{i1})}}{\frac{P(y^*_{tlji1}=0|x_{T(t-1)lji}=1,x_{C(t-1)lji}=0,time_{tlji},\mathbf{X},\delta_{lji1},\lambda_{1j1},\theta_{j1},\lambda_{2i1},\beta_{i1})}{P(y^*_{tlji1}=0|x_{T(t-1)lji}=-1,x_{C(t-1)lji}=0,time_{tlji},\mathbf{X},\delta_{lji1},\lambda_{1j1},\theta_{j1},\lambda_{2i1},\beta_{i1})}}. \tag{6}$$

The λ_{Cji1} is the model-based conditional log odds ratio between y^*_{tlji1} and $x_{C(t-1)lji}$ at Node 1:

$$\lambda_{Tji1} = \frac{\frac{P(y^*_{tlji1}=1|x_{T(t-1)lji}=0,x_{C(t-1)lji}=1,time_{tlji},\mathbf{X},\delta_{lji1},\lambda_{1j1},\theta_{j1},\lambda_{2i1},\beta_{i1})}{P(y^*_{tlji1}=1|x_{T(t-1)lji}=0,x_{C(t-1)lji}=-1,time_{tlji},\mathbf{X},\delta_{lji1},\lambda_{1j1},\theta_{j1},\lambda_{2i1},\beta_{i1})}}{\frac{P(y^*_{tlji1}=0|x_{T(t-1)lji}=0,x_{C(t-1)lji}=1,time_{tlji},\mathbf{X},\delta_{lji1},\lambda_{1j1},\theta_{j1},\lambda_{2i1},\beta_{i1})}{P(y^*_{tlji1}=0|x_{T(t-1)lji}=0,x_{C(t-1)lji}=-1,time_{tlji},\mathbf{X},\delta_{lji1},\lambda_{1j1},\theta_{j1},\lambda_{2i1},\beta_{i1})}}. \tag{7}$$

The λ_{Tji2} is the model-based conditional log odds ratio between y^*_{tlji1} and $x_{T(t-1)lji}$ at Node 2:

$$\lambda_{Tji1} = \frac{\frac{P(y^*_{tlji2}=1|x_{T(t-1)lji}=1,x_{C(t-1)lji}=0,time_{tlji},\mathbf{X},\delta_{lji2},\lambda_{1j2},\theta_{j2},\lambda_{2i2},\beta_{i2})}{P(y^*_{tlji2}=1|x_{T(t-1)lji}=-1,x_{C(t-1)lji}=0,time_{tlji},\mathbf{X},\delta_{lji2},\lambda_{1j2},\theta_{j2},\lambda_{2i2},\beta_{i2})}}{\frac{P(y^*_{tlji2}=0|x_{T(t-1)lji}=1,x_{C(t-1)lji}=0,time_{tlji},\mathbf{X},\delta_{lji2},\lambda_{1j2},\theta_{j2},\lambda_{2i2},\beta_{i2})}{P(y^*_{tlji2}=0|x_{T(t-1)lji}=-1,x_{C(t-1)lji}=0,time_{tlji},\mathbf{X},\delta_{lji2},\lambda_{1j2},\theta_{j2},\lambda_{2i2},\beta_{i2})}}. \tag{8}$$

The λ_{Cji2} is the model-based conditional log odds ratio between y^*_{tlji1} and $x_{C(t-1)lji}$ at Node 2:

$$\lambda_{Tji1} = \frac{\frac{P(y^*_{tlji2}=1|x_{T(t-1)lji}=0,x_{C(t-1)lji}=1,time_{tlji},\mathbf{X},\delta_{lji2},\lambda_{1j2},\theta_{j2},\lambda_{2i2},\beta_{i2})}{P(y^*_{tlji2}=1|x_{T(t-1)lji}=0,x_{C(t-1)lji}=-1,time_{tlji},\mathbf{X},\delta_{lji2},\lambda_{1j2},\theta_{j2},\lambda_{2i2},\beta_{i2})}}{\frac{P(y^*_{tlji2}=0|x_{T(t-1)lji}=0,x_{C(t-1)lji}=1,time_{tlji},\mathbf{X},\delta_{lji2},\lambda_{1j2},\theta_{j2},\lambda_{2i2},\beta_{i2})}{P(y^*_{tlji2}=0|x_{T(t-1)lji}=0,x_{C(t-1)lji}=-1,time_{tlji},\mathbf{X},\delta_{lji2},\lambda_{1j2},\theta_{j2},\lambda_{2i2},\beta_{i2})}}. \tag{9}$$

In λ_{Tjir} and λ_{Cjir} ($r = 1, 2$), own-lag (T $\rightarrow$ T; C $\rightarrow$ C; O $\rightarrow$ O) and cross-lag (e.g., T $\rightarrow$ O; O $\rightarrow$ T&C) effects were considered, presented in the following diagram. Paths from time point $t - 1$ to time point t indicate the comparison structure in the log odds ratio.

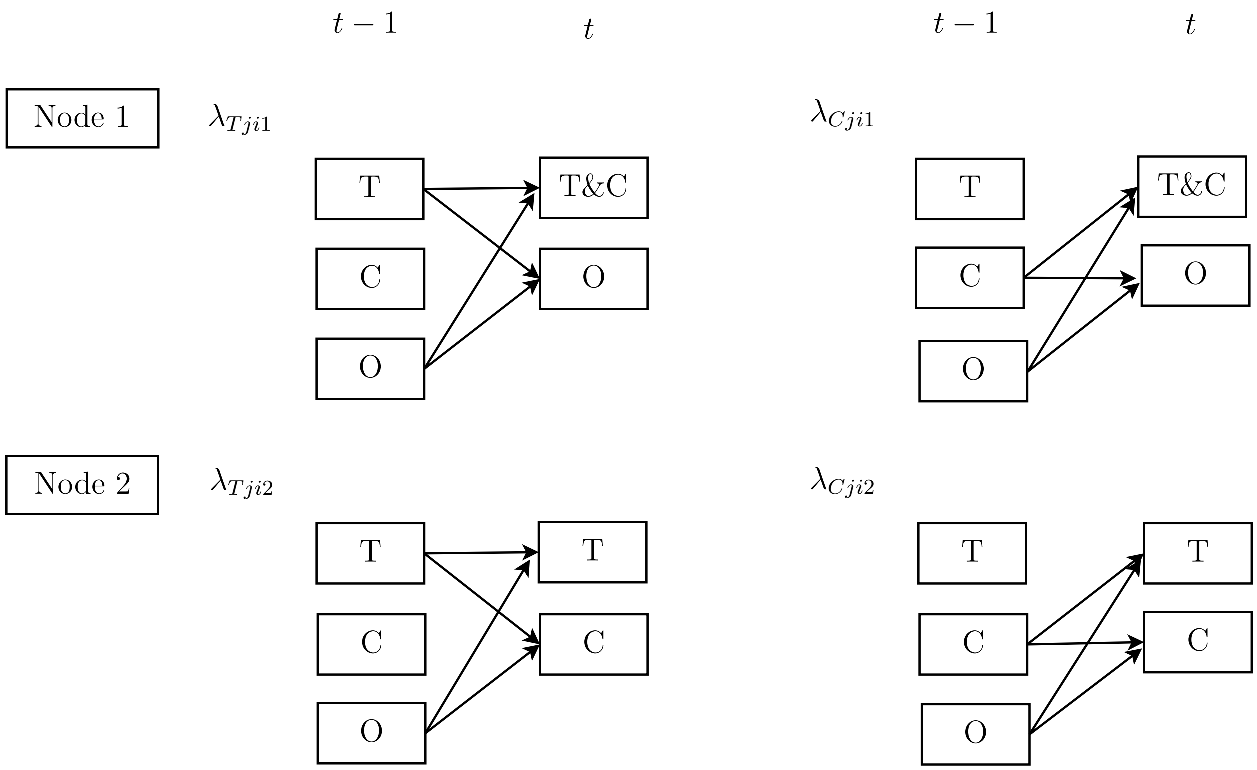


Figure A.1-2 Graphical representation for λ_{Tjir} and λ_{Cjir}.
Note. T, C, and O indicates "Target", "Competitor", and "Unrelated Objects" respectively.

Trend and Autocorrelations in an Empirical Study

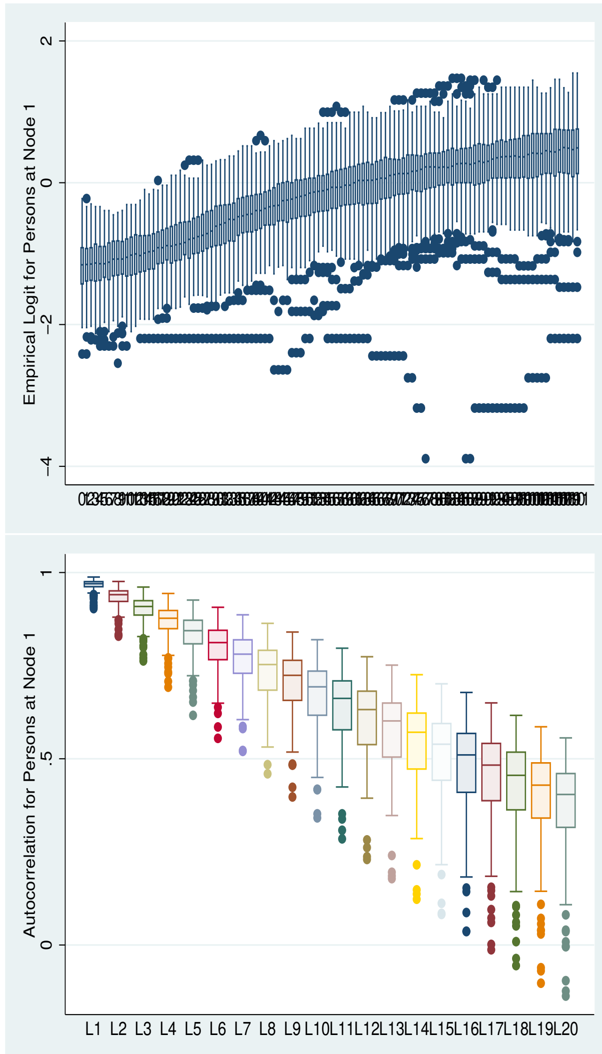


Figure A.2 Trend over time (indicated on x-axis) (top) and autocorrelations of empirical logit at Node 1 as a function of lag (indicated on x-axis) (bottom)

Linear and Polynomial Trends in an Empirical Study

Fitted lines over time are presented below for the linear function and Kernel-weighted local polynomial smoothing function:

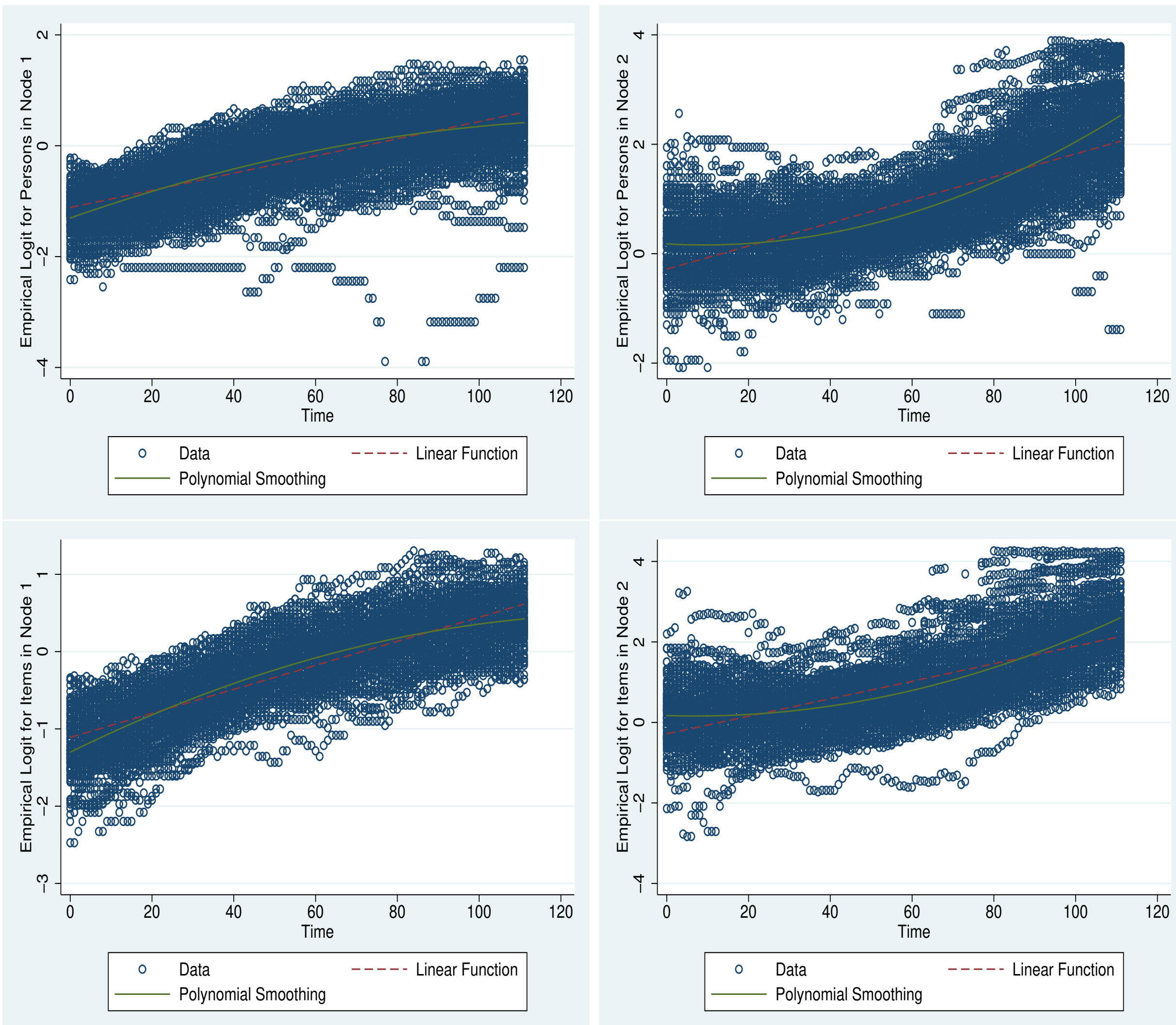


Figure A.3 Linear and polynomial trends over time.

Fitted lines over time were similar between the linear function and Kernel-weighted local polynomial smoothing function and small deviations from the linear trend were observed.

Trend by Trials in an Empirical Study

To explore whether the trend pattern is similar across 288 trials graphically, we plot the logit-transformed proportion measures for *each* trial l at each time point t ($ln\frac{P_{tlr}}{1-P_{tlr}}$; $P_{tlr} = (\sum_{j=1}^{J}\sum_{i=1}^{I} y^*_{tljir})/JI$) against time at each node. As shown in Figures A.3, the linear trend pattern is similar across the 288 trials in each node.

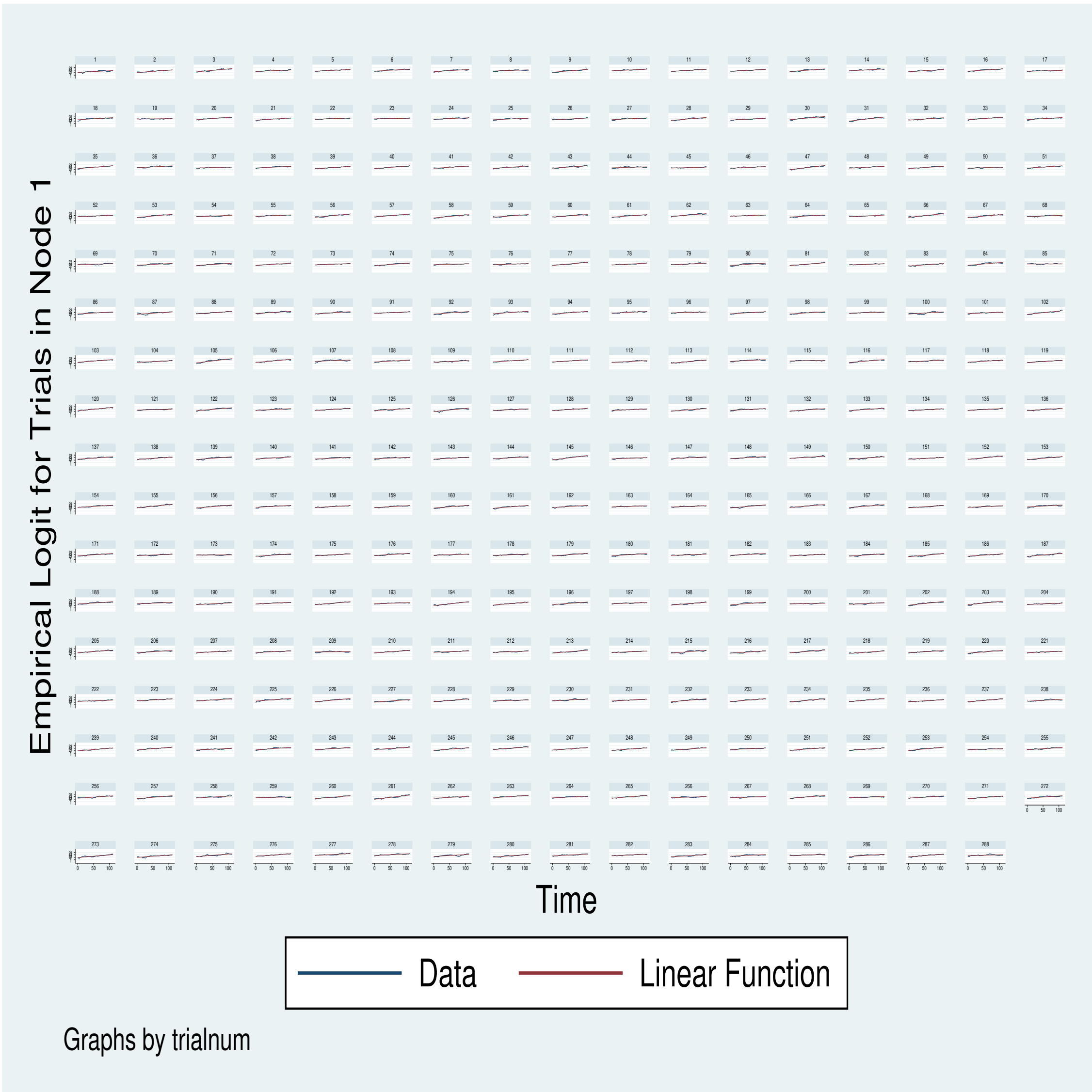


Figure A.4-1 Linear trend by trials and data (empirical logit) over time at Node 1.

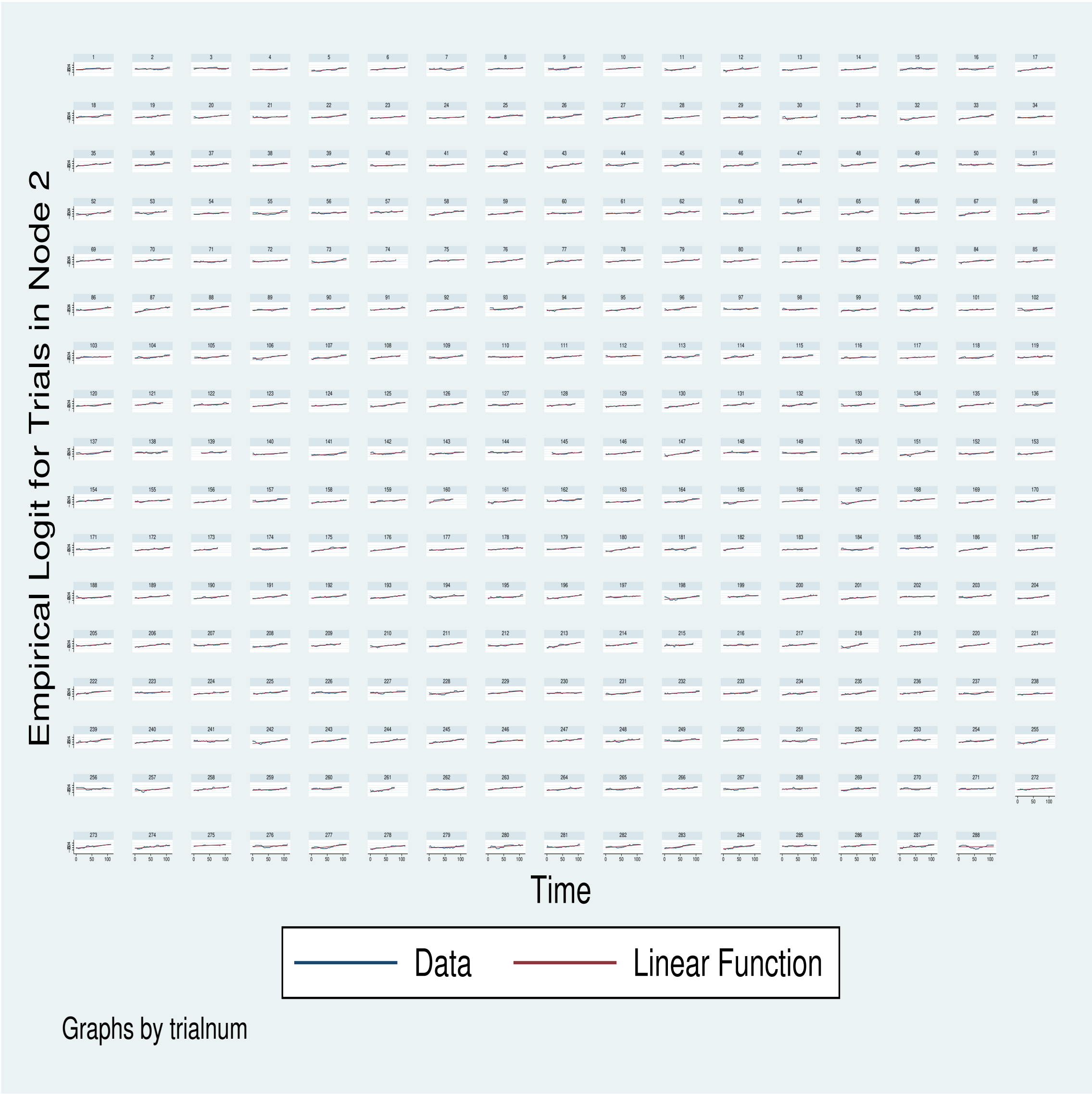


Figure A.4-2 Linear trend by trials and data (empirical logit) over time at Node 2.

Models Considered in Model Selection Regarding Random Effects in an Empirical Study (Shown in Table 3)

Models with $y^*_{(t-1)lji}$

• Model B*

$$\eta_{tljir} = \gamma_{1r} + y^{*'}_{(t-1)lji}\lambda_r + time^{'}_{tlji}\zeta_r + \delta_{lji2} + \theta_{jr} + \beta_{ir}, \tag{10}$$

where δ_{lji2} is a random trial effect at Node 2.

• Model B*-Person

$$\eta_{tljir} = \gamma_{1r} + y^{*'}_{(t-1)ljir}\lambda_r + time^{'}_{tlji}\zeta_r + \delta_{lji2} + y^{*'}_{(t-1)ljir}\lambda_{1jr} + \theta_{jr} + \beta_{ir} \tag{11}$$

• Model B*-Item

$$\eta_{tljir} = \gamma_{1r} + y^{*'}_{(t-1)ljir}\lambda_r + time^{'}_{tlji}\zeta_r + \delta_{lji2} + \theta_{jr} + y^{*'}_{(t-1)ljir}\lambda_{2ir} + \beta_{ir} \tag{12}$$

• Model B*-Person&Item

$$\eta_{tljir} = \gamma_{1r} + y^{*'}_{(t-1)ljir}\lambda_r + time^{'}_{tlji}\zeta_r + \delta_{lji2} + y^{*'}_{(t-1)ljir}\lambda_{1jr} + \theta_{jr} + y^{*'}_{(t-1)ljir}\lambda_{2ir} + \beta_{ir} \tag{13}$$

Models with $x_{T(t-1)lji}$ and $x_{T(t-1)lji}$

- Model B*

$$\eta_{tljir} = \gamma_{1r} + x^{'}_{T(t-1)lji}\lambda_{Tr} + x^{'}_{C(t-1)lji}\lambda_{Cr} + time^{'}_{tlji}\zeta_r + \delta_{lji2} + \theta_{jr} + \beta_{ir}, \tag{14}$$

where δ_{lji2} is a random trial effect at Node 2.

- Model B*-Person

$$\eta_{tljir} = \gamma_{1r} + x^{'}_{T(t-1)lji}\lambda_{Tr} + x^{'}_{C(t-1)lji}\lambda_{Cr} + time^{'}_{tlji}\zeta_r + \delta_{lji2} + y^{*'}_{(t-1)ljir}\lambda_{1jr} + \theta_{jr} + \beta_{ir} \tag{15}$$

- Model B*-Item

$$\eta_{tljir} = \gamma_{1r} + x^{'}_{T(t-1)lji}\lambda_{Tr} + x^{'}_{C(t-1)lji}\lambda_{Cr} + time^{'}_{tlji}\zeta_r + \delta_{lji2} + y^{*'}_{(t-1)ljir}\lambda_{1jr} + \theta_{jr} + \beta_{ir} \tag{16}$$

- Model B*-Person&Item

$$\eta_{tljir} = \gamma_{1r} + x^{'}_{T(t-1)lji}\lambda_{Tr} + x^{'}_{C(t-1)lji}\lambda_{Cr} + time^{'}_{tlji}\zeta_r + \delta_{lji2} + y^{*'}_{(t-1)ljir}\lambda_{1jr} + \theta_{jr} + y^{*'}_{(t-1)ljir}\lambda_{2ir} + \beta_{ir} \tag{17}$$

R Code for the Dynamic IRTree Model

In this section, we describe how to estimate the dynamic IRTree model using the `glmer` function in `lme4` version 1.1.15 R package. The following is the R code for Model 2 in the paper for which we interpreted parameter estimates (see results in Table 4 of the paper):

```
1. data <- read.table("C:\\data.txt",header=T,fill=T)
2. data <- na.omit(data)
3. data$item <- as.factor(data$item)
   data$subject <- as.factor(data$subject)
   data$trialnum <- as.factor(data$trialnum)
   data$node <- as.factor(data$node)
   data$node2 <- as.factor(data$node2)
   data$ctime1 <- as.numeric(data$ctime1)
   data$clag1 <- as.numeric(data$clag1)
   data$privileged1 <- as.numeric(data$privileged1)
   data$contrast <- as.numeric(data$contrast)
4. Model1 <- glmer(y ~
5. -1 + node + cylag1:node +  privileged1:node + contrast:node + ctime1:node +
6. (-1+node2|trialnum) + (-1+cylag1:node+node|subject) + (-1+node|item),
7. family = binomial,
8. data = data)
9. summary(Model1)
```

Each line is explained in more detail below:

Line 1. A file, data.txt, is read in table format and a data frame is created from it.

Line 2. Missing values coded in `NA` in the data are deleted in the data. As we described in the paper, the lagged response at the first time point ($t = 0$), y^*_{0lji}, was treated as a missing variable and its subsequent response variable y^*_{1lji} was not modelled. Note that there are no missing values in the other variables including the outcome variable y^*_{tlji} in our illustrative data.

Line 3. The experimental factors (`item`, `subject`, and `trialnum`) and nodes in the tree model (`node` and `node2`) are coded as factors, and the covariates (the lagged response `cylag1` and trend `ctime1`) and two experimental condition contrasts `privileged1` and `contrast`) are coded as numeric.

Line 4. The binary variable called `y` (y^*_{tlji}) is specified in the `glmer` function and the model name is assigned as `Model 2`.

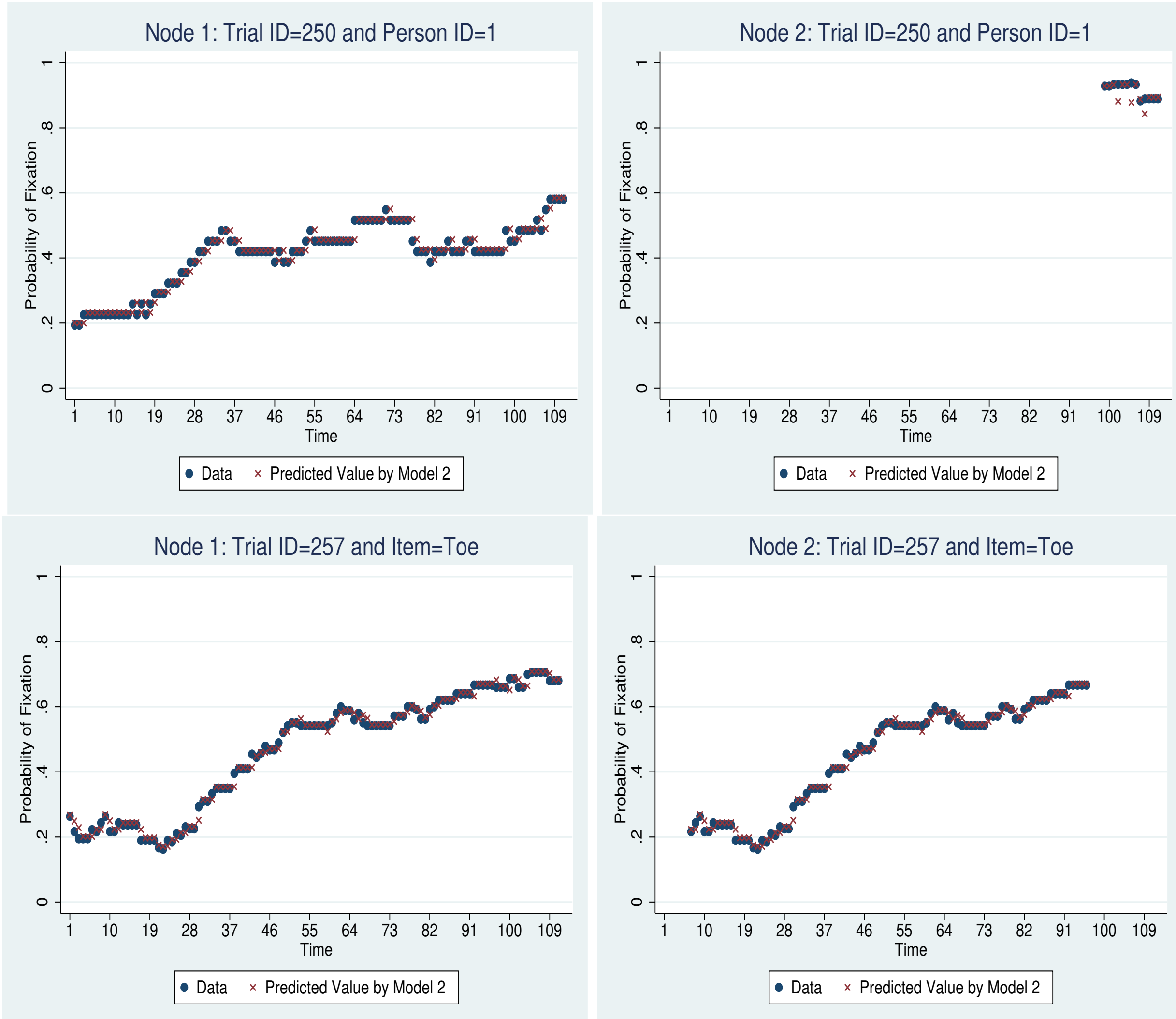
Line 5. The fixed effects of the model are specified: `node` is for the intercept γ, `cylag1` is for the fixed lagged effect λ, `ctime1` is for the fixed trend effect ζ, and `privileged1` and `contrast` are for the two experimental condition effects $\boldsymbol{\gamma}$.

Line 6. The random effects of the model are specified: `(-1+node2|trialnum)` is for the trial random effect at Node 2 δ_{lji2}, `(-1+cylag1:node+node|subject)` are for person random effects $[\theta_{j1}, \theta_{j2}, \lambda_{1j1}, \lambda_{1j2}]'$, and `(-1+node|item)` are for item random effects $[\beta_{i1}, \beta'_{i2}$.

Line 7. The linking component for binomial data is specified as `family = binomial`. Because the logistic link is the default, the specification `‘‘logit’’` argument of `family = binomial()` is omitted.

Line 8. The data set called `data` is specified.

Line 9. The results of `Model2` are provided.



Model-Data Fit for Model 2 in an Empirical Study

Figure A.5 Model prediction from Model 2 over 111 time-point data, for a person (top) and an item (bottom) at Node 1 (lexico-semantic processing) and at Node 2 (ambiguity resolution).

Model Comparisons regrading Different Change Processes in an Empirical Study

For comparison purposes, Model 2 (also reported in Table 4 of the manuscript), Model 2 without a trend effect, and Model 2 without $AR(1)$ effects were fit to the empirical data. Results are presented in Table A.1. Statistical inference for the experimental condition effects did not differ between Model 2 and Model 2 without the trend effect, although Model 2 (AIC=180262; BIC=180554) fits better than Model 2 without the trend effect (AIC=182397; BIC=182664). However, statistical inference differs between Model 2 and Model 2 without the AR effects and Model 2 (AIC=180262; BIC=180554) fits much better than Model 2 without the AR effects (AIC=1711679; BIC=1711861). Compared to the results of Model 2, Model 2 without AR effects exhibited larger effects of trend and experimental condition, and a significant effect of the second condition contrast (*Privileged* covariate).

Table A.1 *Estimates (Standard Errors) of the Dynamic IRTree Model for an Empirical Study*

	Model 2		Model 2 without trend		Model 2 without $AR(1)$	
	Node 1	Node 2	Node 1	Node 2	Node 1	Node 2
Fixed Effects						
Intercept[γ_1]	**0.095**(0.021)	**1.313**(0.052)	**0.100**(0.020)	**1.150**(0.038)	**-0.259**(0.041)	**1.211**(0.071)
$AR(1)$ylag[λ]	**4.181**(0.013)	**5.173**(0.031)	**4.210**(0.013)	**5.070**(0.028)	-	
Trend[ζ]	**0.006**(0.000)	**0.031**(0.001)	-	-	**0.016**(0.000)	**0.021**(0.000)
Privileged[γ_2]	0.005(0.021)	0.071(0.046)	0.005(0.021)	0.073(0.046)	**0.018**(0.005)	**0.188**(0.009)
Contrast[γ_3]	**0.050**(0.013)	**-0.386**(0.034)	**0.048**(0.013)	**-0.314**(0.033)	**0.132**(0.003)	**-0.607**(0.007)

	Model 1				Model 2				Model 3			
	SD	Corr			SD	Corr			SD	Corr		
Random Effects												
Trial (Σ_1)												
Node 1(δ_{lji1})	-				-				-			
Node 2(δ_{lji2})	0.094	-			0.000	-			0.436			
Person (Σ_2)												
Node 1:Intercept[θ_{j1}])	0.172				0.167				0.356			
Node 2:Intercept[θ_{j2}])	0.267	0.705			0.167	0.945			0.368	0.358		
Node 1:AR(1)ylag[λ_{1j1}]	0.115	-0.360	-0.180		0.112	-0.279	-0.045		-	-	-	
Node 2:AR(1)ylag[λ_{1j2}]	0.227	-0.141	-0.246	0.927	0.204	0.033	0.265	0.950	-	-	-	-
Item (Σ_3)												
Node 1:Intercept[β_{i1}])	0.127				0.120				0.280			
Node 2:Intercept[β_{i2}])	0.383	0.410			0.256	0.439			0.578	0.305		
AIC	180287				182397				1711679			
BIC	180579				182664				1711861			

Note. - indicates that an effect is not modelled; Values in bold indicates significance at the 5% level for fixed effects.

Model Comparisons regarding Linear and Quadratic Trend Effects in an Empirical Study

We added a quadratic effect to Model 2 (called Model 2-Quadratic) to investigate unmodelled trend effects with the linear trend effect only. As presented in Table A.2, results of Model 2 and Model 2-Quadratic are similar and the fixed quadratic trend estimate is near 0.

Table A.2 *Estimates (Standard Errors) of the Dynamic IRTree Model for an Empirical Study with a Quadratic Trend Effect*

	Model 2		Model 2-Quadratic	
	Node 1	Node 2	Node 1	Node 2
Fixed Effects				
Intercept[γ_1]	**0.095**(0.021)	**1.313**(0.052)	**0.190**(0.023)	**1.135**(0.055)
AR(1)ylag[λ]	**4.181**(0.013)	**5.173**(0.031)	**4.181**(0.013)	**5.151**(0.031)
LinearTrend[ζ_1]	**0.006**(0.000)	**0.031**(0.001)	**0.006**(0.000)	**0.030**(0.001)
QuadraticTrend[ζ_2]	-	-	**-0.00009**(0.00001)	**0.00023**(0.00002)
Privileged[γ_2]	0.005(0.021)	0.071(0.046)	0.005(0.021)	0.064(0.047)
Contrast[γ_3]	**0.050**(0.013)	**-0.386**(0.034)	**0.050**(0.013)	**-0.386**(0.034)

	Model 2				Model 2-Quadratic			
	SD	Corr			SD	Corr		
Random Effects								
Trial (Σ_1)								
Node 1(δ_{lji1})	-				-			
Node 2(δ_{lji2})	0.094				0.126			
Person (Σ_2)								
Node 1:Intercept[θ_{j1}])	0.172				0.172			
Node 2:Intercept[θ_{j2}])	0.267	0.71			0.275	0.67		
Node 1:AR(1)ylag[λ_{1j1}]	0.115	-0.36	-0.18		0.115	-0.37	-0.15	
Node 2:AR(1)ylag[λ_{1j2}]	0.227	-0.14	-0.25	0.93	0.233	-0.14	-0.26	0.91
Item (Σ_3)								
Node 1:Intercept[β_{i1}])	0.127				0.127			
Node 2:Intercept[β_{i2}])	0.383	0.41			0.389	0.38		

Note. - indicates that an effect is not modelled; Values in bold indicates significance at the 5% level for fixed effects.

Results of the Simulation Study

Table A.3 *Results of the Dynamic IRTree Model for the Simulation Study: Question (c)*

	Model 3 (True)				Model 2 (Misspecified)			
	Bias	RMSE	SD	M(SE)	Bias	RMSE	SD	M(SE)
Fixed Effects								
Node 1:Intercept[γ_{11}]	0.000	0.019	0.019	0.019	0.000	0.019	0.019	0.019
Node 2:Intercept[γ_{12}]	0.006	0.046	0.046	0.045	0.004	0.045	0.045	0.045
Node 1:$AR(1)$ylag[λ_1]	-				0.000	0.002	0.002	0.002
Node 2:$AR(1)$ylag[λ_2]	-				0.000	0.005	0.005	0.004
Node 1:Trend[ζ_1]	-				0.000	0.000	0.000	0.000
Node 2:Trend[ζ_2]	-				0.000	0.000	0.000	0.000
Node 1:Privileged[γ_{21}]	0.001	0.005	0.005	0.005	0.001	0.005	0.005	0.005
Node 2:Privileged[γ_{22}]	0.001	0.009	0.009	0.009	0.001	0.009	0.009	0.009
Node 1:Contrast[γ_{31}]	0.000	0.003	0.003	0.003	0.001	0.003	0.003	0.003
Node 2:Contrast[γ_{32}]	0.001	0.007	0.007	0.007	0.001	0.007	0.007	0.007
Random Effects								
Trial (Σ_1)								
Node 2:Intercept[δ_{lji2}]	-0.085	0.085			-0.085	0.085		
Person (Σ_2)								
Node 1:Intercept[θ_{j1}])	0.000	0.004			0.000	0.004		
Node 2:Intercept[θ_{j2}])	-0.001	0.009			0.000	0.009		
Node 1:AR(1)ylag[λ_{1j1}]	-				0.000	0.000		
Node 2:AR(1)ylag[λ_{1j2}]	-				0.000	0.000		
Covariance(s)	0.000	0.005			0.000	0.001		
Item (Σ_3)								
Node 1:Intercept[β_{i1}])	0.000	0.002			0.000	0.002		
Node 2:Intercept[β_{i2}])	0.000	0.018			-0.001	0.018		
Covariance	0.000	0.005			0.000	0.005		

Note. - indicates that an effect is not modelled; *SD* indicates the standard deviations of the estimates across 200 replications; *SE* indicates the mean standard error estimates across 200 replications, which are available for the fixed effects; average bias and RMSE across covariances of random person effects are reported.

Table A.4 *Results of the Dynamic IRTree Model for the Simulation Study: Question (d)*

	Model 4 (True)				Model 2 (Misspecified)			
	Bias	RMSE	SD	M(SE)	Bias	RMSE	SD	M(SE)
Fixed Effects								
Node 1:Intercept[γ_{11}]	-0.001	0.025	0.025	0.023	-0.106	0.108	0.022	0.021
Node 2:Intercept[γ_{12}]	-0.003	0.055	0.056	0.054	0.182	0.191	0.058	0.051
Node 1:$AR(1)$ylag[λ_1]	-0.002	0.012	0.012	0.013	-0.016	0.020	0.012	0.012
Node 2:$AR(1)$ylag[λ_2]	-0.002	0.033	0.033	0.034	0.052	0.063	0.035	0.034
Node 1:LinearTrend[ζ_{11}]	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Node 2:LinearTrend[ζ_{12}]	0.000	0.001	0.001	0.001	0.003	0.003	0.001	0.001
Node 1:QuadraticTrend[ζ_{21}]	0.000	0.000	0.000	0.000	-			
Node 2:QuadraticTrend[ζ_{22}]	0.000	0.000	0.000	0.000	-			
Node 1:Privileged[γ_{21}]	0.003	0.020	0.020	0.020	0.003	0.018	0.018	0.020
Node 2:Privileged[γ_{22}]	0.008	0.033	0.032	0.035	0.006	0.034	0.034	0.034
Node 1:Contrast[γ_{31}]	0.001	0.013	0.013	0.013	0.001	0.013	0.013	0.013
Node 2:Contrast[γ_{32}]	0.002	0.022	0.022	0.022	0.003	0.026	0.022	0.022
Random Effects								
Trial (Σ_1)	-0.086	0.086			-0.087	0.087		
Node 2:Intercept[δ_{lji2}]								
Person (Σ_2)								
Node 1:Intercept[θ_{j1}])	0.000	0.005			0.000	0.006		
Node 2:Intercept[θ_{j2}])	0.001	0.021			-0.007	0.021		
Node 1:AR(1)ylag[λ_{1j1}]	0.000	0.003			0.000	0.003		
Node 2:AR(1)ylag[λ_{1j2}]	0.007	0.020			0.010	0.024		
Covariances	0.000	0.007			0.001	0.006		
Item (Σ_3)								
Node 1:Intercept[β_{i1}])	0.000	0.003			0.000	0.003		
Node 2:Intercept[β_{i2}])	-0.003	0.026			-0.006	0.024		
Covariance	0.001	0.007			0.000	0.006		

Note. - indicates that an effect is not modelled; *SD* indicates the standard deviations of the estimates across 200 replications; *SE* indicates the mean standard error estimates across 200 replications, which are available for the fixed effects; average bias and RMSE across covariances of random person effects are reported.

Patterns of Trends, Autocorrelations, and Partial Autocorrelations

We provided the patterns of the trend, autocorrelation, and partial autocorrelation in the presence of trend and $AR(1)$ (Model 2 in Table 4 of the manuscript), trend only (Model 2 without the $AR(1)$ effects), and $AR(1)$ only (Model 2 without the trend effect) using 50 simulated data sets under the dynamic IRTree model. Estimates reported in Table A.1 were considered true parameters. In order to explore change processes, logit-transformed proportion measures for *each* person j at a time point t ($ln\frac{P_{tjr}}{1-P_{tjr}}$) and logit-transformed measures (called *empirical logit*) for *each* item i at a time point t ($ln\frac{P_{tir}}{1-P_{tir}}$) were calculated based on binary response y^*_{tljir} for each node in the tree. The P_{tjr} and P_{tir} were calculated as follows: $P_{tjr} = (\sum_l^L \sum_{i=1}^I y^*_{tljir})/LI$ and $P_{tir} = (\sum_l^L \sum_{j=1}^J y^*_{tljir})/LJ$. We found similar patterns in the trend, autocorrelation, and partial autocorrelation for persons, items, and nodes, across 50 replications. Thus, below, we present the patterns for persons at Node 1 from one replication data set. Individual differences in the trend, autocorrelation, and partial autocorrelation were presented using box plots on the figure. For example, in the figures on the top panel, there are 112 box plots (for 112 time points).

When there is trend in time series, the autocorrelations for small lags tend to be large and positive because observations nearby in time are also close by in size (Chatfield, 2004). Thus, they have positive values that slowly decrease as the lags increase. We observed the same pattern in our study. The *partial* autocorrelations can be used to investigate the order of AR. As shown in Figure A.5, there are distinct patterns in the trend, autocorrelation, and partial autocorrelation in the presence of trend and $AR(1)$, trend only, and $AR(1)$ only.

• When there are trend and $AR(1)$ (as in our empirical study), the following is observed: (a) the linear pattern is observed in the time series plot (although there is some deviance from the linear function in the first few time points), (b) the autocorrelations for small lags are large and positive and they slowly decreased, and (c) the partial autocorrelations with the order of 1 are clearly larger than 0 and those with a larger lag are nearly 0.

• When there is trend only, the following patterns are evident: (a) the linear pattern is observed in the time series plot, (b) the autocorrelations are large and positive and slowly decreases, and (c) the partial autocorrelations are large and positive for small lags (unlike in the presence of AR).

• When there is AR only, the patterns are: (a) although there is some increasing pattern in first few time points, overall pattern is that there is no clear increasing or decreasing pattern over time, (b) the autocorrelation exponentially decreases to 0 as the lag increases (unlike in the presence of trend), and (c) the partial autocorrelations with the order of 1 are clearly larger than 0 and those with a larger lag are nearly 0 (unlike in the presence of trend).

Although these results are based on a limited condition similar to our empirical study, similar patterns in the autocorrelation and partial autocorrelations were found regarding the presence of trend and AR (e.g., Chatfield, 2004) corroborating our diagnostic approach. In the time series plot, the shape of the change pattern can be observed. As shown in the simulation study, ignoring small deviations from the overall trend pattern (i.e., the linear pattern) did not lead to biased results for the experimental condition effects.

	Trend and $AR(1)$	Trend only	$AR(1)$ only
Time Series Plot (x-axis: Time)			
Autocorrelation (x-axis: Lag)			
Partial Autocorrelation (x-axis: Lag)			

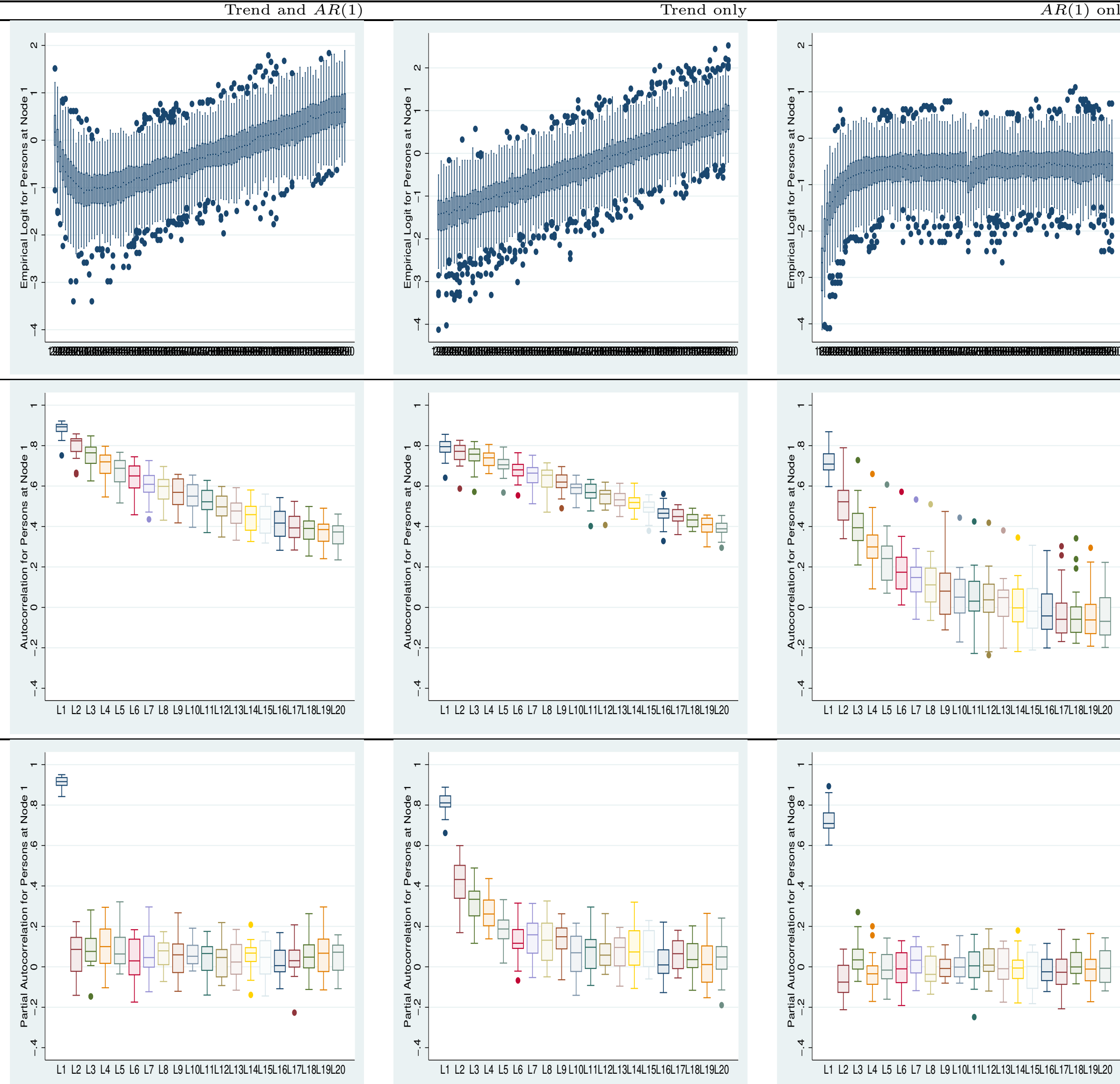


Figure A.6 Patterns of trends, autocorrelations, and partial autocorrelations.

Comparability between Laplace Approximation and Bayesian Analysis for the Dynamic IRTree Model

In this section, we provide comparability of estimates and statistical inference between Laplace approximation implemented in the `glmer` function and Bayesian analysis using `Stan` (Carpenter et al., 2017).

Bayesian analysis. The `rStan` (an `R` package that interfaces with `Stan` in `R`) is recently developed software implementing the no-U-turn sampler (Hoffman & Gelman, 2014), which is an extension to the Hamiltonian Monte Carlo (HMC; Neal, 2011) algorithm. Prior and hyper-prior distributions were specified in `rStan` as follows:

$$\begin{aligned}
\lambda_r &\sim N(0, 1, 000), \\
\zeta_r &\sim N(0, 1, 000), \\
\gamma_r &\sim N(0, 1, 000), \\
\Sigma_{1(1\times1)} &\sim Cauchy(0, 5), \\
\Sigma_{2(4\times4)} &\sim Inverse-Wishart(4, I_4),
\end{aligned}$$

and

$$\Sigma_{3(2\times2)} \sim Inverse-Wishart(2, I_2).$$

In the inverse-Wishart distributions, I_D indicates the unit matrix of size D and the degrees of freedom ν is set to D as the rank of the random effects to represent vague prior knowledge. `Stan` code for Model 2 is written as follows:

```
data {
  int R;  // number of observations
  int T; // number of trials
  int J; // number of persons
  int I; // number of items

  int trialnum[R];  // trial indicator
  int subject[R];   // person indicator
```

```
  int item1[R];     // item indicator
  int node1[R];     // Node 1 indicator
  int node2[R];     // Node 2 indicator

  real privileged1[R];   // independent variable
  real contrast[R];      // independent variable
  real ctime1[R];         // independent variable
  int  clag1[R];         // lag
  int<lower=0, upper=1> c[R];  // dependent variable

  vector[4] Zero1;
  matrix[4,4] Omega1;

  vector[2] Zero2;
  matrix[2,2] Omega2;
}

parameters {
//fixed
   vector[2] zeta;
   vector[2] gamma1;
   vector[2] gamma2;
   vector[2] gamma3;
   vector[2] gamma4;

//random
  real delta[T];
  real<lower=0> sigmat;

  vector[4] theta[J];  // [J,4] dim matrix for theta
  cov_matrix[4] Rth;

  vector[2] beta[I];   // [I,2] dim matrix for beta
  cov_matrix[2] Rbe;
}

model {
   //priors
  zeta   ~ normal(0,1000);
  gamma1 ~ normal(0,1000);
  gamma2 ~ normal(0,1000);
  gamma3 ~ normal(0,1000);
  gamma4 ~ normal(0,1000);

  sigmat ~ cauchy(0,5);
  Rth ~  inv_wishart(4, Omega1);
  Rbe ~  inv_wishart(2, Omega2);

  //random effects

  for (t in 1:T) delta[t] ~ normal(0, sigmat);

  for (j in 1:J) {
    theta[j] ~ multi_normal(Zero1, Rth);
  }

  for (i in 1:I){
beta[i] ~ multi_normal(Zero2, Rbe);
  }


for (r in 1:R){
```

```
  c[r]  ~
  bernoulli_logit((
(gamma1[1]+gamma2[1]*clag1[r]+gamma3[1]*privileged1[r]+gamma4[1]*contrast[r]+zeta[1]*ctime1[r])*node1[r]+
(gamma1[2]+gamma2[2]*clag1[r]+gamma3[2]*privileged1[r]+gamma4[2]*contrast[r]+zeta[2]*ctime1[r])*node2[r]+
(                     theta[subject[r],1]+theta[subject[r],3]*clag1[r]+beta[item1[r],1])*node1[r]  +
(delta[trialnum[r]]  +theta[subject[r],2]+theta[subject[r],4]*clag1[r]+beta[item1[r],2])*node2[r]));
  }
}
```

For convergence diagnostics, the potential scale reduction factor (PSRF; Gelman & Rubin, 1992) was considered with two chains, and the PSRF value of 1.01 was used as a threshold to indicate model convergence (Gelman et el., 2014). In the selected model, significance of fixed effects was tested using a 95% highest posterior density (HPD) interval. When the HPD interval did not include 0, the fixed effects were considered significantly different from 0.

Results. 3,000 iterations were run and the first 100 iterations were discarded as a burn-in period. About 172 hours (user time in R) were required on a 2.81GHz computer with 16.0 GB of RAM to obtain the 3,000 iterations with the two chains. As posterior moment, the posterior mean and the standard deviation are reported because the posterior distribution is symmetric. Because Stan output provides results up to the second decimal points, results from glmer rounded up to two decimal points are reported. As shown in Table 1, estimates and statistical inference of fixed effects from Bayesian analysis and Laplace approximation are comparable.

Table A.5 *Comparability of Estimates (Standard Error) [HPD Interval] and Statistical Inference between Bayesian Analysis and Laplace Approximation for Model 2 in Table 4*

	Bayesian		Laplace	
	Node 1	Node 2	Node 1	Node 2
Fixed Effects				
Intercept[γ_1]	**0.10**(0.02)[0.05,0.15]	**1.34**(0.05)[1.23,1.44]	**0.10**(0.02)	**1.31**(0.05)
$AR(1)$ylag[λ_Y]	**4.19**(0.02)[4.16,4.22]	**5.23**(0.03)[5.16,5.29]	**4.18**(0.01)	**5.17**(0.03)
Trend[ζ]	**0.01**(0.00)[0.01,0.01]	**0.03**(0.00)[0.03,0.03]	**0.01**(0.00)	**0.03**(0.00)
Privileged[γ_2]	0.00(0.02)[-0.04,0.04]	0.07(0.05)[-0.02,0.16]	0.01(0.02)	0.07(0.05)
Contrast[γ_3]	**0.05**(0.01)[0.03,0.08]	**-0.38**(0.03)[-0.45,-0.32]	**0.05**(0.01)	**-0.39**(0.03)

	Bayesian				Laplace			
	SD	Corr			SD	Corr		
Random Effects								
Trial (Σ_1)								
Node 1(δ_{lji1})	-				-			
Node 2(δ_{lji2})	0.11				0.09			
Person (Σ_2)								
Node 1:Intercept[θ_{j1}])	0.20				0.17			
Node 2:Intercept[θ_{j2}])	0.30	0.67			0.27	0.71		
Node 1:AR(1)ylag[λ_{1j1}]	0.17	-0.29	-0.20		0.12	-0.36	-0.18	
Node 2:AR(1)ylag[λ_{1j2}]	0.26	-0.19	-0.13	0.90	0.23	-0.14	-0.25	0.93
Item (Σ_3)								
Node 1:Intercept[β_{i1}])	0.15				0.13			
Node 2:Intercept[β_{i2}])	0.41	0.42			0.38	0.41		

Note. - indicates that an effect is not modelled; Values in bold indicates significance at the 5% level for fixed effects.

References

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