Supplementary Material 1: How social network influences human behavior: An integrated latent space approach for differential social influence

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1 Existing Social Influence Models

The network autocorrelation model is a widely used method to study social influence (Ord, 1975; Doreian, 1989; Leenders, 2002, Frank et al., 2004; Zheng et al., 2010; Scott et al., 2012; Fujimoto et al., 2013; Carr and Zube, 2015; Sewell, 2017, Dittrich et al., 2019). This model quantifies the strength of a peer effect on a network (i.e., social influence) while controlling for individuals' characteristics and network autocorrelations.

Suppose $\boldsymbol{y}$ is a vector containing measures on a response variable. A linear network autocorrelation model can be given as follows:

$$y = \rho W y + \epsilon, \tag{1}$$

where ρ is a network autocorrelation parameter and $W = \{w_{ij}\}$ is a matrix of influence coefficients that indicate the influence of actor j on actor i; the error term $\epsilon \sim N(0, \sigma^2 I)$ (Leenders, 2002).

This model is extended by including covariates X:

$$y = \rho W y + X\beta + \epsilon, \tag{2}$$

where $\epsilon \sim N(0, \sigma^2 I)$. This extended model is also referred to as the regressive-autoregressive model (Ord, 1975) or the network effects model (Doreian, 1989). Other extensions models are also available (e.g., Ord, 1975; Doreian, 1980; Dow et al., 1982; Doreian, 1982, 1989; Rietveld and Wintershoven, 1998).

Manski (1993) proposed a linear-in-means model (LIMM) to analyze social interaction and peer effects. The LIMM estimates covariate effects, exogenous effects (the influence of peer characteristics), and endogenous effects (the influence of peer outcomes). Suppose y_k and x_k are the outcome and the covariate of respondent k, respectively, and M_k is the number of friends of respondents k. Then, LIMM is given by

$$y_k = \alpha + \beta_x x_k + \beta_w \frac{1}{M_k} \sum_{l=1}^{N} w_{kl} y_k + \beta_{\bar{x}} \frac{1}{M_k} \sum_{l=1}^{N} w_{kl} x_k + \epsilon_k, \tag{3}$$

where β_x, β_w, and $\beta_{\bar{x}}$ indicate a covariate effect, an endogenous peer effect, and an exogenous peer effect, respectively. The LIMM and the network autocorrelation model are used for comparison with our proposed model in the empirical and simulation studies.

In addition, researchers utilized network formation process for dealing with network endogeneity or unobserved heterogeneity problems in the above social interaction models (Goldsmith-Pinkham and Imbens, 2013; Hsieh and Lee, 2015). For example, Goldsmith-Pinkham and Imbens (2013) combined network formation process with the linear-in-means model to identify and estimate peer effects, i.e., social influence, taking into account potential network endogeneity. Hsieh and Lee (2015) extended a spatial autoregressive model using network formation to capture social influences based on unobserved characteristics.

Another model for social influence is the autologistic actor attribute model (ALAAM; Robins et al. 2001; Daraganova and Robins 2013; Parker et al. 2021). This model leverages exponential random graph models (ERGM; Holland and Leinhardt 1981; Frank and Strauss 1986; Robins et al. 2007; Hunter 2007; Fienberg 2012) for network configuration. The autologistic actor attribute model specifies the probability of observing attributes $\boldsymbol{y}$ given network $\boldsymbol{x}$ (Parker et al., 2021):

$$P(\mathbf{y}|\mathbf{x}) = \frac{1}{\kappa(\boldsymbol{\theta})} \exp\Big(\sum_{i=1}^{p} \theta_i s_i(\mathbf{y}, \mathbf{x}, \mathbf{w})\Big),$$

where $s_i(\mathbf{y}, \mathbf{x}, \mathbf{w})$ is a network statistic that involves interactions among attributes $\mathbf{y}$, network data $\mathbf{w}$, and other actor-specific characteristics $\mathbf{x}$. θ_i is the corresponding parameter for $s_i(\mathbf{y}, \mathbf{x}, \mathbf{w})$, and $\kappa(\boldsymbol{\theta})$ is a normalizing constant to ensure the proper probability distribution. The autologistic actor attribute model is designed for cross-sectional network data analysis. Koskinen and Daraganova (2022) extended the autologistic actor attribute model by presenting a comprehensive Bayesian inference scheme that supports the testing of dependencies across subsets of data and the presence of missing data.

The stochastic actor-oriented model (SAOM) (Snijders, 2001; Snijders et al., 2010; Block et al., 2019) is a widely recognized method to analyze social networks and social behavior from longitudinal panel data. SAOM views the observed panel data as snapshots of continuous changes in both network and behavior, and the changes are modeled as a Markov process of sequential mini-steps. The probability of a node changing its connections in a mini-step depends on the current network (Snijders, 2017).

Let $\mathbf{y}^{\pm ij}$ represent the network in which actor i changes the connection with actor j from network $\mathbf{y}$. Let $\mathbf{x}$ denote the behavior responses for the nodes in the network $\mathbf{y}$. Then, the probability of changing from network $\mathbf{y}$ to network $\mathbf{y}^{\pm ij}$ is

$$p\left(\mathbf{y} \to \mathbf{y}^{\pm ij}, \mathbf{x}; \beta\right) = \rho_i \times \frac{\exp\Big(\sum_k \beta_k \Delta s_{k,i}\left(\mathbf{y}, \mathbf{y}^{\pm ij}, \mathbf{x}\right)\Big)}{\sum_h \exp\Big(\sum_k \beta_k \Delta s_{k,i}\left(\mathbf{y}, \mathbf{y}^{\pm ih}, \mathbf{x}\right)\Big)}$$

where $\Delta s_{k,i}(\mathbf{y}, \mathbf{y}^{\pm ij}, \mathbf{z})$ is the difference in network statistics between network $\mathbf{y}$ and $\mathbf{y}^{\pm ij}$, β_k is the corresponding parameter for $s_{k,i}(\mathbf{y}^{\pm ij}, \mathbf{z})$, and ρ_i is the rate function for the actor i.

In addition, latent space modeling (LSMs) approaches, introduced in Section 2.1, have also been used to study social influence. For example, Sweet and Adhikari (2020) proposed how the latent space model for network data can be used in the social diffusion model (Valente, 2005), which is a temporal version of the NAM. Specifically, they proposed replacing the weight matrix W of the network autocorrelation model (1) and (2) with the affinity matrix calculated from the latent positions of people estimated based on the LSM. This model can also be applied to cross-sectional data. The model explained above is related to our proposed model because both utilize latent space modeling approaches for identifying social relationships between respondents from the network data of interest. However, the two models are different in terms of how respondents' social relationships are used for understanding and estimating their impacts on the respondents' behavioral outcomes. The model above is based on the autocorrelation model for behaviors, whereas our approach is based on the latent space model for behavior data.

Other research on social influence is available in the literature. For example, McFowland and Shalizi (2021) demonstrated that the estimation of social influence could be improved and yield consistent results under the latent network models when latent homogeneous attributes are controlled. Cui and Chen (2023) proposed a methodology to test the degree of social influence in terms of how much influence can pass through individuals in dynamic social networks. They used sequential hypothesis testing procedures that describe multiple observations of the same objects over different periods of time using generalized estimation equations.

Another line of related models is joint latent network models that focus on modeling respondents' social networks and attributes jointly. For example, Wang et al. (2019) proposed a joint attribute and person latent space model (APLSM) that merges information from the social network and the multivariate person covariates. Fosdick and Hoff (2015) proposed a joint model to test the dependencies between a network and person attributes. Similarly, joint latent network models and stochastic block models for multilayer networks or multidimensional networks (Gollini and Murphy, 2016; Salter-Townshend and McCormick, 2017; D'Angelo et al., 2019) are related to this class of models in the sense that multiple networks are involved in the model.

These joint models are similar to our approach in the sense that respondents' social networks and attributes (behaviors or item responses) or multiple networks (for social relations and relations with items) are involved in the models. However, the most important differences are that these models do not directly measure or quantify social influence. Although the relationships between social homophily and attributes may be indirectly inferred from the estimated results, these relationships are likely to be confounded with social selection. In contrast, our model enables us to evaluate the presence (vs. absence) and the size of social influence with a specific model parameter. In addition, since the social network is measured before the attributes in our analysis, the confounding with social selection is less of a concern in our approach.

2 Update step of MCMC for the LSIRM for social influence

The MCMC sampler involves updating all parameters of the LSM and then update all parameters of the adapted LSIRM updated given **Z** from the LSM. The MCMC at the iteration l for the LSIRM for social influence is given as follows:

1. **Update the latent position of respondents, $\mathbf{z}_k^{(l)}$ in LSM.** For $k = 1, \cdots, n$,

$$r\Big(\mathbf{z}_k^*, \mathbf{z}_k^{(l)}\Big) = \frac{\pi\Big(\mathbf{z}_k^*\Big)}{\pi\Big(\mathbf{z}_k^{(l)}\Big)} \frac{P\Big(\mathbf{Y}^t \mid \mathbf{z}_k^*, \mathbf{Z}_{-k}, \mathbf{\Theta}_{-\mathbf{Z}}^n\Big)}{P\Big(\mathbf{Y}^t \mid \mathbf{z}_k^{(l)}, \mathbf{Z}_{-k}, \mathbf{\Theta}_{-\mathbf{Z}}^n\Big)} \frac{q\Big(\mathbf{z}_k^* \to \mathbf{z}_k^{(l)}\Big)}{q\Big(\mathbf{z}_k^{(l)} \to \mathbf{z}_k^*\Big)},$$

where $\mathbf{Z}_{-k} = (\mathbf{z}_1, \cdots, \mathbf{z}_{k-1}, \mathbf{z}_{k+1}, \cdots, \mathbf{z}_n)$ and $\mathbf{\Theta}_{-\mathbf{Z}}^n$ is the parameter sets for LSM except $\mathbf{Z}$, respectively.

2. **Update the intercept parameter, $\boldsymbol{\alpha}^{(l)}$ in LSM.**

$$r\Big(\alpha^*, \alpha^{(l)}\Big) = \frac{\pi\Big(\alpha^*\Big)}{\pi\Big(\alpha^{(l)}\Big)} \frac{P\Big(\mathbf{Y}^t \mid \alpha^*, \mathbf{\Theta}_{-\alpha}^n\Big)}{P\Big(\mathbf{Y}^t \mid \alpha^{(l)}, \mathbf{\Theta}_{-\alpha}^n\Big)} \frac{q\Big(\alpha^* \to \alpha^{(l)}\Big)}{q\Big(\alpha^{(l)} \to \alpha^*\Big)}.$$

where $\mathbf{\Theta}_{-\alpha}^n$ is the parameter set for LSM except α.

3. **Update the weight parameter of the distance term, $\boldsymbol{\gamma}^{(l)}$ in LSM.**

$$r\Big(\gamma^*, \gamma^{(l)}\Big) = \frac{\pi\Big(\gamma^*\Big)}{\pi\Big(\gamma^{(l)}\Big)} \frac{P\Big(\mathbf{Y}^t \mid \gamma^*, \mathbf{\Theta}_{-\gamma}^n\Big)}{P\Big(\mathbf{Y}^t \mid \gamma^{(l)}, \mathbf{\Theta}_{-\gamma}^n\Big)} \frac{q\Big(\gamma^* \to \gamma^{(l)}\Big)}{q\Big(\gamma^{(l)} \to \gamma^*\Big)}$$

where $\mathbf{\Theta}_{-\gamma}^n$ is the parameter set for LSM except γ.

4. **Update the latent position of items, $\mathbf{w}_i^{(l)}$ in LSIRM, given $\mathbf{Z}^{(l)}$.** For $i = 1, \cdots, p$,

$$r\Big(\mathbf{w}_i^*, \mathbf{w}_i^{(l)}\Big) = \frac{\pi\Big(\mathbf{w}_i^*\Big)}{\pi\Big(\mathbf{w}_i^{(l)}\Big)} \frac{P\Big(\mathbf{X}^{t+1} \mid \mathbf{w}_i^*, \mathbf{W}_{-i}, \mathbf{\Theta}_{-\mathbf{W}}^{r*}, \mathbf{Z}^{(l)}\Big)}{P\Big(\mathbf{X}^{t+1} \mid \mathbf{w}_i^{(l)}, \mathbf{W}_{-i}, \mathbf{\Theta}_{-\mathbf{W}}^{r*}, \mathbf{Z}^{(l)}\Big)} \frac{q\Big(\mathbf{w}_i^* \to \mathbf{w}_i^{(l)}\Big)}{q\Big(\mathbf{w}_i^{(l)} \to \mathbf{w}_i^*\Big)},$$

where $\mathbf{W}_{-i} = (\mathbf{w}_1, \cdots, \mathbf{w}_{i-1}, \mathbf{w}_{i+1}, \cdots, \mathbf{w}_p)$ and $\mathbf{\Theta}_{-\mathbf{W}}^{r*}$ is the parameter set for LSIRM except $\mathbf{W}$.

5. **Update the item difficulty parameter, $\boldsymbol{\beta}_i^{(l)}$ in LSIRM, given $\mathbf{Z}^{(l)}$.** For $i = 1, \cdots, p$,

$$r\Big(\beta_i^*, \beta_i^{(l)}\Big) = \frac{\pi\Big(\beta_i^*\Big)}{\pi\Big(\beta_i^{(l)}\Big)} \frac{P\Big(\mathbf{X}^{t+1} \mid \beta_i^*, \boldsymbol{\beta}_{-i}, \mathbf{\Theta}_{-\boldsymbol{\beta}}^{r*}, \mathbf{Z}^{(l)}\Big)}{P\Big(\mathbf{X}^{t+1} \mid \beta_i^{(l)}, \boldsymbol{\beta}_{-i}, \mathbf{\Theta}_{-\boldsymbol{\beta}}^{r*}, \mathbf{Z}^{(l)}\Big)} \frac{q\Big(\beta_i^* \to \beta_i^{(l)}\Big)}{q\Big(\beta_i^{(l)} \to \beta_i^{(l)}\Big)},$$

where $\boldsymbol{\beta}_{-i} = (\beta_1, \cdots, \beta_{i-1}, \beta_{i+1}, \cdots, \beta_p)$ and $\mathbf{\Theta}_{-\boldsymbol{\beta}}^{r*}$ is the parameter set for LSIRM except $\boldsymbol{\beta}$.

6. **Update the person trait parameter, $\boldsymbol{\theta}_k^{(l)}$ in LSIRM, given $\mathbf{Z}^{(l)}$.** For $k = 1, \cdots, n$,

$$r\left(\theta_k^*, \theta_k^{(l)}\right) = \frac{\pi\left(\theta_k^*\right)}{\pi\left(\theta_k^{(l)}\right)} \frac{P\left(\mathbf{X}^{t+1} \mid \theta_k^*, \boldsymbol{\theta}_{-k}, \boldsymbol{\Theta}_{-\theta}^{r*}, \mathbf{Z}^{(l)}\right)}{P\left(\mathbf{X}^{t+1} \mid \theta_k^{(l)}, \boldsymbol{\theta}_{-k}, \boldsymbol{\Theta}_{-\theta}^{r*}, \mathbf{Z}^{(l)}\right)} \frac{q\left(\theta_k^* \to \theta_k^{(l)}\right)}{q\left(\theta_k^{(l)} \to \theta_k^*\right)},$$

where $\boldsymbol{\theta}_{-k} = (\theta_1, \cdots, \theta_{k-1}, \theta_{k+1}, \cdots, \theta_n)$ and $\boldsymbol{\Theta}_{-\boldsymbol{\theta}}^{r*}$ is the parameter set for LSIRM except $\boldsymbol{\theta}$.

7. **Update the variance parameter of the person trait parameter, $\sigma^{2(l)}$ in LSIRM, given $\mathbf{Z}^{(l)}$ using Gibbs sampler.**

$$\pi\left(\sigma^2 \mid \cdot\right) \sim \mathsf{IG}\left(a + \frac{1}{2}n, b + \frac{1}{2}\sum_{k=1}^{n} \theta_k^2\right)$$

8. **Update the social influence parameter, $\delta^{(l)}$ in LSIRM, given $\mathbf{Z}^{(l)}$**

$$r\left(\delta^*, \delta^{(l)}\right) = \frac{\pi\left(\delta^*\right)}{\pi\left(\delta^{(l)}\right)} \frac{P\left(\mathbf{X}^{t+1} \mid \delta^*, \boldsymbol{\Theta}_{-\delta}^{r*}, \mathbf{Z}^{(l)}\right)}{P\left(\mathbf{X}^{t+1} \mid \delta^{(l)}, \boldsymbol{\Theta}_{-\delta}^{r*}, \mathbf{Z}^{(l)}\right)} \frac{q\left(\delta^* \to \delta^{(l)}\right)}{q\left(\delta^{(l)} \to \delta^*\right)}.$$

where $\boldsymbol{\Theta}_{-\delta}^{r*}$ is the parameter set for LSIRM except δ.

3 Jumping Rule

$$\begin{aligned}
\beta_i' &\sim \textsf{Norma}\Big(\beta_i^{(t)}, 0.5^2\Big),\\
\theta_k' &\sim \textsf{Normal}\Big(\theta_k^{(t)}, 1.0^2\Big),\\
\alpha' &\sim \textsf{Normal}\Big(\alpha^{(t)}, 0.25^2\Big),\\
\gamma' &\sim \textsf{log-Normal}\Big(\log(\gamma^{(t)}), 0.2^2\Big),\\
\delta' &\sim \textsf{Normal}\Big(\delta^{(t)}, 0.2^2\Big),\\
\mathbf{z}_k' &\sim \textsf{Normal}\Big(\mathbf{z}_k^{(t)}, 0.2^2\Big),\\
\mathbf{w}_i' &\sim \textsf{Normal}\Big(\mathbf{w}_i^{(t)}, 0.2^2\Big).
\end{aligned}$$

4 The List of Questions for School Activities

Item number	Description
1	I do participate in sports at school. (1: Yes, 0: No)
2	I do participate in sports outside of school. (1: Yes, 0: No)
3	I do participate in theater/drama. (1: Yes, 0: No)
4	I do participate in music. (1: Yes, 0: No)
5	I do participate in other arts. (1: Yes, 0: No)
6	I do participate in other school clubs. (1: Yes, 0: No)
7	I do date people at this school. (1: Yes, 0: No)
8	I do lots of homework. (1: Yes, 0: No)
9	I do read books for fun. (1: Yes, 0: No)
10	I play video games. (1: Yes, 0: No)

Table 1: The list of questions for school activities

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