Supplementary Material to
"Restricted Recalibration of Item Response Theory Models"

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In this supplementary document, we illustrate how to calculate adjusted standard errors (SEs) for focal parameters and perform goodness-of-fit (GOF) assessment using the R code (`RR-GRM.R`) enclosed in the zip archive. The code can accommodate the graded response model (GRM), which subsumes the two-parameter logistic model as a special case; it does assume that all items have the same number of categories.

1 Data and Parameter Estimates

The example data were generated under the null condition of the simulation study (see Section 3 of the paper for details). The previous sample ($n' = 1000$) was used to calibrate items 1–9. The estimated item slopes and intercepts can be found in the M*plus* (Muthén & Muthén, 1998–2017) output file `original.out` and are imported to R (R Core Team, 2018) as[1]

```
alpha <- c(0.318, 0.177, 0.585, -1.100, 0.572, -2.331, 0.014, -1.001, 0.252)
beta <- c(1.222, 1.634, 1.644, 1.791, 1.890, 1.575, 0.740, 0.747, 1.688)
```

Restricted recalibration (RR) was conducted using the current sample ($n = 1000$) to estimate item 10's parameters as well as the latent variable (LV) mean and variance conditional on the estimates obtained in the original fitting. The RR results can be found in the M*plus* output file `updated.out`. In particular, the respective estimates of the LV mean and standard deviation (SD) are

```
mu <- -1.178
sigma <- sqrt(1.694)
```

The estimated slope and intercept parameters for item 10 are also appended to the previously defined R vectors `beta` and `alpha`

```
alpha <- c(alpha, -0.293)
beta <- c(beta, 0.982)
```

[1]Note that the threshold parameters in M*plus* are minus intercepts.

2 Adjusted Standard Errors for Focal Parameters

Upon loading the source code (`RR-GRM.R`), we can calculate the expected Fisher information matrices using the function `calc.info`:

```
source("RR-GRM.R")
info.xi <- calc.info(NULL, matrix(alpha[-10]), beta[-10], 0, 1, T, F)
omega.xi <- solve(info.xi)
info <- calc.info(NULL, matrix(alpha), beta, mu, sigma, T, T)
```

in which `info.xi` is the information matrix of nuisance parameters based on the original calibration, `omega.xi` is the corresponding ACM, and `info` is the information matrix for all parameters based on the recalibration. The intercept vector `alpha` must be converted to a one-column matrix before it is passed to `calc.info`. Also note that the order of item parameters in the information matrix is[2]

```
beta[1], ..., beta[10], alpha[1], ..., alpha[10], mu, sigma^2
```

The adjusted asymptotic covariance matrix (ACM) for focal parameters can be computed by Equation 7 in the paper:

```
n0 <- 1000
n1 <- 1000
c <- n1 / n0
fo <- c(10, 20, 21, 22)
info.eta.inv <- solve(info[fo, fo])
A <- info.eta.inv %*% info[fo, -fo]
omega.eta <- info.eta.inv + tcrossprod(c * A %*% omega.xi, A)
omega.eta
#           [,1]      [,2]      [,3]      [,4]
# [1,] 11.420886  5.895445  2.334796 -7.949920
# [2,]  5.895445 12.069071 -1.522955 -1.422745
# [3,]  2.334796 -1.522955  6.159625 -8.419250
# [4,] -7.949920 -1.422745 -8.419250 34.596693
```

in which `c` is the sample size ratio, `fo` collects the indices for focal parameters, `info.eta.inv` is the inverse of $\boldsymbol{\mathcal{I}}_{\eta}$ (Equation 9 in the paper), and `A` equals to $\boldsymbol{\mathcal{I}}_{\eta}^{-1}\boldsymbol{\mathcal{I}}_{\eta\xi}$. Then the adjusted SEs can be obtained by

```
se <- sqrt( diag(omega.eta) / n1 )
se
# [1] 0.10686855 0.10985933 0.07848328 0.18600186
```

It is remarked that direct calculations of matrix inverses are not efficient and should be avoided when the number of parameters is large; they are used here only for illustration. In addition, setting `exp = F` in the function `calc.info` yields the observed information in the cross-product form (i.e., `estimator = MLF` in M*plus*).

[2]Be careful that the Fisher information is calculated for the LV variance instead of SD.

3 Goodness-of-Fit Assessment

We now import the current item response data:

```
dat <- as.matrix( read.table("n1000m10k2-rep5000.dat") )
```

The function `gof` in the source code (`RR-GRM.R`) computes the global M_2 and R_2 statistics, as well as the standardized residual means and cross-products. It takes a list of arguments, all of which have been defined previously:

```
result <- gof(dat, matrix(alpha), beta, mu, sigma, omega.xi, info, fo, c)
result
# $m2
# [1] 38.48495
#
# $df.m2
# [1] 35
#
# $r2
# [1] 55.76742
#
# $df.r2
# [1] 55
#
# $z
#  [1]  0.92174578 -1.51078900  0.33616564 -0.08263367 -0.50581534
#  [6]  0.90252704  0.70517079 -0.83412173  0.36476412 -0.05837639
# [11] -0.66743235  0.57140966 -0.28493400 -0.03999390  1.71208244
# [16]  1.05651311 -0.06172658  1.82108339 -0.08633720 -0.74241726
# [21]  0.01994707 -1.22643522 -0.33967977  0.52249997 -0.77218745
# [26] -1.05072453 -0.39875740  0.22957772 -0.24547358  0.95279521
# [31]  1.86460763 -0.30523226 -0.27437852  0.39329722 -0.05784819
# [36] -0.12123147  0.95102655  0.39006607  0.04613103  0.54045770
# [41]  0.25671110  1.46998103 -0.19041309  0.74094167  0.40189410
# [46]  2.13112938 -0.96865117  0.40027909  0.80199443 -0.21611262
# [51]  2.01624962 -0.10405551 -0.48330319 -0.93017764  1.21414068
```

In `result$z`, the first ten elements are standardized residual means for items 1–10, followed by 45 standardized residual cross-products that fill the lower-triangular part of a 10×10 matrix in the column-major order.

References

Muthén, L. K., & Muthén, B. O. (1998–2017). Mplus user's guide [Computer software manual]. Los Angeles, CA.

R Core Team. (2018). R: A language and environment for statistical computing [Computer software manual]. Vienna, Austria. Retrieved from `https://www.R-project.org/`