Supplementary Material for "Variational Estimation for Multidimensional Generalized Partial Credit Model"

This supplement contains additional material for the article "Variational Estimation for Multidimensional Generalized Partial Credit Model". In Appendix A, we provide the detailed derivation of the pGVEM algorithm. Additional numerical results for Section 4 are presented in Appenix B. In Appendix C, we provide the item information for the datasets in Section 5, and in Appendix D we display the detailed estimation results of the TIMSS dataset.

Appendix A: Derivation of pGVEM for Generalized Partial Credit Model

In this appendix we give the detailed derivation for the pGVEM update Equation (3.6), (3.7) and (3.10)-(3.13) for the Generalized Partial Credit Model.

For the zero step, all item parameters are randomly generated within their domain, denoted by $\boldsymbol{a}_j^{(0)}, b_{jk}^{(0)}$ and $\xi_{ijk}^{(0)}$ for $i = 1, \cdots, N,\ j = 1, \cdot, J$ and $k = 1, \cdot, K_j - 1$. Recall b_{j0} are fixed as 0 for all j. For the t^{th} step of minimizing the KL divergence $KL\{q_i^{(t)}(\boldsymbol{\theta}_i)||P(\boldsymbol{\theta}_i|Y_i, M_p^{(t-1)})\} = KL\{q_i^{(t)}(\boldsymbol{\theta}_i)|P(\boldsymbol{\theta}_i, Y_i|M_p^{(t-1)})\}$ with density $q_i^{(t)}(\boldsymbol{\theta}_i)$ selected from a Gaussian distribution, we can derive its mean and variance extracted from Equation (3.5):

$$\big[\boldsymbol{\Sigma}_i^{(t)}\big]^{-1} = \boldsymbol{\Sigma}_\theta^{-1} + 2\sum_{j=1}^{J}\sum_{k=0}^{K_j-1}\eta\big(\xi_{ijk}^{(t-1)}\big)(k-k_{ij})^2\boldsymbol{a}_j^{(t-1)}\boldsymbol{a}_j^{(t-1)\prime};$$

$$\boldsymbol{\mu}_i^{(t)} = \boldsymbol{\Sigma}_i^{(t)} \times \Big\{\sum_{j=1}^{J}\sum_{k=0}^{K_j-1}(k-k_{ij})\big[2\eta\big(\xi_{ijk}^{(t-1)}\big)(b_{jk}^{(t-1)} - b_{jk_{ij}}^{(t-1)}) - 0.5\big]\boldsymbol{a}_j^{(t-1)\prime}\Big\}.$$

Then we can compute the lower bound

$$\begin{aligned}\underline{E}^{(t)}(M_p,\boldsymbol{\xi}) =& \sum_{i=1}^{N}\sum_{j=1}^{J}\Big\{\log 2 + \sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{k=0}^{K_j-1}\Big[-\eta(\xi_{ijk})(k-k_{ij})^2\boldsymbol{a}_j'(\boldsymbol{\Sigma}_i^{(t)} + (\boldsymbol{\mu}_i^{(t)})(\boldsymbol{\mu}_i^{(t)})')\boldsymbol{a}_j \\ &+ (k-k_{ij})\big(2\eta(\xi_{ijk})(b_{jk}-b_{jk_{ij}}) - 0.5\big)\boldsymbol{a}_j'\boldsymbol{\mu}_i^{(t)} - \eta(\xi_{ijk})(b_{jk}-b_{jk_{ij}})^2 + \frac{1}{2}(b_{jk}-b_{jk_{ij}}) \\ &+ \eta(\xi_{ijk}){\xi_{ijk}}^2 + \frac{1}{2}\xi_{ijk} - \log(1+e^{\xi_{ijk}})\Big]\Big\} \\ &- \frac{N}{2}\log|\boldsymbol{\Sigma}_\theta^{(t)}| - \sum_{i=1}^{N}\frac{1}{2}Tr\Big\{(\boldsymbol{\Sigma}_\theta^{(t)})^{-1}\big[\boldsymbol{\Sigma}_i^{(t)} + (\boldsymbol{\mu}_i^{(t)})\big(\boldsymbol{\mu}_i^{(t)}\big)'\big]\Big\}.\end{aligned}$$

The cross term makes a simultaneous maximization rather difficult, therefore we instead adopt a Gauss-Seidel approach. First by

$$\begin{aligned}\frac{\partial\underline{E}^{(t)}(M_p,\boldsymbol{\xi})}{\partial\boldsymbol{a}_j} =& \sum_{i=1}^{N}\sum_{k=0}^{K_j-1}\Big\{-2\eta\big(\xi_{ijk}^{(t-1)}\big)(k-k_{ij})^2\big[\boldsymbol{\Sigma}_i^{(t)} + (\boldsymbol{\mu}_i)^{(t)}\big(\boldsymbol{\mu}_i^{(t)}\big)'\big]\boldsymbol{a}_j \\ &+ (k-k_{ij})\big[2\eta\big(\xi_{ijk}^{(t-1)}\big)(b_{jk}^{(t-1)} - b_{jk_{ij}^{(t-1)}}) - 0.5\big]\boldsymbol{\mu}_i^{(t)}\Big\},\end{aligned}$$

therefore by plugging in the value of b_{jk} and ξ_{ijk} from previous iteration, we have

$$\begin{aligned}\boldsymbol{a}_j^{(t)} =& \frac{1}{2}\Big[\sum_{i=1}^{N}\sum_{k=0}^{K_j-1}\eta\big(\xi_{ijk}^{(t-1)}\big)(k-k_{ij})^2\big(\boldsymbol{\Sigma}_i^{(t)} + (\boldsymbol{\mu}_i^{(t)})\big(\boldsymbol{\mu}_i^{(t)}\big)'\big)\Big]^{-1} \\ &\Big[\sum_{i=1}^{N}\sum_{k=0}^{K_j-1}(k-k_{ij})\big(2\eta\big(\xi_{ijk}^{(t-1)}\big)(b_{jk}^{(t-1)} - b_{jk_{ij}}^{(t-1)}) - 0.5\big)\boldsymbol{\mu}_i^{(t)}\Big].\end{aligned}$$

For the threshold parameters, the derivatives are given as

$$\frac{\partial \underline{E}^{(t)}(M_p, \boldsymbol{\xi})}{\partial b_{jk}} = \sum_{i=1}^{N} \Big[-2\eta\big(\xi_{ijk}^{(t-1)}\big) b_{jk} + 2\eta\big(\xi_{ijk}^{(t-1)}\big)(k - k_{ij}) \boldsymbol{a}_j^{(t)'} \boldsymbol{\mu}_i^{(t)} + 2\eta\big(\xi_{ijk}^{(t-1)}\big) b_{jk_{ij}}^{(t-1)} + \frac{1}{2}\Big]$$
$$+ \sum_{i=1}^{N} \sum_{v=0}^{K_j-1} \Big[I_{(k=k_{ij})}(-2\eta(\xi_{ijv}^{(t-1)}) - 2\eta(\xi_{ijv}^{(t-1)})(v - k_{ij}) \boldsymbol{a}_j^{(t)'} \boldsymbol{\mu}_i^{(t)} + 2\eta\big(\xi_{ijk}^{(t-1)}\big) b_{jv}^{(t-1)} - \frac{1}{2}\Big],$$

and similarly we have update for $b_{jk}^{(t)}$:

$$b_{jk}^{(t)} = \frac{\sum_{i=1}^{N} \big[\mathrm{B}_1^{(t)}(i,j,k) I_{(k \neq k_{ij})} + I_{(k=k_{ij})} \sum_{v=0, v\neq k}^{K_j-1} \mathrm{B}_2^{(t)}(i,j,v,k)\big]}{2\sum_{i=1}^{N} \big(\eta\big(\xi_{ijk}^{(t-1)}\big) I_{(k\neq k_{ij})} + I_{(k=k_{ij})} \sum_{v=0, v\neq k}^{K_j-1} \eta\big(\xi_{ijv}^{(t-1)}\big)\big)}$$

with

$$\mathrm{B}_1^{(t)}(i,j,k) = 2\eta\big(\xi_{ijk}^{(t-1)}\big)(k - k_{ij}) \boldsymbol{a}_j^{(t)'} \boldsymbol{\mu}_i^{(t)} + 0.5 + 2\eta\big(\xi_{ijk}^{(t-1)}\big);$$
$$\mathrm{B}_2^{(t)}(i,j,v,k) = -2\eta\big(\xi_{ijk}^{(t-1)}\big)(v - k) \boldsymbol{a}_j^{(t)'} \boldsymbol{\mu}_i^{(t)} - 0.5 + 2\eta\big(\xi_{ijv}^{(t-1)}\big).$$

For the variational parameter, notice that the function

$$g(x) = \eta(x)(x^2 - c) + \frac{1}{2}x - \log(1 + e^x)$$

is maximized for $x^2 = \max\{c, 0\}$, so the update for ξ_{ijk} is

$$\xi_{ijk}^{(t)\,2} = \Big[(k - k_{ij}) \boldsymbol{a}_j^{(t)'} \boldsymbol{\mu}_i^{(t)} - \big(b_{jk}^{(t)} - b_{jk_{ij}}^{(t)}\big)\Big]^2 + (k - k_{ij})^2 \boldsymbol{a}_j^{(t)'} \boldsymbol{\Sigma}_i^{(t)} \boldsymbol{a}_j^{(t)}.$$

The derivative with respect to $\boldsymbol{\Sigma}_\theta^{-1}$ is

$$\frac{\partial \underline{E}^{(t)}(M_p, \boldsymbol{\xi})}{\partial \boldsymbol{\Sigma}_\theta^{-1}} = \frac{N}{2} \boldsymbol{\Sigma}_\theta - \frac{1}{2} \sum_{i=1}^{N} \Big[\boldsymbol{\Sigma}_i^{(t)} + (\boldsymbol{\mu}_i^{(t)})\big(\boldsymbol{\mu}_i^{(t)}\big)'\Big]$$

Therefore the update when covariance is given by

$$\boldsymbol{\Sigma}_\theta = \frac{1}{N} \sum_{i=1}^{N} \Big[\boldsymbol{\Sigma}_i^{(t)} + (\boldsymbol{\mu}_i^{(t)})\big(\boldsymbol{\mu}_i^{(t)}\big)'\Big].$$

Note again that the update for $\boldsymbol{\Sigma}_\theta$ is only performed in confirmatory factor analysis and fixed as identity for exploratory factor analysis.

Appendix B: Additional Numerical Results

Recovery of the latent ability

In this subsection, we present the result of recovery of the latent ability. In pGVEM framework, we can only obtain the estimated correlation of the latent factors along with the variational distribution of the factor for each individuals. Here we used the mean of each distribution $\hat{q}_i^{(t)}(\boldsymbol{\theta}_i)$ (i.e., see Equation (3.6)) as point estimates of the latent ability to compute the estimated item response functions, i.e., the probability of presence of each partial credit score. Because $\hat{q}_i^{(t)}(\boldsymbol{\theta}_i)$ is the minimizer of the KL divergence $KL\{q_i^{(t)}(\boldsymbol{\theta}_i)||\Pr(\boldsymbol{Y}_i|M_p, \boldsymbol{\theta}_i)\}$ selected from the Gaussian family, *it is reasonable to use the mean value of* $\hat{q}_i^{(t)}(\boldsymbol{\theta}_i)$ *as an approximation to the latent ability.* And for the methods implemented in 'mirt' package, we use the built-in function to obtain the estimations. The results are presented in Figure B1 and B2.

Comparison with GVEM

In this subsection we compare the results between GVEM and pGVEM. These two algorithms are similar in their derivation but differ in the detailed implementation. We present the comparison here to demonstrate that our proposed pGVEM is still efficient in estimation of M2PL model with GVEM as benchmark. The following Figures B3 and B4 provide comparison of GVEM and pGVEM for 2-category MGPCM (i.e., MGPCM with binary responses). Here the discrimination parameters are simulated from $Unif(1, 2)$. For the convergence criterion of GVEM, we stop the

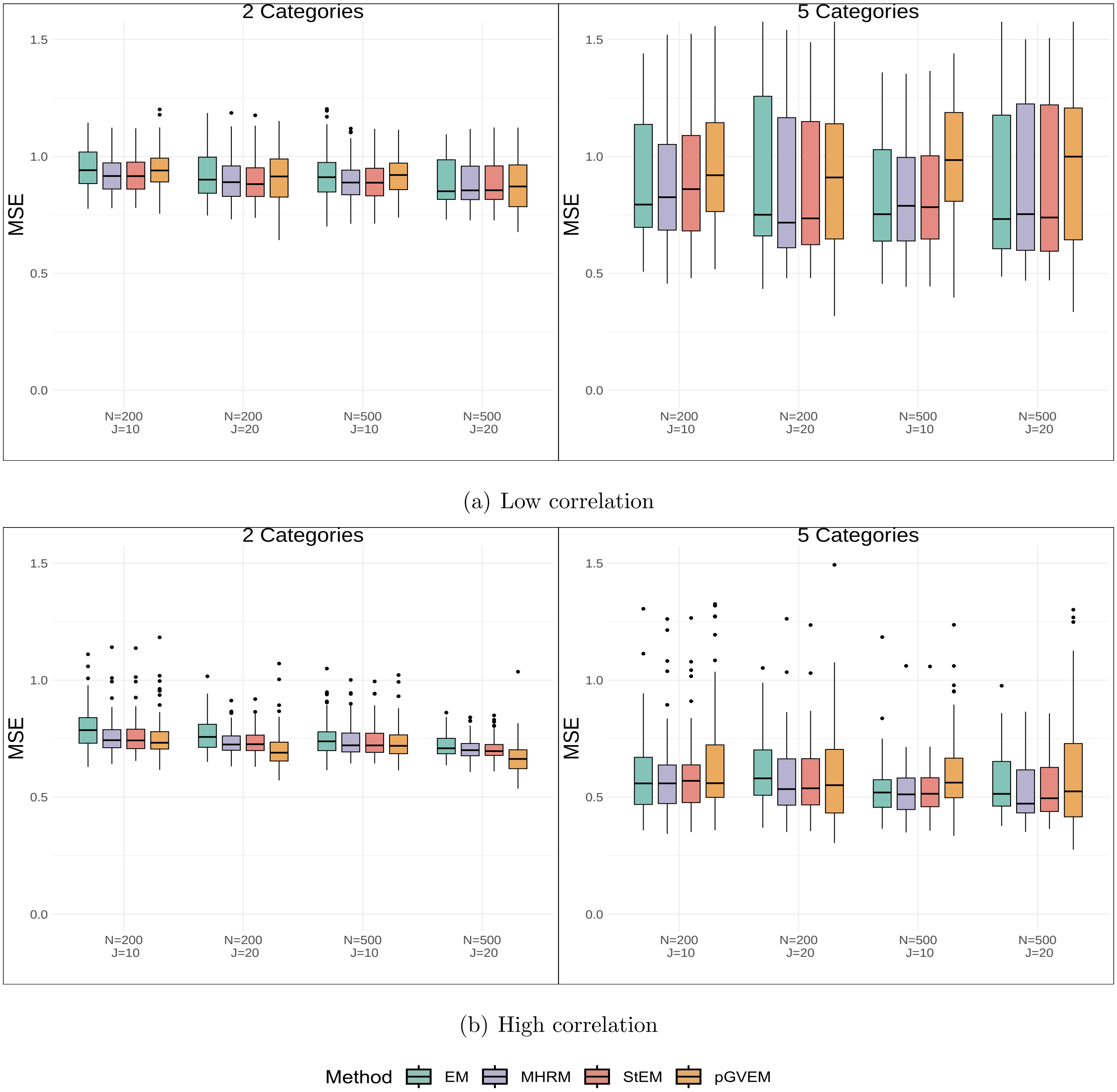
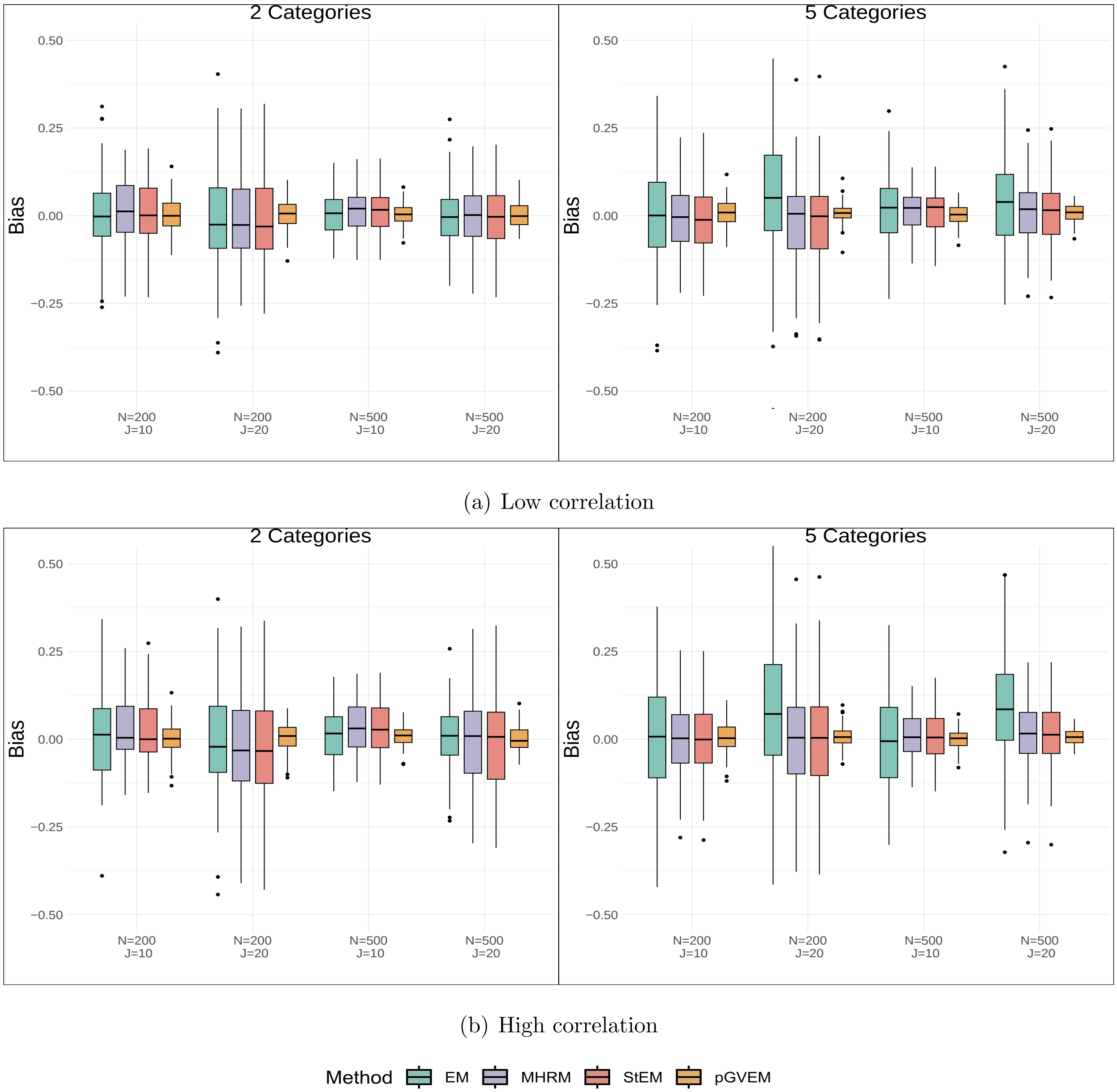


Figure B1: Mean Squared Error of the $\boldsymbol{\theta}$ estimation for the Partial Credit Model from exploratory factor analysis using different methods.



(a) Low correlation

(b) High correlation

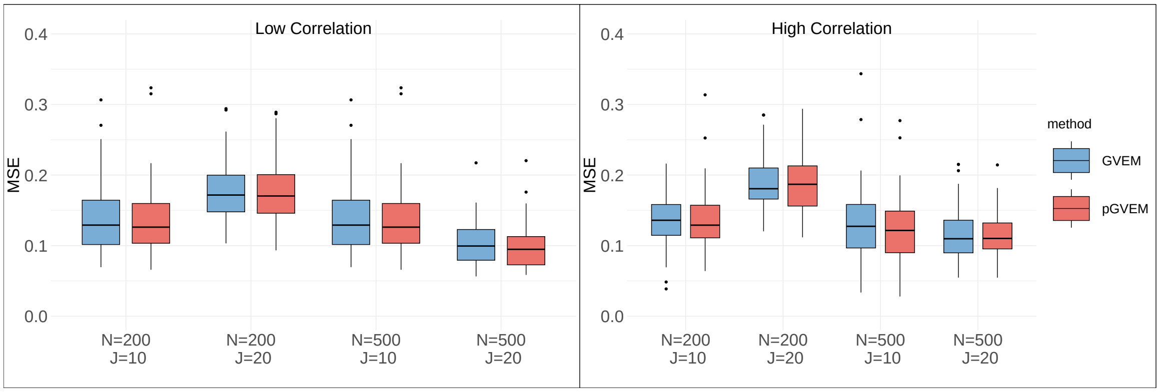
Figure B2: Bias of the $\boldsymbol{\theta}$ estimation for the Partial Credit Model from exploratory factor analysis using different methods.

iteration when the average change of all parameters falls below a threshold:

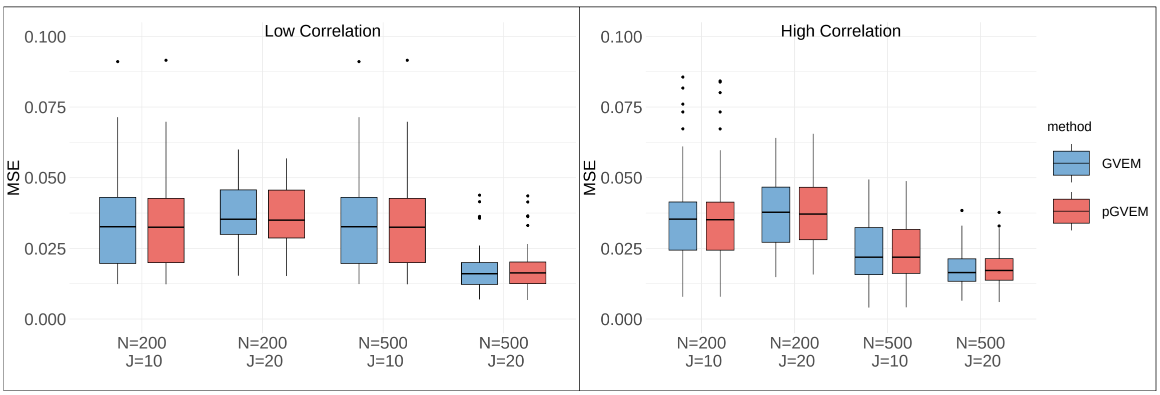
$$\frac{1}{J \times K + J} \sum_{j=1}^{J} \left(\|\hat{\boldsymbol{a}}_j - \boldsymbol{a}_j\|^2 + (\hat{b}_j - b_j)^2 \right) < 1.0^{-5}$$

Full Version of Figure in Simulation Study I

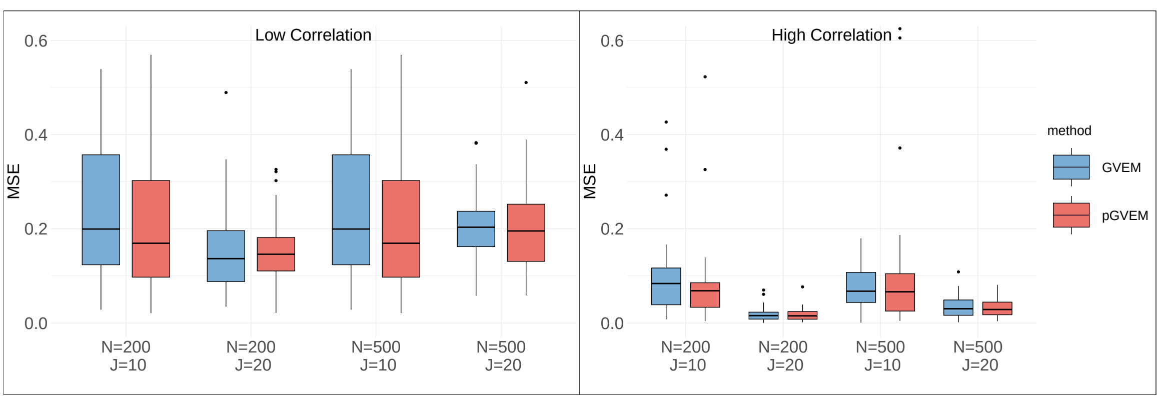
This section is the full version of Figures presented in Simulation Study I. Specifically, we present the MSE for the case of $N = 200$ in Figure 1 and 2 in Figure B5 and the results of MSE and bias in the setting of large loading in Figure B6-B9



(a) MSE of discrimination parameter

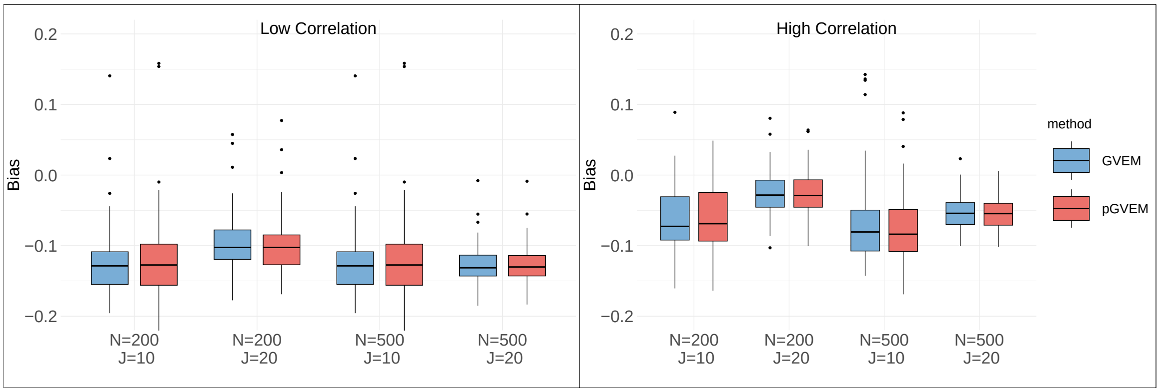


(b) MSE of difficulty parameter

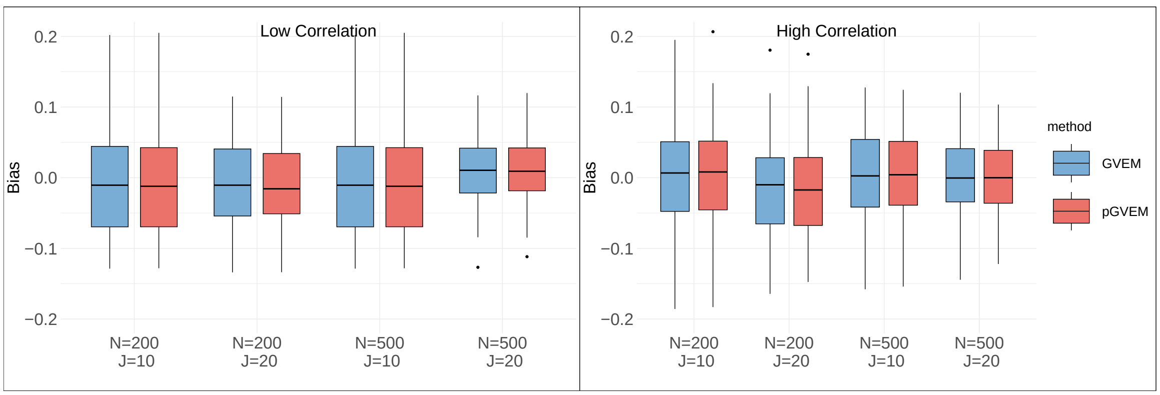


(c) MSE of correlation

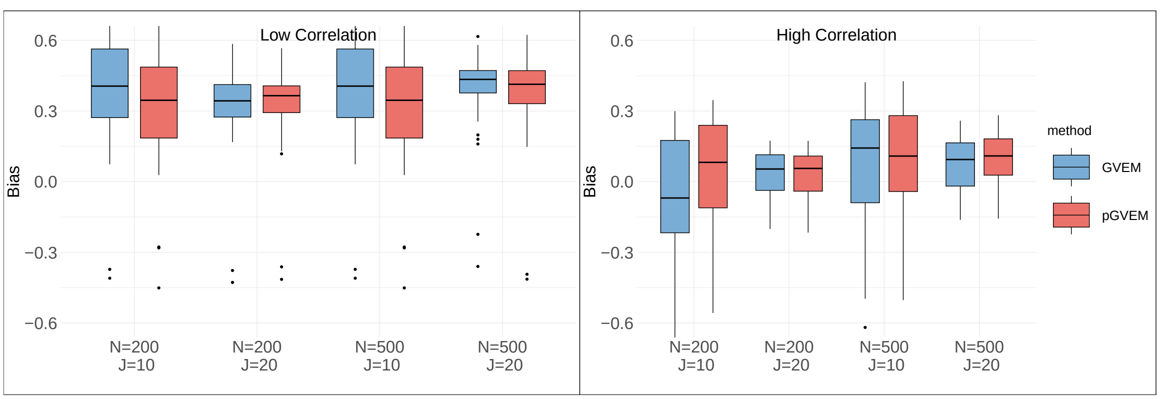
Figure B3: Mean Squared Error of estimation for the M2PL Model from exploratory factor analysis using GVEM and pGVEM method.



(a) Bias of discrimination parameter



(b) Bias of difficulty parameter



(c) Bias of correlation

Figure B4: Bias of estimation for the M2PL Model from exploratory factor analysis using GVEM and pGVEM method.

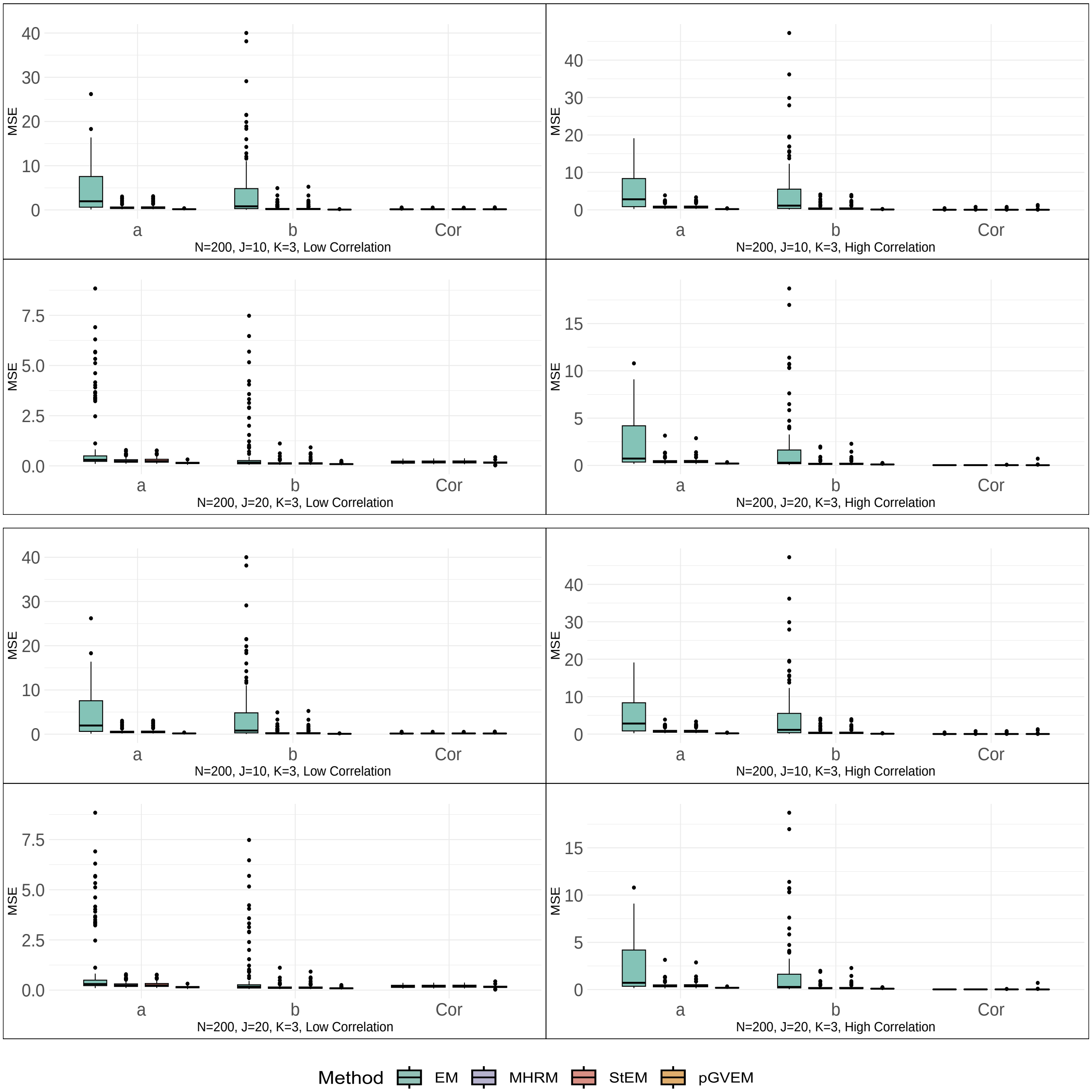


Figure B5: Full version of Figure 1 and 2 for $N = 200$.

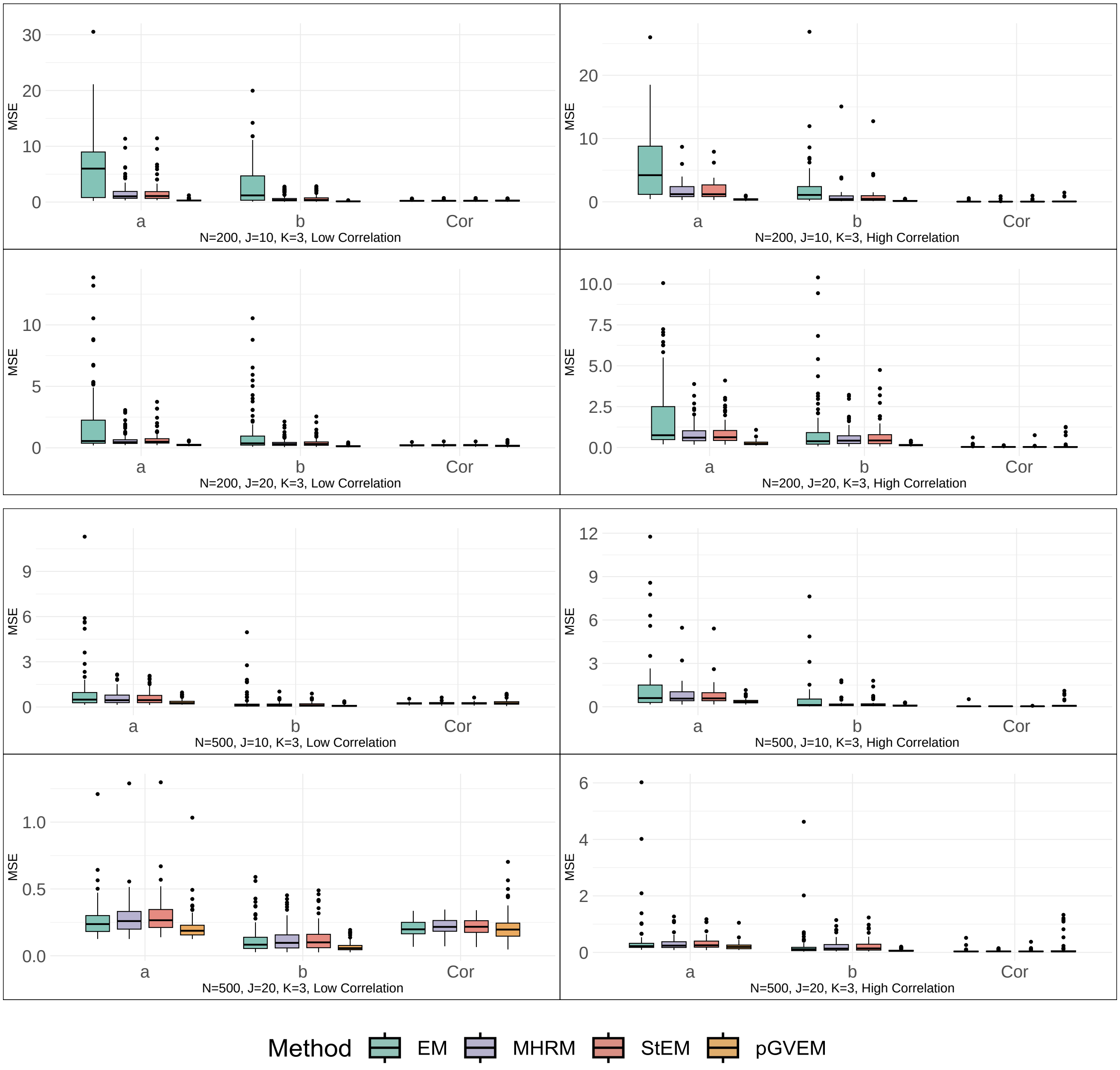


Figure B6: Full version of Figure 5.

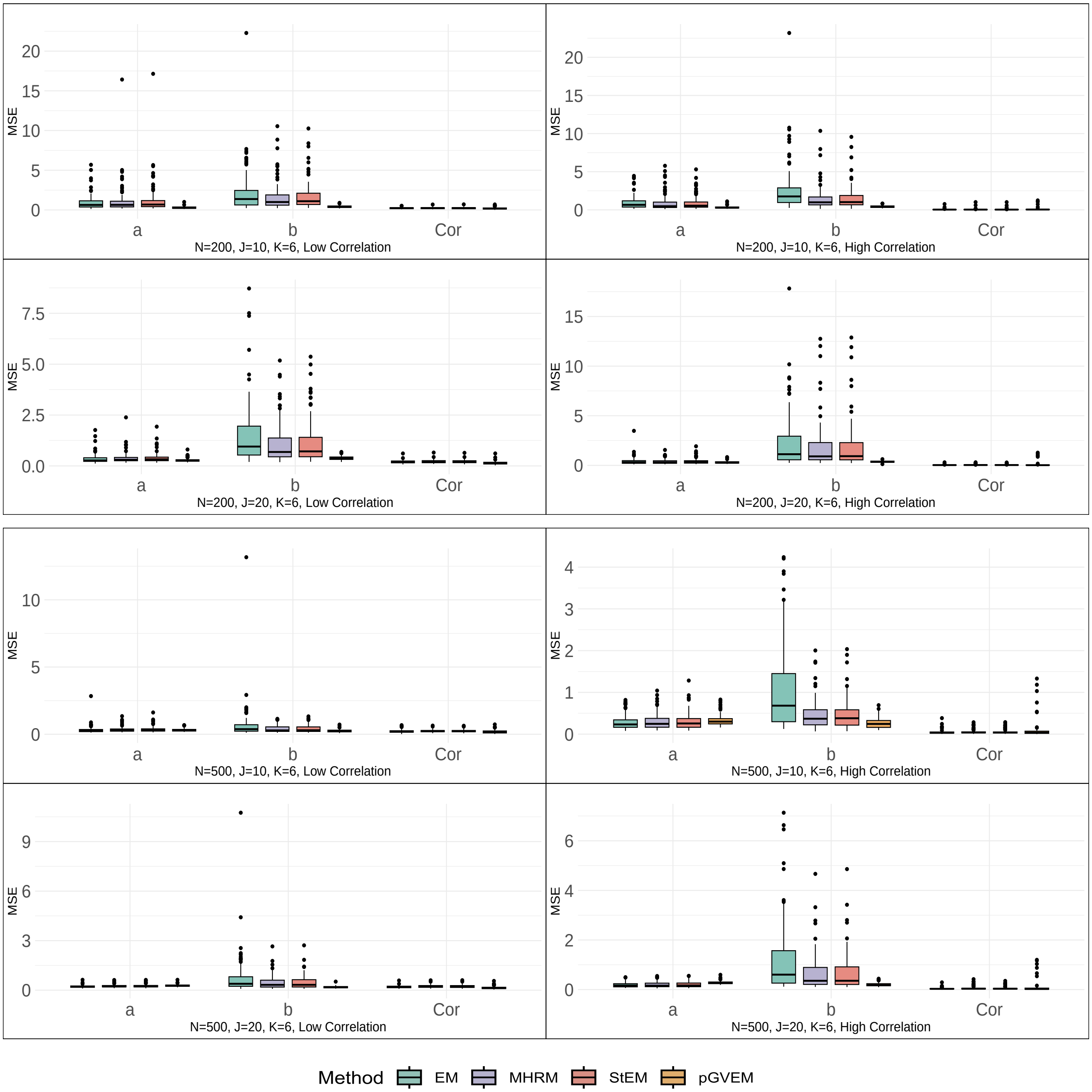


Figure B7: Full version of Figure 6.

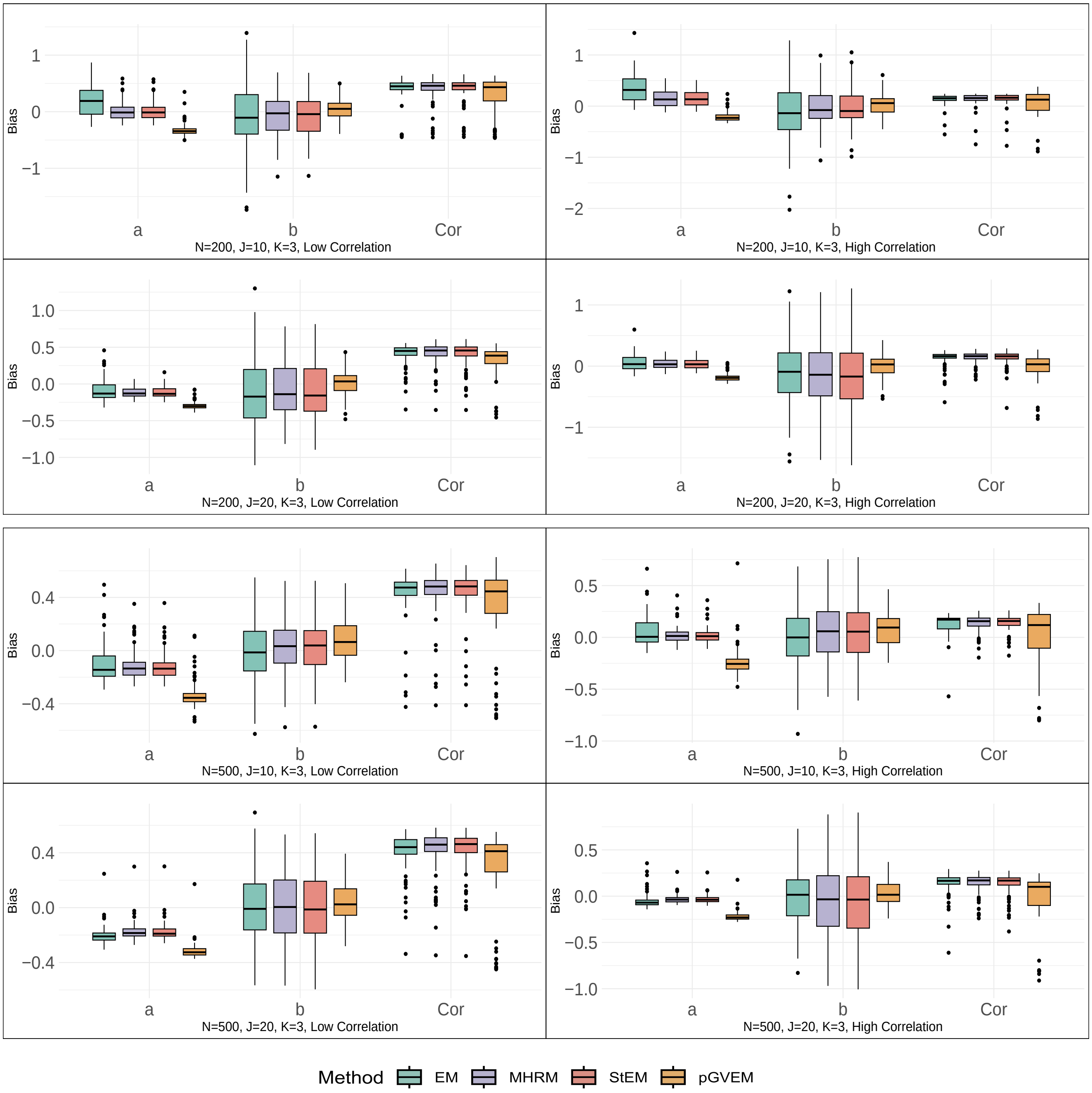


Figure B8: Full version of Figure 7.

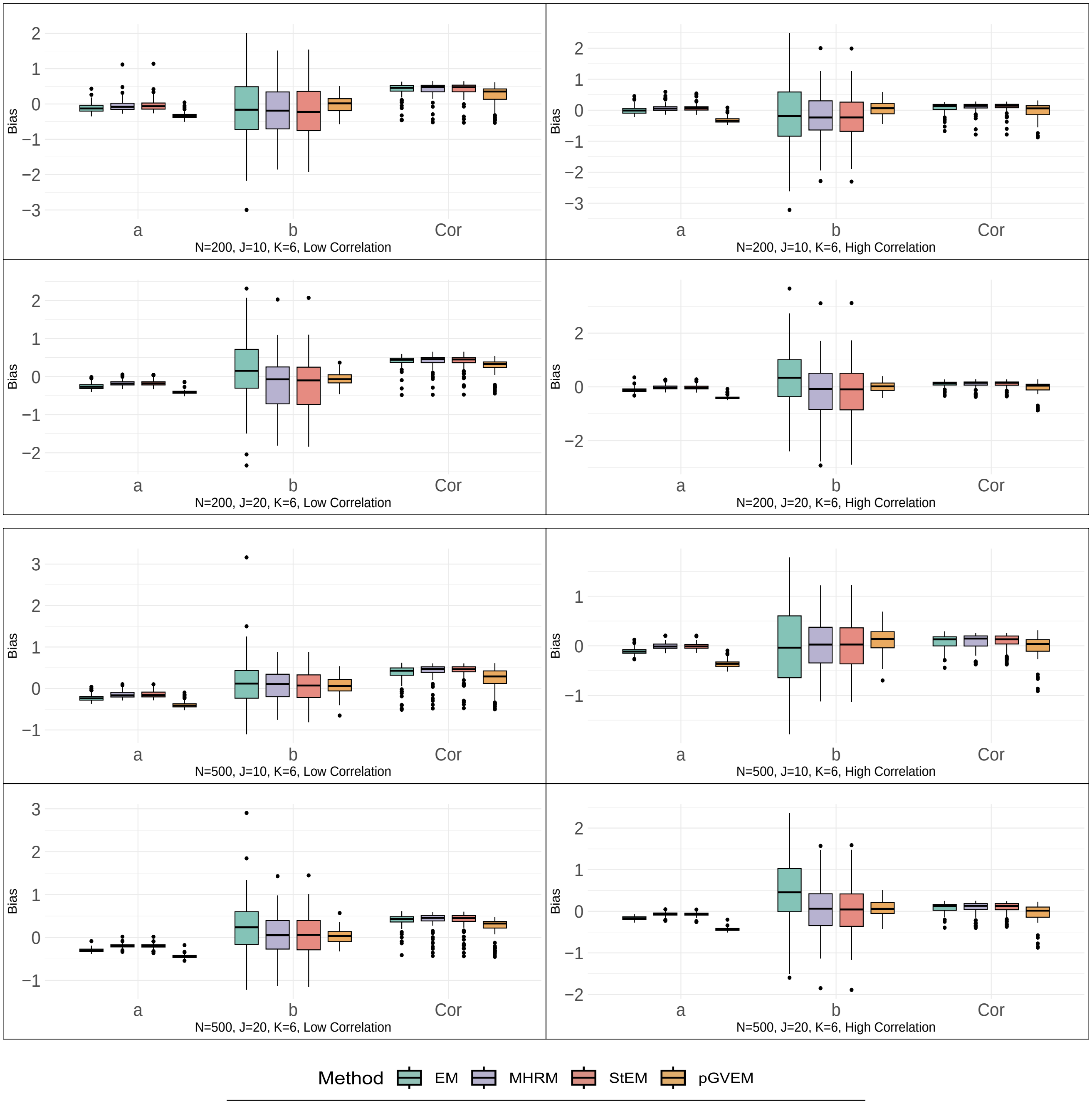


Figure B9: Full version of Figure 8.

Appendix C: Test Items of the Datasets

The following tables include the item codes in the TIMSS dataset and the questions. In the bracket is category information for the items. For details, see Martin and Mullis (2019).

mathematics test items and their code in TIMSS database	
Code	Questions
me62150	DIFFERENCE BETWEEN LOW TEMPERATURE IN CITY X AND Y (1)
me62335	SELECT EQUIVALENT RATIO TO 3:2 (B)
me62219	KATY ENLARGES A PHOTO - NEW HEIGHT (A)
me62002	FILL IN BOXES TO MAKE THE SMALLEST PRODUCT (1)
me62149	IDENTIFY EXPRESSION TO CALCULATE ROBIN'S EARNINGS (D)
me62241	ROY'S PHONE BUSINESS - EQUATION FOR Y (1)
me62105	AREA OF RECTANGLE WITH SIDES X AND 2X + 1 (2)
me52040	ESTIMATE AREA OF IRREGULAR SHAPE ON 1 CM GRID (C)
me62288	FIND VERTICES OF TRAPEZOIDS M AND N (DERIVED) (2)
me62173	FIND ANGLE X ON A FOLDED PIECE OF PAPER (1)
me62133	BLACK AND WHITE MARBLES IN A BAG WITH REPLACEMENT (D)
me62123a	RELAY RACE - MEAN TIME OF RUNNERS (C)
me62123b	RELAY RACE - MEAN TIME WHEN 2 RUNNERS IMPROVE (B)

science test items and their code in TIMSS database	
Code	Questions
se62090	WATER CYCLE IN FOREST ECOSYSTEM (C)
se62274	RAW MATERIALS FOR PHOTOSYNTHESIS (2)
se62284	HAIR COLOR OF YOUNG RABBITS (B)
se62098B	PLANT AND ANIMAL CELLS DIFFERENT (2)
se62098A	PLANT AND ANIMAL CELLS SIMILAR (2)
se62032	HOT METAL BALL ON BALANCE (C)
se62043	ELECTROMAGNET AND PAPER CLIPS (1)
se62158	GRAPHS OF MUSICAL NOTES (B)
se62159	BOX PULLED BY THREE FORCES (D)
se62004	BLOCK POUNDED INTO FLAT SHEET (B)
se62075	HUGO'S CHEMICAL REACTION (D)
se62004	BLOCK POUNDED INTO FLAT SHEET (B)
se62175	POWER PLANT GEOGRAPHIC FACTOR (1)
se62173A	TEMPERATURE AND GEOGRAPHY (DERIVED) (1)
se62173B	TEMPERATURE AND GEOGRAPHY: OCTOBER (B)

The following tables include the item codes in the Big-Five dataset and the corresponding questions.

Items of Extraversion and their code in Big-Five dataset	
Code	Questions
E1	I am the life of the party.
E2	I don't talk a lot.
E3	I feel comfortable around people.
E4	I keep in the background.
E5	I start conversations.
E6	I have little to say.
E7	I talk to a lot of different people at parties.
E8	I don't like to draw attention to myself.
E9	I don't mind being the center of attention.
E10	I am quiet around strangers.

Items of Neuroticism and their code in Big-Five dataset	
Code	Questions
N1	I get stressed out easily.
N2	I am relaxed most of the time.
N3	I worry about things.
N4	I seldom feel blue.
N5	I am easily disturbed.
N6	I get upset easily.
N7	I change my mood a lot.
N8	I have frequent mood swings.
N9	I get irritated easily.
N10	I often feel blue.

Items of Agreeableness and their code in Big-Five dataset	
Code	Questions
A1	I feel little concern for others.
A2	I am interested in people.
A3	I insult people.
A4	I sympathize with others' feelings.
A5	I am not interested in other people's problems.
A6	I have a soft heart.
A7	I am not really interested in others.
A8	I take time out for others.
A9	I feel others' emotions.
A10	I make people feel at ease.

Items of Conscientiousness and their code in Big-Five dataset	
Code	Questions
C1	I am always prepared.
C2	I leave my belongings around.
C3	I pay attention to details.
C4	I make a mess of things.
C5	I get chores done right away.
C6	I often forget to put things back in their proper place.
C7	I like order.
C8	I shirk my duties.
C9	I follow a schedule.
C10	I am exacting in my work.

Items of Openness and their code in Big-Five dataset	
Code	Questions
O1	I have a rich vocabulary.
O2	I have difficulty understanding abstract ideas.
O3	I have a vivid imagination.
O4	I am not interested in abstract ideas.
O5	I have excellent ideas.
O6	I do not have a good imagination.
O7	I am quick to understand things.
O8	I use difficult words.
O9	I spend time reflecting on things.
O10	I am full of ideas.

Appendix D: Detailed Report on Parameter Estimation of the TIMSS Datset

Item	Slope a_j	Location b_j	Guessing c_j	Step 1 D_{j1}	Step 2 D_{j2}
me62150	1.111	-0.193			
me62335	1.377	0.004	0.175		
me62219	2.050	0.961	0.218		
me62002	0.447	0.846			
me62149	1.089	0.617	0.111		
me62241	1.708	0.743			
me62105	0.757	0.960		-1.718	1.718
me62040	0.769	1.057	0.224		
me62288	0.776	1.250		-0.880	0.880
me62173	1.119	0.922			
me62133	1.315	0.726	0.214		
me62123A	1.562	0.464	0.306		
me62123B	1.444	0.814	0.138		
se62090	1.011	0.180	0.304		
se62274	0.577	0.879		1.149	-1.149
se62284	0.375	0.478	0.172		
se62098A	0.639	0.500		-0.050	0.050
se62098B	0.798	1.337		-0.091	0.091
se62032	1.742	1.504	0.287		
se62043	0.907	0.981			
se62158	0.679	0.678	0.299		
se62159	0.983	0.400	0.204		
se62005	1.250	0.666			
se62075	0.990	0.770	0.314		
se62004	1.806	0.885	0.173		
se62175	0.739	0.674			
se62173A	0.647	0.253			
se62173B	0.808	1.862	0.393		

Table D1: IRT parameters from eTIMSS Adjusted Model Calibration provided in official TIMSS file of IRT item parameter downloaded from TIMSS 2019 International Database

Item	Slope a_j	Location b_{j1}	Location b_{j1}
me62150	1.603	-1.306	
me62335	2.010	-2.089	
me62219	1.647	0.514	
me62002	0.705	0.181	
me62149	1.453	0.271	
me62241	2.587	0.393	
me62105	1.183	2.192	1.539
me62040	0.448	0.109	
me62288	1.108	1.712	1.786
me62173	1.379	1.944	
me62133	1.370	-0.400	
me62123A	1.477	-0.770	
me62123B	1.400	0.231	
se62090	0.839	-1.409	
se62274	0.646	-1.063	1.164
se62284	0.777	-1.018	
se62098A	0.754	-0.962	-0.575
se62098B	0.883	0.748	2.055
se62032	0.702	0.476	
se62043	0.947	1.767	
se62158	0.482	-0.649	
se62159	1.150	-0.499	
se62005	1.466	0.923	
se62075	0.024	1.411	
se62004	1.512	-0.352	
se62175	0.440	0.298	
se62173A	1.036	-0.146	
se62173B	0.677	0.906	

Table D2: IRT parameters Estimated from EM

Item	Slope a_j	Location b_{j1}	Location b_{j1}
me62150	1.465	-1.258	
me62335	1.735	-1.927	
me62219	1.455	0.477	
me62002	0.702	0.177	
me62149	1.317	0.251	
me62241	1.986	0.328	
me62105	1.032	2.183	1.493
me62040	0.454	0.107	
me62288	0.993	1.656	1.699
me62173	1.241	1.856	
me62133	1.281	-0.398	
me62123A	1.385	-0.758	
me62123B	1.302	0.215	
se62090	0.754	-1.363	
se62274	0.706	-1.078	1.193
se62284	0.738	-0.997	
se62098A	1.041	-1.178	-0.673
se62098B	1.002	0.696	2.169
se62032	0.613	0.460	
se62043	0.749	1.660	
se62158	0.493	-0.649	
se62159	0.923	-0.462	
se62005	1.065	0.797	
se62075	0.051	1.411	
se62004	1.135	-0.312	
se62175	0.433	0.295	
se62173A	0.904	-0.142	
se62173B	0.599	0.880	

Table D3: IRT parameters Estimated from pGVEM

Item	Slope $a_j^{(1)}$	Slope $a_j^{(2)}$	Location b_{j1}	Location b_{j1}
me62150	1.449	0.222	-1.307	
me62335	2.033	0.005	-2.122	
me62219	1.598	0.052	0.510	
me62002	0.704	-0.61	0.179	
me62149	1.649	-0.385	0.265	
me62241	2.432	0.260	0.397	
me62105	1.160	-0.117	2.193	1.472
me62040	0.530	-0.137	0.108	
me62288	1.105	0.127	1.714	1.866
me62173	1.595	-0.357	1.977	
me62133	1.324	0.087	-0.406	
me62123A	1.435	-0.075	-0.753	
me62123B	1.312	0.087	0.227	
se62090	0.438	0.505	-1.404	
se62274	0.551	0.186	-1.073	1.174
se62284	0.438	0.497	-1.031	
se62098A	0.102	1.058	-1.204	-0.677
se62098B	0.022	1.842	0.773	3.153
se62032	0.626	0.055	0.468	
se62043	0.451	0.444	1.693	
se62158	0.071	0.438	-0.649	
se62159	0.996	0.216	-0.508	
se62005	1.252	0.221	0.890	
se62075	-0.093	0.404	1.450	
se62004	0.988	0.325	-0.330	
se62175	0.109	0.380	0.298	
se62173A	1.140	0.100	-0.169	
se62173B	0.697	0.019	0.910	

Table D4: IRT parameters Jointly Estimated from EM

Item	Slope $a_j^{(1)}$	Slope $a_j^{(2)}$	Location b_{j1}	Location b_{j1}
me62150	1.377	0.204	-1.324	
me62335	1.802	0.051	-2.047	
me62219	1.479	0.034	0.432	
me62002	0.680	-0.026	0.154	
me62149	1.451	-0.251	0.206	
me62241	1.984	0.269	0.273	
me62105	1.013	0.012	2.181	1.399
me62040	0.557	-0.170	0.093	
me62288	1.045	0.107	1.627	1.735
me62173	1.371	-0.182	1.827	
me62133	1.220	0.153	-0.446	
me62123A	1.336	-0.021	-0.783	
me62123B	1.213	0.133	0.173	
se62090	0.479	0.413	-1.410	
se62274	0.580	0.186	-1.097	1.172
se62284	0.0491	0.405	-1.046	
se62098A	-0.07	1.455	-1.510	-0.881
se62098B	0.169	1.104	0.649	2.350
se62032	0.628	0.059	0.442	
se62043	0.489	0.345	1.639	
se62158	0.088	0.414	-0.658	
se62159	1.068	0.074	-0.545	
se62005	1.220	0.142	0.814	
se62075	-0.296	0.303	1.431	
se62004	1.047	0.201	-0.372	
se62175	0.155	0.305	0.281	
se62173A	1.105	0.094	-0.210	
se62173B	0.683	0.034	0.879	

Table D5: IRT parameters Jointly Estimated from GVEM

References

Martin, M. O. and Mullis, I. V. (2019). TIMSS 2015: Illustrating advancements in large-scale international assessments. *Journal of Educational and Behavioral Statistics*, 44(6):752–781.